

iChiral phases in two-flavor quark matter under magnetic field

Hiroaki Abuki
Aichi University of Education

Refs: H. Abuki, PLB728 (2014),
H. Abuki, et al, in preparation

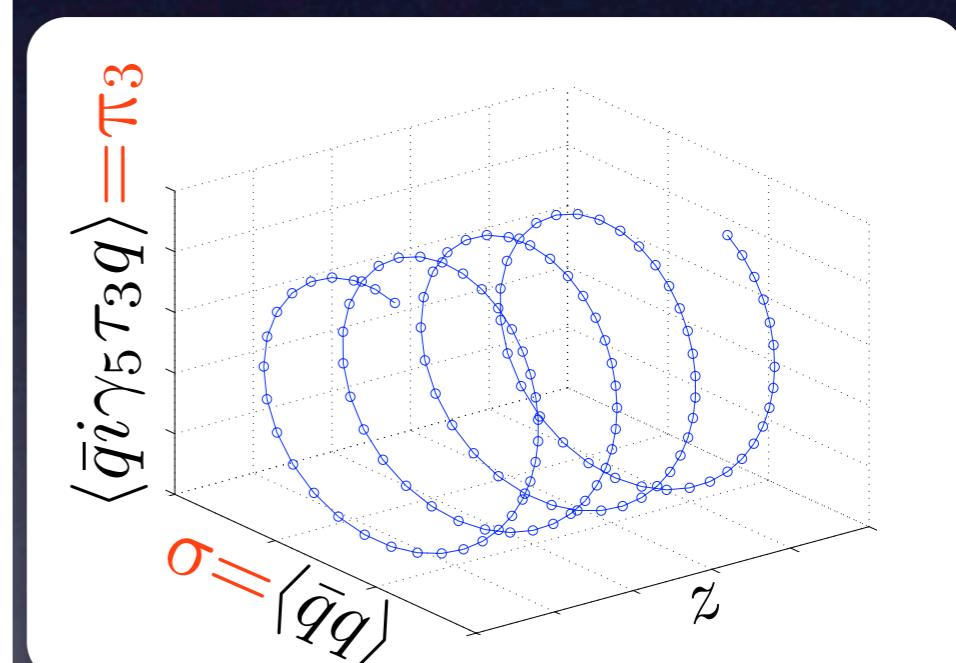
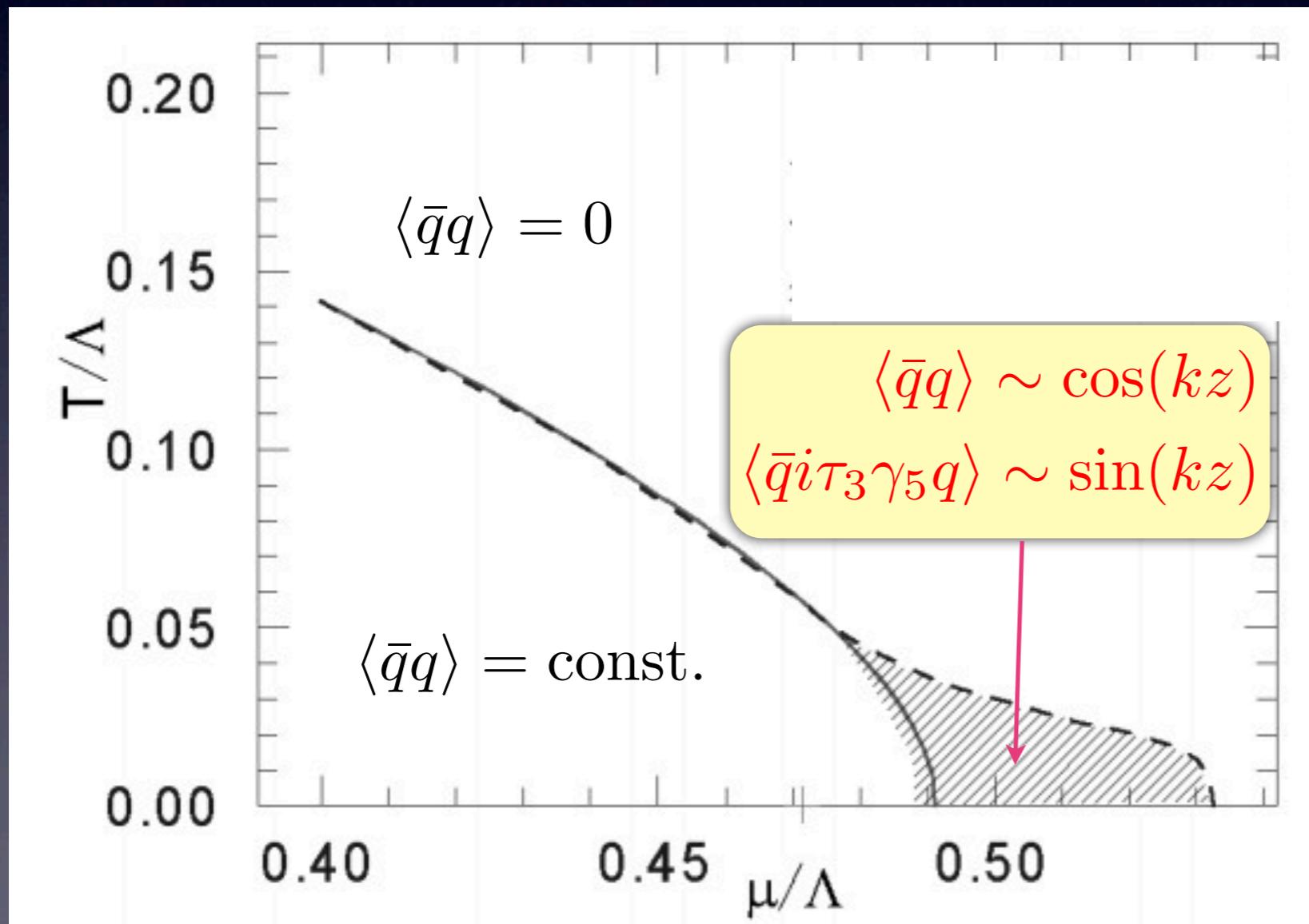
QCD@work 2016: International Workshop on QCD; Theory and Experiment
Martina Franca, Italia 2016年6月26日-29日

Plan

- What is iChiral phase?
- generalized Ginzburg-Landau (gGL) expansion and the effect of quark mass
- Effect of magnetic field
- Summary and Outlook

Chiral Spiral Phase

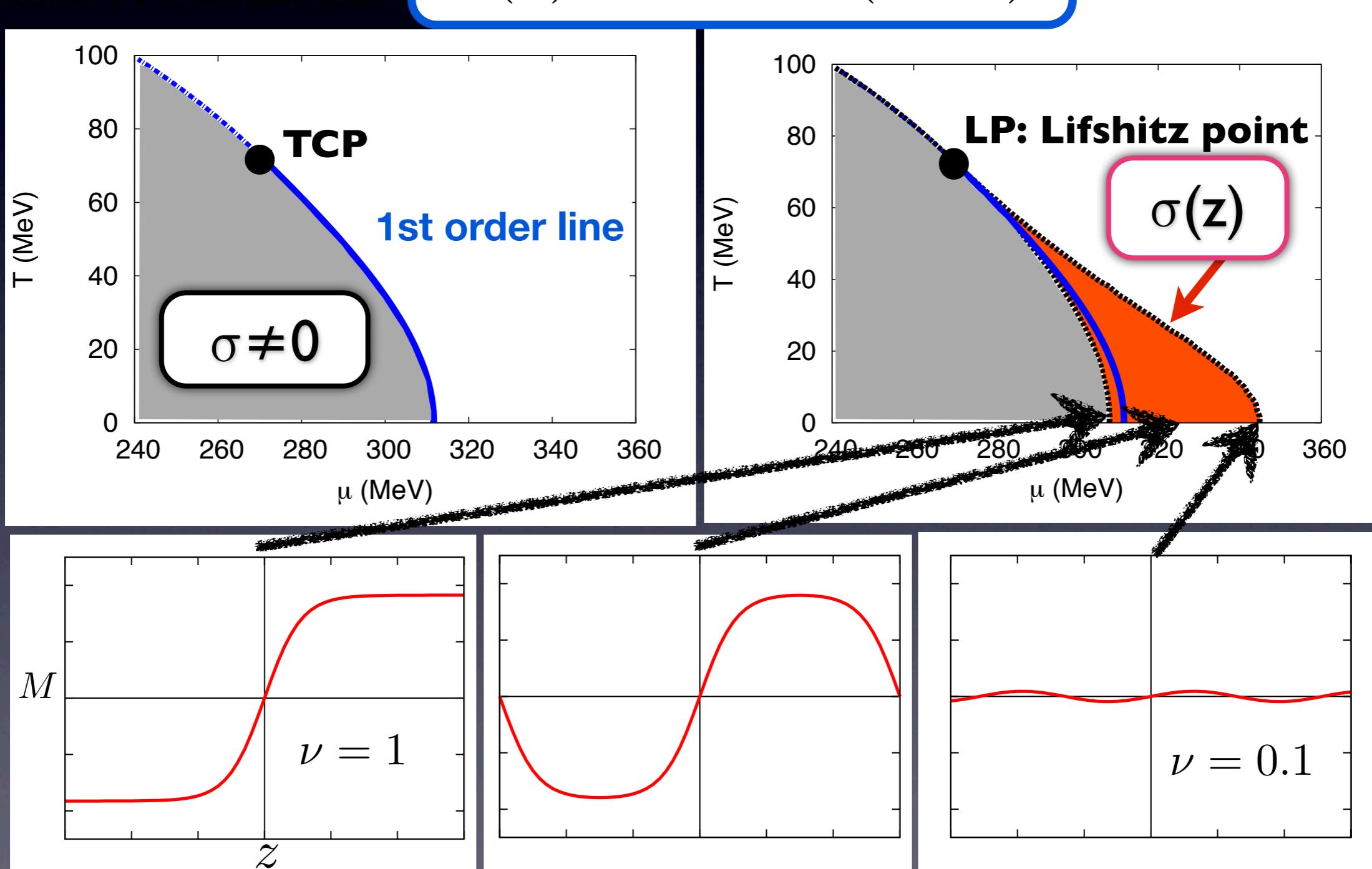
Nakano, Tatsumi, PRD71(2005) 114006



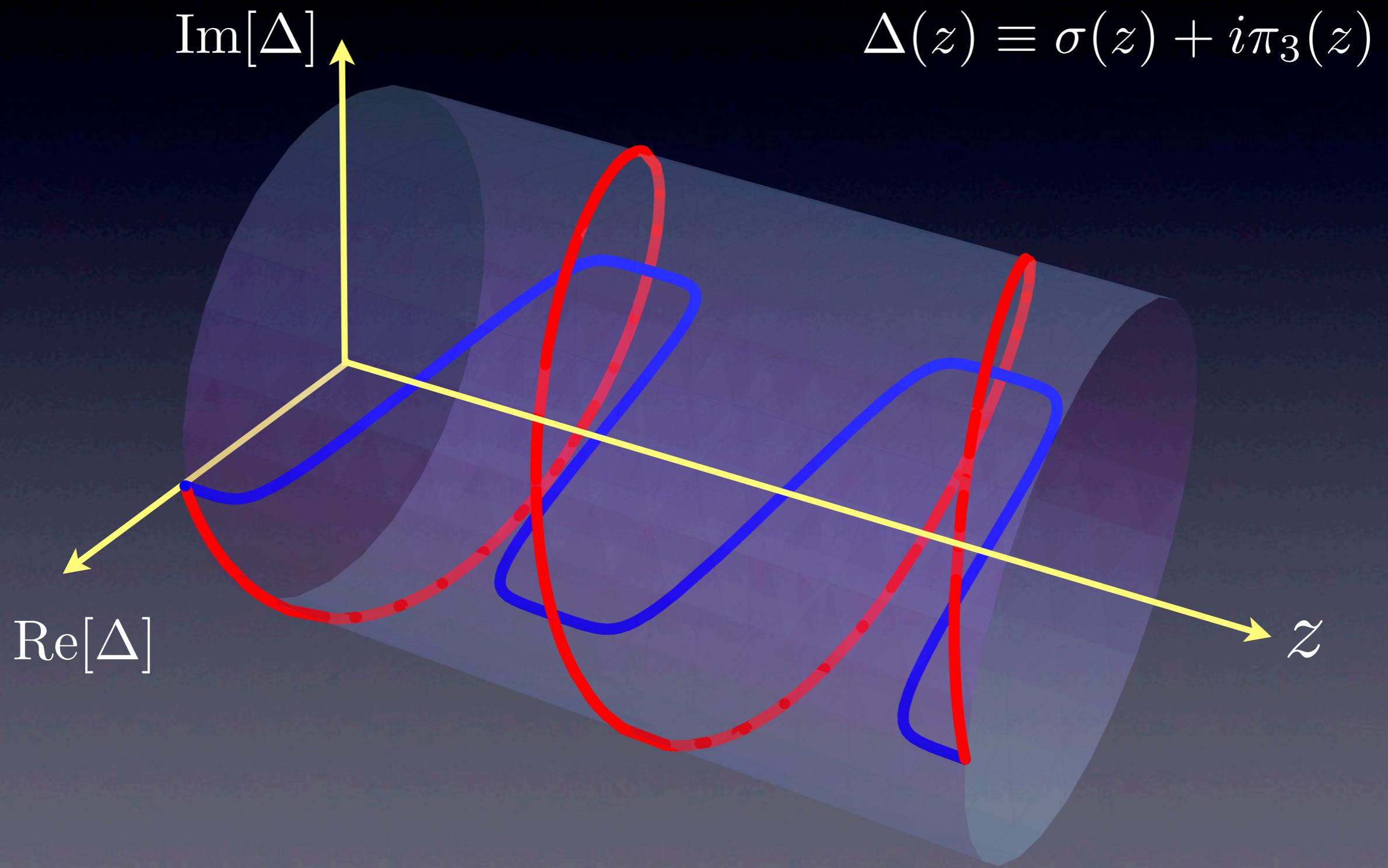
Real Kink Crystal (RKC)

$$\sigma(z) = \sqrt{\nu} q \operatorname{sn}(qz, \nu)$$

D. Nickel, PRL09, PRD09



Spiral or RKC?



$1+1$ dim. NJL/GN model

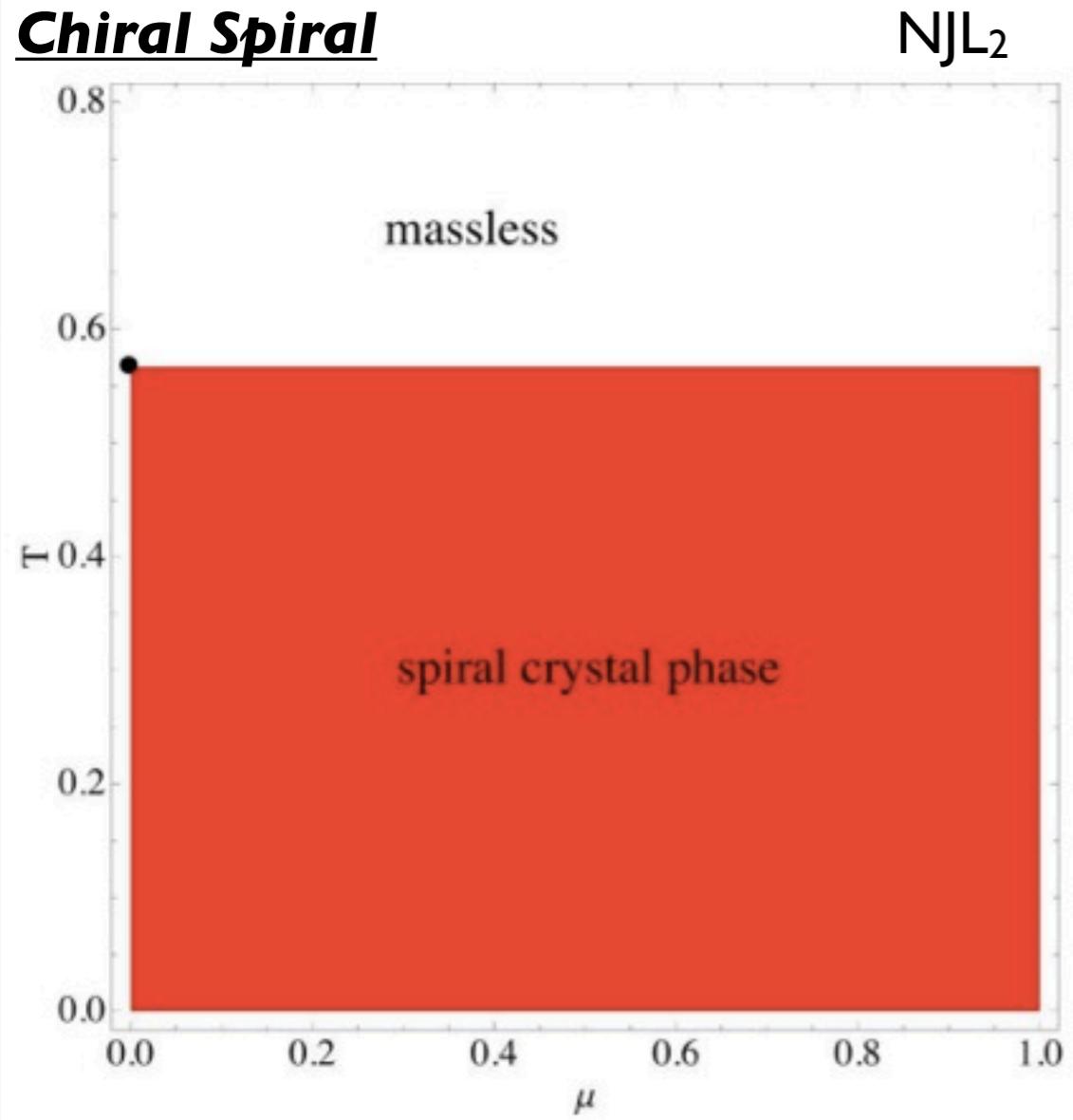
$$\sigma = \langle \bar{\varphi} \varphi \rangle = \cos(2\mu z) \Delta$$

$$\pi = \langle \bar{\varphi} i \gamma_5 \varphi \rangle = \sin(2\mu z) \Delta$$

M.Thies, J. Phys. A2006

$$\sigma = \langle \bar{\varphi} \varphi \rangle = \sqrt{\nu} q \text{sn}(qz, \nu)$$

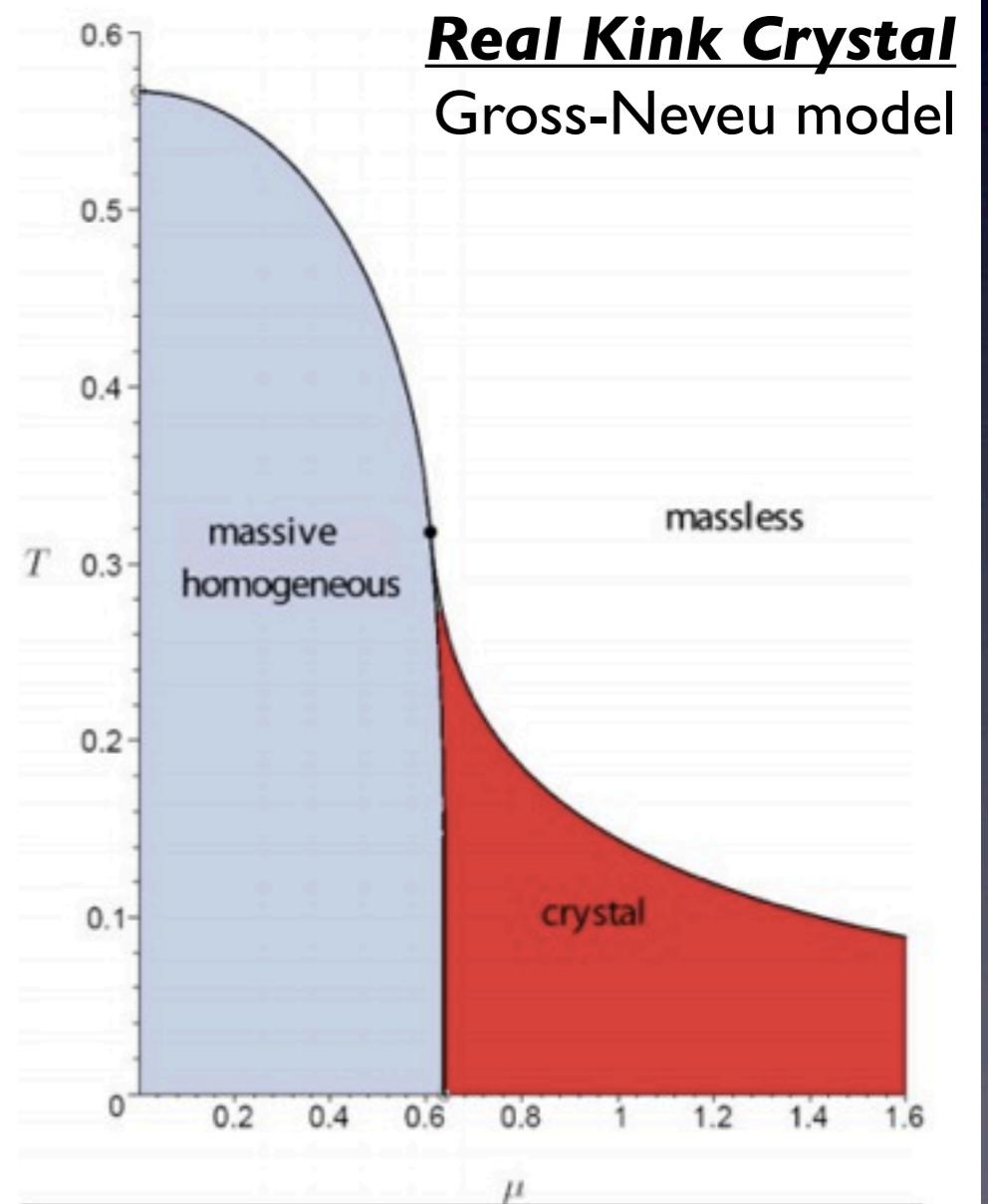
Chiral Spiral



NJL₂

Real Kink Crystal

Gross-Neveu model



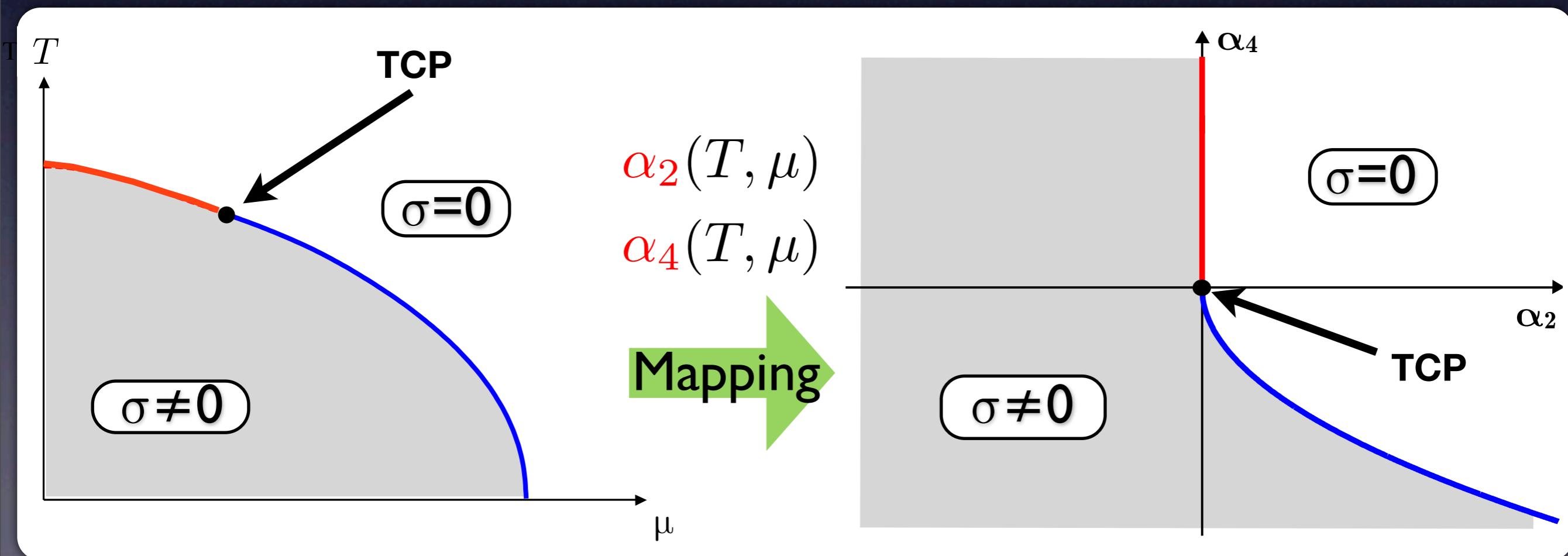
Plan

- What is iChiral phase?
- generalized Ginzburg-Landau (gGL)
approach and the effect of quark mass
- Effect of magnetic field
- Summary and Outlook

Ginzburg-Landau (GL) approach (homogeneous)

Minimal GL to describe the tricritical point (TCP)

$$\Omega_{\text{GL}} = \frac{\alpha_2}{2}\sigma(\mathbf{x})^2 + \frac{\alpha_4}{4}\sigma(\mathbf{x})^4 + \frac{1}{6}\sigma(\mathbf{x})^6$$



generalized Ginzburg-Landau (gGL) expansion

D. Nickel, PRL09

mass: *External field* : $O(4) \mapsto O(3)$ *isospin*

$$\Omega_{\text{GL}} = \boxed{-h\sigma(\mathbf{x}) + \frac{\alpha_2}{2}\sigma(\mathbf{x})^2} \quad \text{gradient terms}$$
$$+ \frac{\alpha_4}{4}\sigma(\mathbf{x})^4 + \frac{\alpha_{4b}}{4}(\nabla\sigma(\mathbf{x}))^2$$
$$+ \frac{\alpha_6}{6}\sigma(\mathbf{x})^6 + \frac{\alpha_{6b}}{6}\sigma^2(\nabla\sigma)^2 + \frac{\alpha_{6c}}{6}(\Delta\sigma)^2$$

$\alpha_{4b} = 0$: Lifshitz Point (LP)

$h = \alpha_2 = \alpha_4 = 0$: Tricritical Point (TCP)

$h = \alpha_{4b} = \alpha_2 = \alpha_4 = 0$: Lifshitz TCP (LTCP)

reduced gGL potential

$$\Omega_{\text{GL}} = -h\sigma + \frac{\alpha_2}{2}\sigma^2 + \frac{\alpha_4}{4}(\sigma^4 + (\nabla\sigma)^2)$$

$$+ \frac{\alpha_6}{6} \left(\sigma^6 + 5\sigma^2(\nabla\sigma)^2 + \frac{1}{2}(\Delta\sigma)^2 \right)$$

Nickel, PRL09
Abuki (2014)

- Four independent GL parameters.

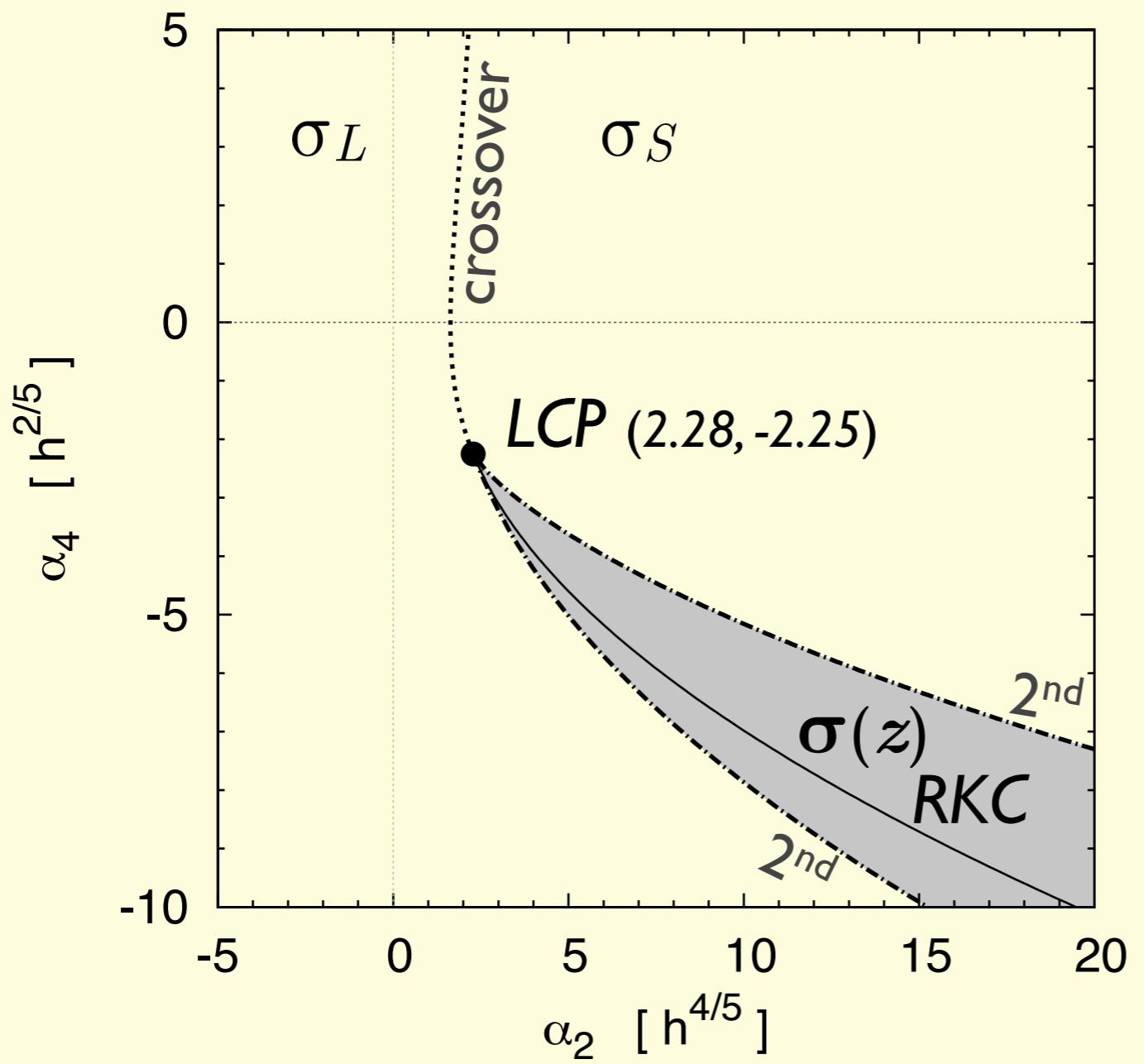
$$[\alpha_6] = \Lambda^{-2}$$

- For thermodynamic stability $\alpha_6 > 0$, so use it to set an energy scale; $\alpha_6 \rightarrow 1$

$$\sigma[h^{1/5}], \quad \alpha_2[h^{4/5}], \quad \alpha_4[h^{2/5}], \quad \mathbf{x}[h^{-1/5}],$$

$$\Omega \rightarrow h^{6/5}\Omega_0$$

Effect of h



(I) CP (2.28, -2.25)
coincides with
Lifshitz point

LCP = Lifshitz CP

(2) RKC stabilized:
Two critical lines
enclosing RKC are
both of 2nd order

Remark

Spiral being always disfavored against RKC in 3D

Boehmer, Thies, Urlichs, PRD75 (2007)

$$\Delta(z) \equiv \sigma(z) + i\pi(z)$$

$$\begin{aligned}\Omega_{\text{NJL}_2} = & \frac{\alpha_2}{2} |\Delta(z)|^2 + \boxed{\frac{\alpha_3}{3} \text{Im} [\Delta^* \Delta']} \\ & + \frac{\alpha_4}{4} (|\Delta(z)|^4 + |\Delta'(z)|^2) \\ & + \boxed{\frac{\alpha_5}{5} \text{Im} ((\Delta'' - 3|\Delta|^2 \Delta) \Delta'^*)} + \frac{1}{6} \Delta(\mathbf{x})^6\end{aligned}$$

For chiral spiral:

$$\Delta(z) \equiv \Delta_0 e^{iqz}$$

$$-\frac{\alpha_5}{5} (q^3 \Delta_0^2 + 3\Delta_0^4 q) \quad -\frac{\alpha_3}{3} \Delta_0^2 q$$

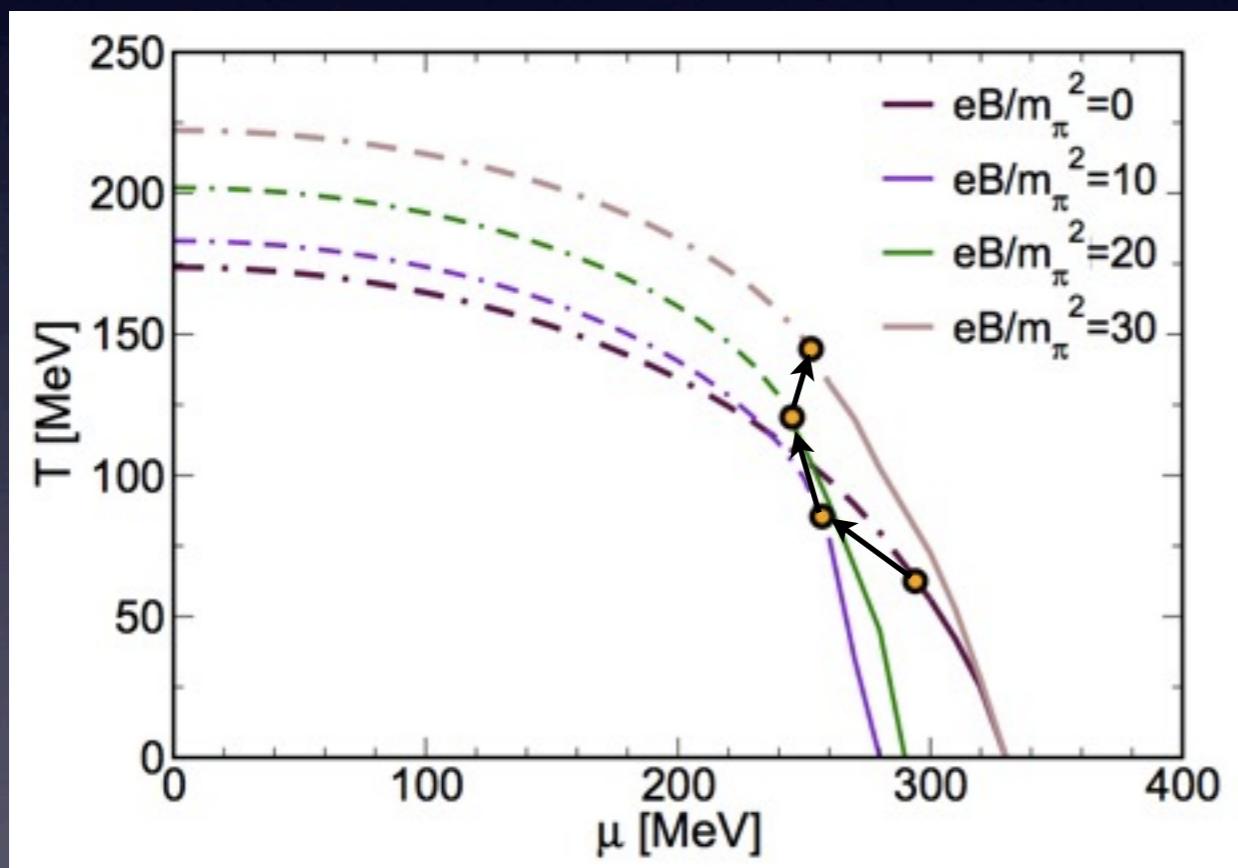
see Tatsumi, Nishiyama, Karasawa, PLB743 (2015)

Plan

- What is iChiral phase?
- generalized Ginzburg-Landau (gGL) expansion and the effect of quark mass
- Effect of magnetic field
- Summary and Outlook

Magnetic Field & Chiral symmetry breaking

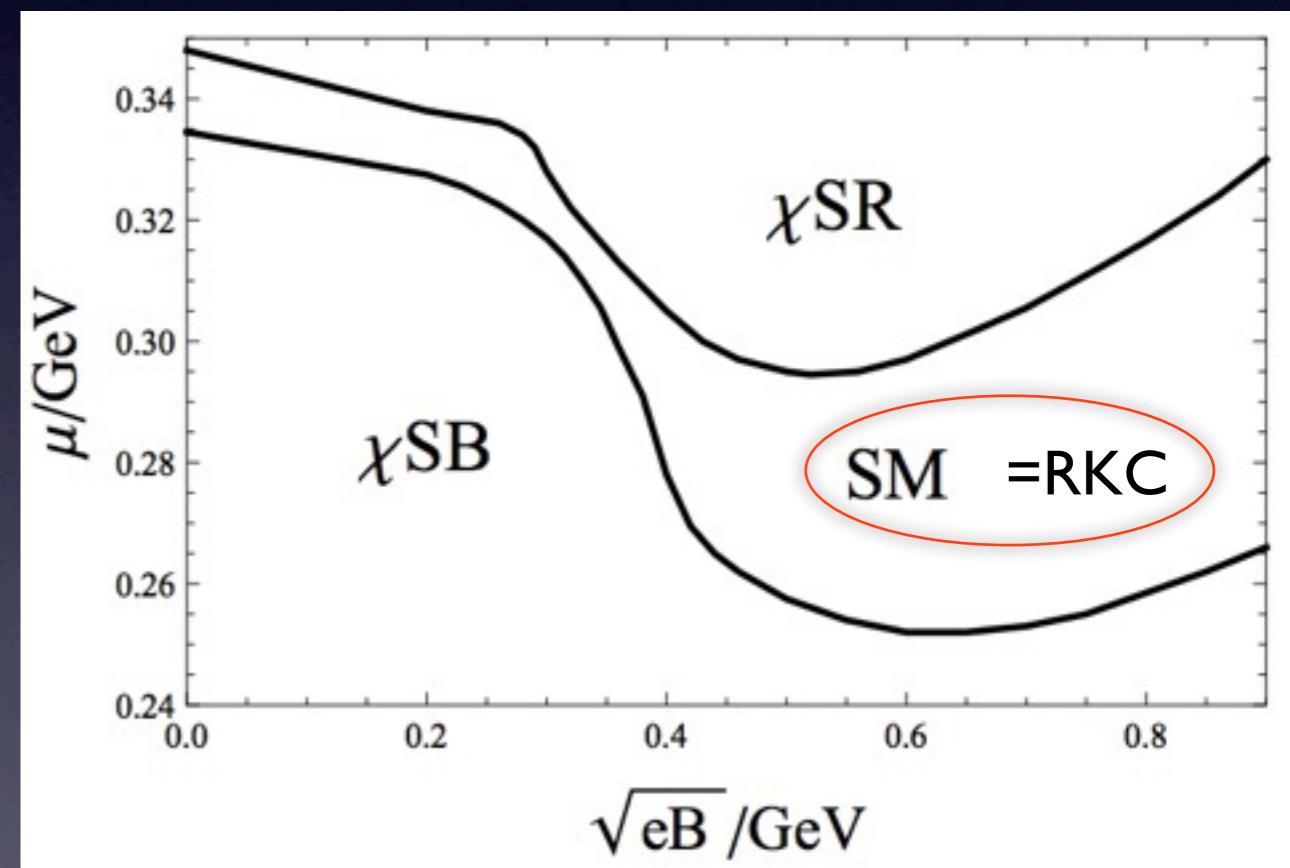
Behavior of the critical point



Chiral limit: Quark-meson model & GL

M. Ruggieri et al., PLB734 (2014)

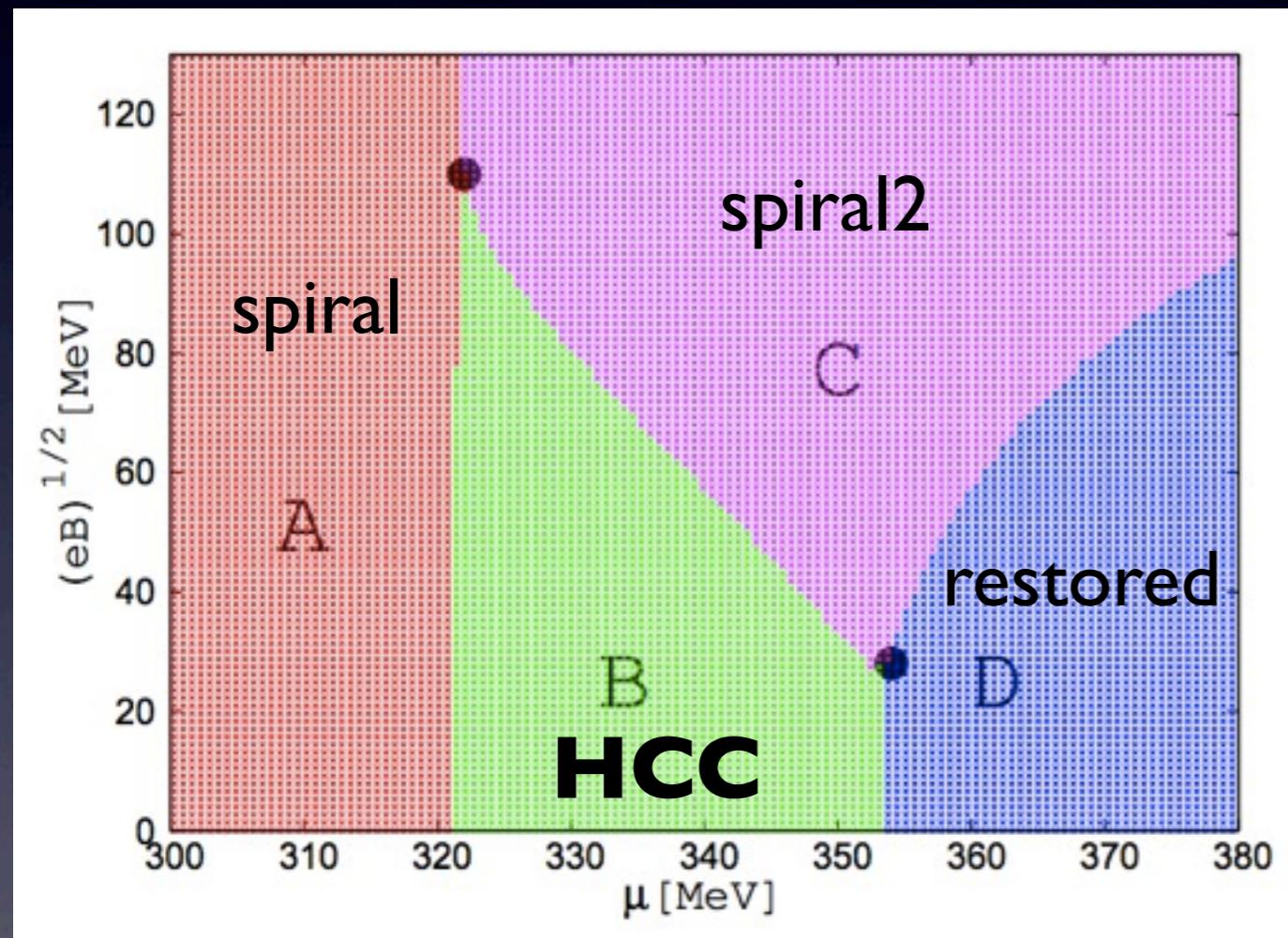
Solitonic modulation (SM) phase



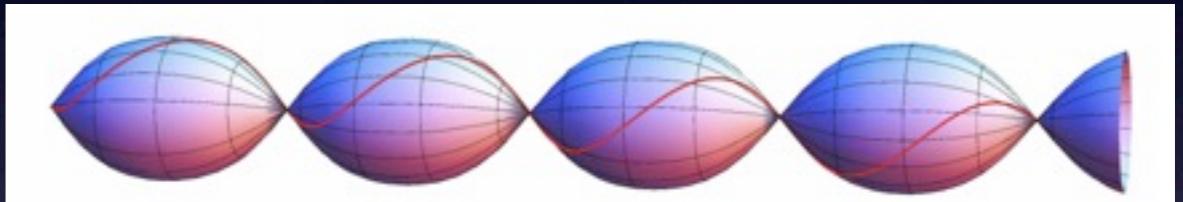
NJL model @ $T=0$

G. Cao and A. Huang., PRD93 (2016)

Magnetic Field & Chiral symmetry breaking



Hybrid Chiral Condensate



$$M(z) = \frac{2m\nu}{1 + \sqrt{\nu}} \text{sn} \left(\frac{2mz}{1 + \sqrt{\nu}}; \nu \right) e^{iqz}$$

K. Nishiyama, S. Karasawa, T. Tatsumi,
PRD92 (2015)

NJL@T=0 & Chiral Limit

gGL with magnetic field

- How does quark propagator modify?

I. Strong Magnetic Field (LLL approximation)

$$-iS(p_0, p_{\parallel}, \mathbf{p}_{\perp}) = 2e^{-\frac{\mathbf{p}_{\perp}^2}{QB}} \frac{p + \mu}{(p_0 + \mu)^2 - p_{\parallel}^2} P_{+}$$

2. Weak Magnetic Field $10\text{GT} \cong 3 \times 10^{-5}(m_{\pi})^2$

$$\begin{aligned} -iS(p) &= \frac{p + \mu + m}{(p_0 + \mu)^2 - \mathbf{p}^2 - m^2} \\ &\quad + (QB) \frac{p_{\parallel} + \mu + m}{((p_0 + \mu)^2 - \mathbf{p}^2 - m^2)^2} (i\gamma^1\gamma^2) \\ &\quad + (QB)^2 \dots \end{aligned}$$

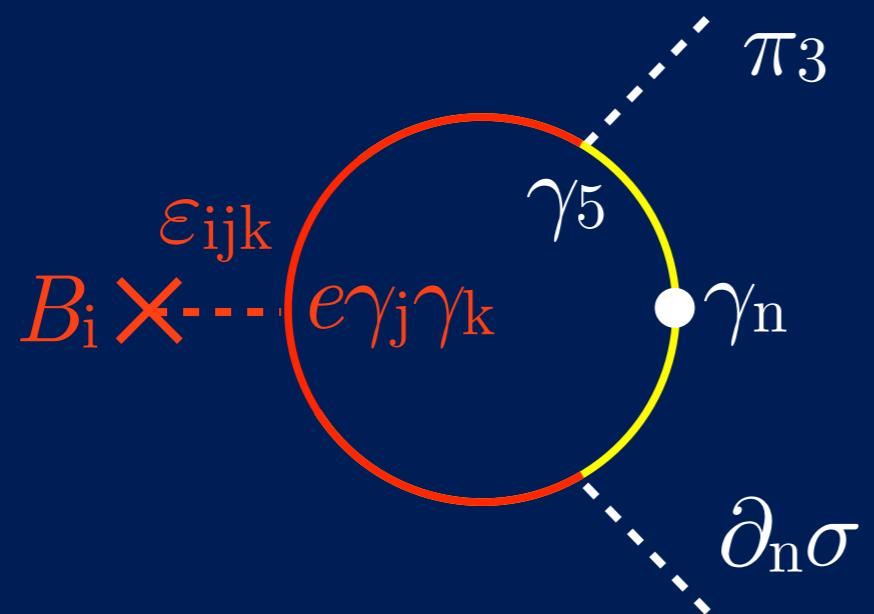
well-established: see for instance, Gobar, Miransky, Shovkovy, PRD88 (2013)

New deriv. coupling

New couplings: 3D rotational symmetry breaking

$$\begin{aligned}\delta\Omega &= \left[-\frac{1}{8} \frac{\partial\alpha_4}{\partial\mu} (e\mathbf{B}) \right] \cdot (\pi_3 \nabla \sigma - \sigma \nabla \pi_3) \\ &= \mathbf{b} \cdot (\pi_3 \nabla \sigma - \sigma \nabla \pi_3)\end{aligned}$$

Feynman graph contributing to new deriv. coupling



$$\begin{aligned}-iS(p) &= \frac{p+\mu}{(p+\mu)^2} \\ &+ (e_q B_i) \frac{i\epsilon_{ijk}\gamma^j\gamma^k}{2} \frac{p_{||}+\mu}{(p+\mu)^4}\end{aligned}$$

Mass vs Magnetic field

$$\delta\Omega = \mathbf{b} \cdot (\pi_3 \nabla \sigma - \sigma \nabla \pi_3) - h\sigma$$

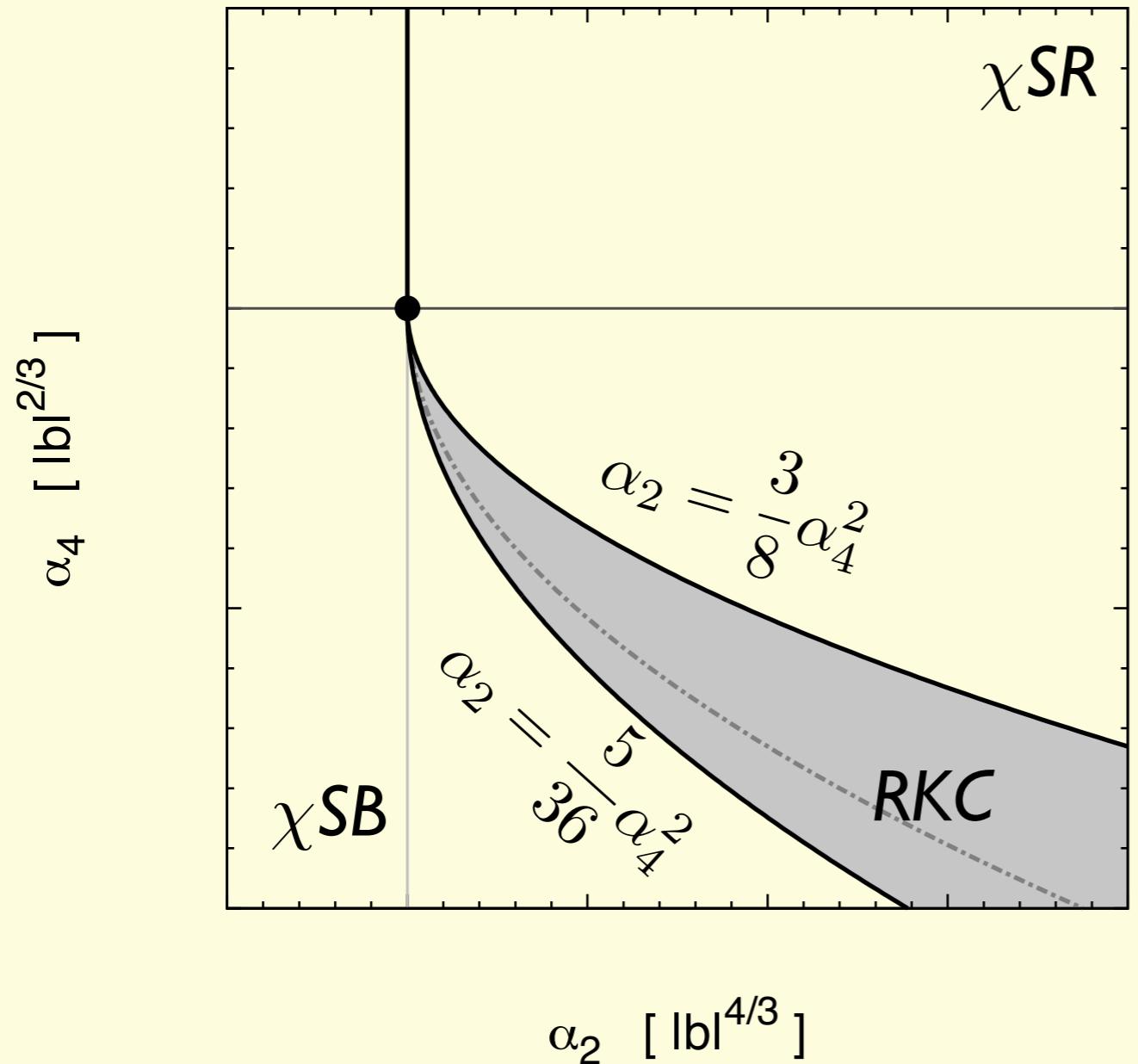
h : mass term

chiral $SU(2) \times SU(2) \Rightarrow$ isospin $SU(2)$

b : magnetic field

Rotational $O(3)$ symmetry
Time reversal symmetry

The fate of LTCP

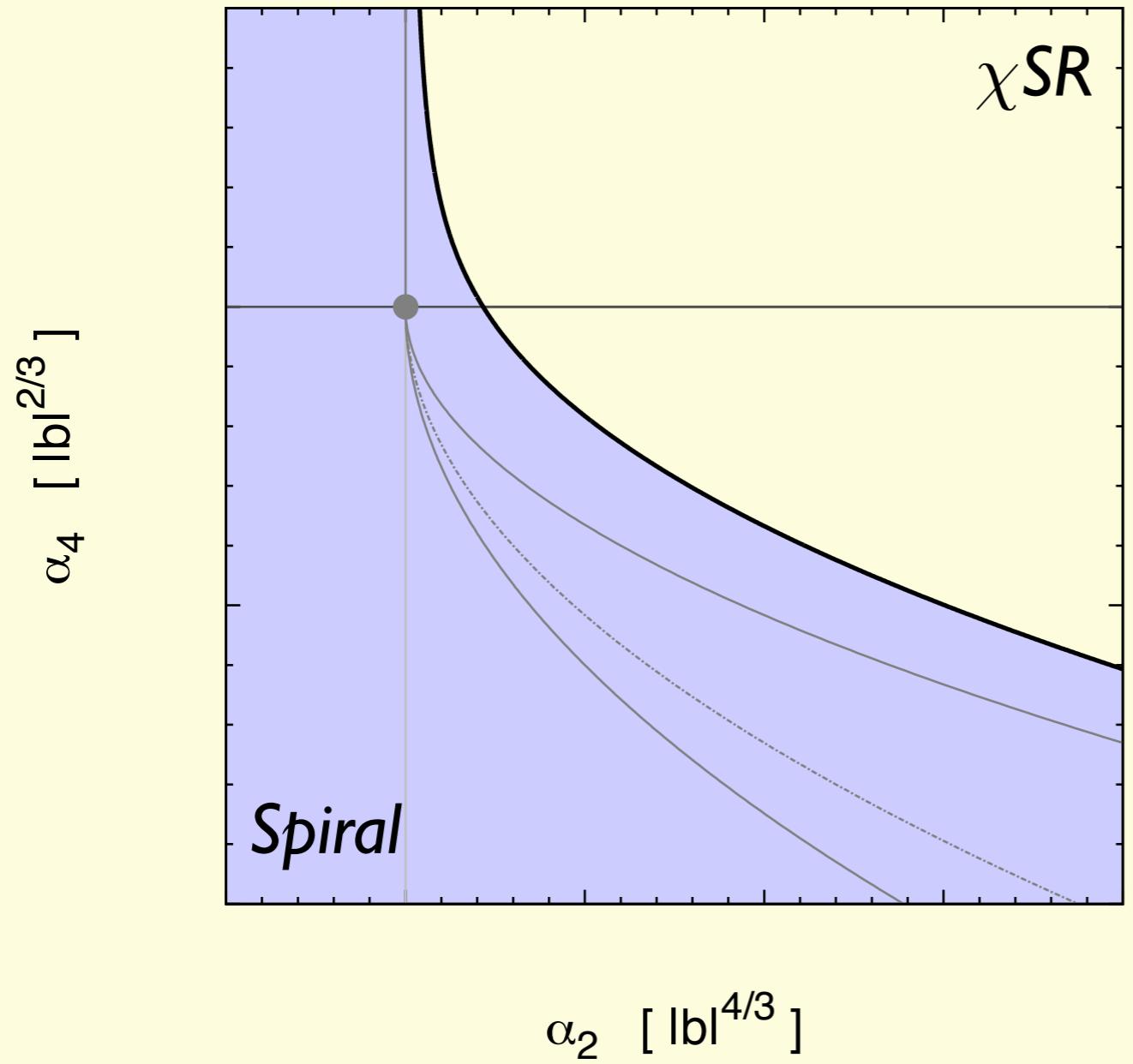


i. No magnetic field:

Lifshitz TCP
& RKC phase

$$\sigma = k\nu s n(kz, \nu)$$

The fate of LTCP



i. No magnetic field:

Lifshitz TCP
& RKC phase

$$\sigma = k\nu sn(kz, \nu)$$

ii. Magnetic field on:

Chiral spiral covers
the whole region

$$\sigma + i\pi_3 = \Delta_0 e^{iqz}$$

Only second order
phase transition!

Mass vs Magnetic field

$$\delta\Omega = \mathbf{b} \cdot (\pi_3 \nabla \sigma - \sigma \nabla \pi_3) - h\sigma$$

h : mass term

chiral $SU(2) \times SU(2) \Rightarrow$ isospin $SU(2)$

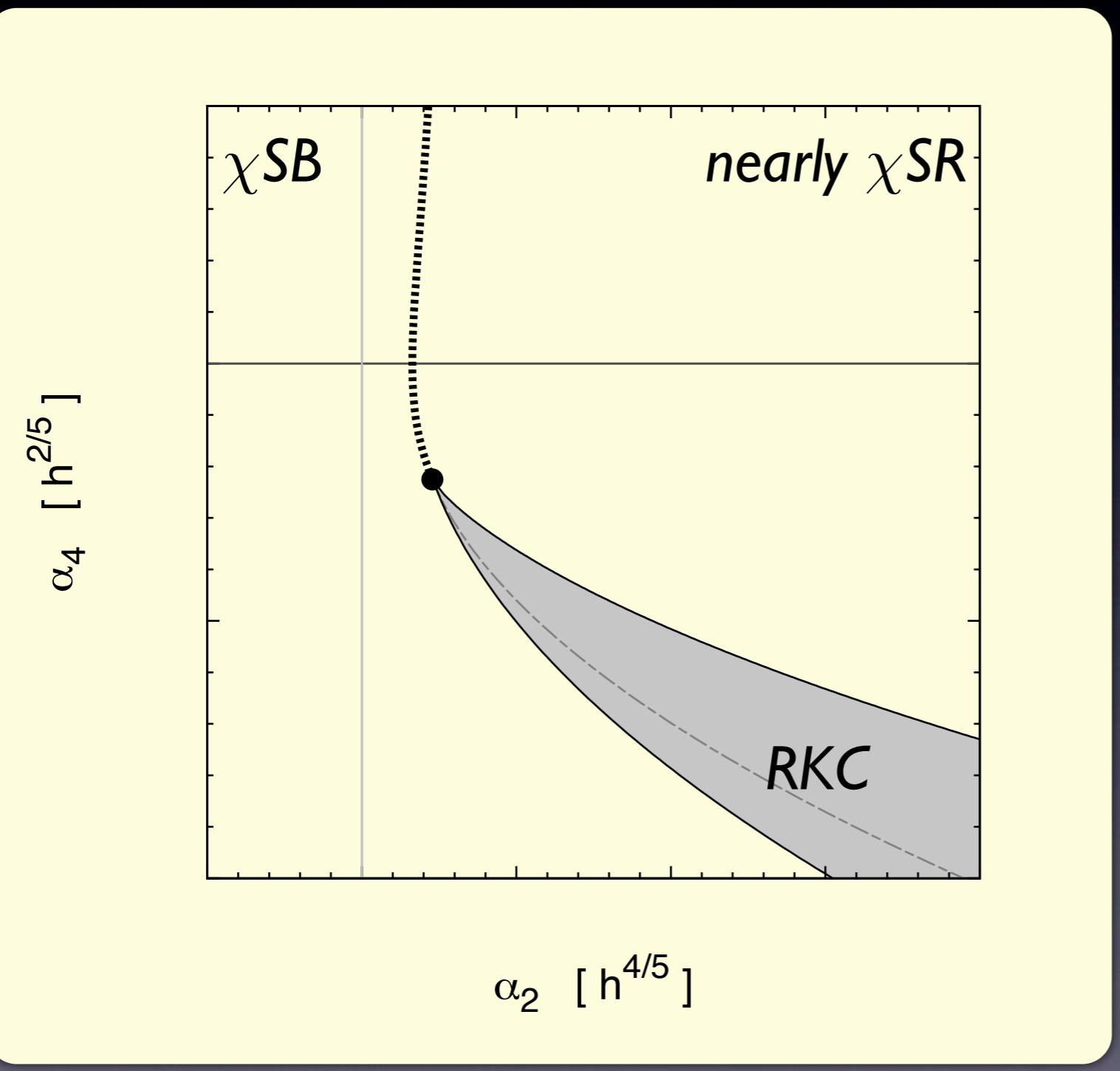
b : magnetic field

Rotational $O(3)$ symmetry

Time reversal symmetry

Isospin symmetry

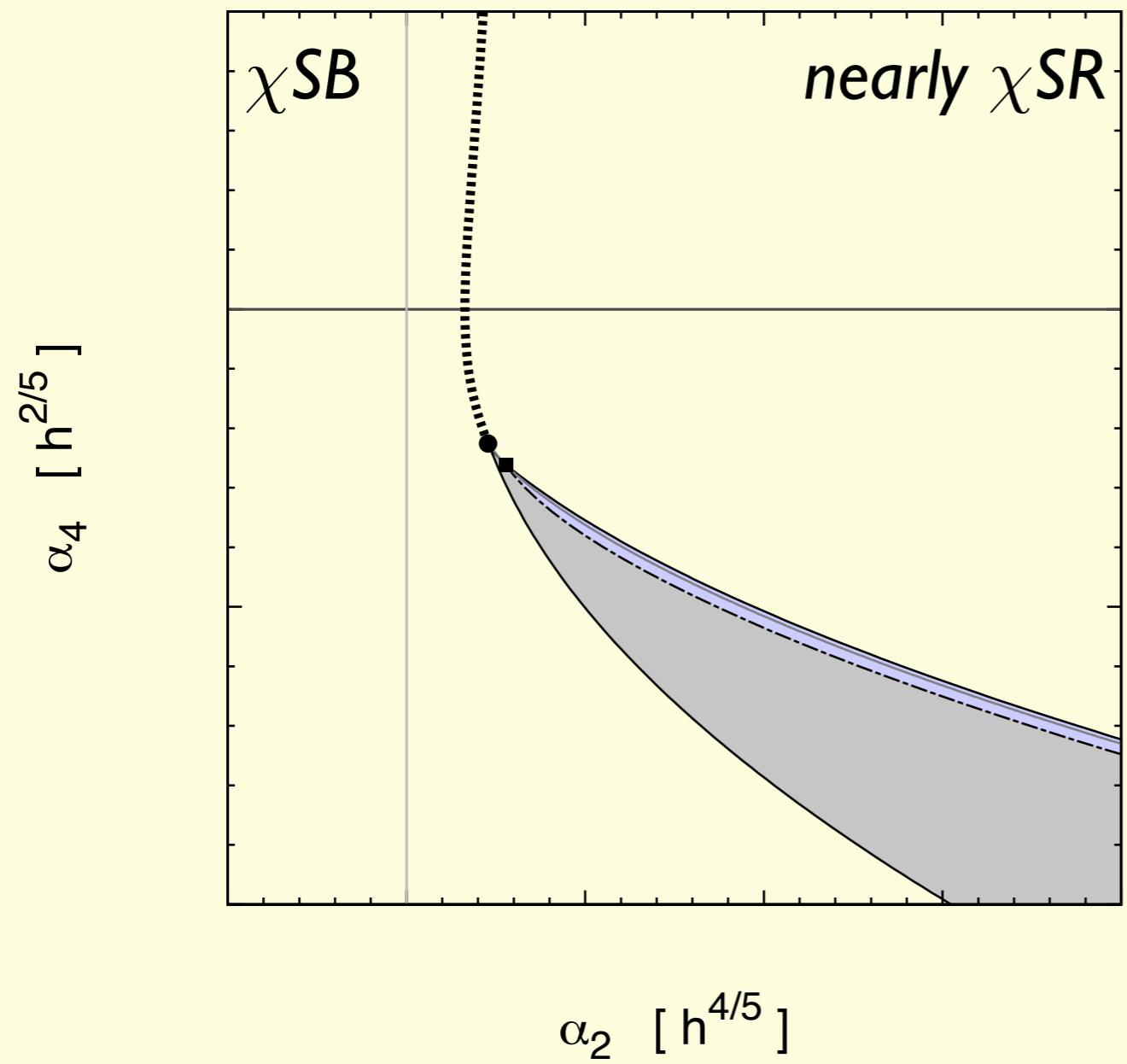
Mass vs Magnetic field



i. Only mass term on:

Crossover,
Lifshitz point &
RKC phase

Mass vs Magnetic field



i. Only mass term on:

Crossover,
Lifshitz point &
RKC phase

ii. Magnetic field on:

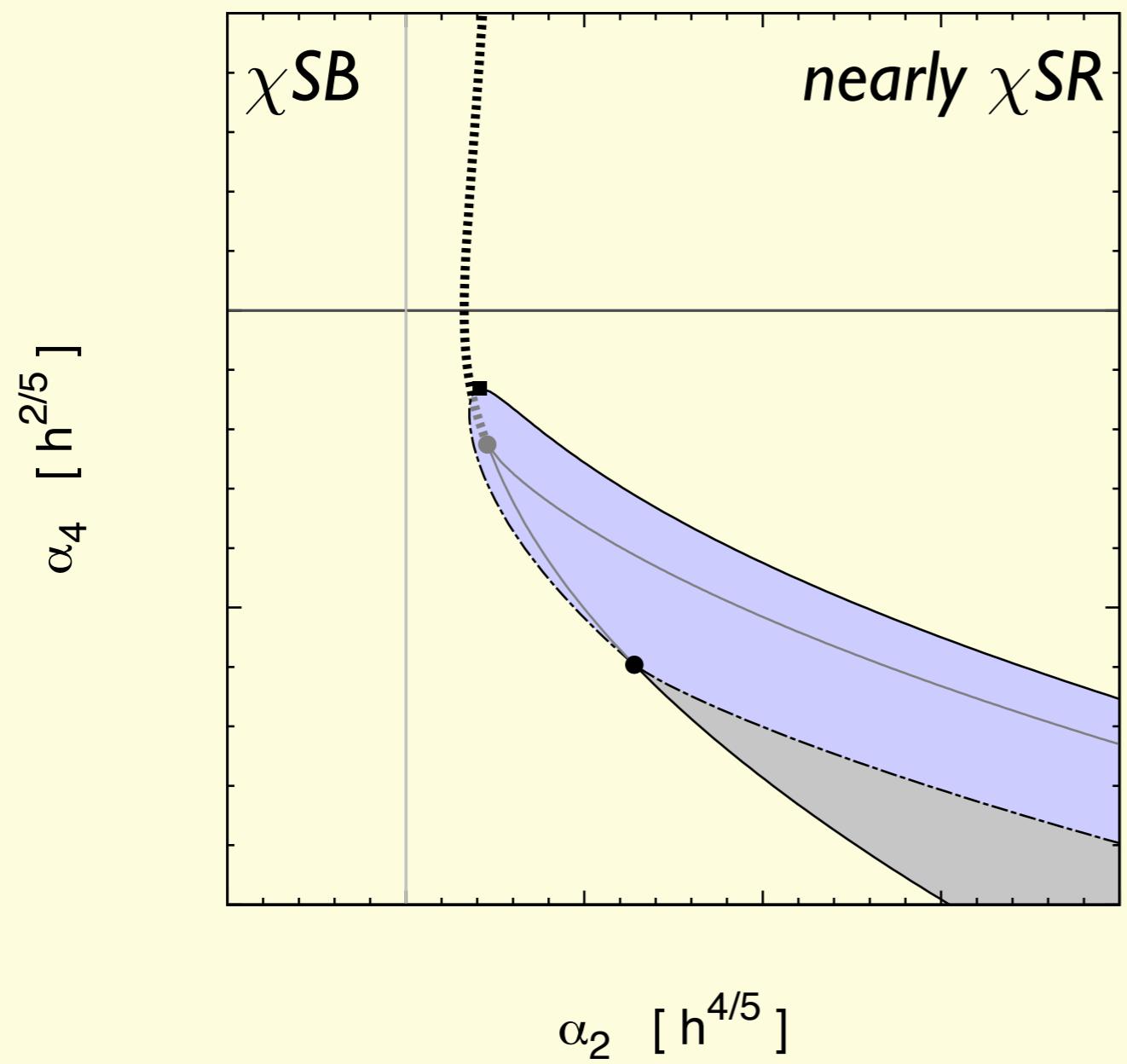
$$8b = 0.5 \times h^{3/5}$$
$$\left(\sqrt{eB} \sim 1.2 \text{ MeV} \right)$$

$$10^9 \text{ T} \leftrightarrow 0.24 \text{ MeV}$$

$$10^{11} \text{ T} \leftrightarrow 2.4 \text{ MeV}$$

$$10^{13} \text{ T} \leftrightarrow 24 \text{ MeV}$$

Mass vs Magnetic field



i. Only mass term on:

Crossover,
Lifshitz point &
RKC phase

ii. Magnetic field on:

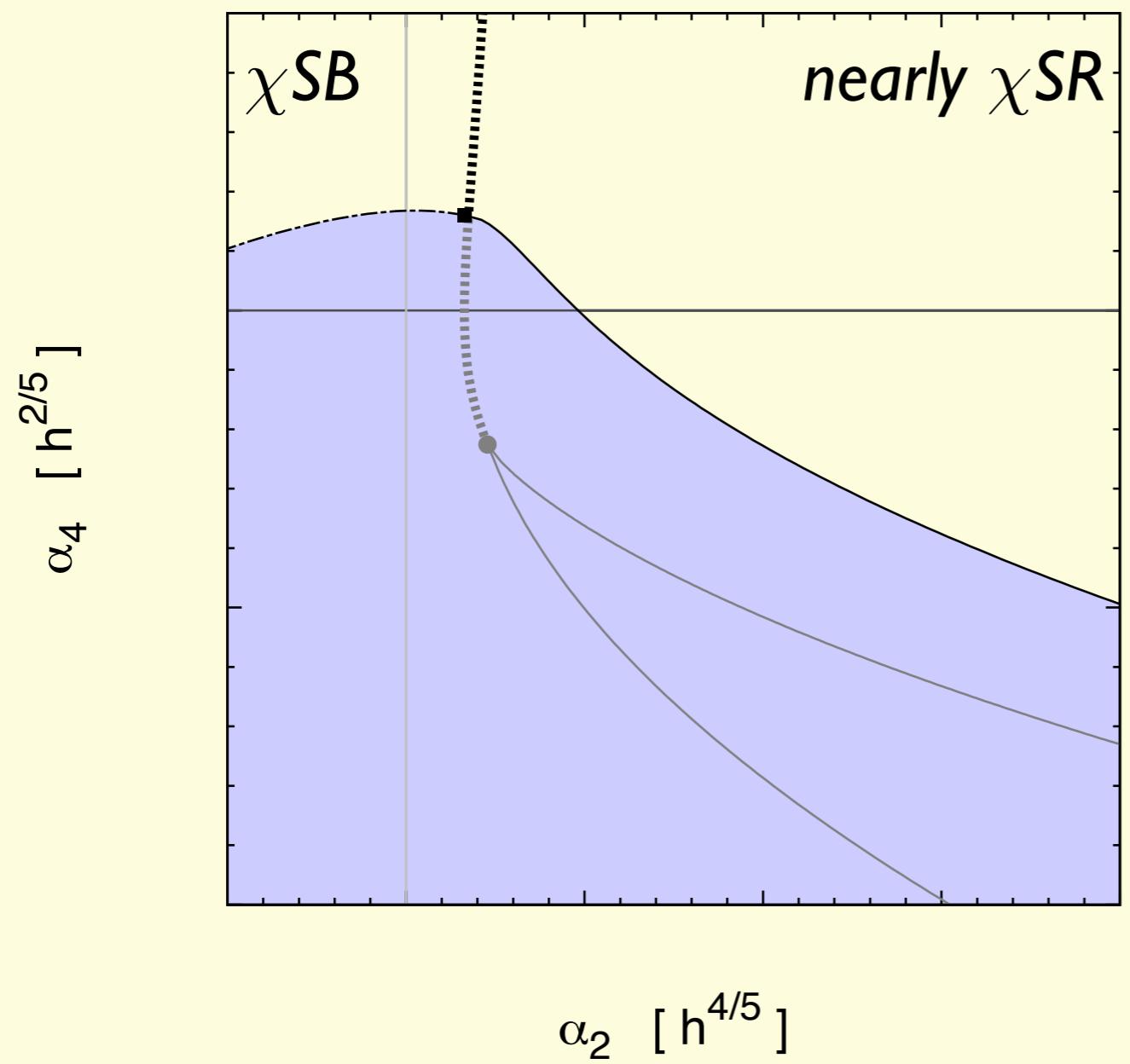
$$8b = 5.0 \times h^{3/5}$$
$$\left(\sqrt{eB} \sim 12 \text{ MeV} \right)$$

$$10^9 T \leftrightarrow 0.24 \text{ MeV}$$

$$10^{11} T \leftrightarrow 2.4 \text{ MeV}$$

$$10^{13} T \leftrightarrow 24 \text{ MeV}$$

Mass vs Magnetic field



i. Only mass term on:

Crossover,
Lifshitz point &
RKC phase

ii. Magnetic field on:

$$8b = 15 \times h^{3/5}$$
$$\left(\sqrt{eB} \sim 35 \text{ MeV} \right)$$

$$10^9 T \leftrightarrow 0.24 \text{ MeV}$$

$$10^{11} T \leftrightarrow 2.4 \text{ MeV}$$

$$10^{13} T \leftrightarrow 24 \text{ MeV}$$

Summary

- Studied the response of TCP against m_q and B within a generalized GL approach
- In the chiral limit: Spiral takes over RKC phase
- Quark mass protect the Lifshitz critical point (LCP) from weak external magnetic field.
- Strong enough magnetic field can bring a drastic change of phase structure; Spiral replaces RKC, and the usual chiral CP may even vanish.

Thank you.