

The 3D Nucleon Structure

Barbara Pasquini

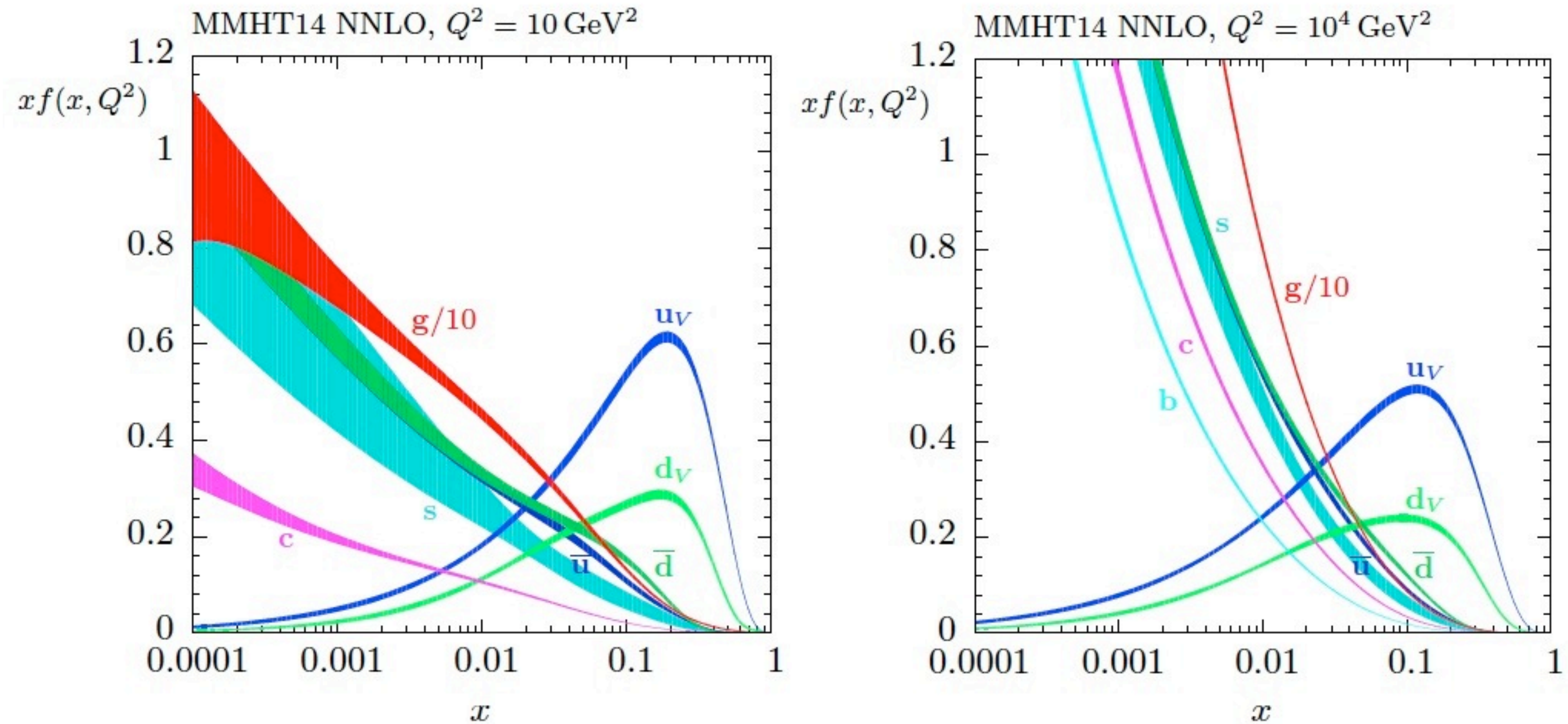
Università di Pavia & INFN

Funded by:



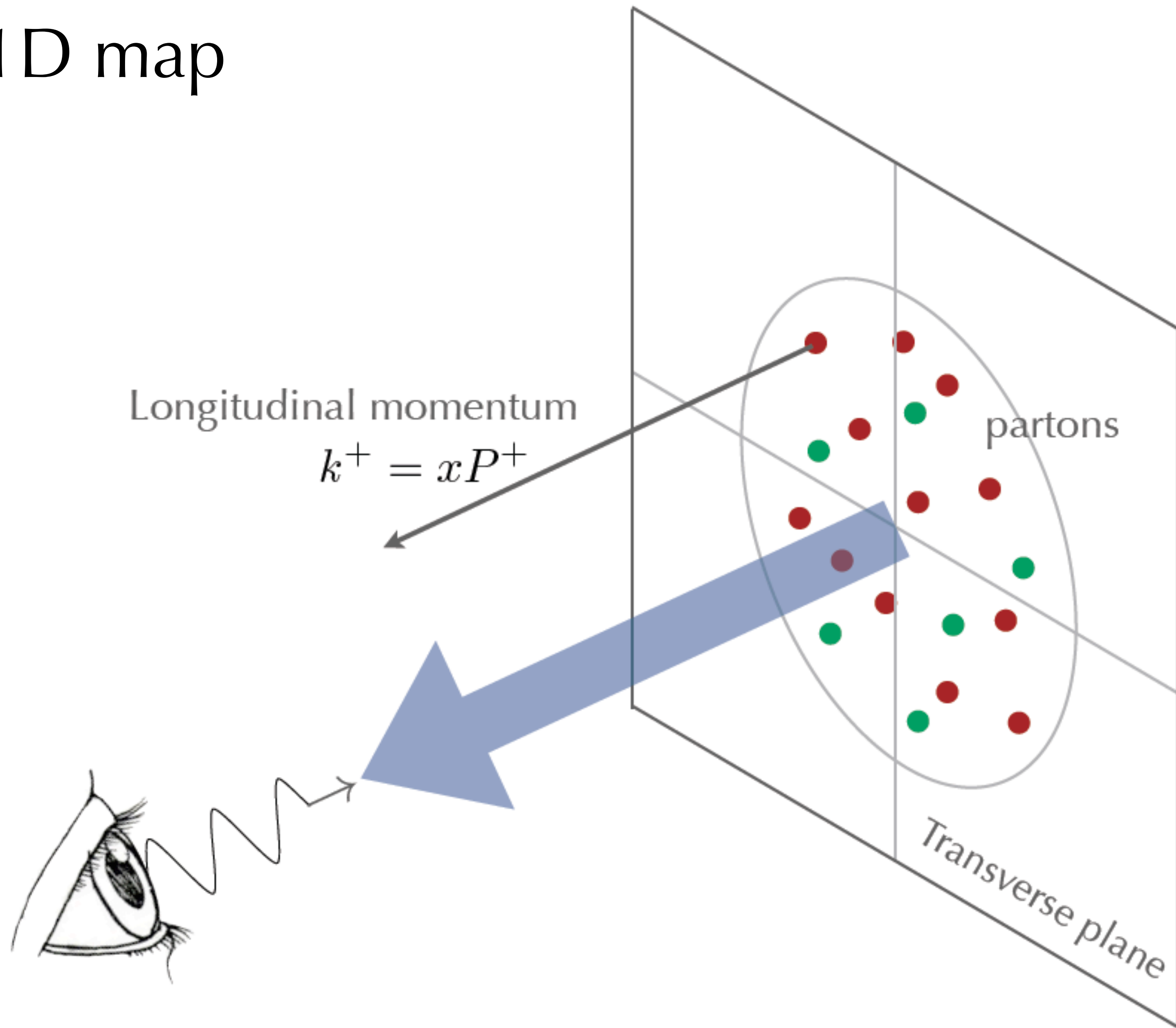
Principal Investigator: A. Bacchetta

Available Maps: Parton Distribution Functions monodimensional (in momentum space)



MMHT2014 Eur. Phys. J. C75 (2015) 204

PDFs: 1D map

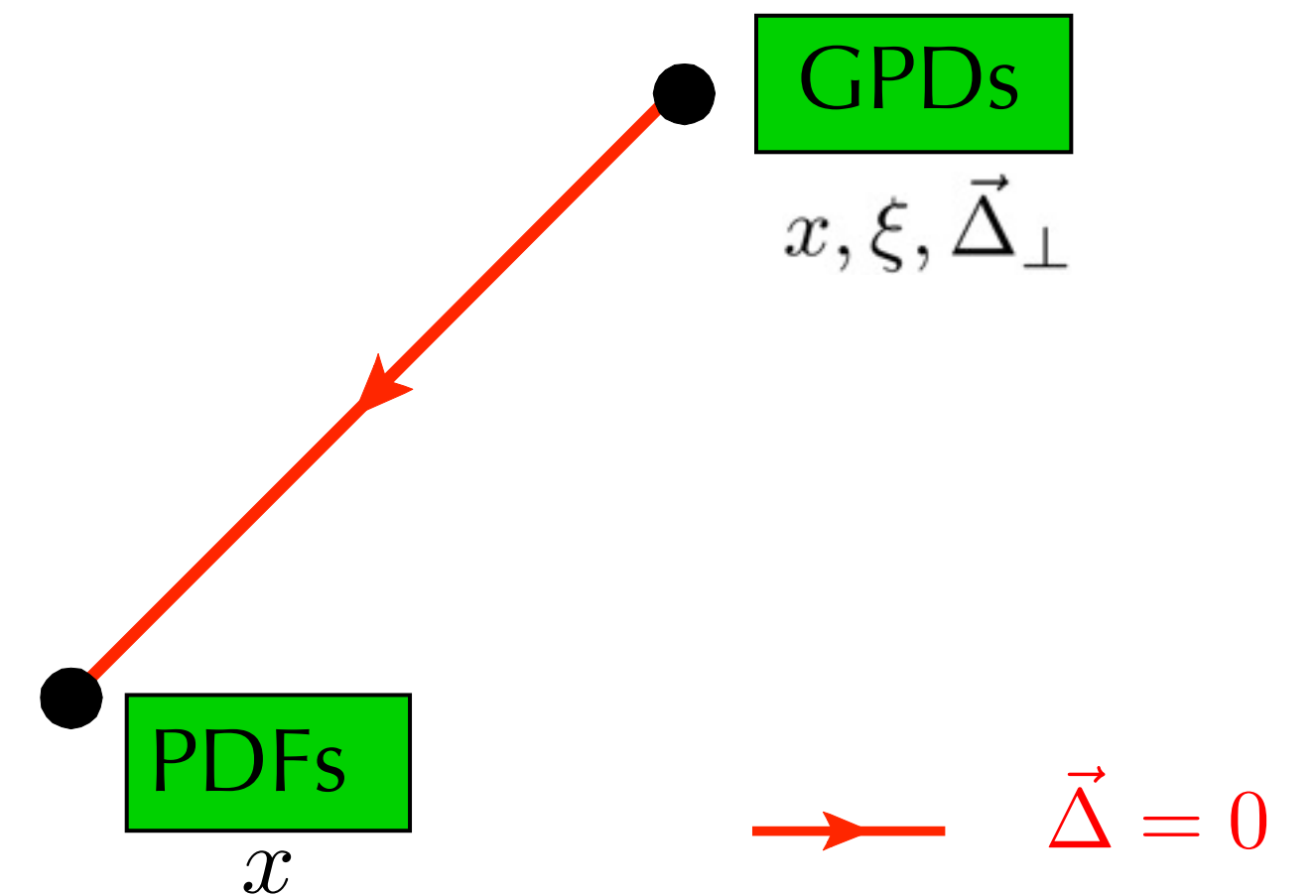


How can we built up
a multidimensional picture
of the nucleon?

Generalized Parton Distributions (GPDs)

$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ik^+ z^-} \langle p'^+, -\frac{\vec{\Delta}_\perp}{2}, \Lambda' | \bar{\psi}(-\frac{z}{2}) \Gamma \mathcal{W} \psi(\frac{z}{2}) | p^+, \frac{\vec{\Delta}_\perp}{2}, \Lambda \rangle_{z^+=0, z_\perp=0}$$

→ non-diagonal matrix elements



Generalized Parton Distributions (GPDs)

$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ik^+ z^-} \langle p'^+, -\frac{\vec{\Delta}_\perp}{2}, \Lambda' | \bar{\psi}(-\frac{z}{2}) \Gamma \mathcal{W} \psi(\frac{z}{2}) | p^+, \frac{\vec{\Delta}_\perp}{2}, \Lambda \rangle_{z^+=0, z_\perp=0} \longrightarrow \text{non-diagonal matrix elements}$$

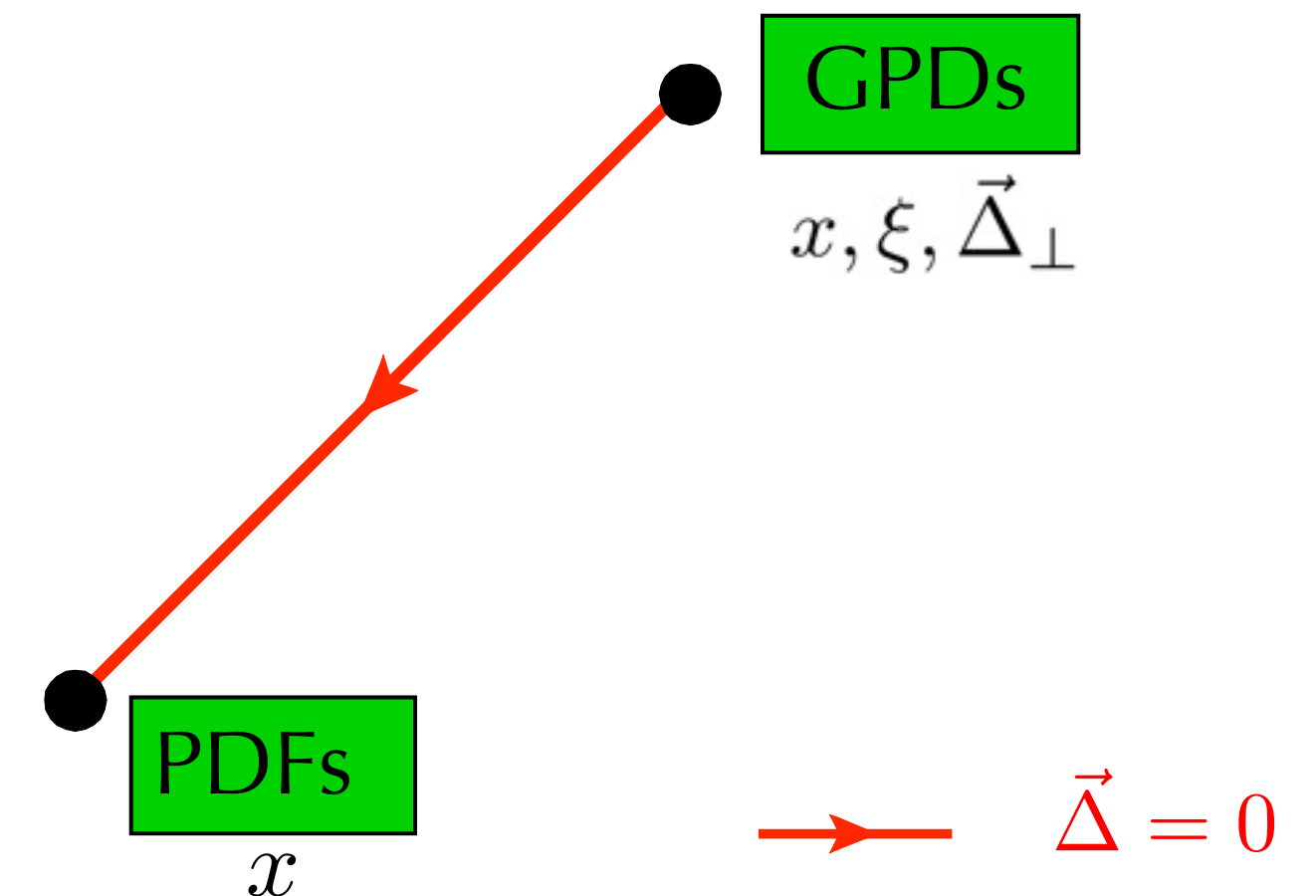
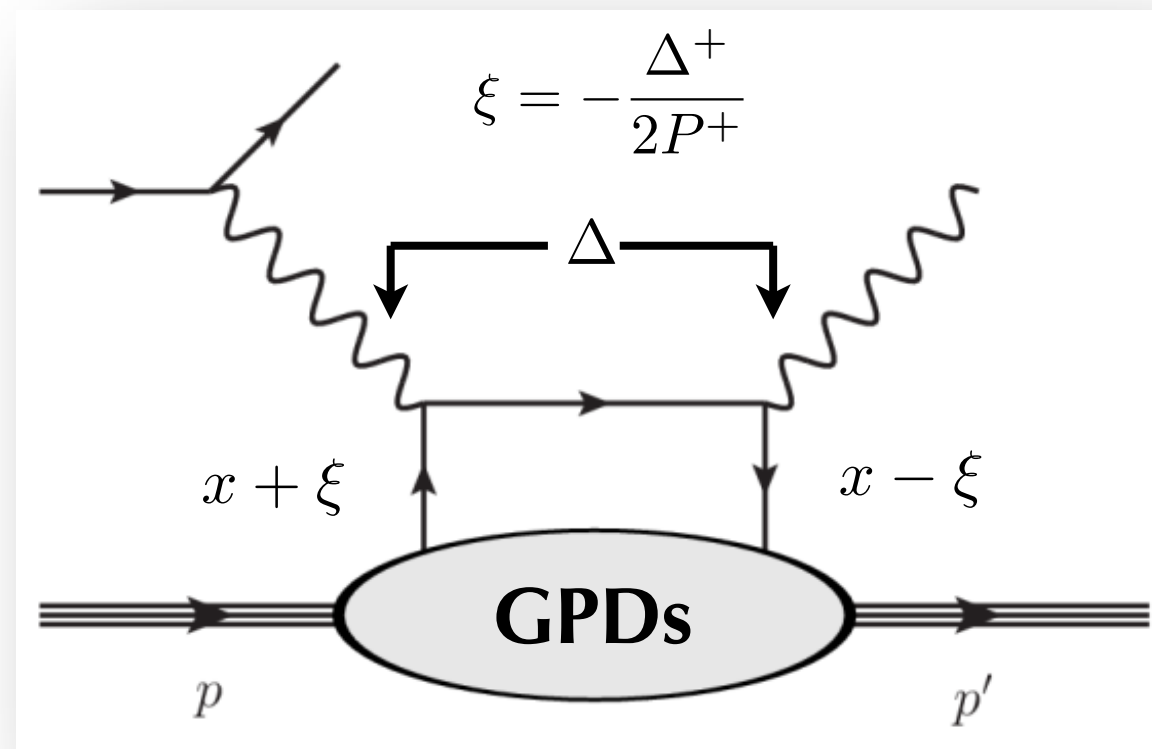
Depend on

$x = \frac{k^+}{P^+}$: longitudinal momentum fraction

Δ : momentum transfer

$\Lambda, \Lambda', \Gamma$: nucleon and quark polarizations

Deeply Virtual Compton Scattering



Generalized Parton Distributions (GPDs)

$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ik^+ z^-} \langle p'^+, -\frac{\vec{\Delta}_\perp}{2}, \Lambda' | \bar{\psi}(-\frac{z}{2}) \Gamma \mathcal{W} \psi(\frac{z}{2}) | p^+, \frac{\vec{\Delta}_\perp}{2}, \Lambda \rangle_{z^+=0, z_\perp=0} \longrightarrow \text{non-diagonal matrix elements}$$

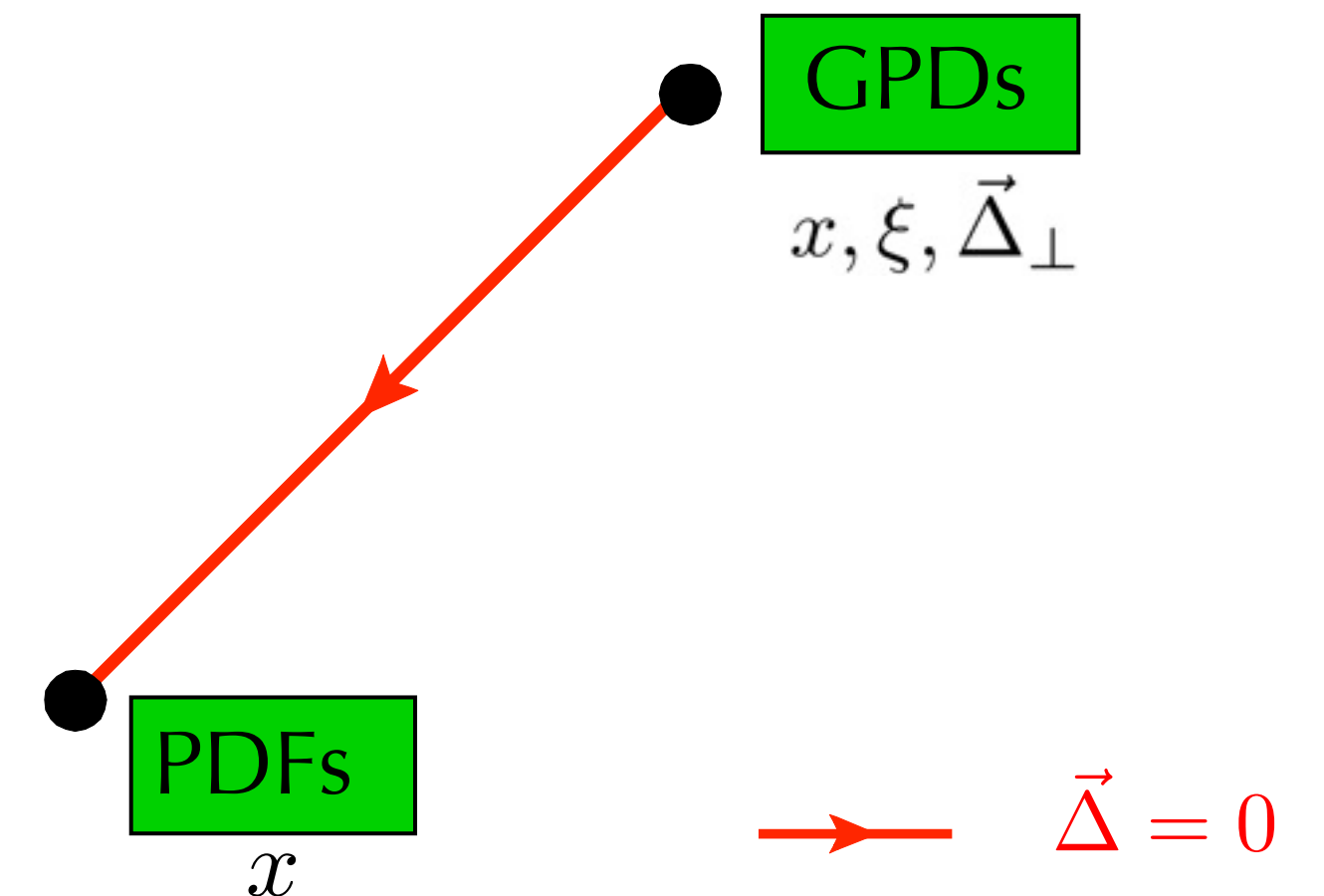
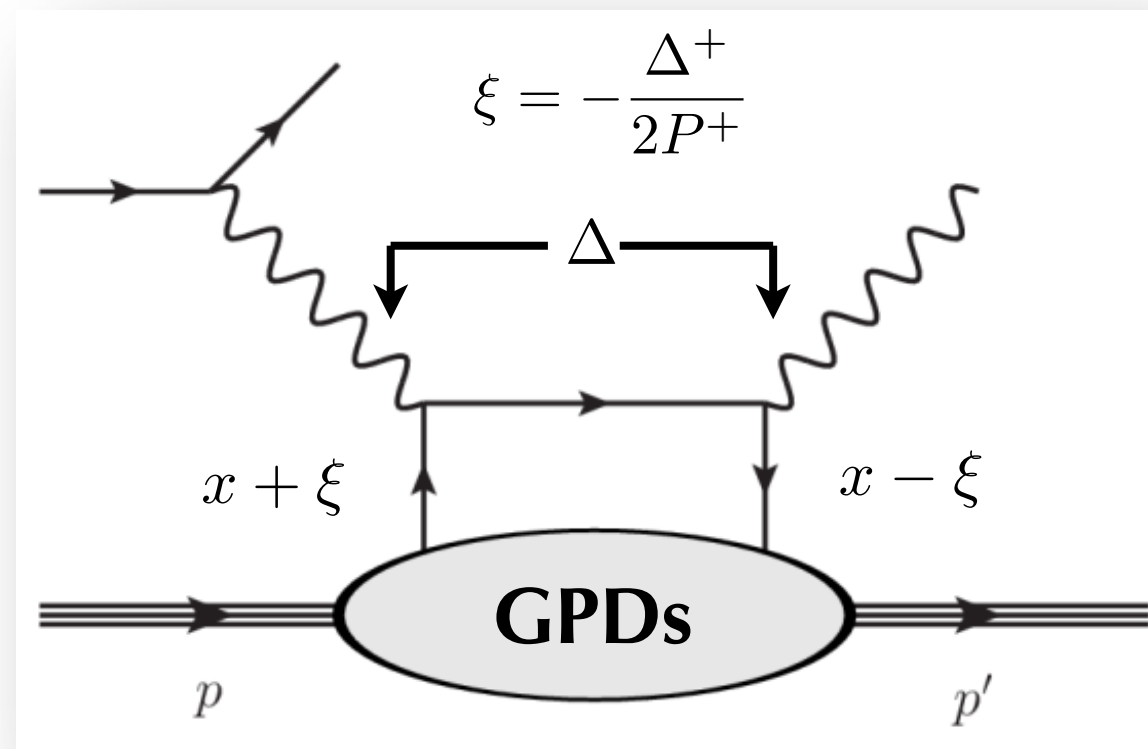
Depend on

$x = \frac{k^+}{P^+}$: longitudinal momentum fraction

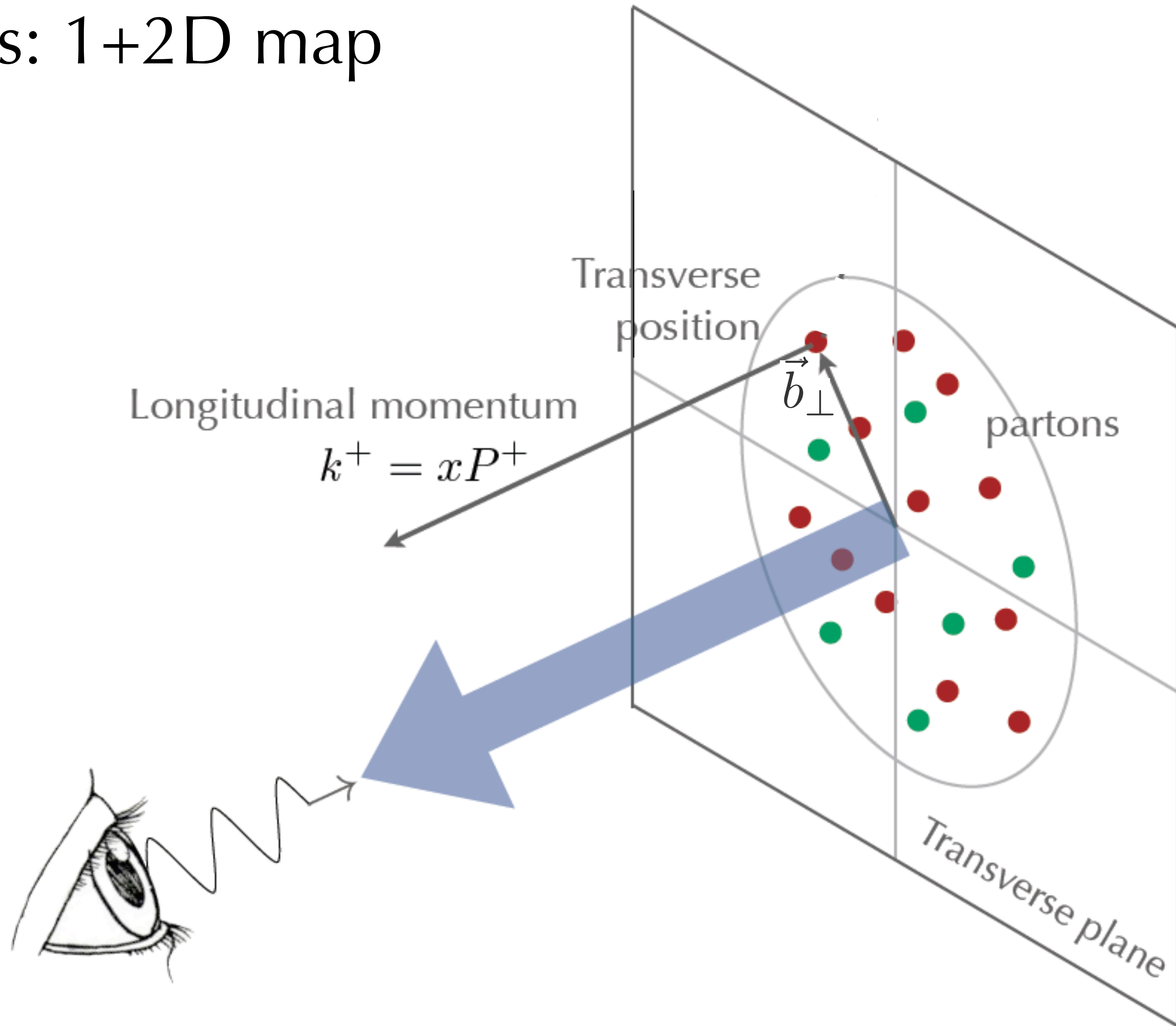
Δ : momentum transfer $\vec{\Delta}_\perp \xleftrightarrow{\text{FT}} \vec{b}_\perp$: impact parameter

$\Lambda, \Lambda', \Gamma$: nucleon and quark polarizations

Deeply Virtual Compton Scattering



GPDs: 1+2D map



Transverse Momentum PDFs (TMDs)

$$\frac{1}{2} \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{ik \cdot z} \langle p^+, \vec{0}_\perp, \Lambda' | \bar{\psi}(-\frac{z}{2}) \Gamma \mathcal{W} \psi(\frac{z}{2}) | p^+, \vec{0}_\perp, \Lambda \rangle_{z^+=0}$$

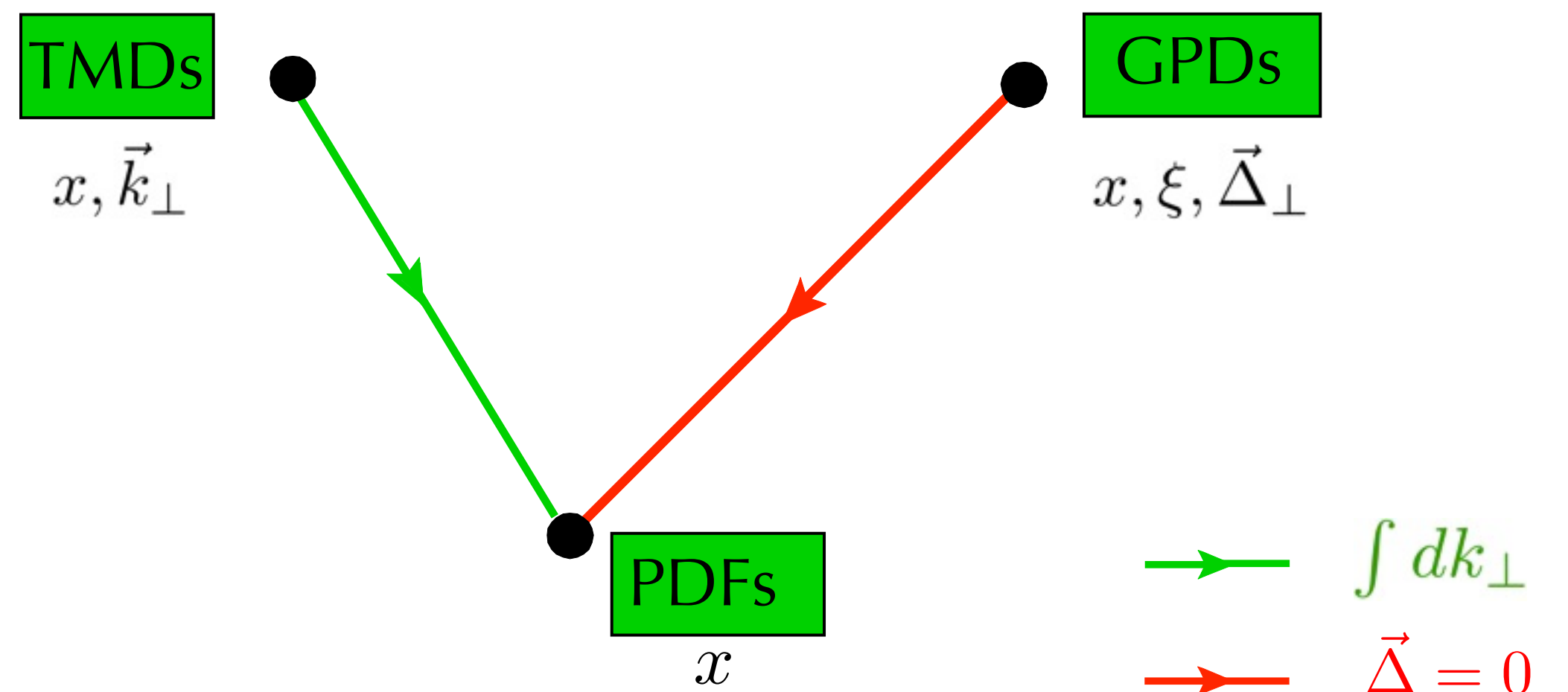
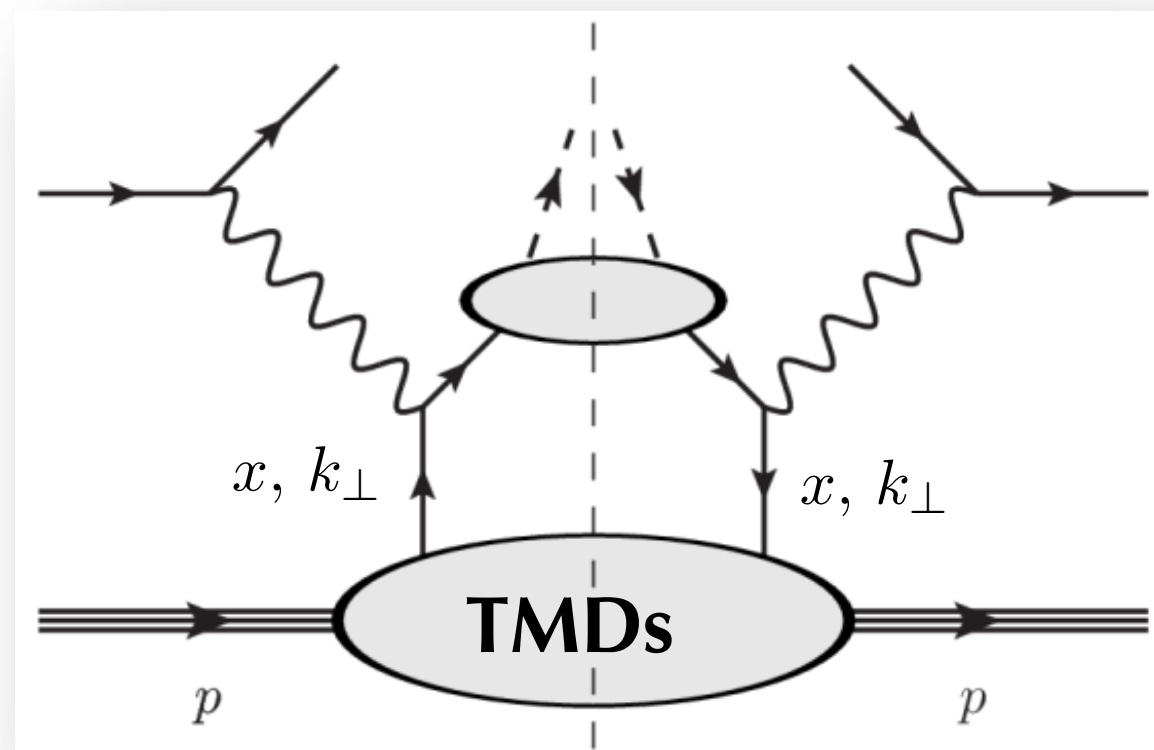
Depend on

$x = \frac{k^+}{P^+}$: longitudinal momentum fraction

k_\perp : parton transverse momentum

$\Lambda, \Lambda', \Gamma$: nucleon and quark polarizations

Semi-Inclusive Deep Inelastic Scattering



Transverse Momentum PDFs (TMDs)

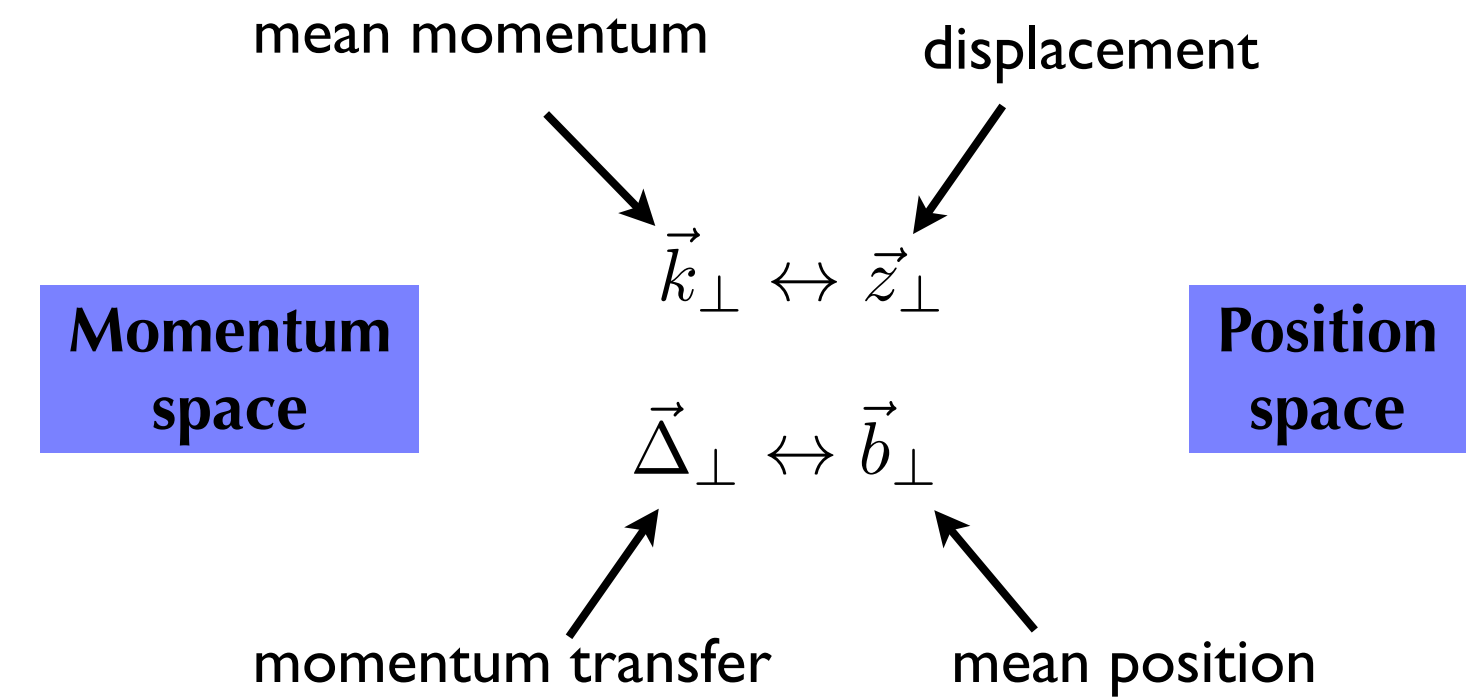
$$\frac{1}{2} \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{ik \cdot z} \langle p^+, \vec{0}_\perp, \Lambda' | \bar{\psi}(-\frac{z}{2}) \Gamma \mathcal{W} \psi(\frac{z}{2}) | p^+, \vec{0}_\perp, \Lambda \rangle_{z^+=0}$$

Depend on

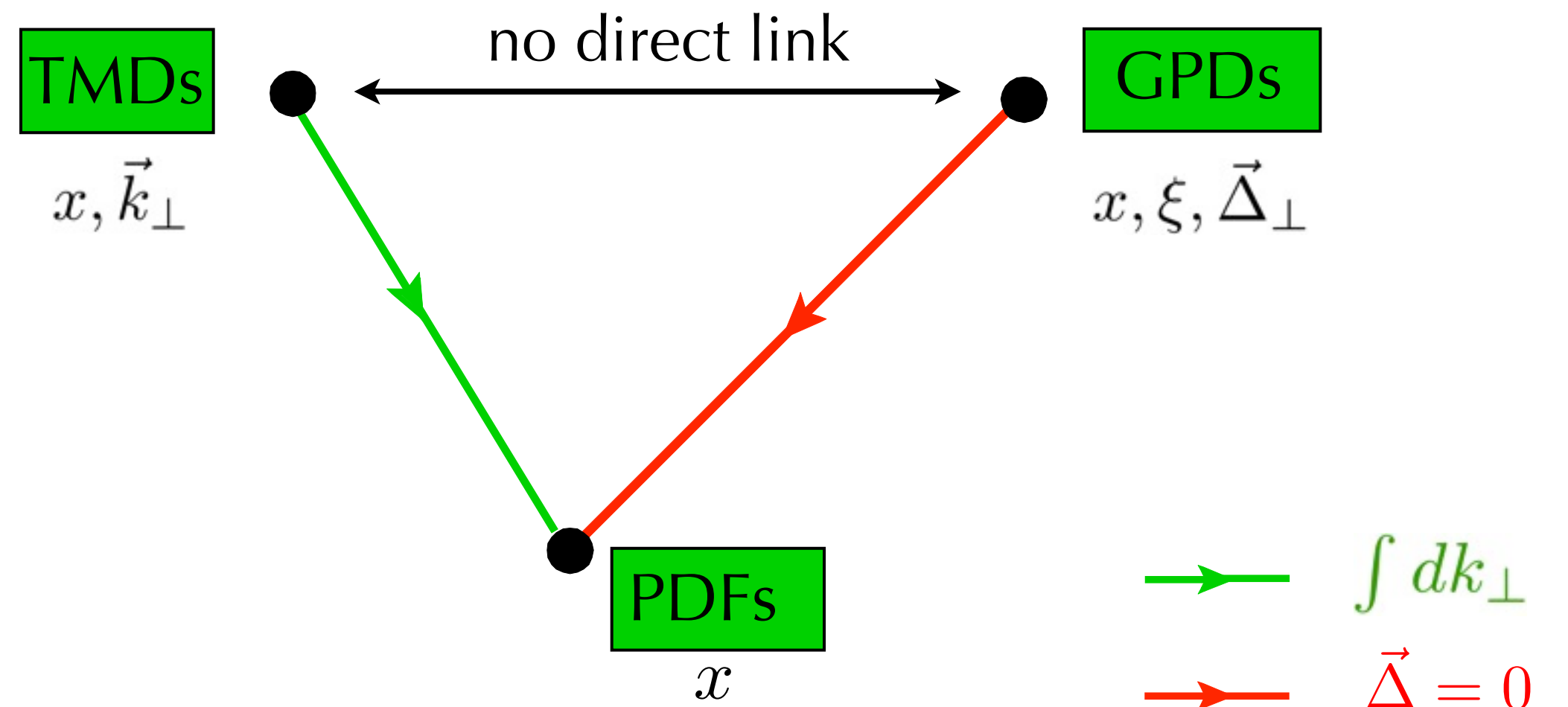
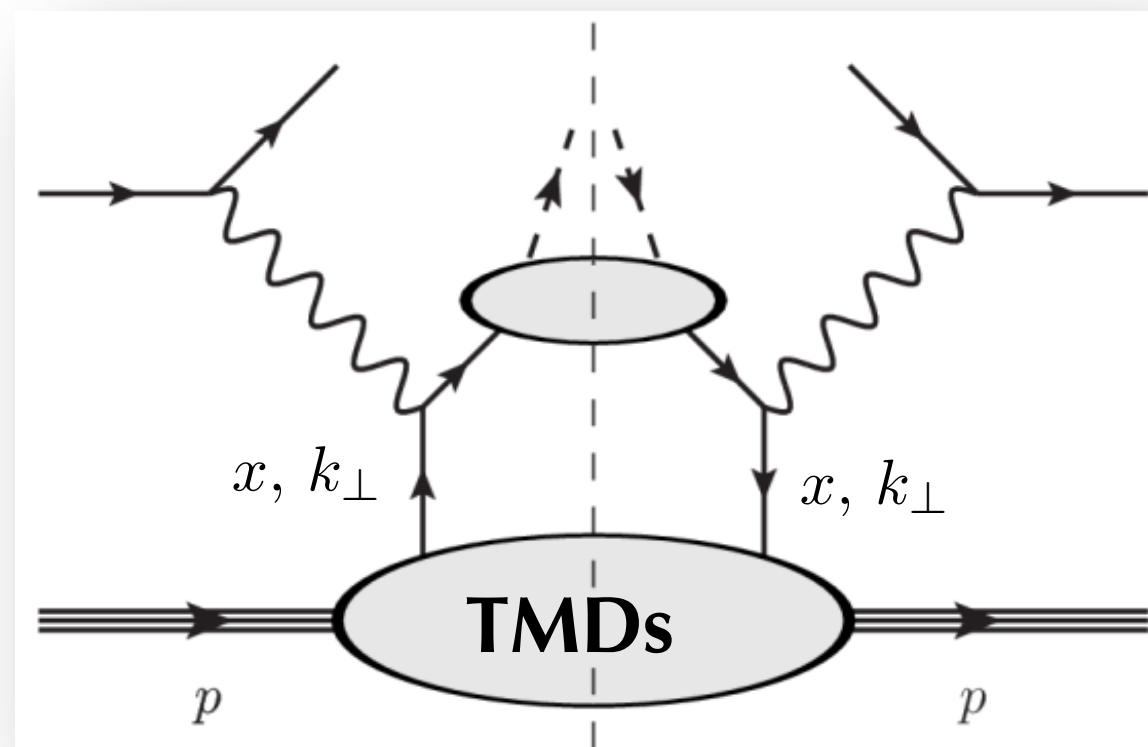
$x = \frac{k^+}{P^+}$: longitudinal momentum fraction

k_\perp : parton transverse momentum

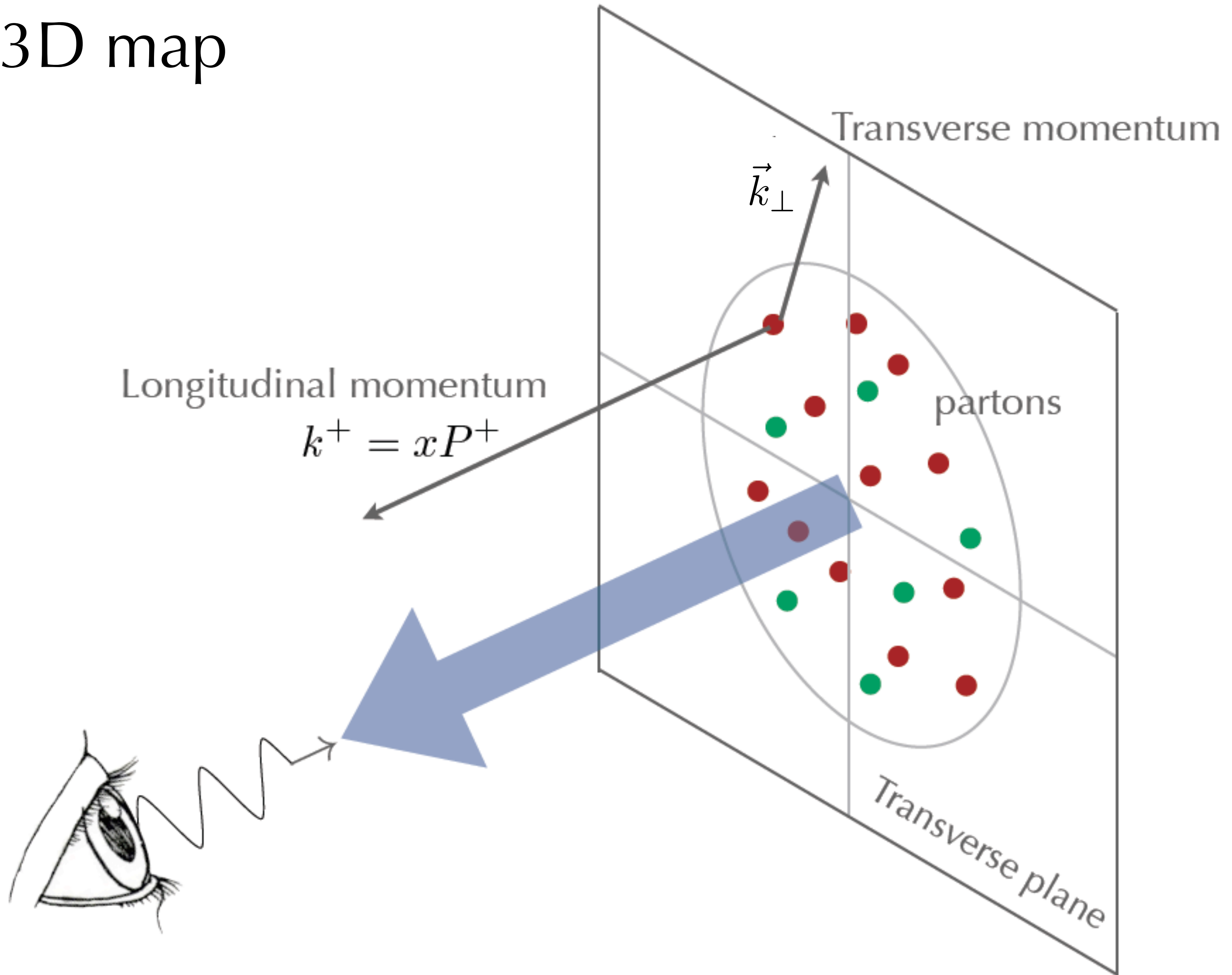
$\Lambda, \Lambda', \Gamma$: nucleon and quark polarizations



Semi-Inclusive Deep Inelastic Scattering



TMDs: 3D map



Generalized TMDs (GTMDs)

$$\frac{1}{2} \int \frac{dz^- d^2z_\perp}{(2\pi)^3} e^{ik \cdot z} \langle p^+, -\frac{\vec{\Delta}_\perp}{2}, \Lambda' | \bar{\psi}(-\frac{z}{2}) \Gamma \mathcal{W} \psi(\frac{z}{2}) | p^+, \frac{\vec{\Delta}_\perp}{2}, \Lambda \rangle_{z^+=0}$$

Depend on

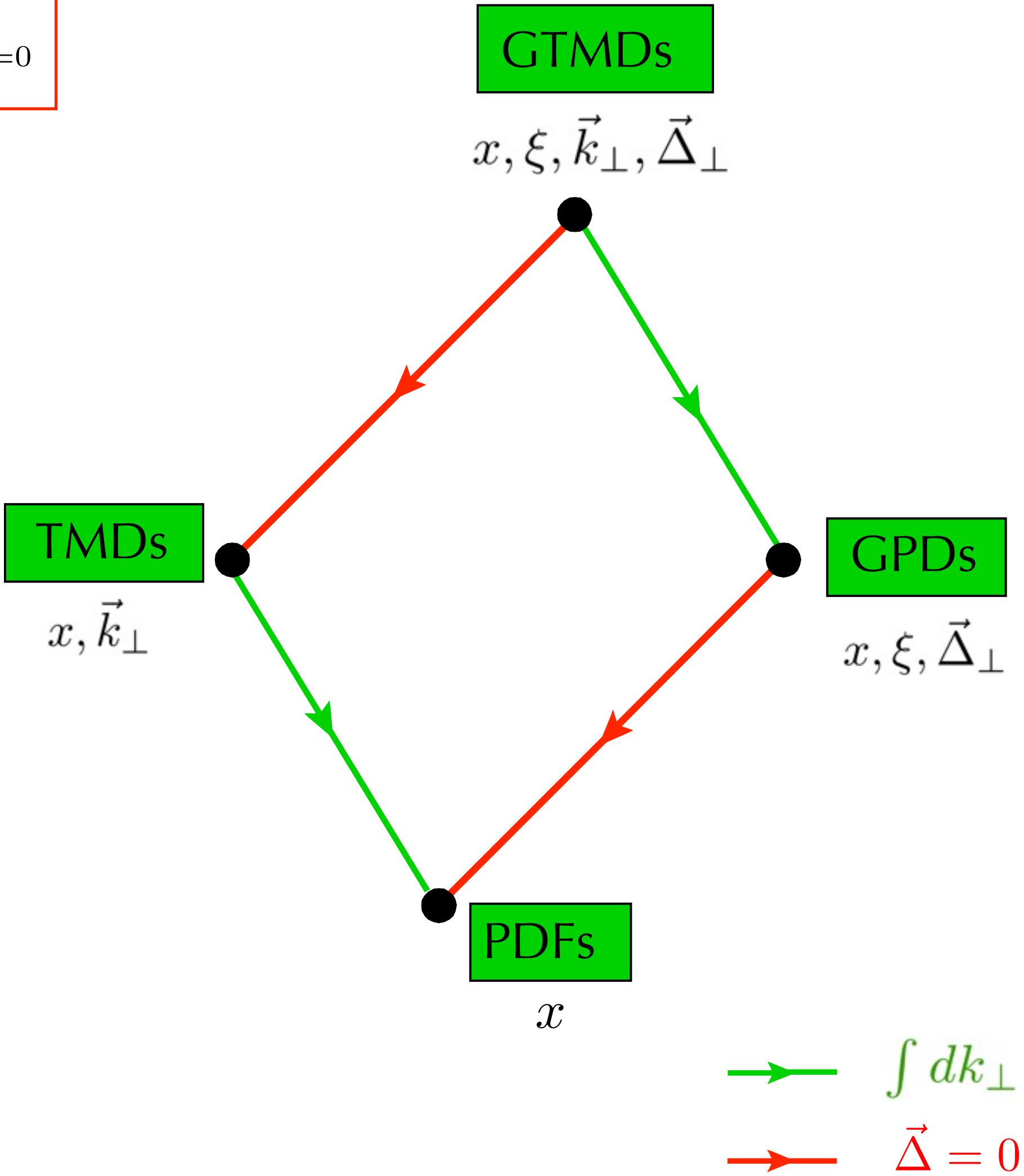
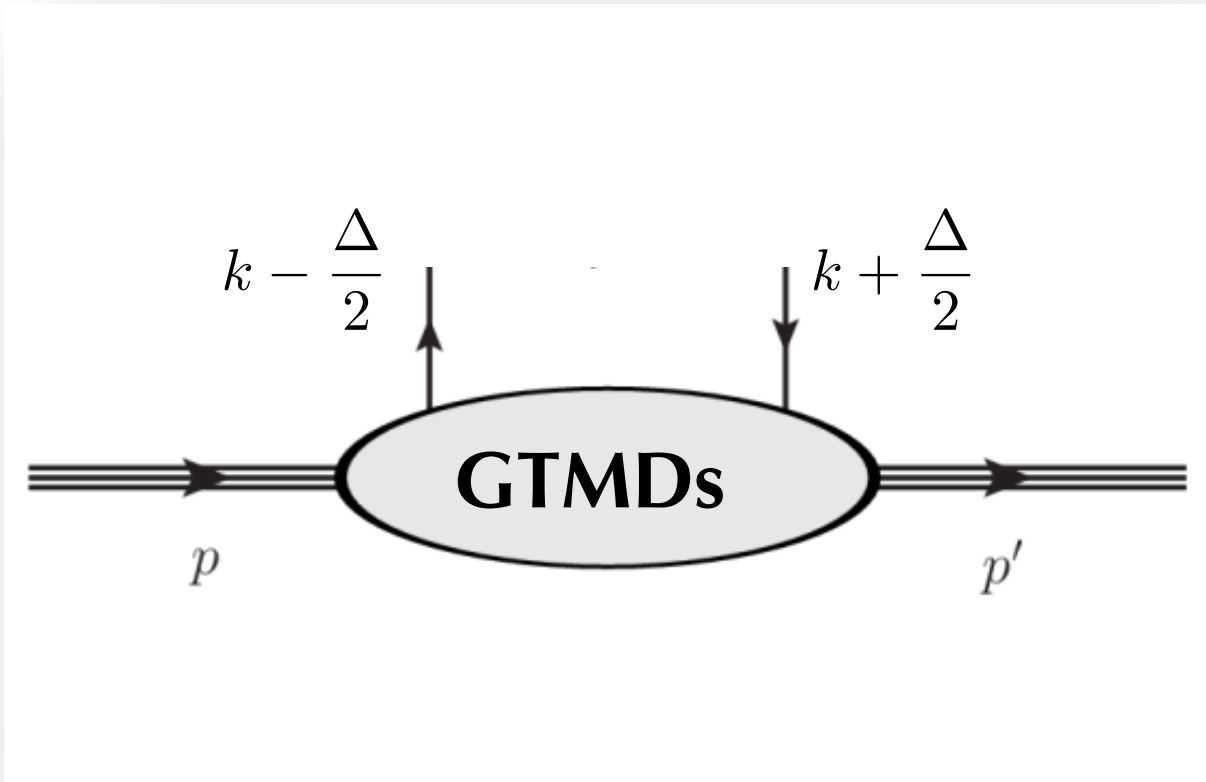
$x = \frac{k^+}{P^+}$: longitudinal momentum fraction

Δ : momentum transfer

k_\perp : parton transverse momentum

$\Lambda, \Lambda', \Gamma$: nucleon and quark polarizations

????????



Generalized TMDs (GTMDs)

$$\frac{1}{2} \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{ik \cdot z} \langle p^+, -\frac{\vec{\Delta}_\perp}{2}, \Lambda' | \bar{\psi}(-\frac{z}{2}) \Gamma \mathcal{W} \psi(\frac{z}{2}) | p^+, \frac{\vec{\Delta}_\perp}{2}, \Lambda \rangle_{z^+=0}$$

Depend on

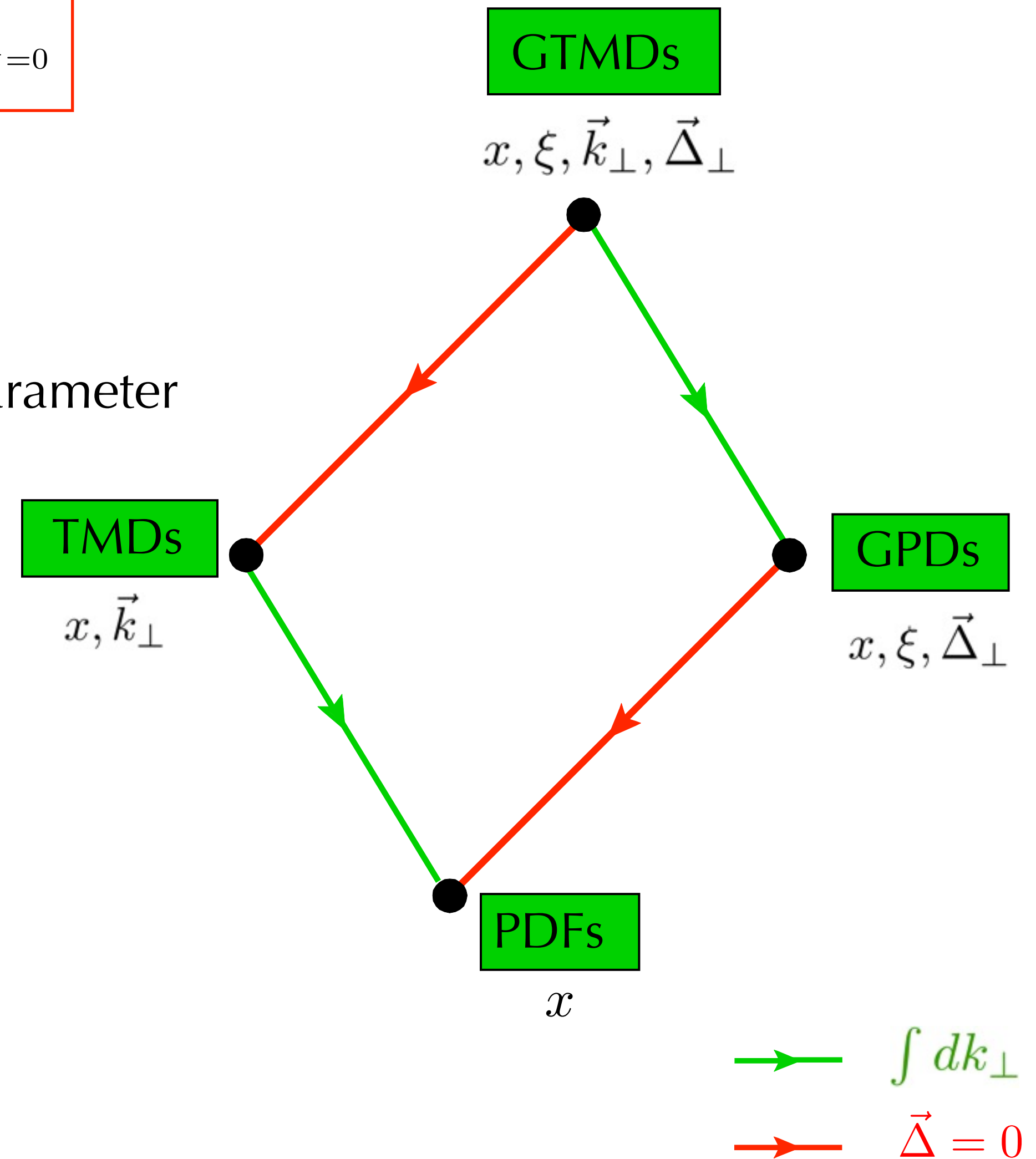
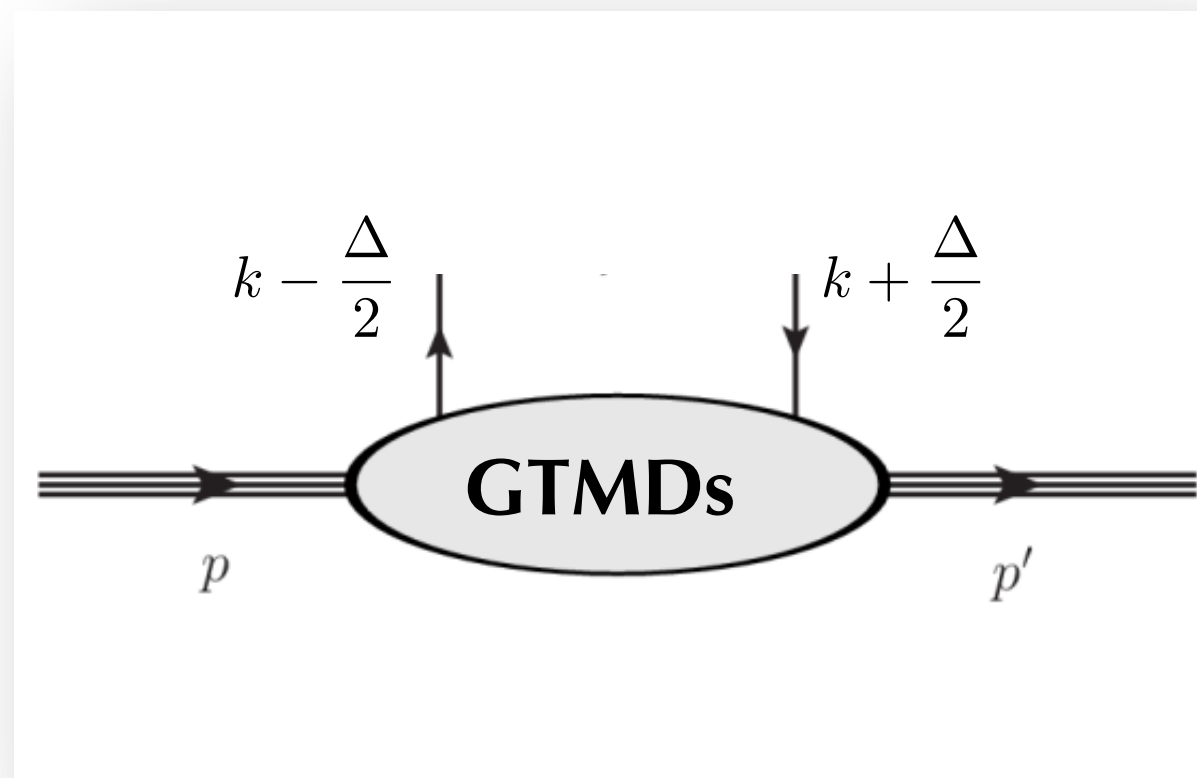
$x = \frac{k^+}{P^+}$: longitudinal momentum fraction

Δ : momentum transfer $\vec{\Delta}_\perp \xleftrightarrow{\text{FT}} \vec{b}_\perp$: impact parameter

k_\perp : parton transverse momentum

$\Lambda, \Lambda', \Gamma$: nucleon and quark polarizations

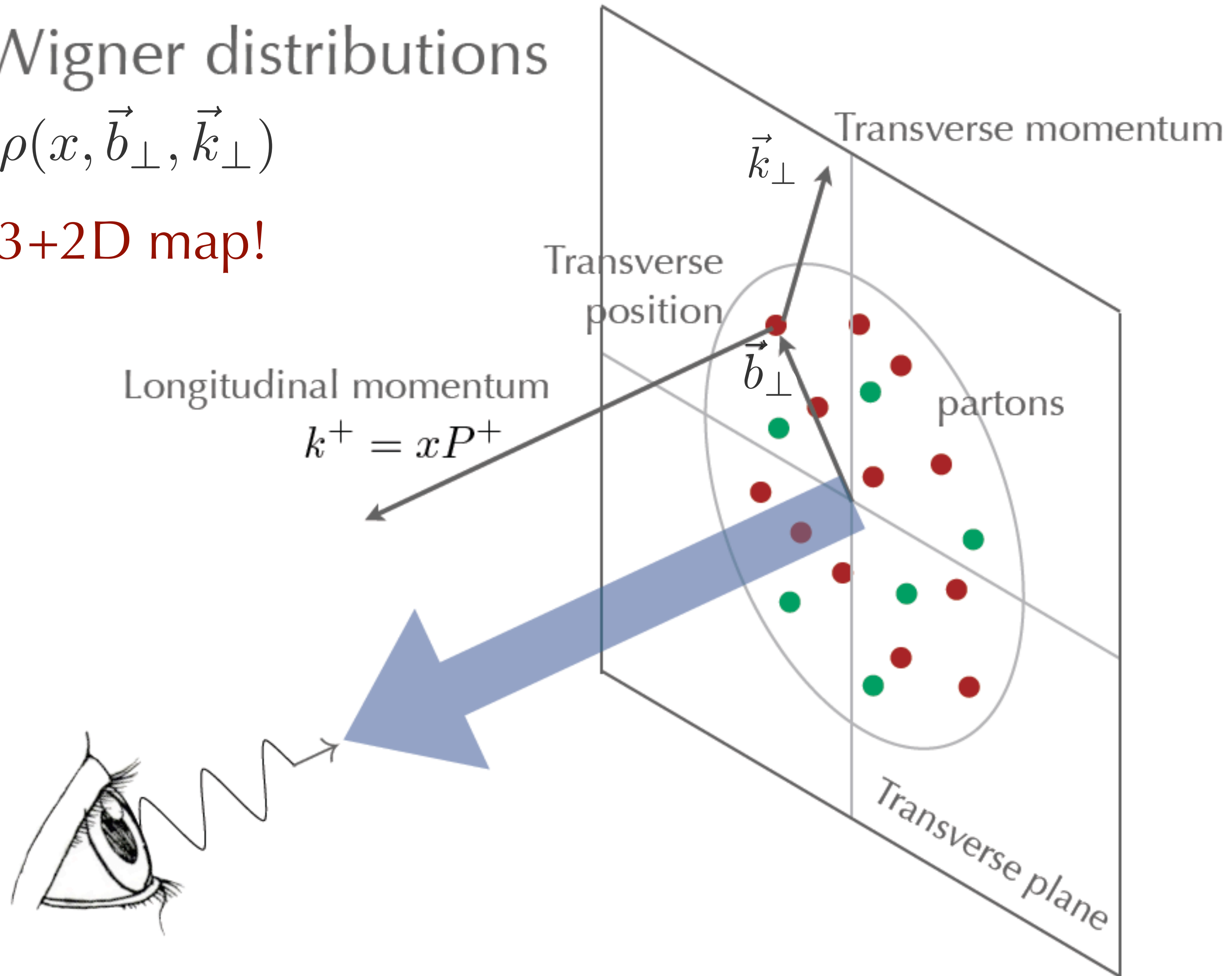
????????

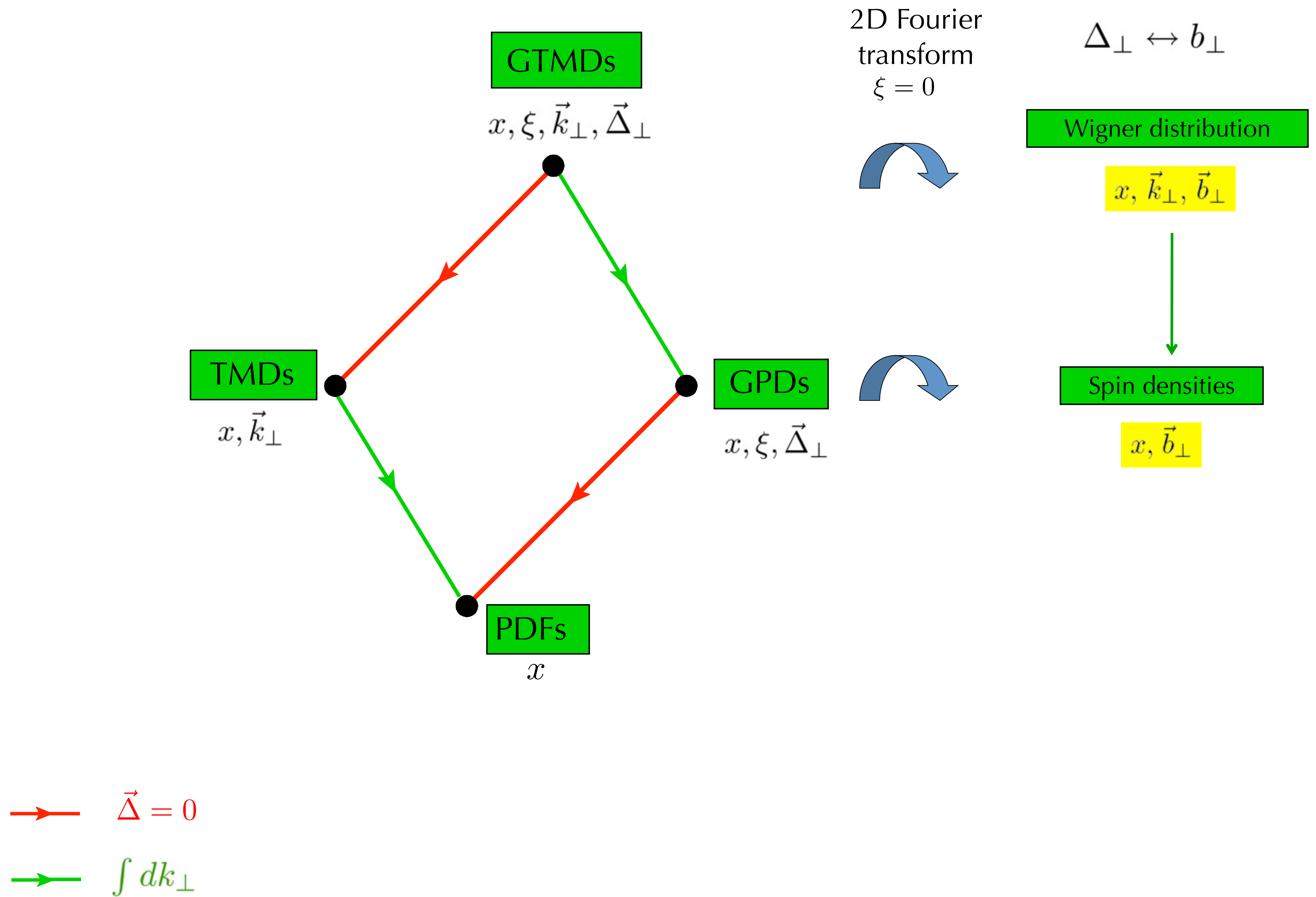


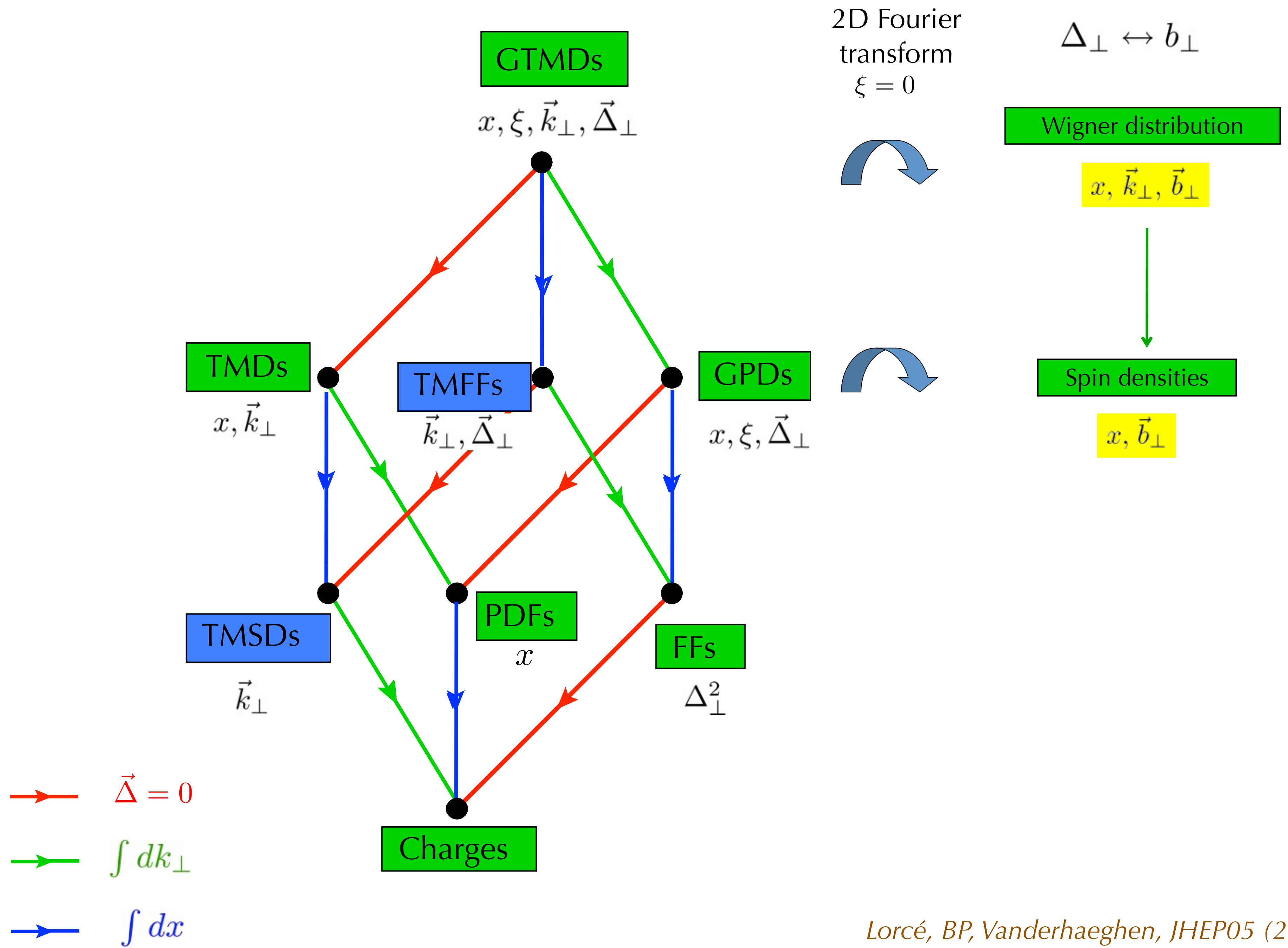
Wigner distributions

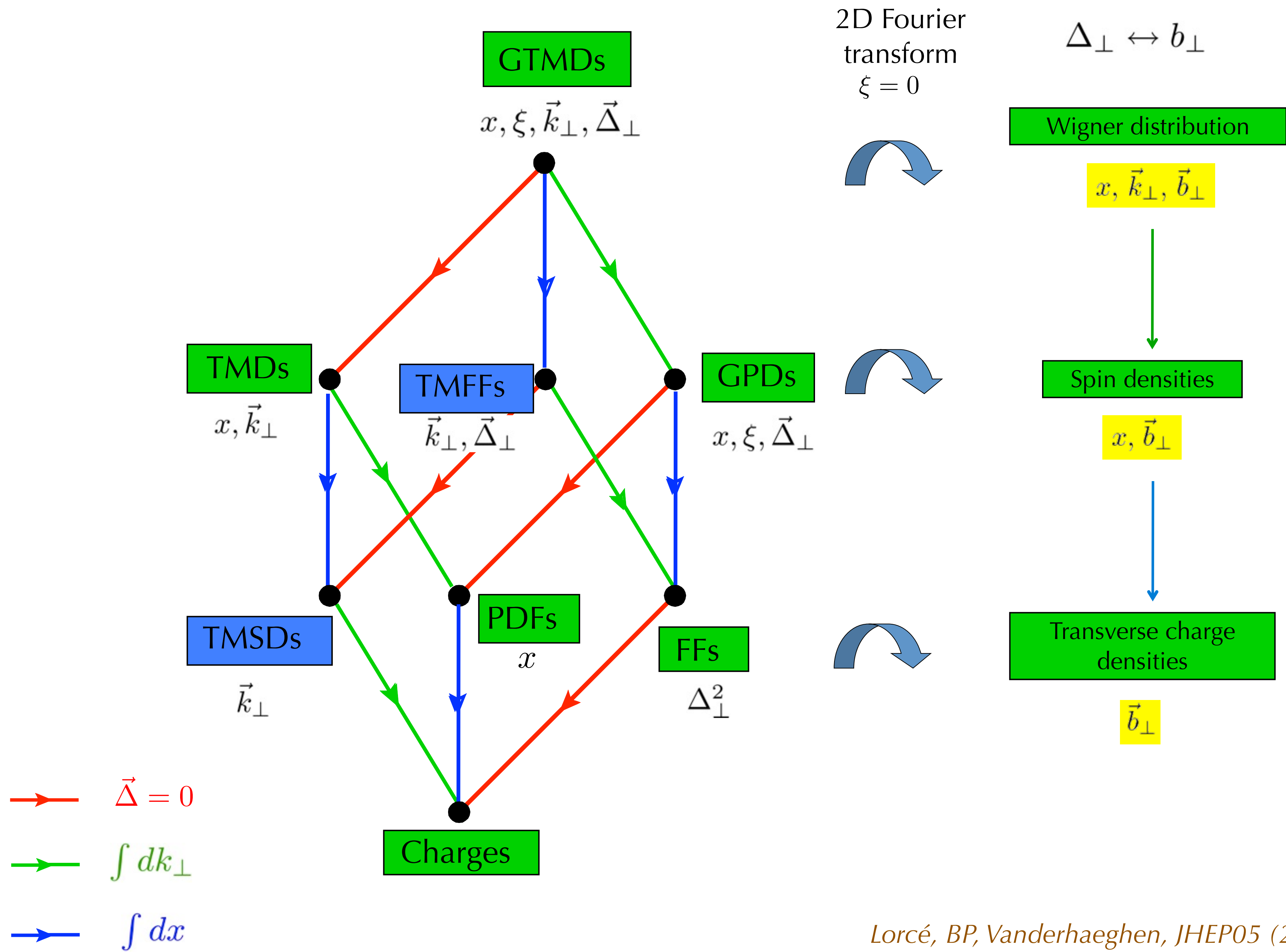
$$\rho(x, \vec{b}_\perp, \vec{k}_\perp)$$

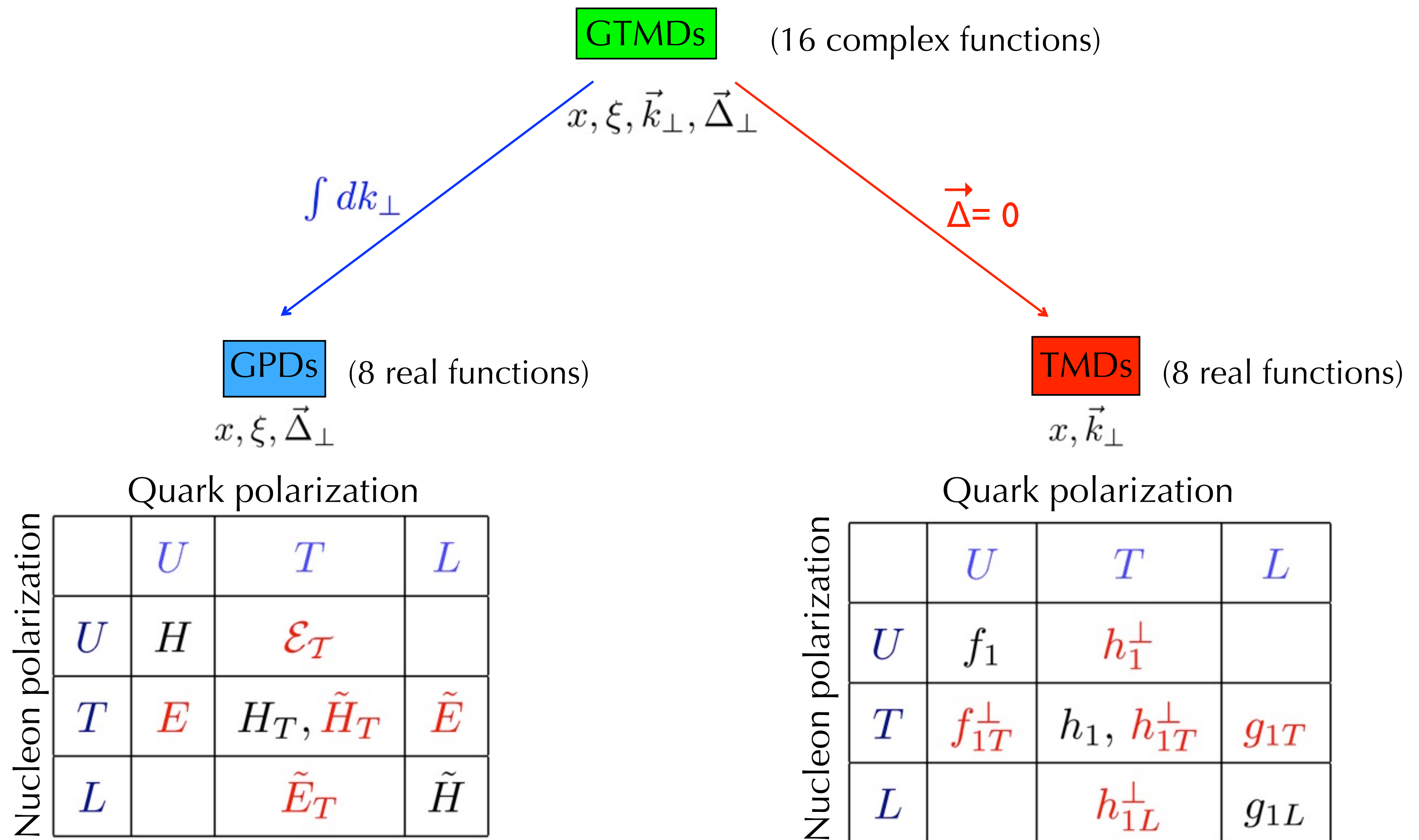
3+2D map!











each distribution contains unique information

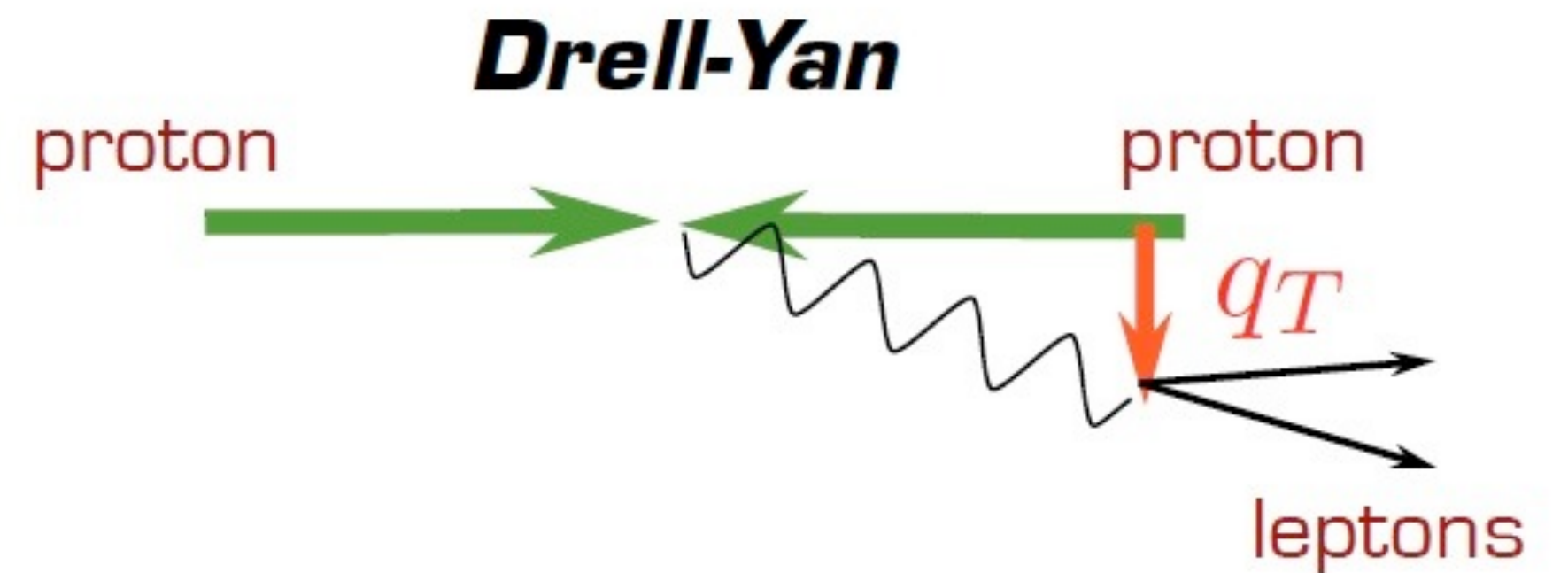
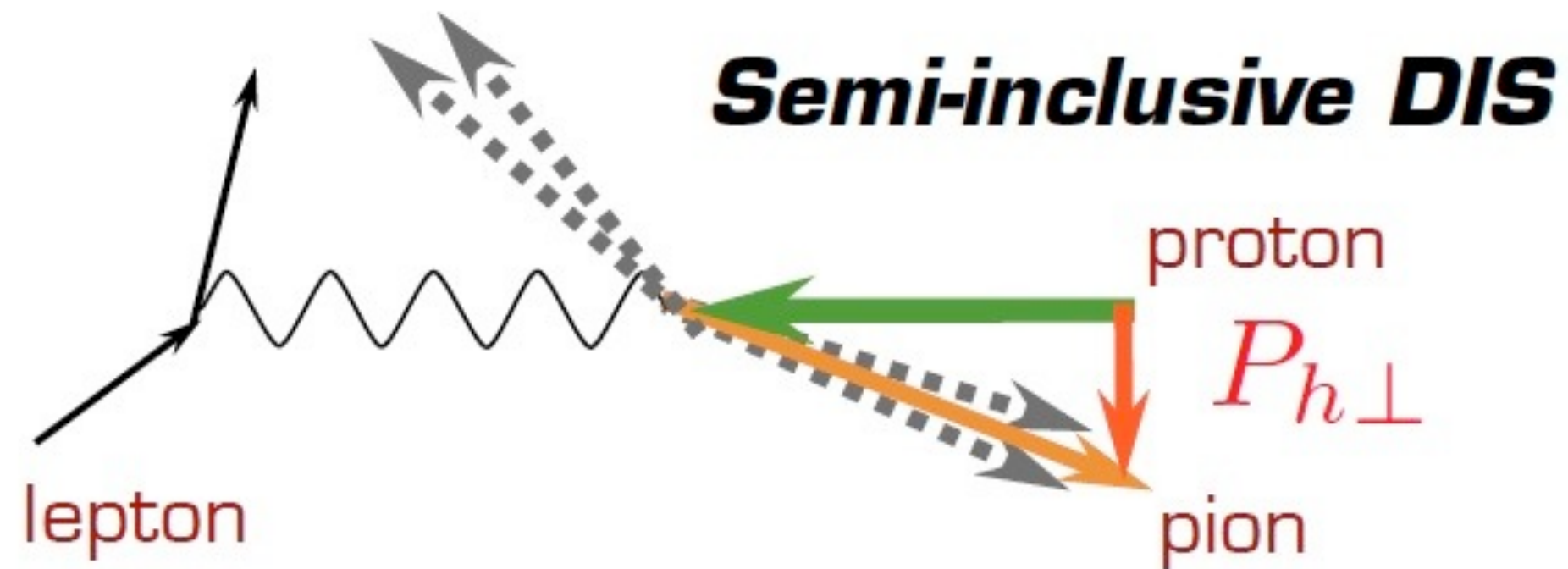
the distributions in **red** vanish if there is no quark orbital angular momentum

the distributions in black survive in the collinear limit

Key information from TMDs

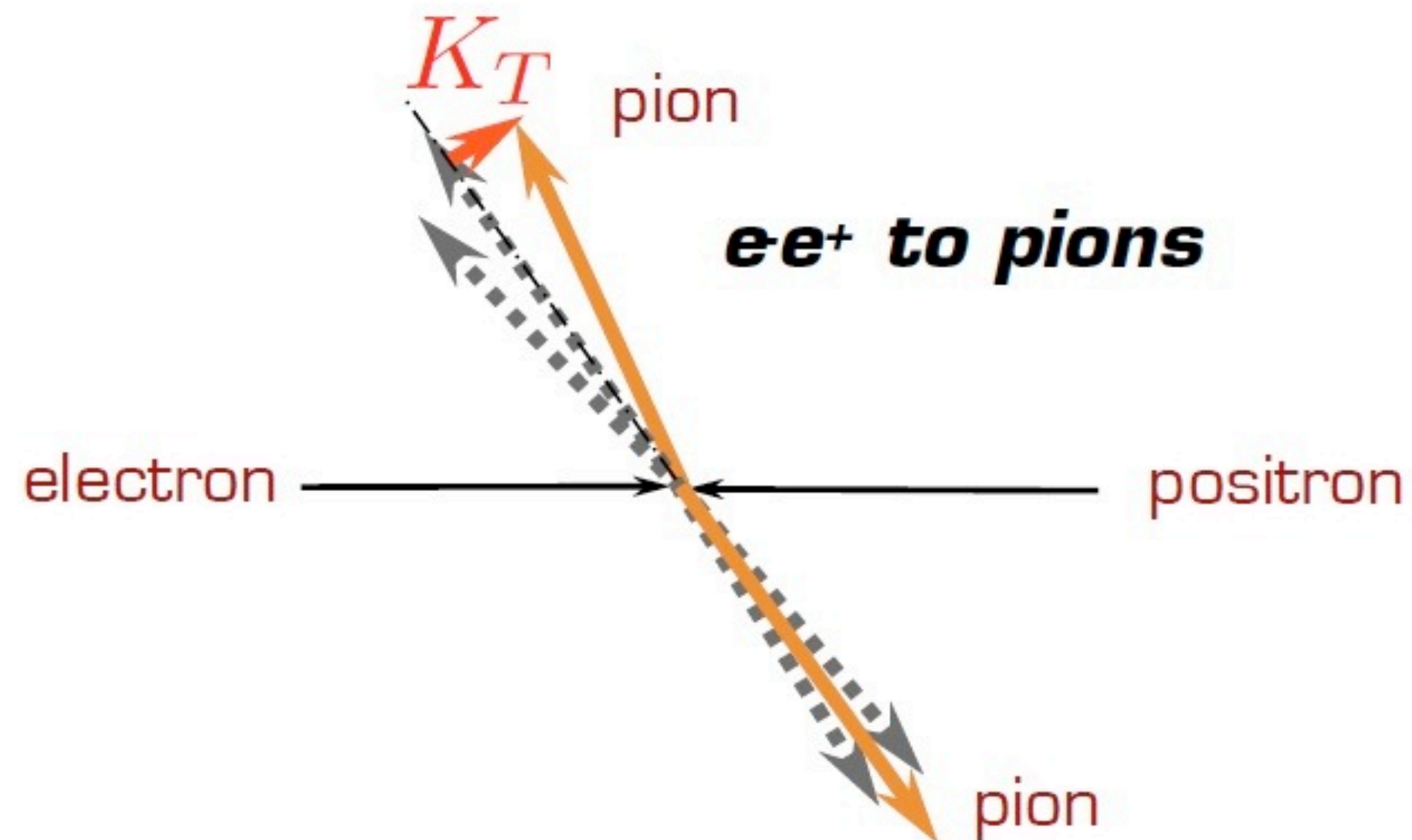
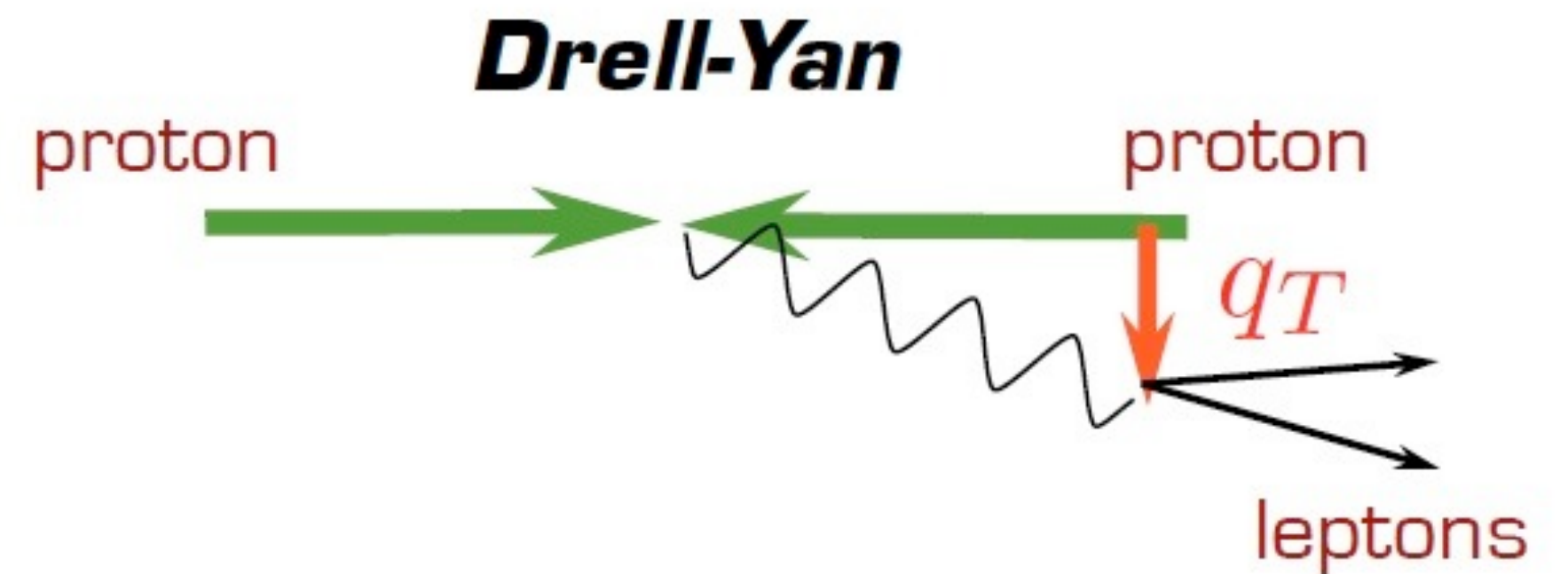
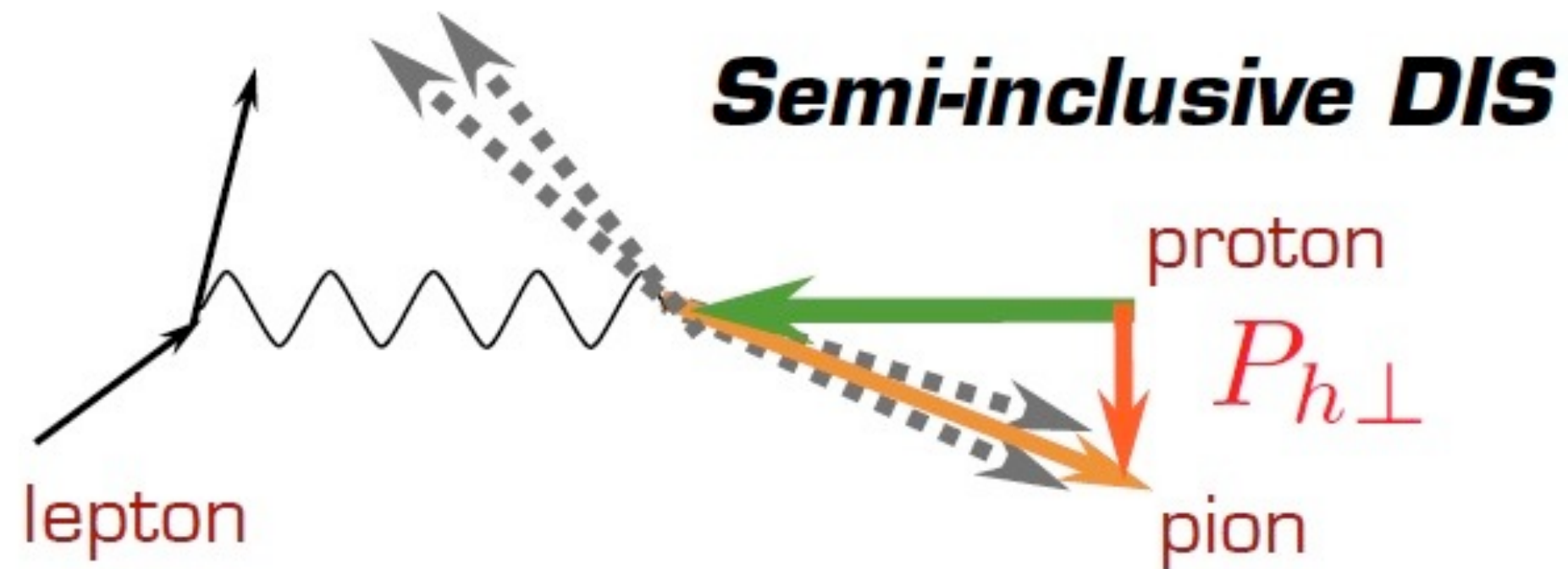
- Spin-Spin and Spin-Orbit Correlations of partons
- Transverse momentum size
- Test what we can calculate with QCD (perturbative and lattice)
- Non-perturbative structure we cannot calculate with QCD

Where can we access TMDs?



we are interested
in the region of small transverse-momenta
sensitive to non-perturbative QCD effects

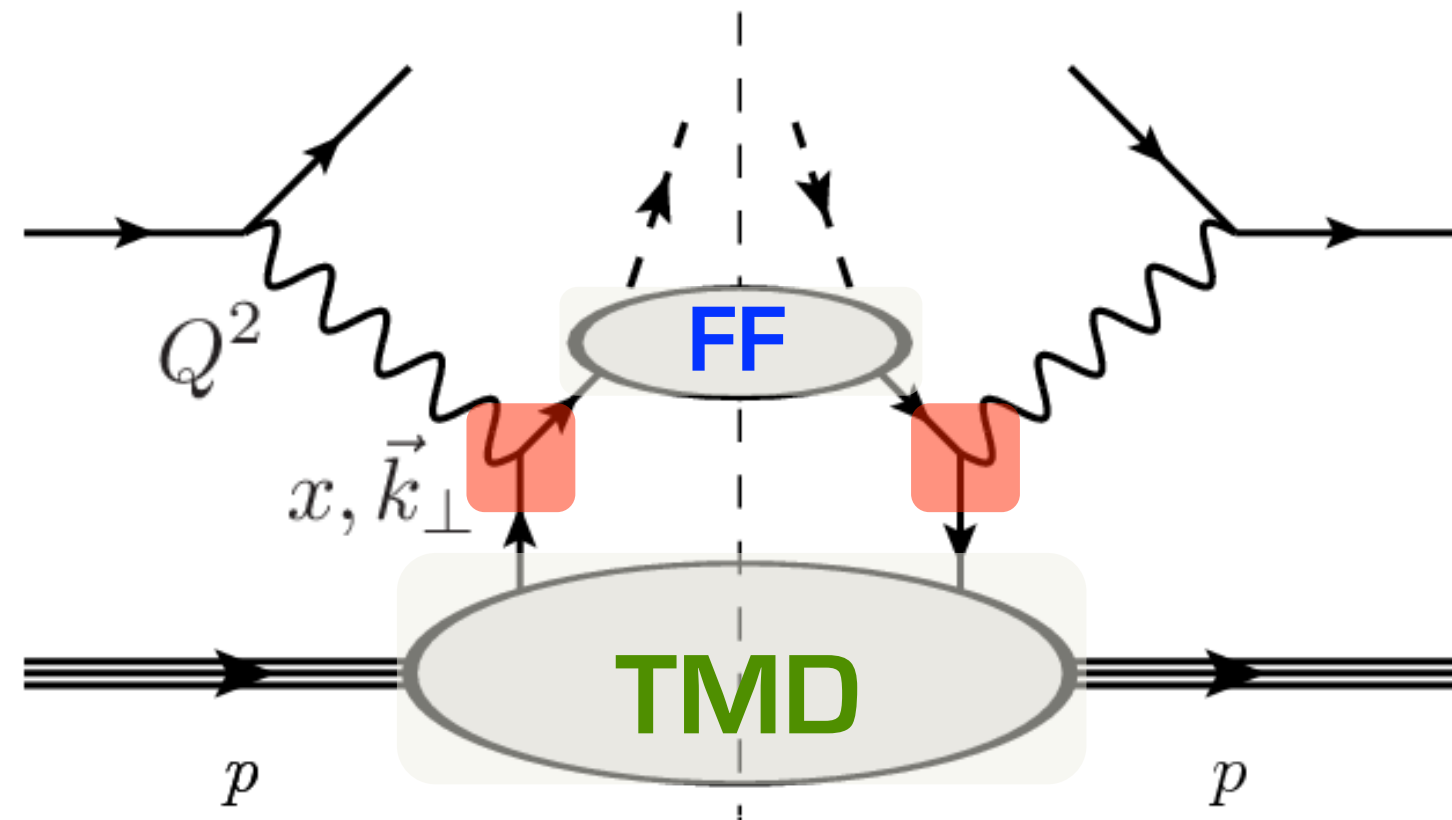
Where can we access TMDs?



we are interested
in the region of small transverse-momenta
sensitive to non-perturbative QCD effects

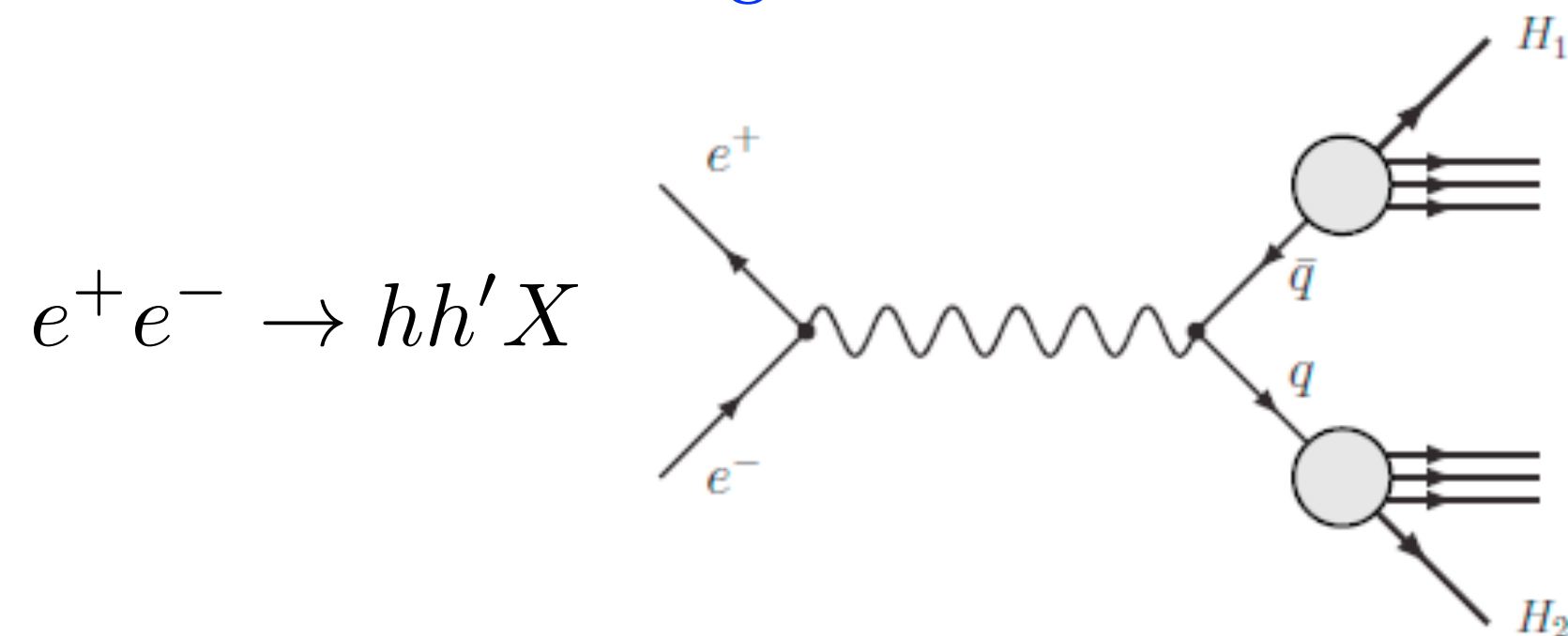
How to measure the TMDs

$$\ell(l) + N(P) \rightarrow \ell(l') + h(P_h) + X$$

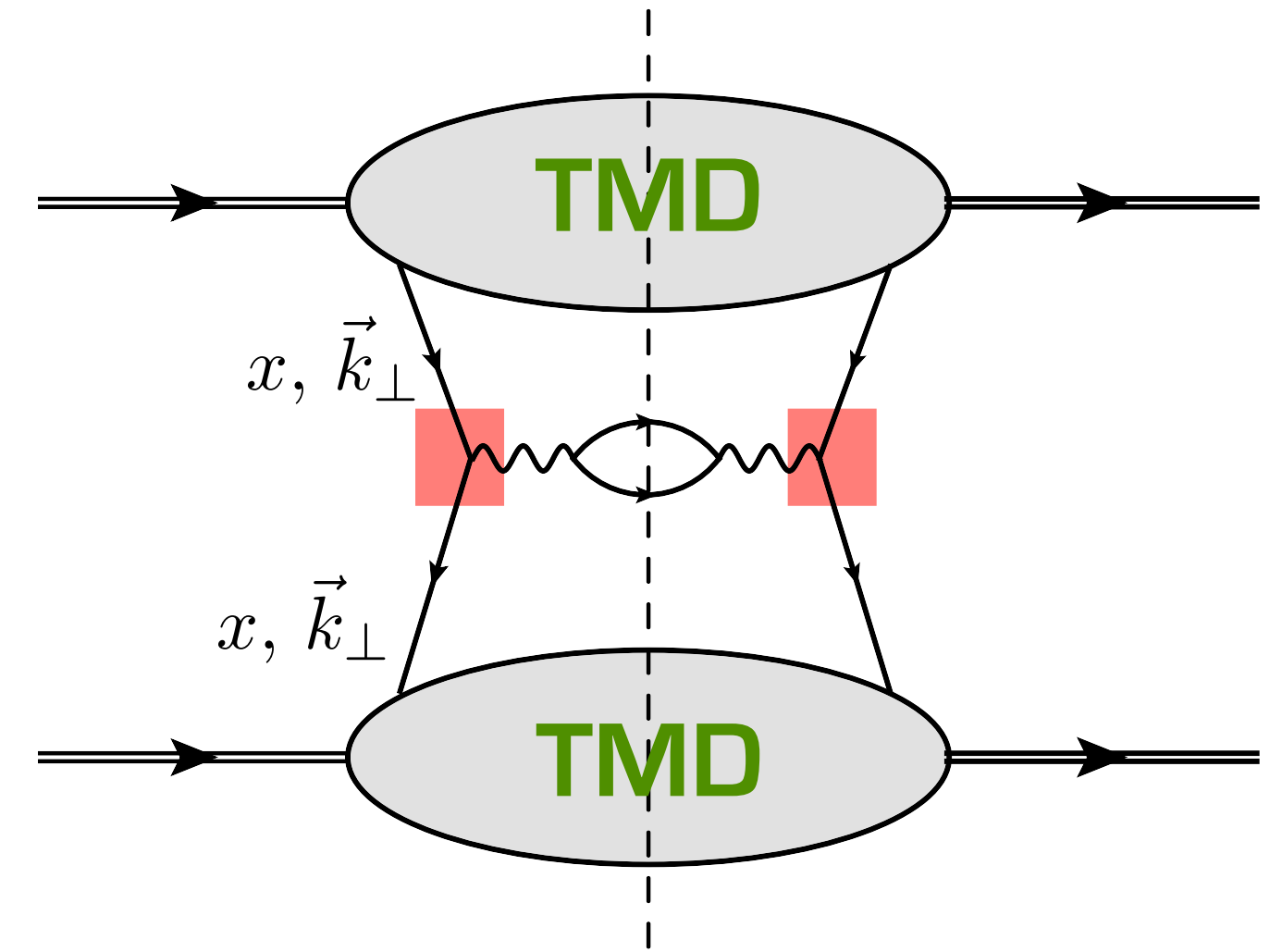


$$d\sigma \sim \sum \text{TMD}(x, \vec{k}_\perp) \otimes d\hat{\sigma}_{hard} \otimes \text{FF}(z, \vec{p}_\perp) + \mathcal{O}(\frac{P_T}{Q})$$

Fragmentation Functions



$$h(P_1) + h(P_2) \rightarrow \ell^+(l) + \ell^-(l')$$



$$d\sigma \sim \sum \text{TMD}(x, \vec{k}_\perp) \otimes \overline{\text{TMD}}(x, \vec{k}_\perp) \otimes d\hat{\sigma}_{hard}$$

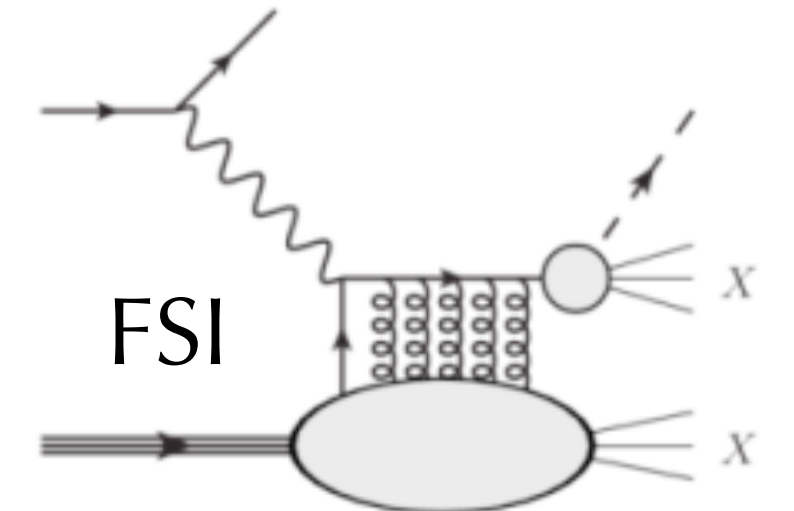
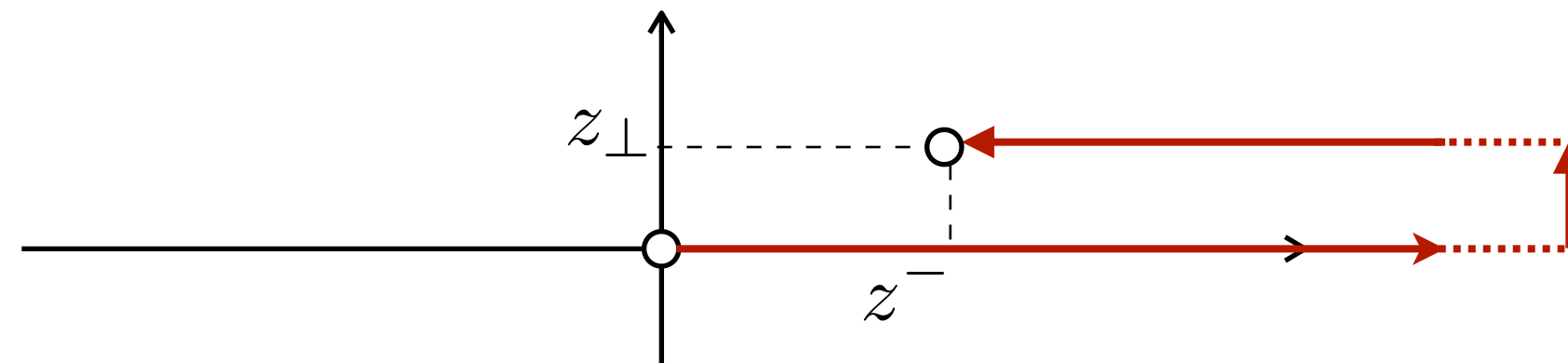
✓ Factorization

✓ Universality

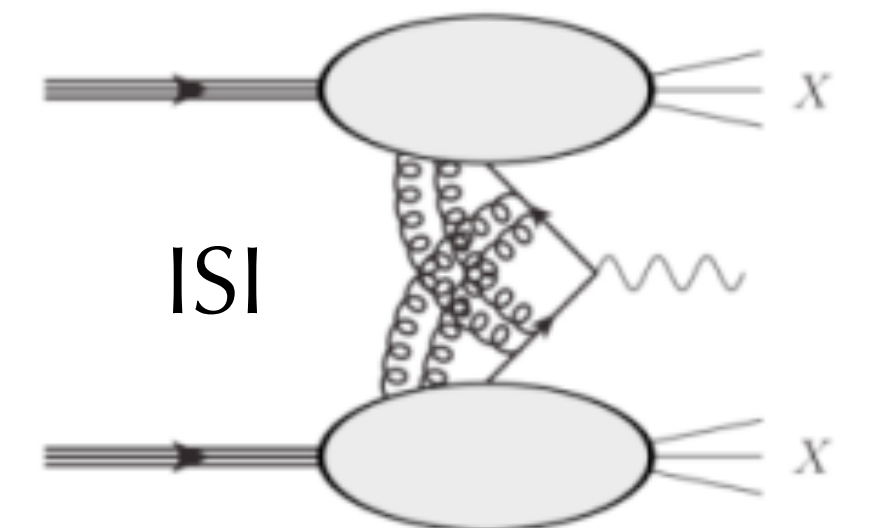
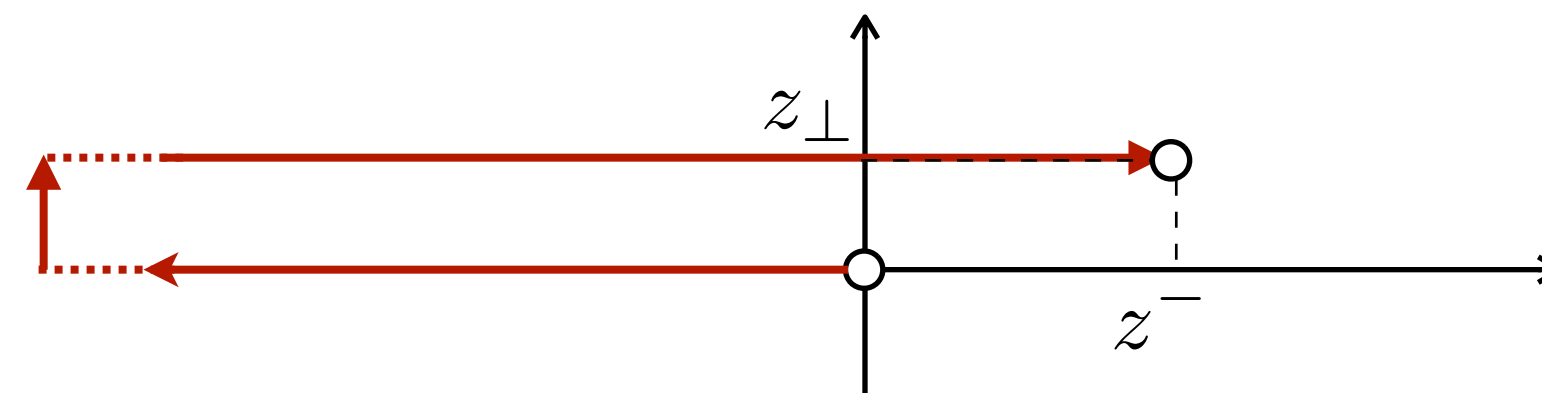
Gauge link dependence of TMDs

$$\frac{1}{2} \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{i(k^+ z^- - \vec{k}_\perp \cdot \vec{z}_\perp)} \langle p^+, 0_\perp, \Lambda' | \bar{\psi}(0) \gamma^+ \text{GaugeLink} \psi(0, z^-, z_\perp) | p^+, 0_\perp, \Lambda \rangle$$

SIDIS



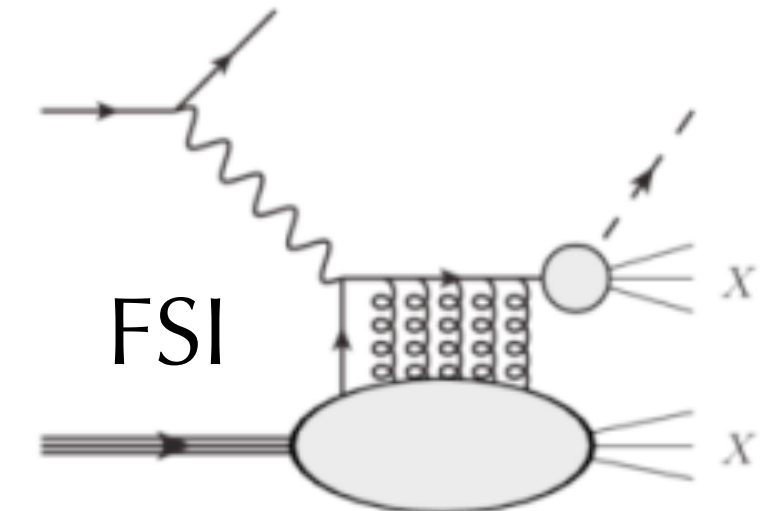
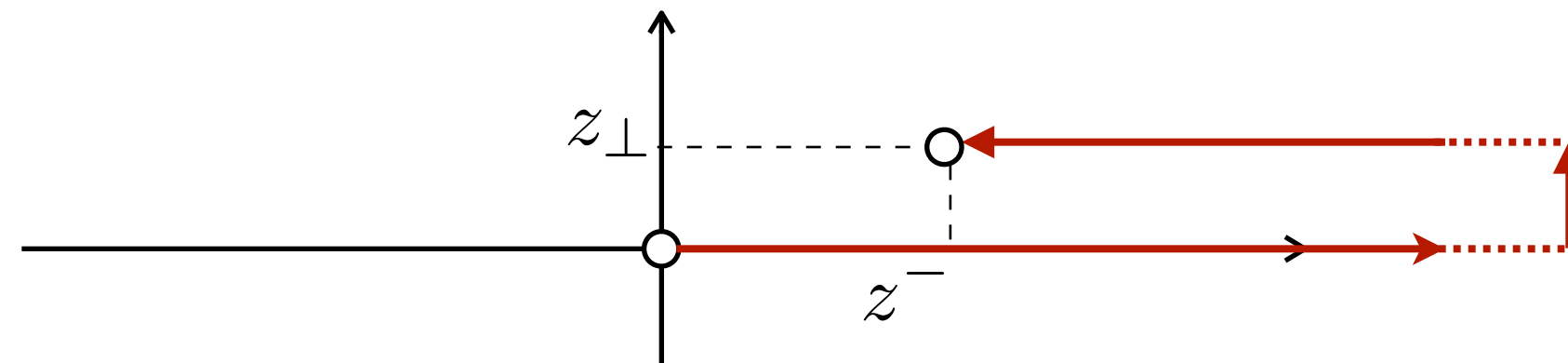
Drell-Yan



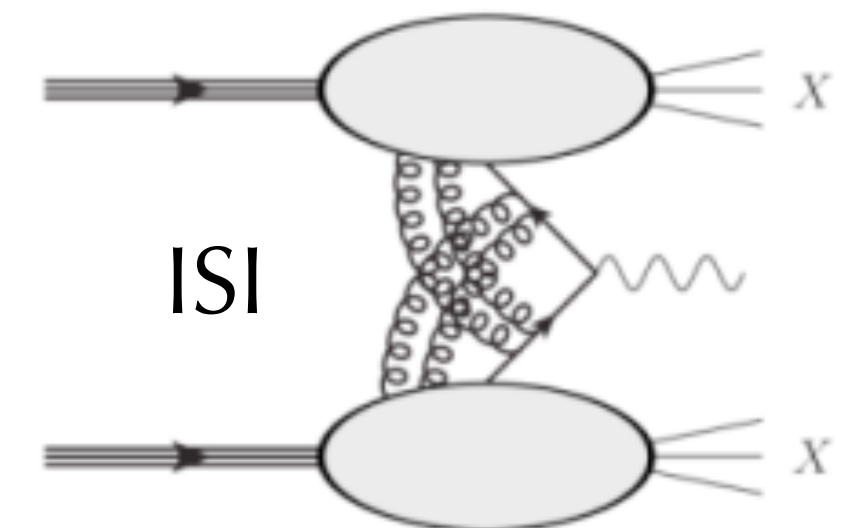
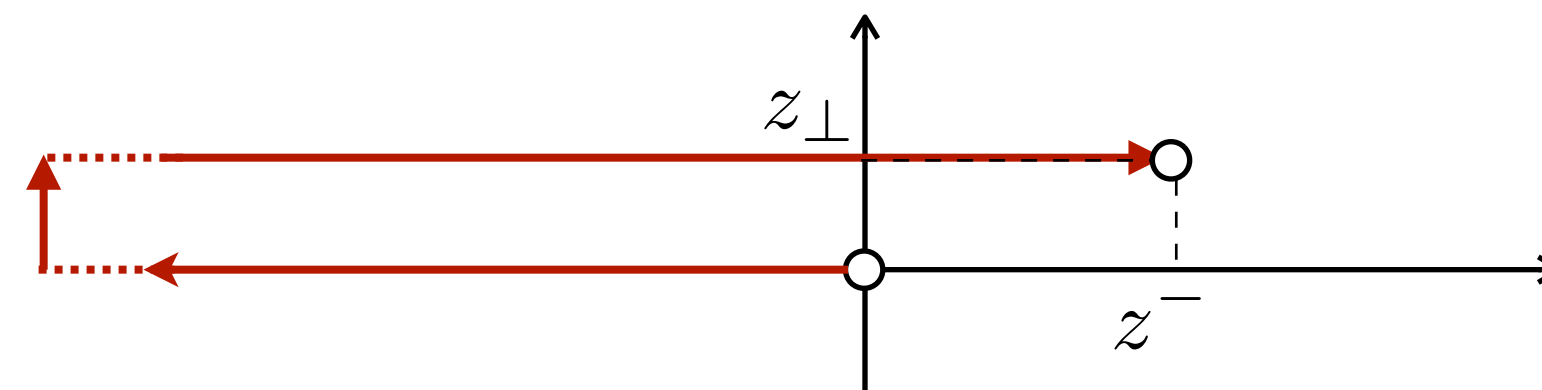
Gauge link dependence of TMDs

$$\frac{1}{2} \int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{i(k^+ z^- - \vec{k}_\perp \cdot \vec{z}_\perp)} \langle p^+, 0_\perp, \Lambda' | \bar{\psi}(0) \gamma^+ \boxed{\text{GaugeLink}} \psi(0, z^-, z_\perp) | p^+, 0_\perp, \Lambda \rangle$$

SIDIS



Drell-Yan



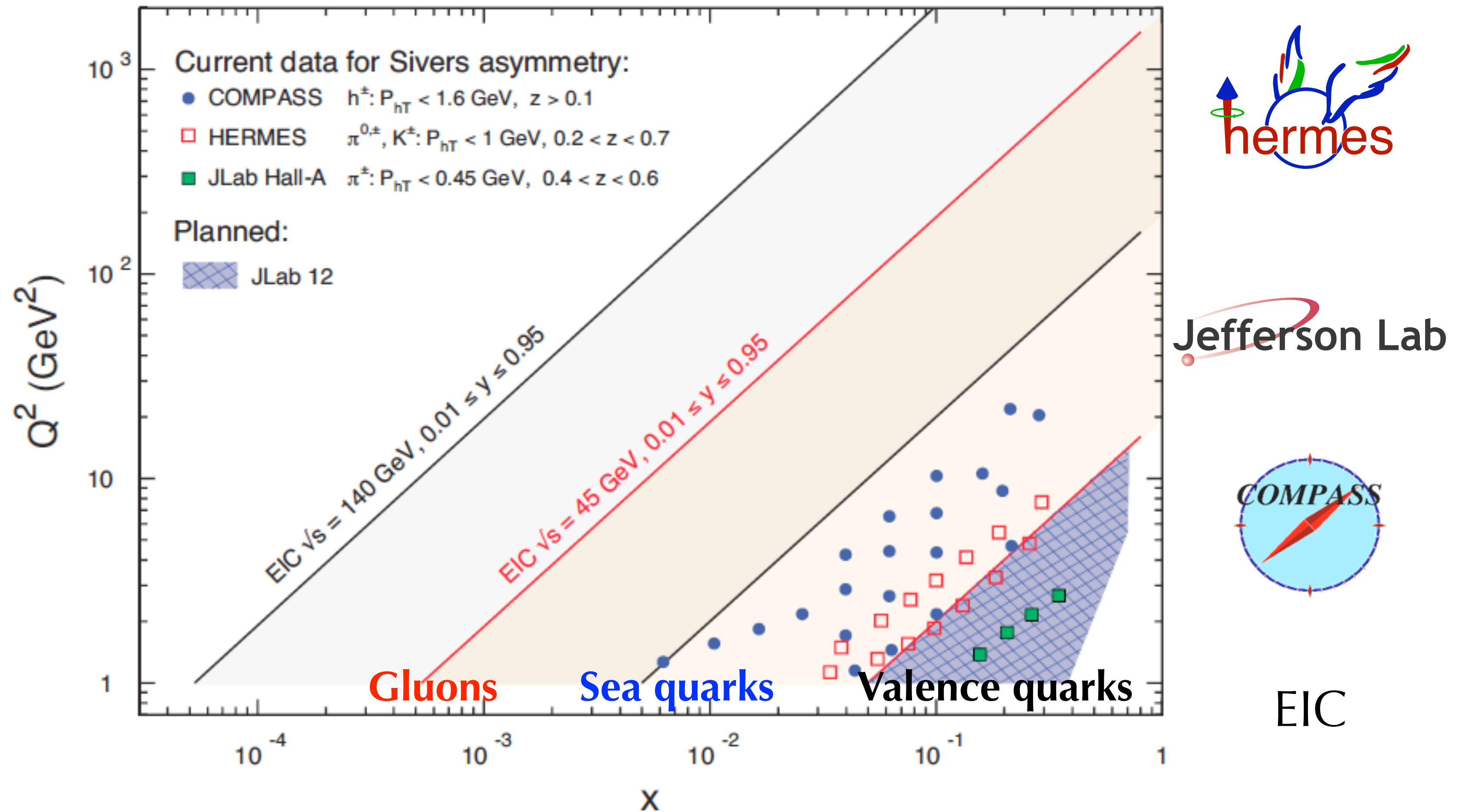
Sivers function_{SIDIS} = − Sivers function_{Drell-Yan}

Boer-Mulders function_{SIDIS} = − Boer-Mulders function_{Drell-Yan}

Strong QCD prediction. Needs to be tested.

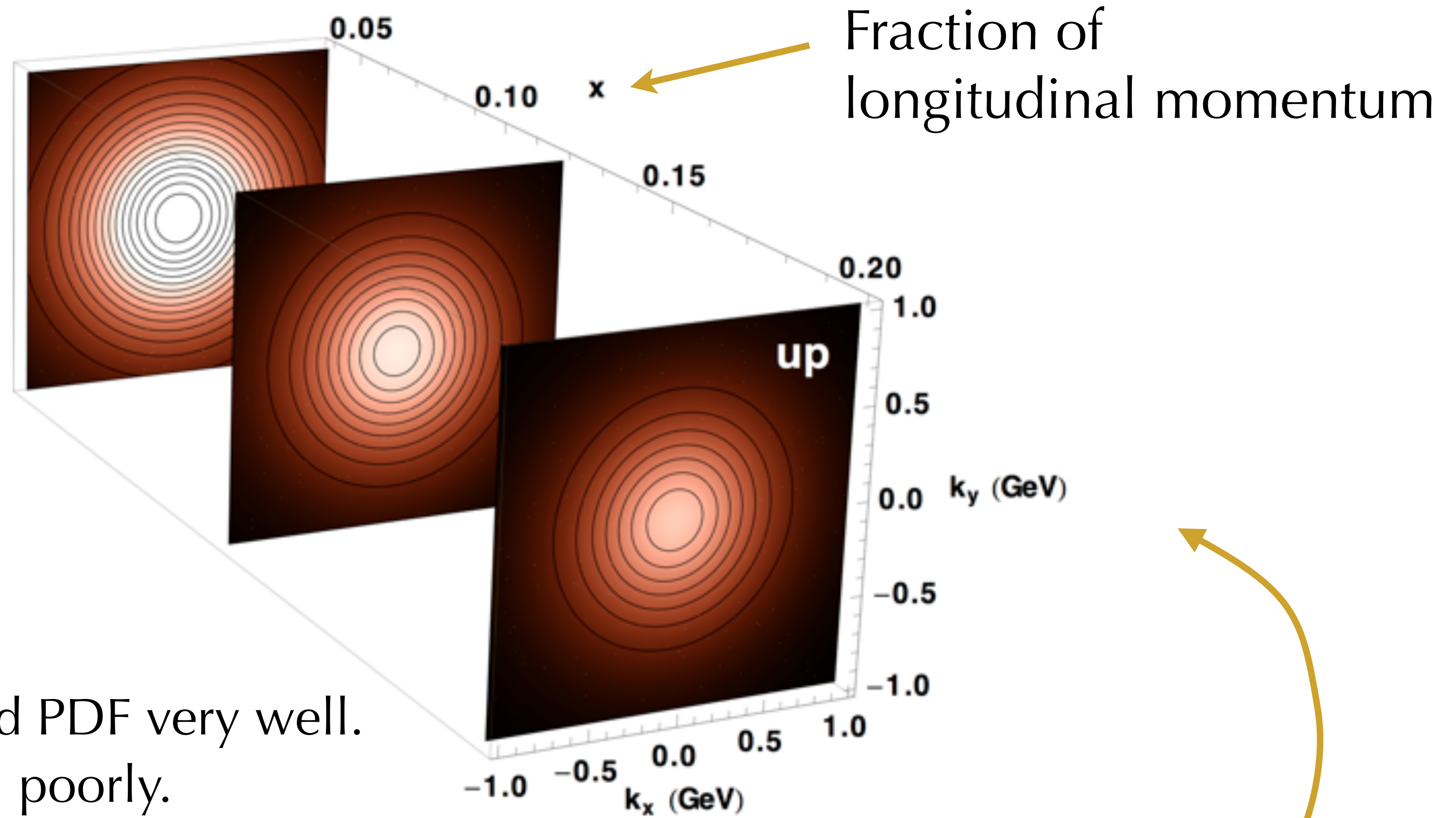


Paste, present and future TMD measurements



Accardi et al., *The Electron Ion Collider: the next QCD Frontier*
arXiv:1212.1701

The unpolarized TMD f_1



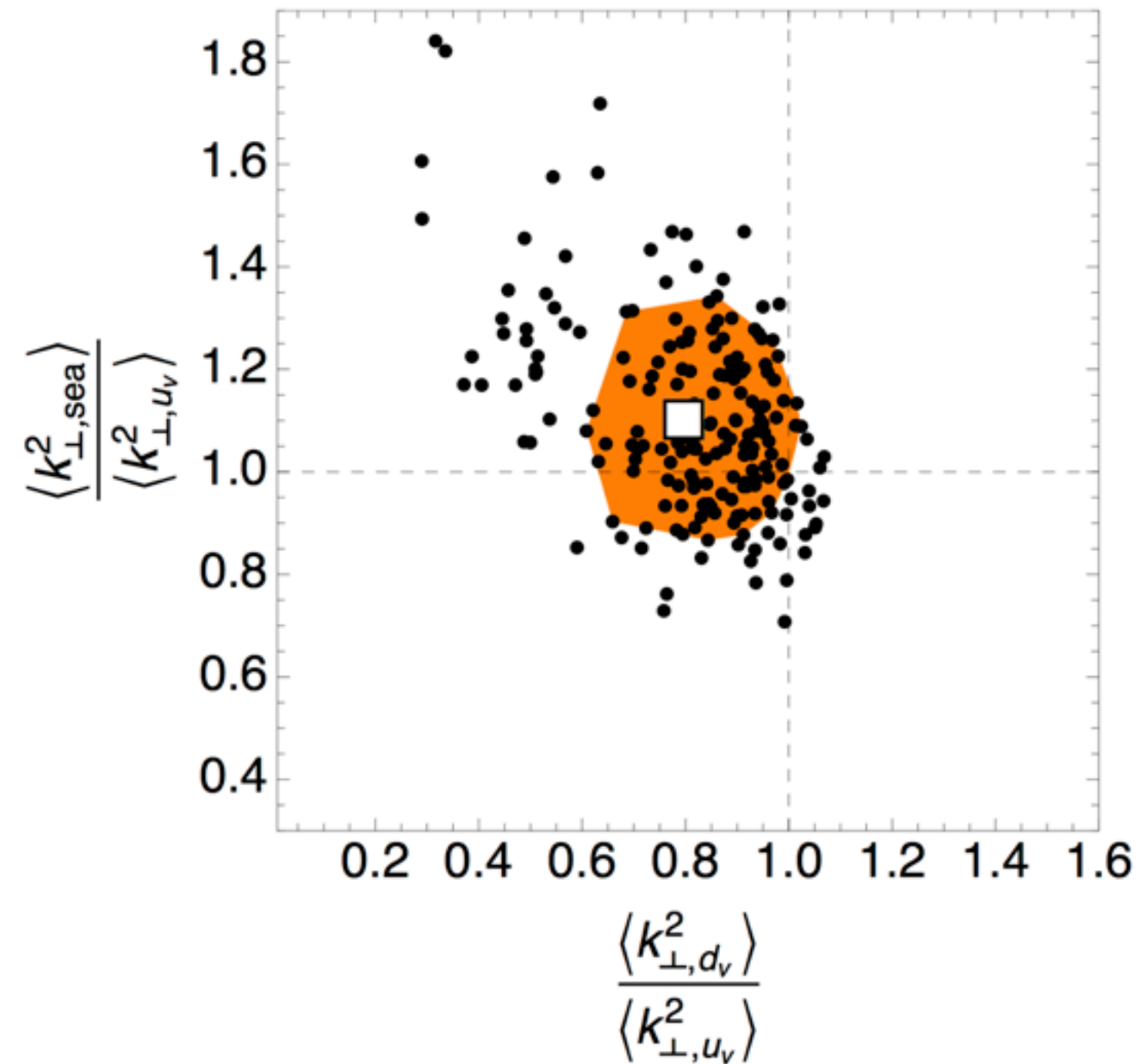
We know the integrated PDF very well.
We know the TMD still poorly.

Correlation between x and k_\perp ?
Flavor dependence of TMDs

Transverse momentum

Flavor structure of TMDs: indications from data

Ratio of width of sea /
width of up valence



Ratio width of down valence/
width of up valence

fit to SIDIS multiplicities from HERMES:

$$\langle k_{\perp, d_v}^2 \rangle < \langle k_{\perp, u_v}^2 \rangle < \langle k_{\perp, sea}^2 \rangle$$

Signori, et.al., JHEP 1311 (13)

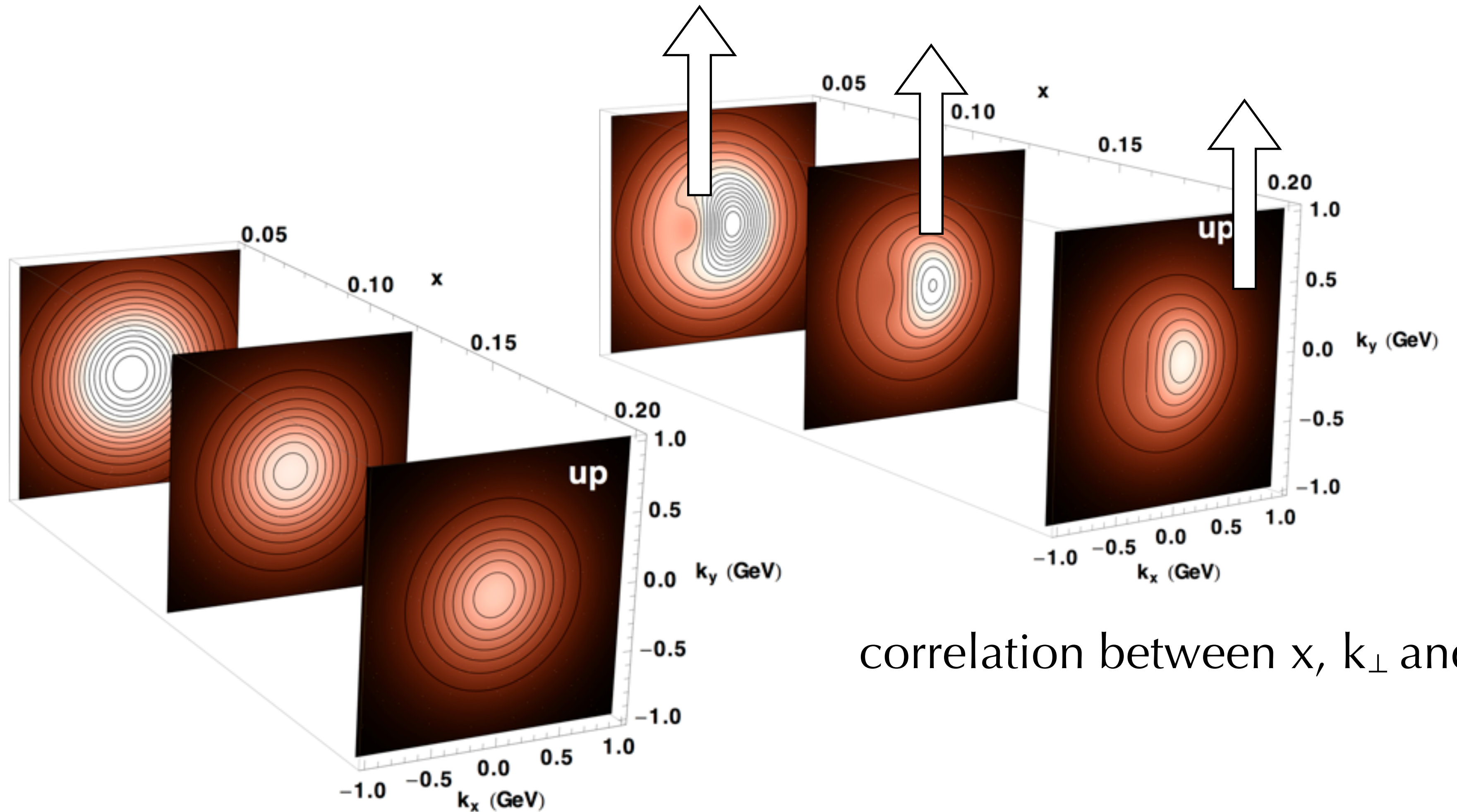
Flavor-independence is not ruled out:

$$0.4 < \langle k_{\perp}^2 \rangle < 0.8 \text{ GeV}^2$$

fit to SIDIS multiplicities from HERMES and COMPASS

Anselmino, et.al., JHEP 1404 (14)

Adding the spin



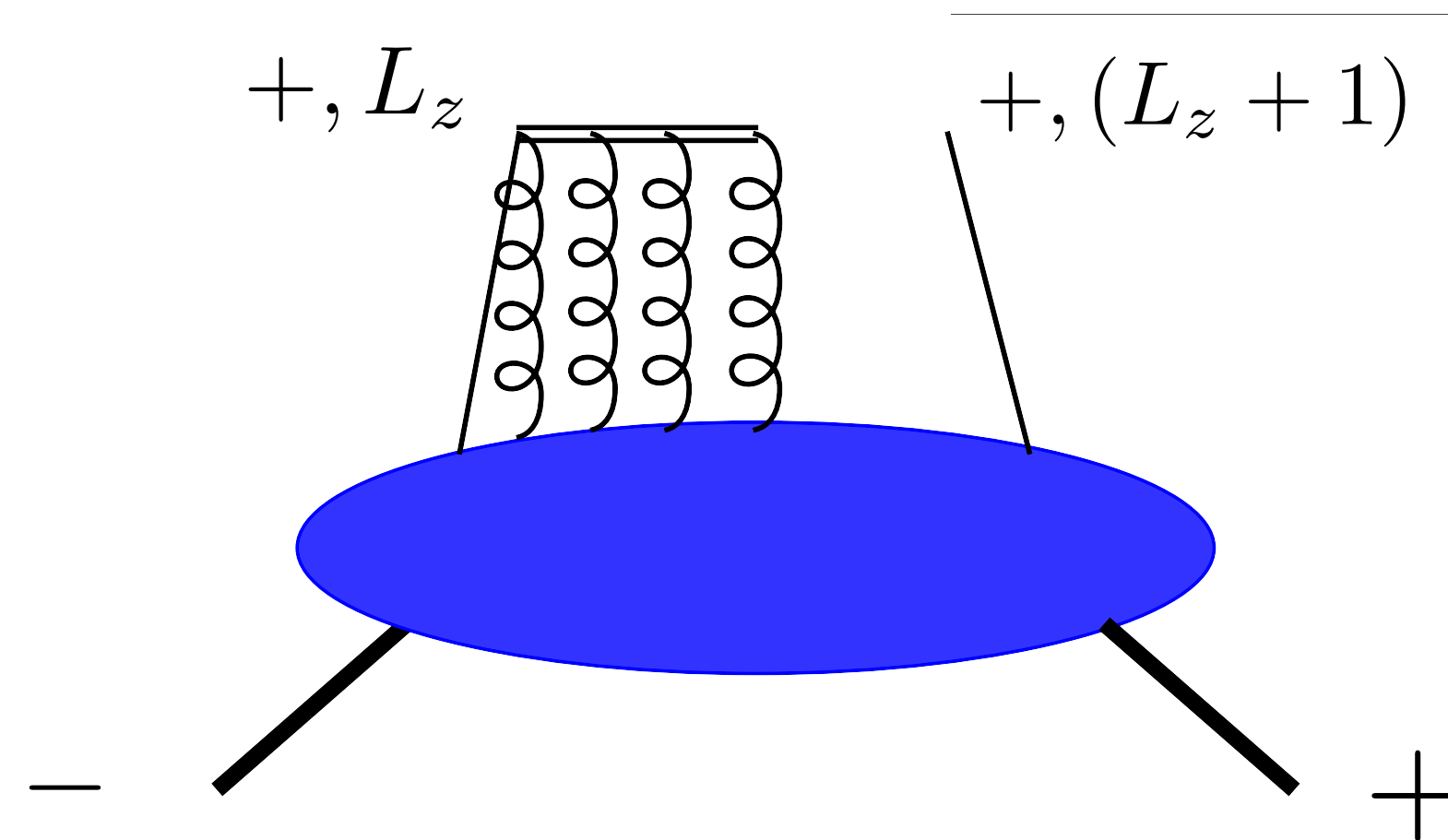
correlation between x , k_{\perp} and spin

correlation between x and k_{\perp}

Sivers function

$$f_{1T}^\perp = \text{[diagram: unpolarized quark line] - \text{[diagram: polarized quark line]}}$$

unpolarized quarks in \perp pol. nucleon



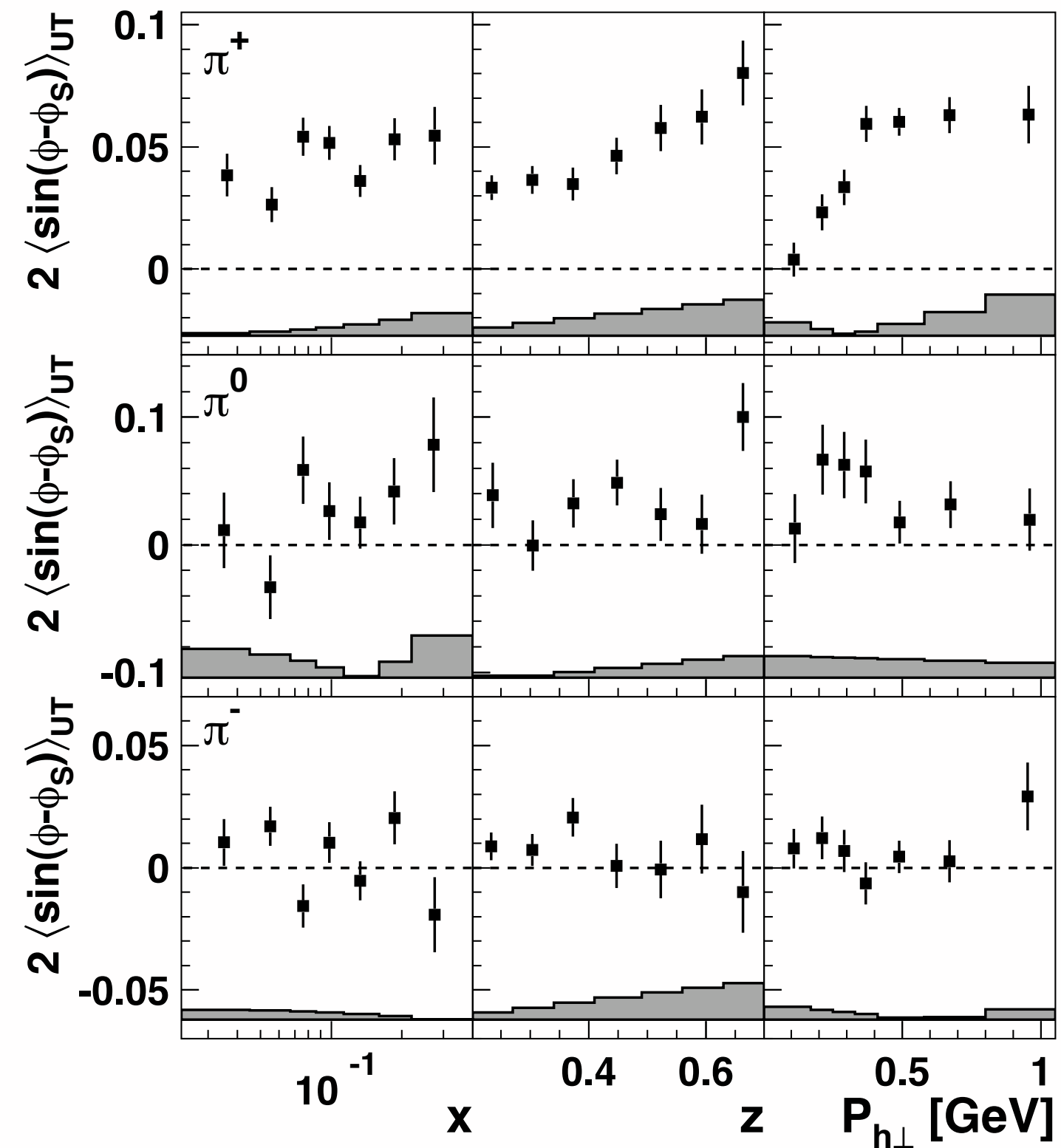
$$f_{1T}^\perp|_{\text{SIDIS}} = -f_{1T}^\perp|_{\text{DY}}$$

non-zero ONLY with final-state interaction

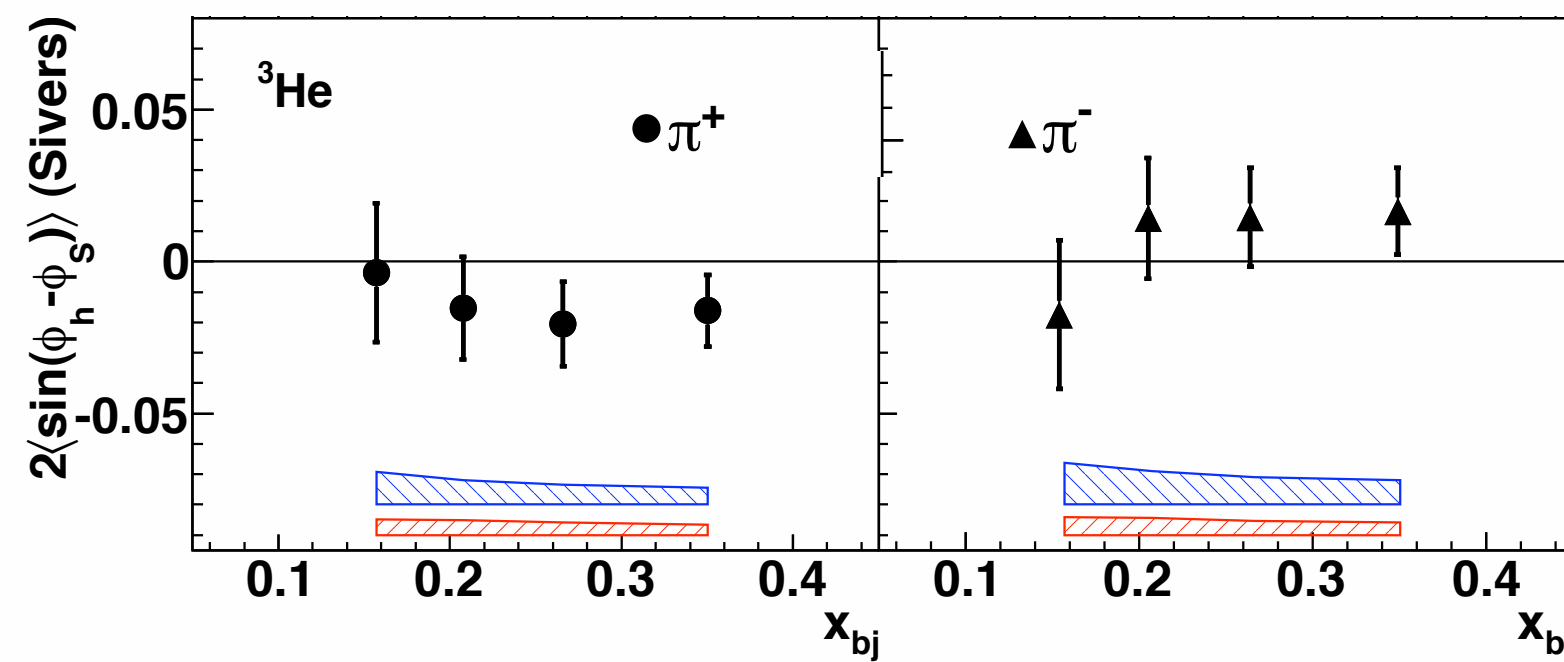
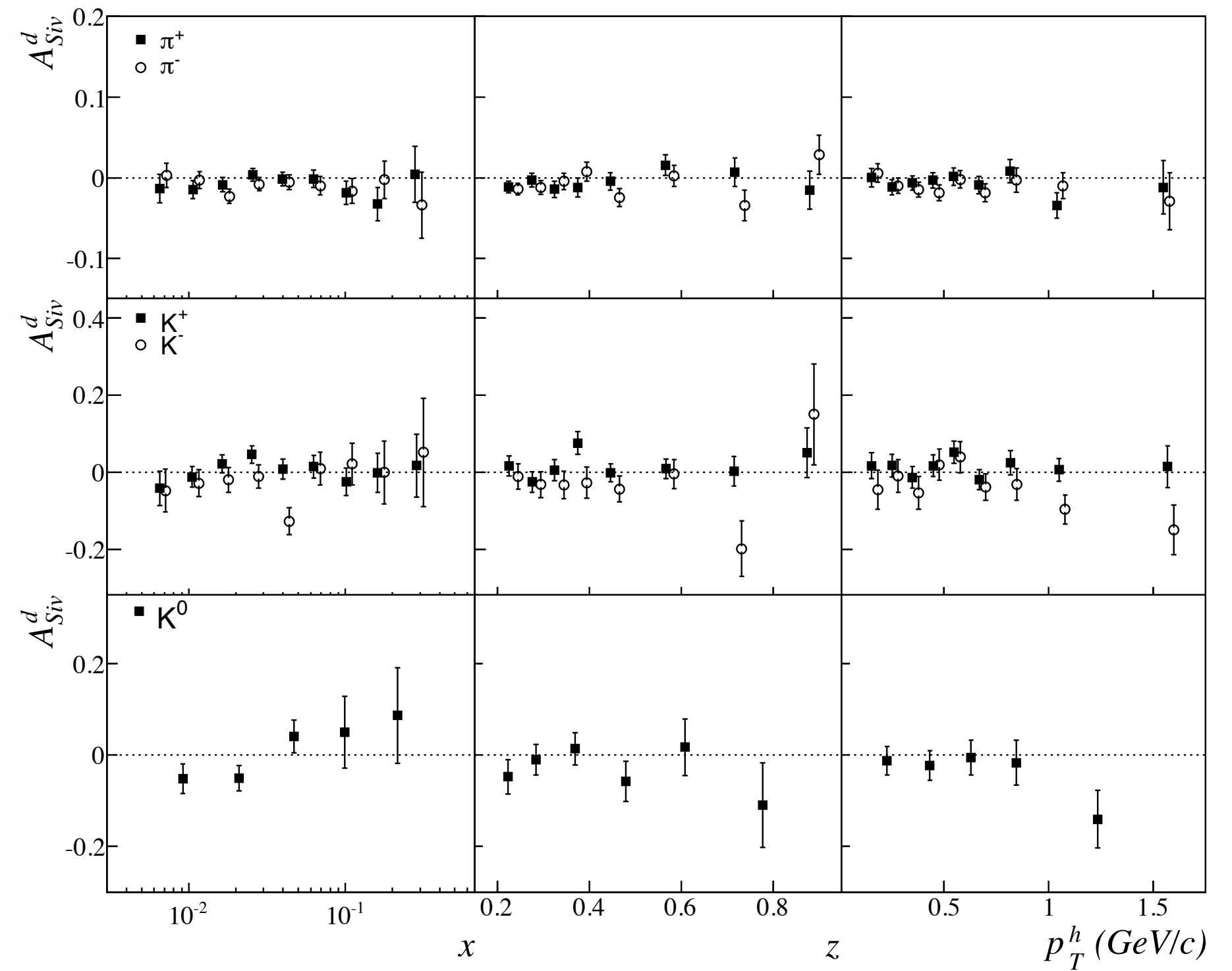
the helicity mismatch requires orbital angular momentum



*PRL***103** (09) 152002



*PL B***673** (09) 127



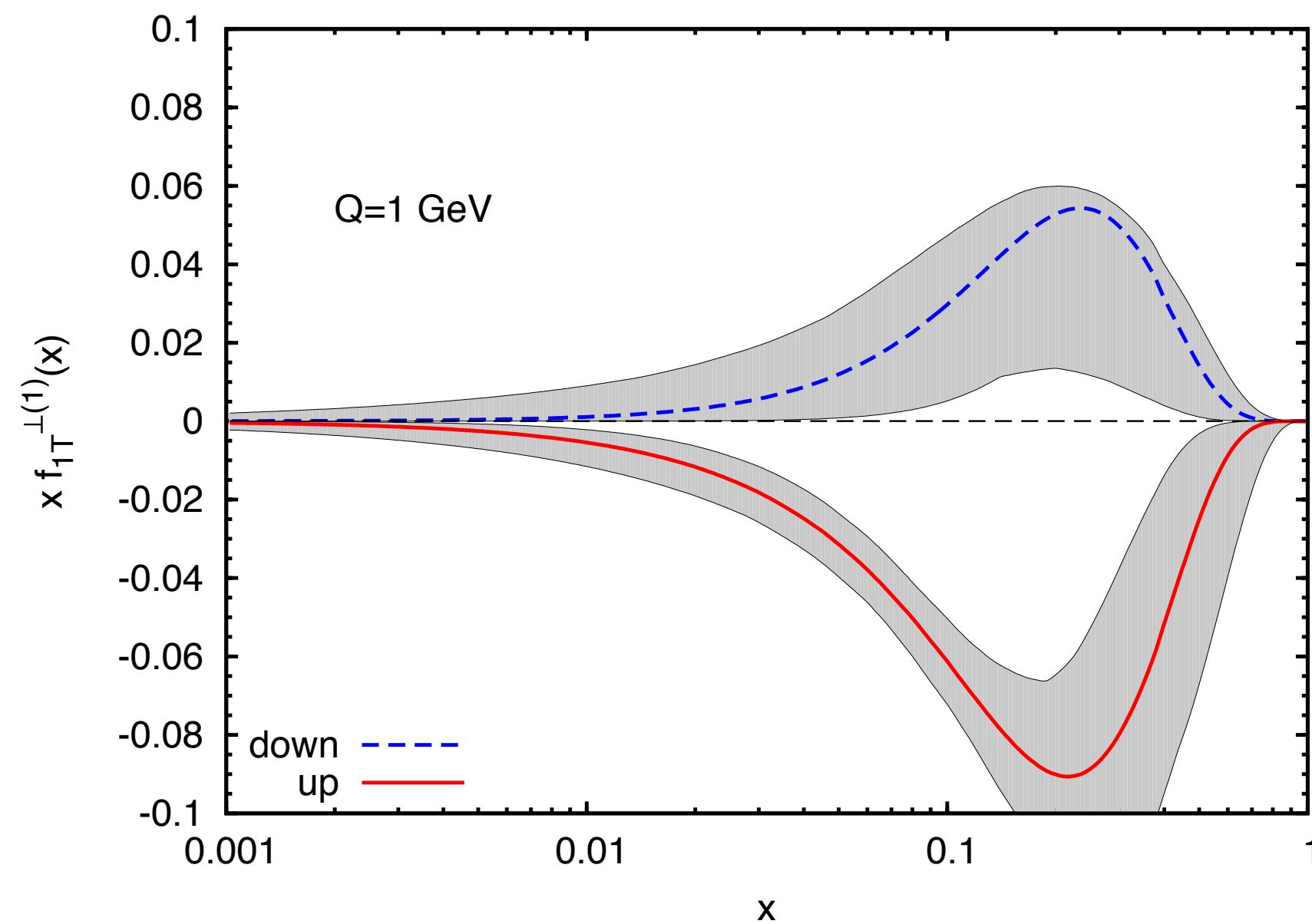
Jefferson Lab
Hall A

*PRL***107** (2011) 072003

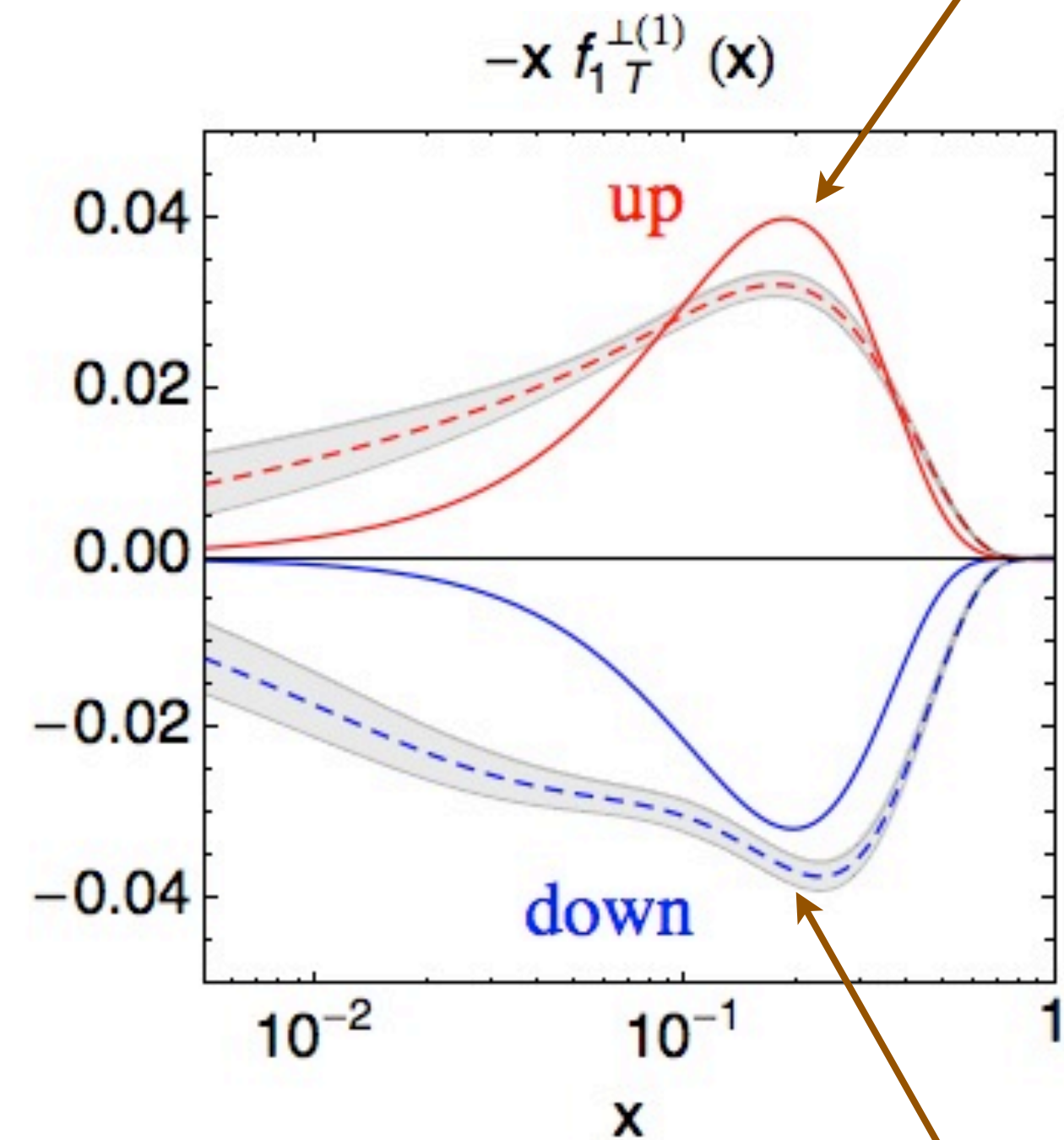
Sivers effect
has been
measured!

Sivers function has been extracted

Anselmino et al., PRD86 (2012) 014028
[no TMD evo]



Echevarria, Idilbi, Kang, Vitev, PRD 89 (14)
[with TMD evo]

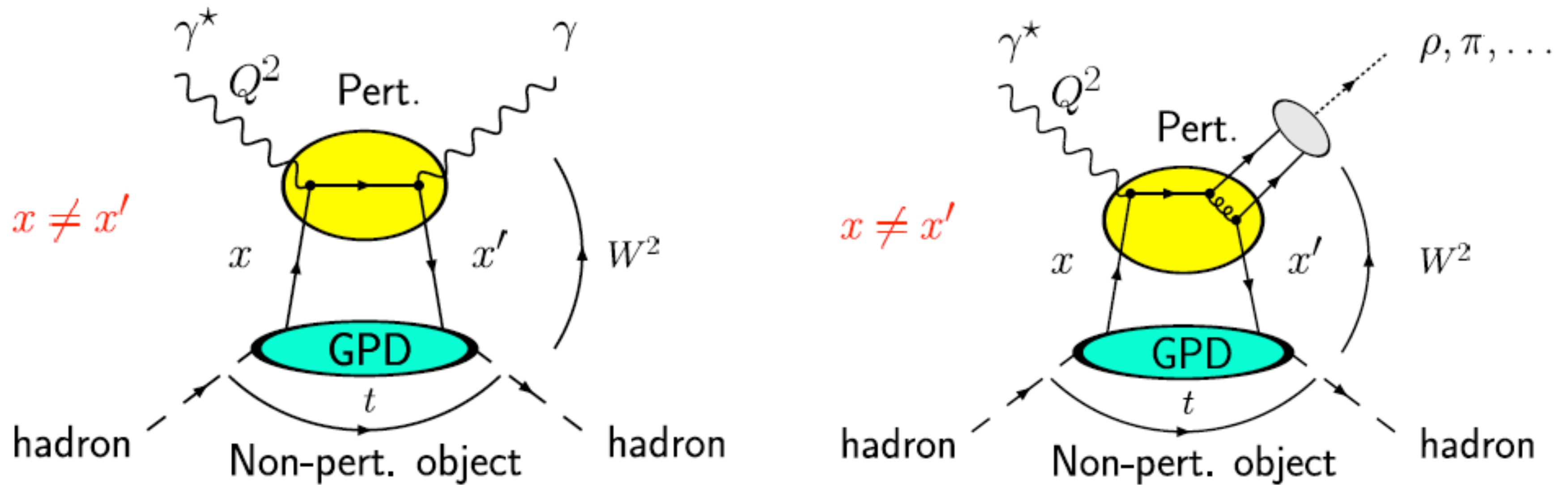


Bacchetta, Radici, PRL 107 (11)
[no TMD evo]

Key information from GPDs

- Transverse position size
- Decomposition of Form Factors w.r.t. x
- Sum rule for Angular Momentum
- Access to Form Factors of Energy Momentum Tensor
→ “mechanical” properties of the nucleon

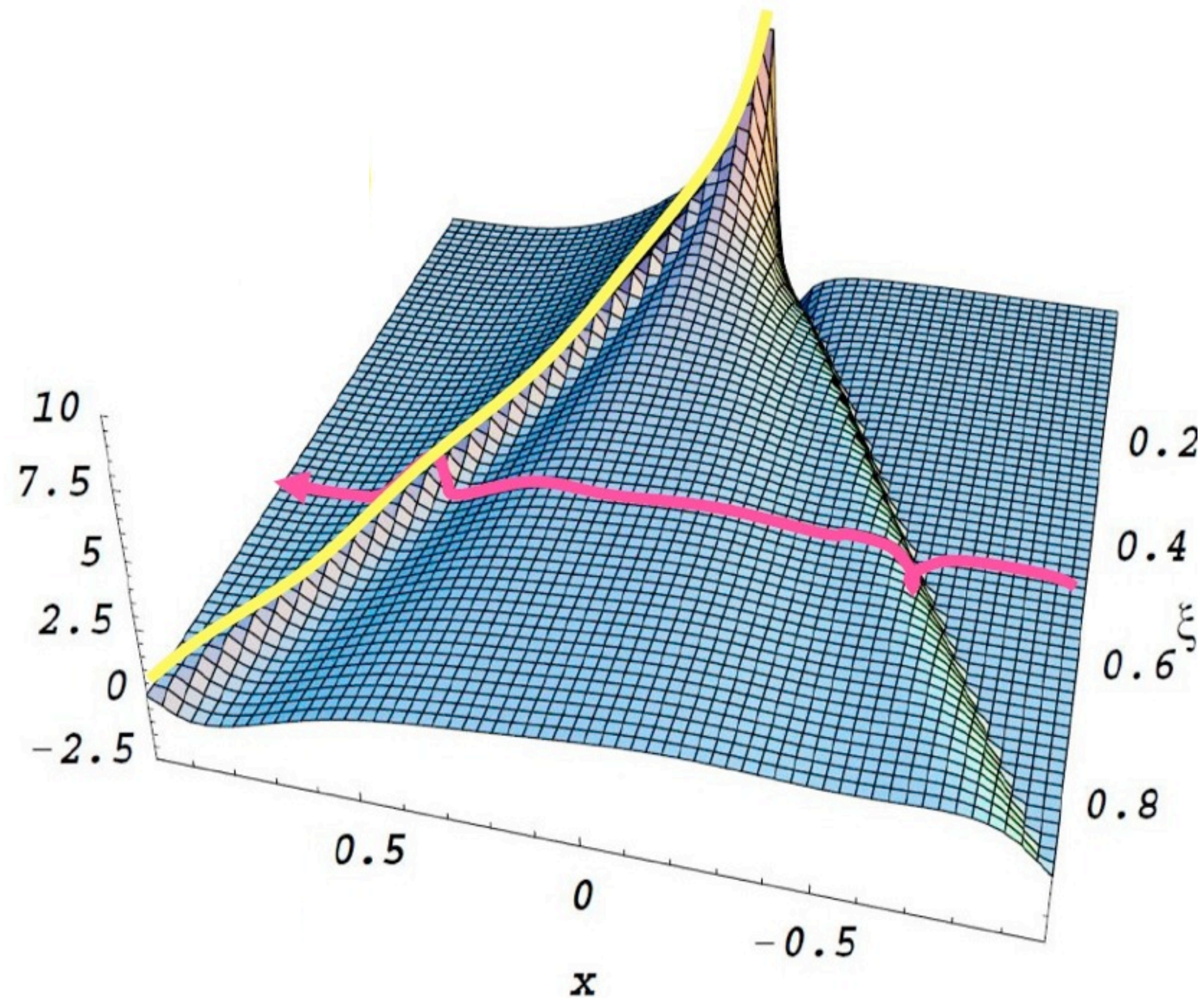
How to measure the GPDs



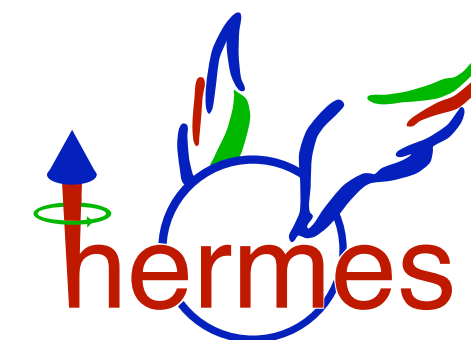
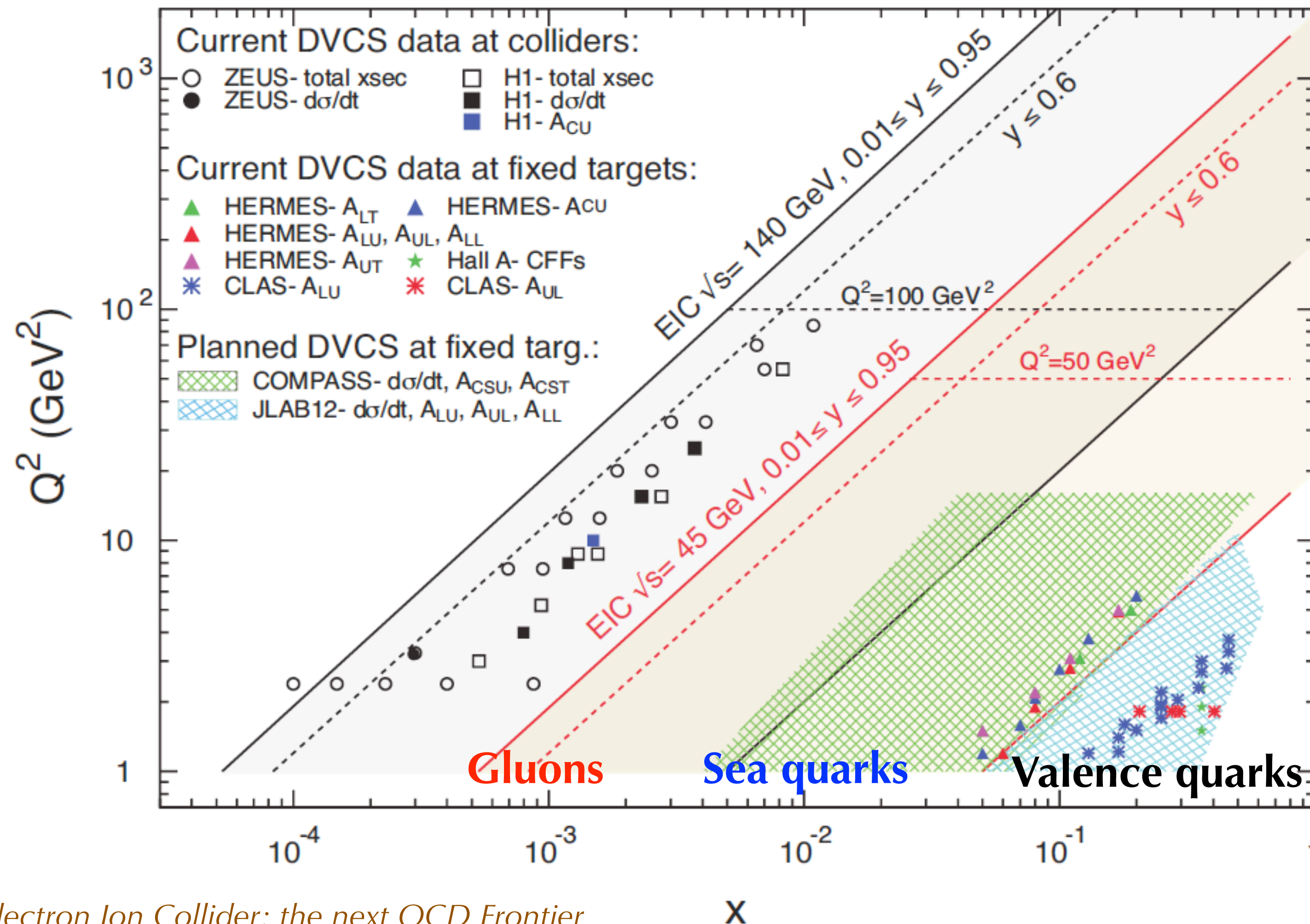
- ▶ accessible in exclusive reactions
- ▶ factorization for large Q^2 , $|t| \ll Q^2$, W^2
- ▶ depend on 3 variables: x, ξ, t

Compton form factors

$$\int_{-1}^{+1} dx \frac{H(x, \xi, t)}{x - \xi + i\epsilon} = \mathcal{P} \int_{-1}^{+1} dx \frac{H(x, \xi, t)}{x - \xi} - i\pi H(\xi, \xi, t)$$



Paste, present and future DVCS experiments



Jefferson Lab



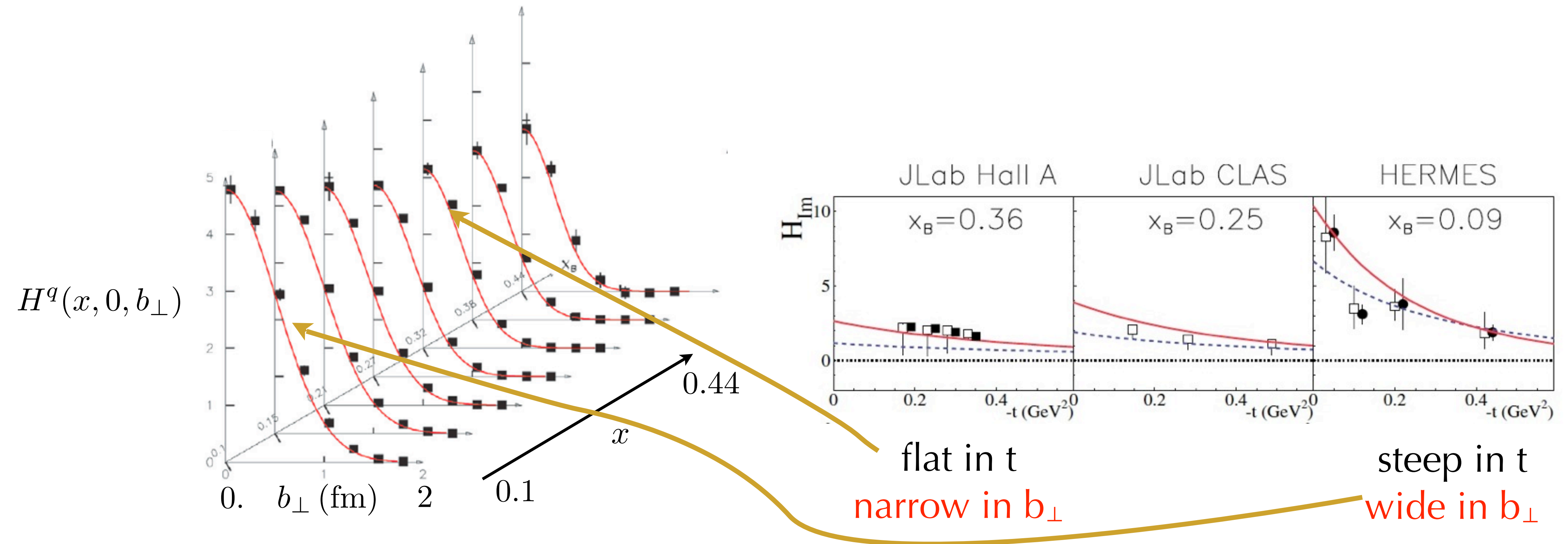
EIC

The unpolarized GPD H

$$F_1(t) = \int dx H(x, 0, t)$$

$$H(x, 0, \vec{b}_\perp) = \int d^2\Delta_\perp H(x, 0, t) e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp}$$

\downarrow extrapolation from data $\swarrow t = -\vec{\Delta}_\perp^2$



The unpolarized GPD H

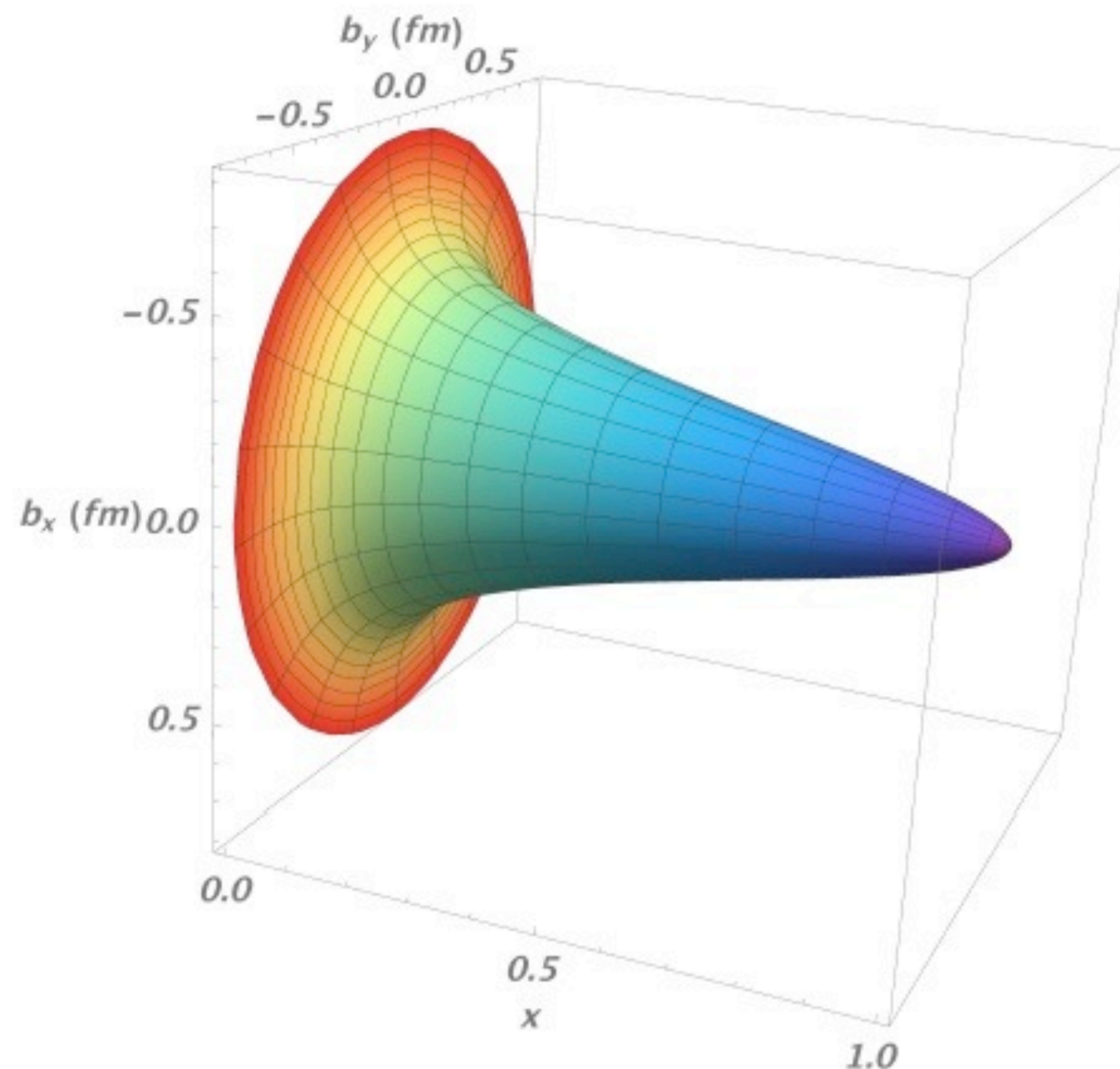
$$F_1(t) = \int dx H(x, 0, t)$$

$$H(x, 0, \vec{b}_\perp) = \int d^2\Delta_\perp H(x, 0, t) e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp}$$

\searrow
 $t = -\vec{\Delta}_\perp^2$

As $x \rightarrow 1$, the active parton carries all the momentum and represents the transverse centre of momentum

$$\langle \vec{b}_\perp^2(x) \rangle = \frac{\int d^2\vec{b}_\perp \vec{b}_\perp^2 H(x, 0, b_\perp)}{\int d^2\vec{b}_\perp H(x, 0, b_\perp)}$$

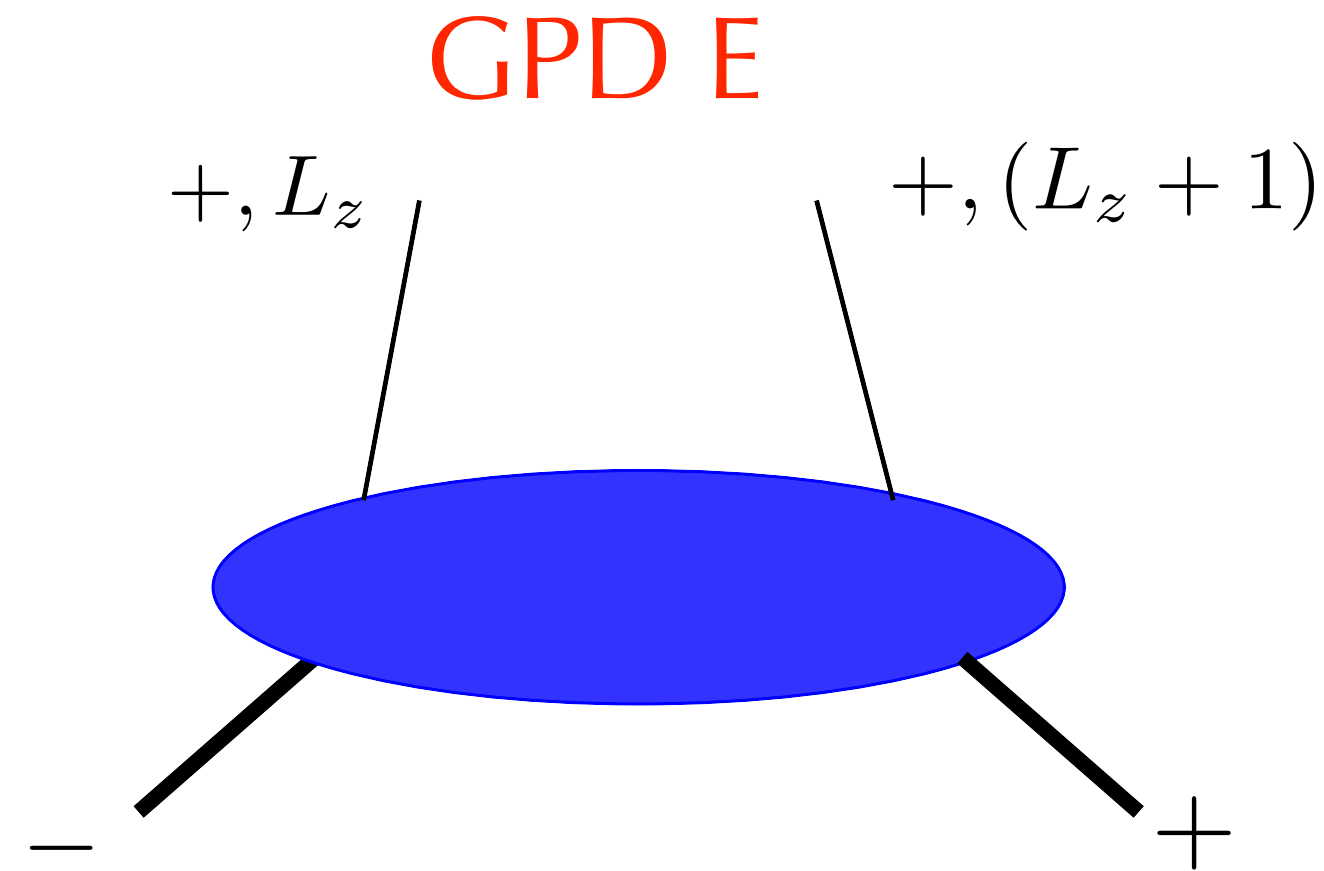


Dupré et al., arXiv:1606.07821

Unpolarized quarks in transversely pol. nucleon

“Helicity mismatch” requires orbital angular momentum

- $F_2(t) = \int dx E(x, \xi, t)$
- no-forward limit to PDF



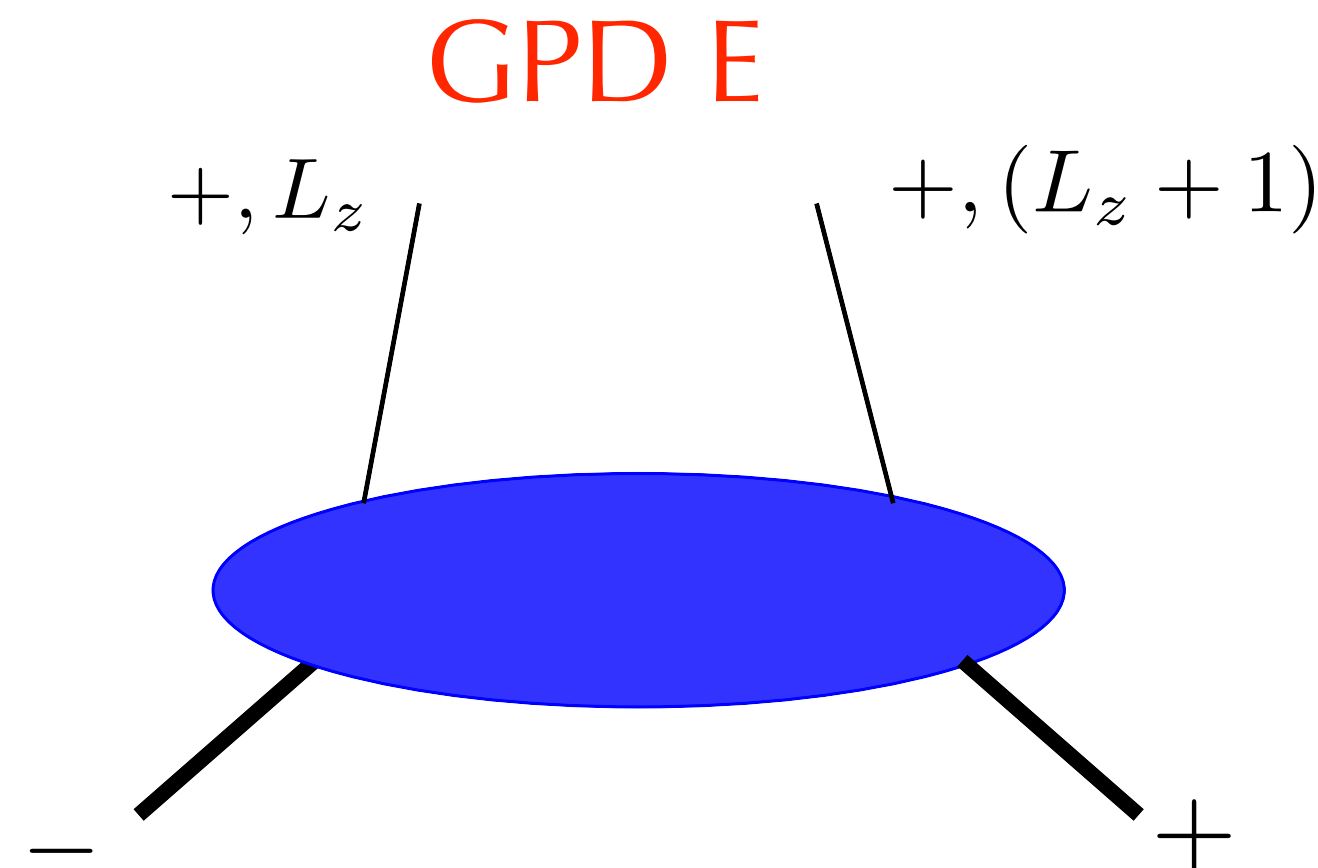
unpolarized quarks
in \perp pol. nucleon

↓
“partner” of Sivers function

Unpolarized quarks in transversely pol. nucleon

“Helicity mismatch” requires orbital angular momentum

- $F_2(t) = \int dx E(x, \xi, t)$
- no-forward limit to PDF



unpolarized quarks
in \perp pol. nucleon
↓
“partner” of Sivers function

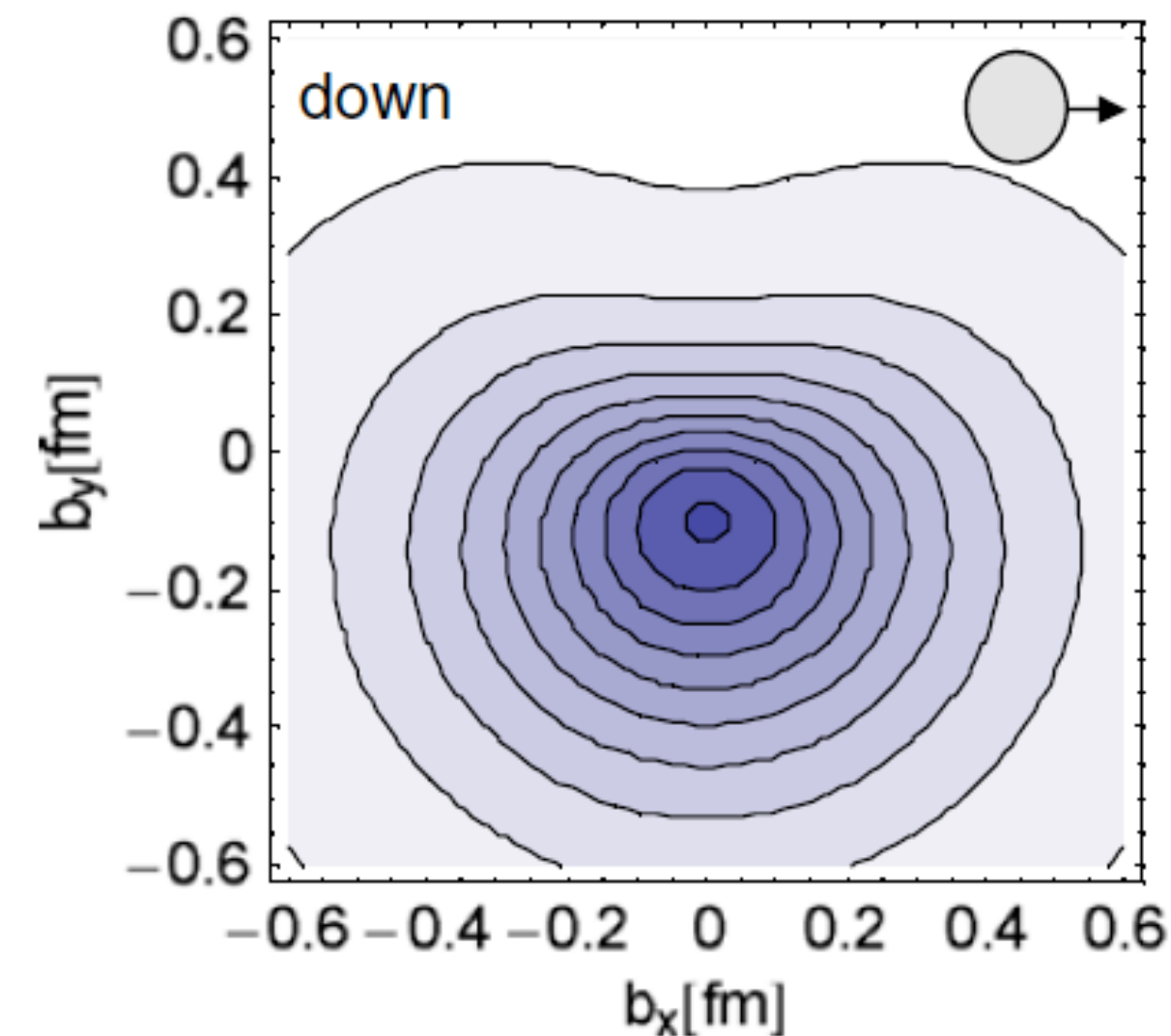
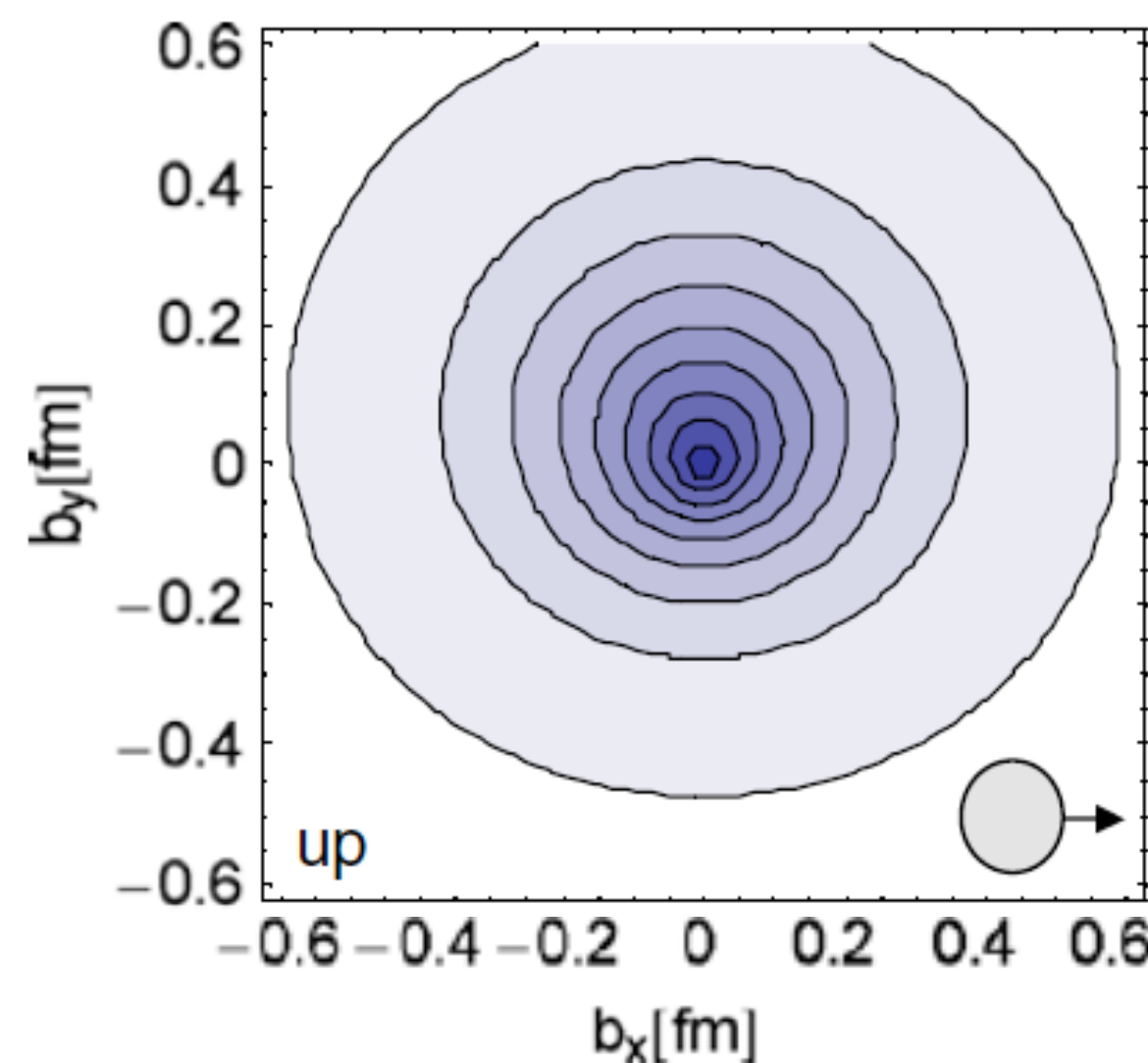
Lattice calculation

Transverse
dipole moment:

$$d_y^q = \frac{\kappa^q}{2M}$$

$$\kappa^u = 1.86 \quad \kappa^d = -1.57$$

quark contribution to
proton anomalous
magnetic moment



Angular Momentum Relation (“Ji’s Sum Rule”)

X. Ji, PRL **78** (1997) 610

quark and gluon contribution to the nucleon spin

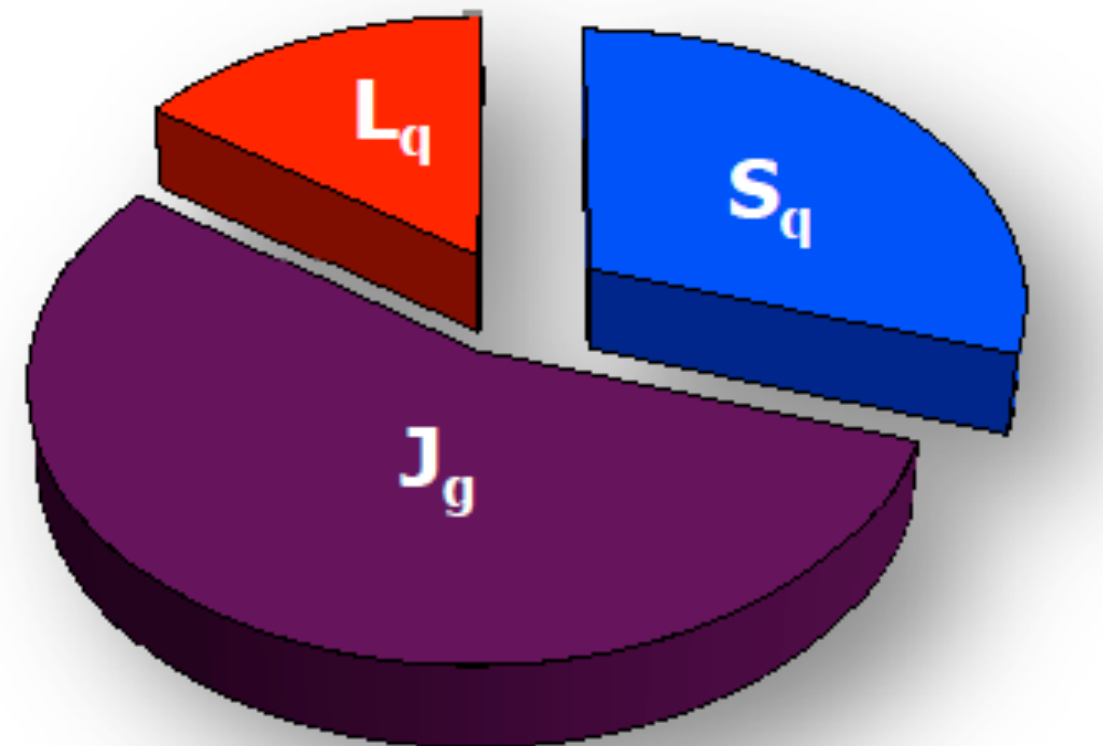
$$J^{q,g} = \frac{1}{2} \int_{-1}^1 dx x \left(\underset{\substack{\downarrow \\ \text{unpolarized PDF}}}{H^{q,g}(x, 0, 0)} + E^{q,g}(x, 0, 0) \right)$$

not directly accessible

Proton spin decomposition

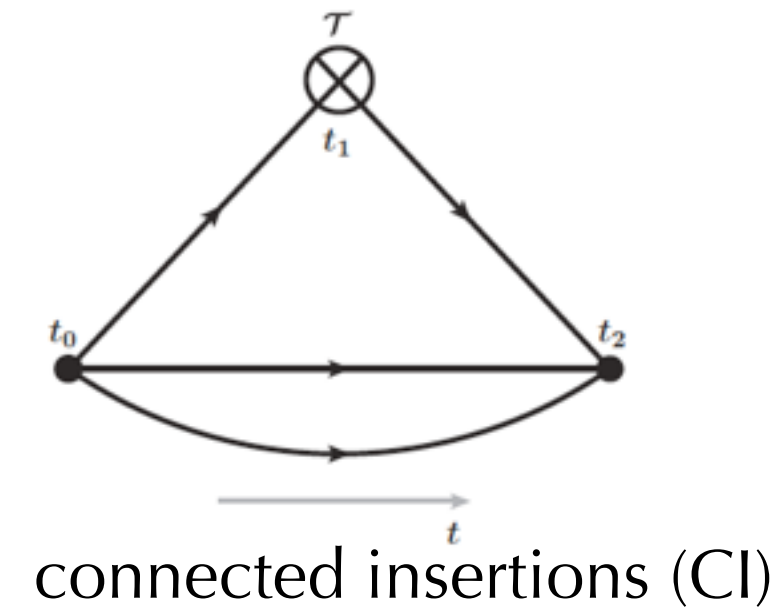
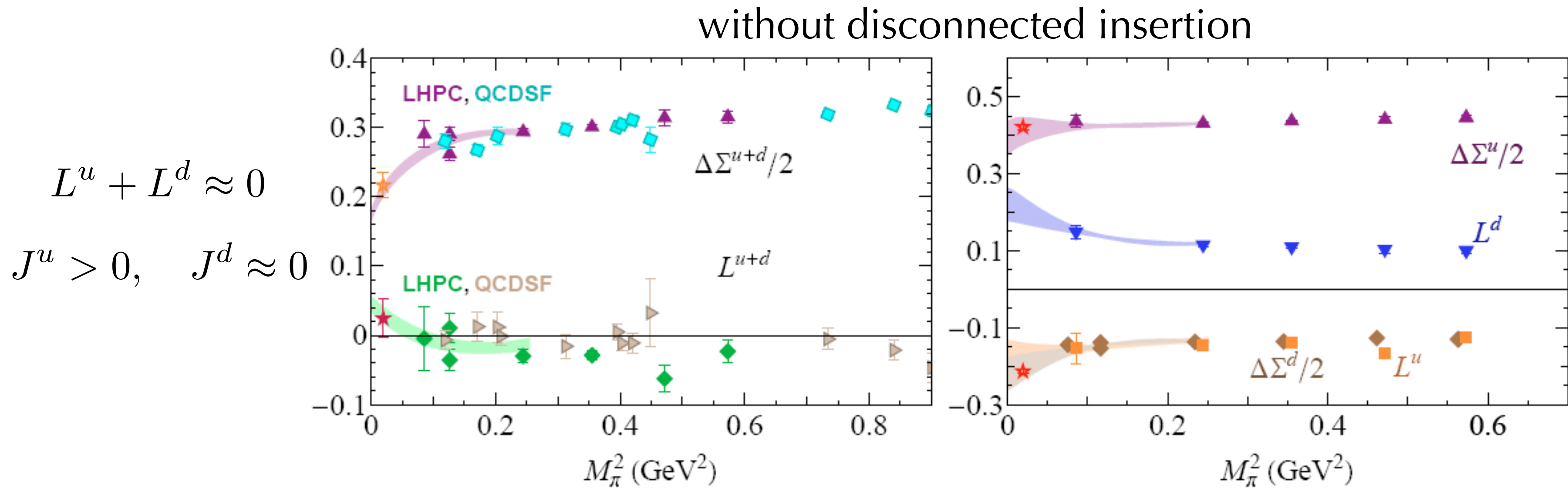
$$J^q = L^q + \overset{\substack{\uparrow \\ \frac{1}{2}\Delta\Sigma \text{ from DIS}}}{S^q}$$

gauge invariant decomposition
sum rule for L^q from twist-3 GPDs



J^g
no further gauge-invariant
decomposition

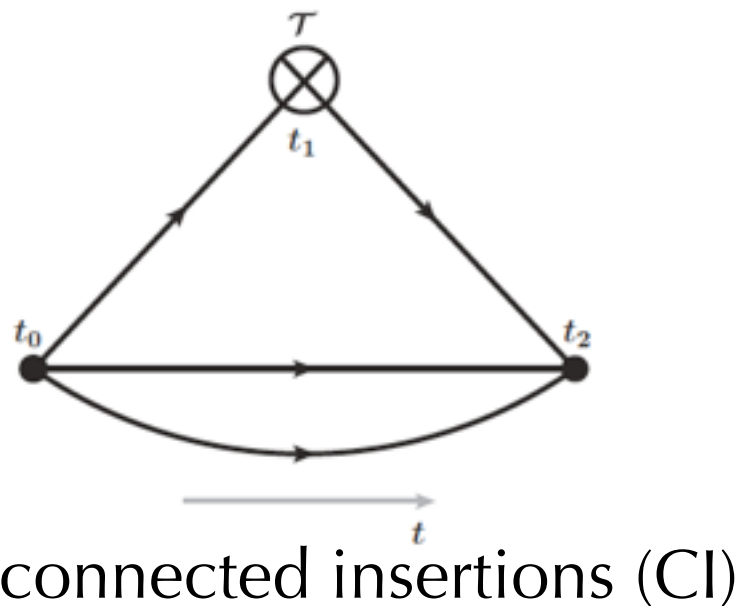
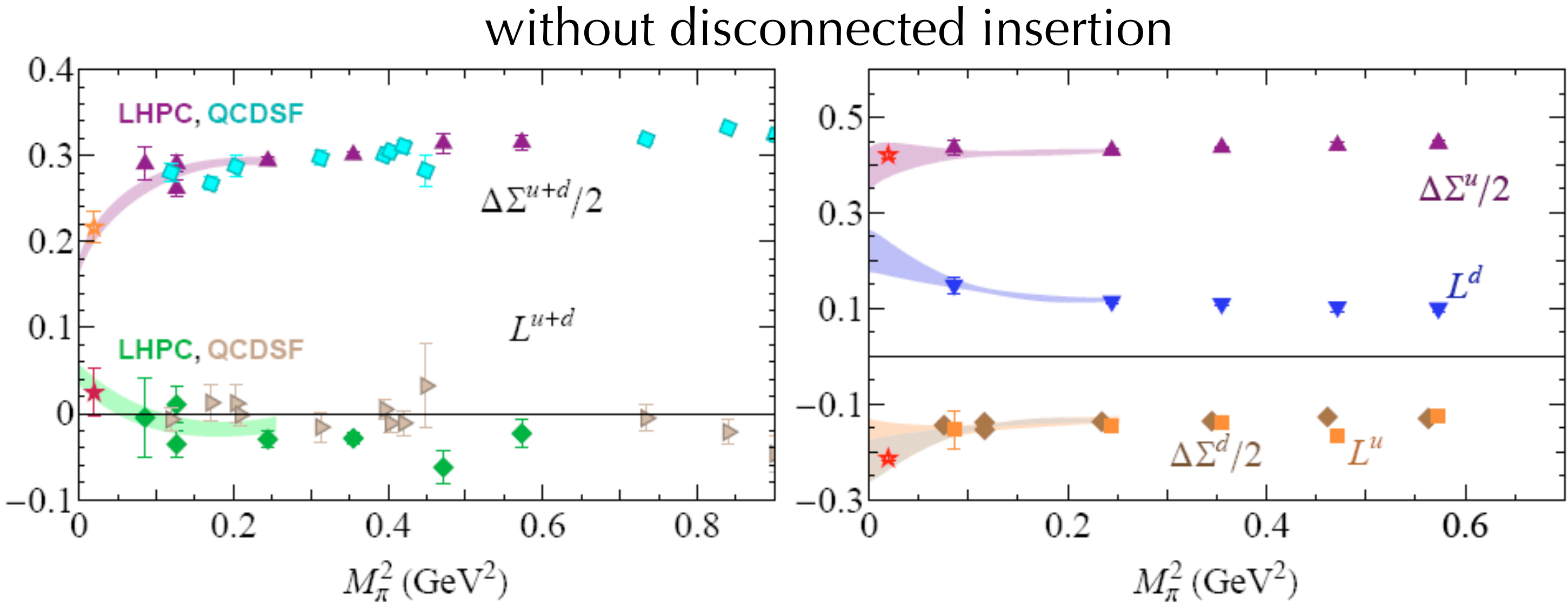
Lattice Calculations of Angular Momentum



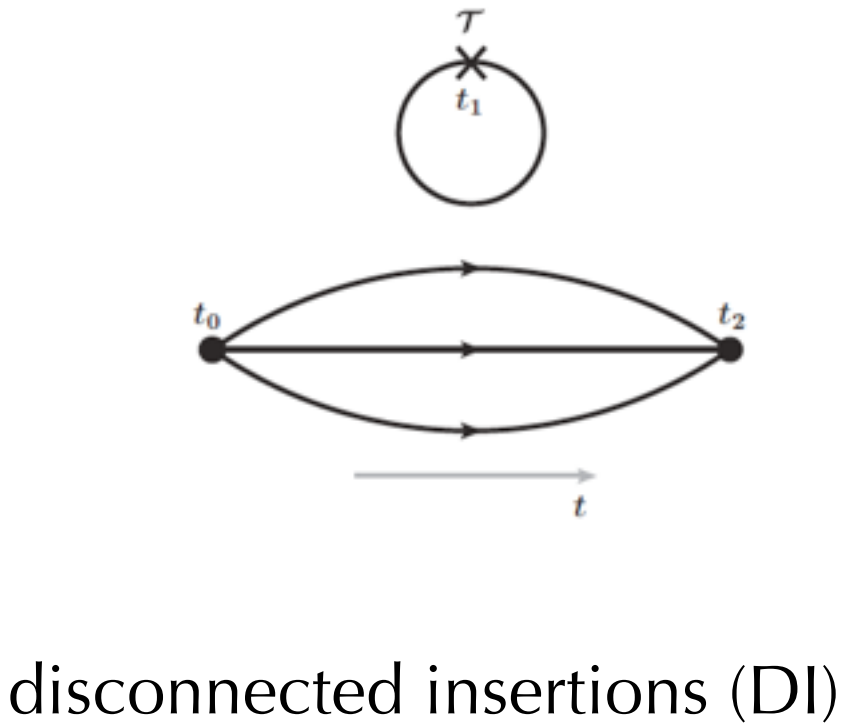
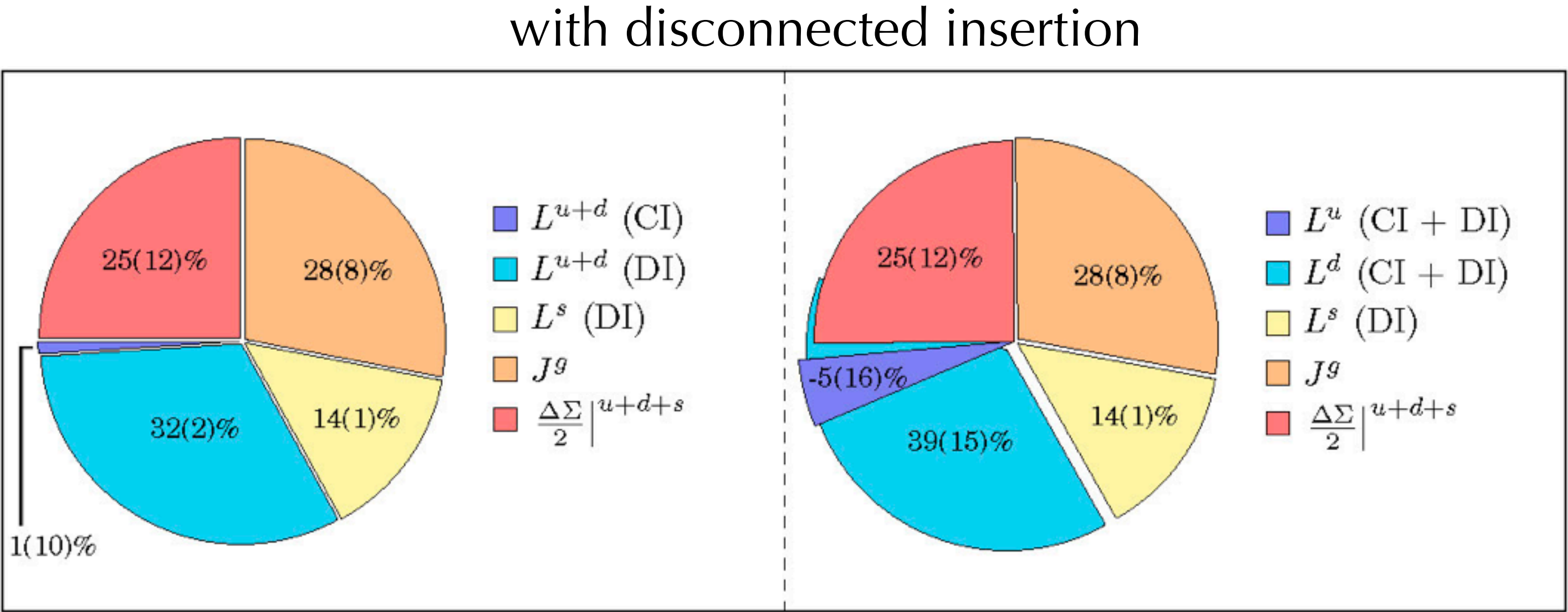
Lattice Calculations of Angular Momentum

$$L^u + L^d \approx 0$$

$$J^u > 0, \quad J^d \approx 0$$

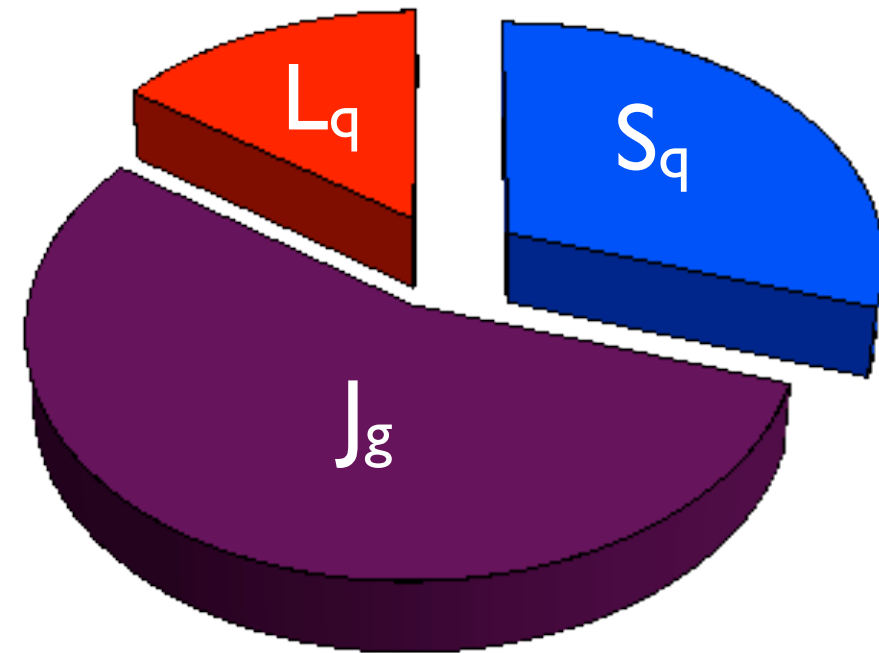


$$L^u + L^d \approx 33\%$$



Different definitions of OAM

Ji's sum rule



Pros:

- Each term is gauge invariant
- Accessible in DIS and DVCS
- Can be calculated in Lattice QCD

Cons:

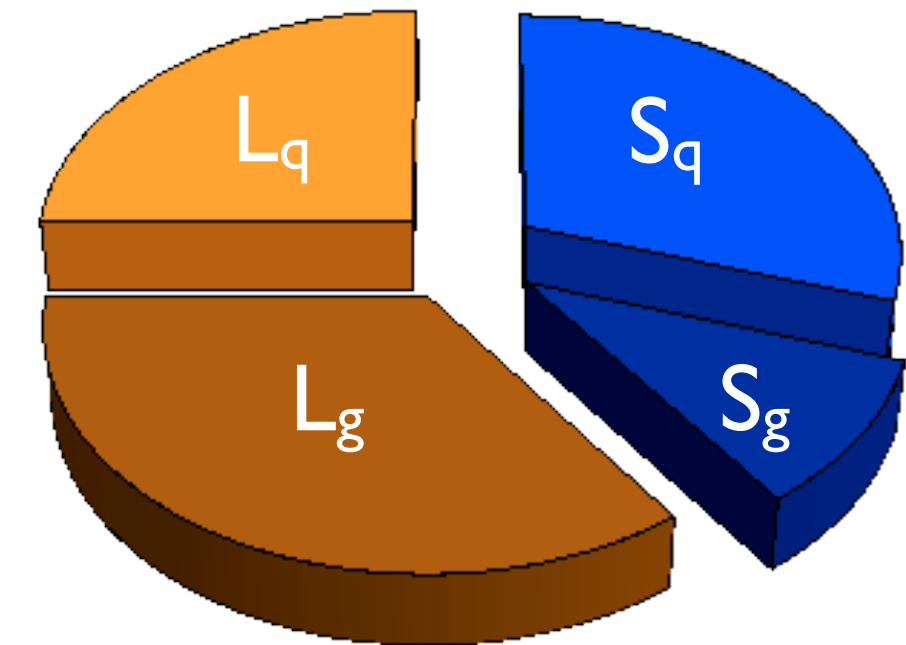
- Does not satisfy canonical commutation relations
- No decomposition of J_g in spin and orbital part

Improvements:

- Complete decomposition

$$J^g = L^g + \Delta g$$

Jaffe-Manohar



Pros:

- Satisfies canonical relations
- Complete decomposition

Cons:

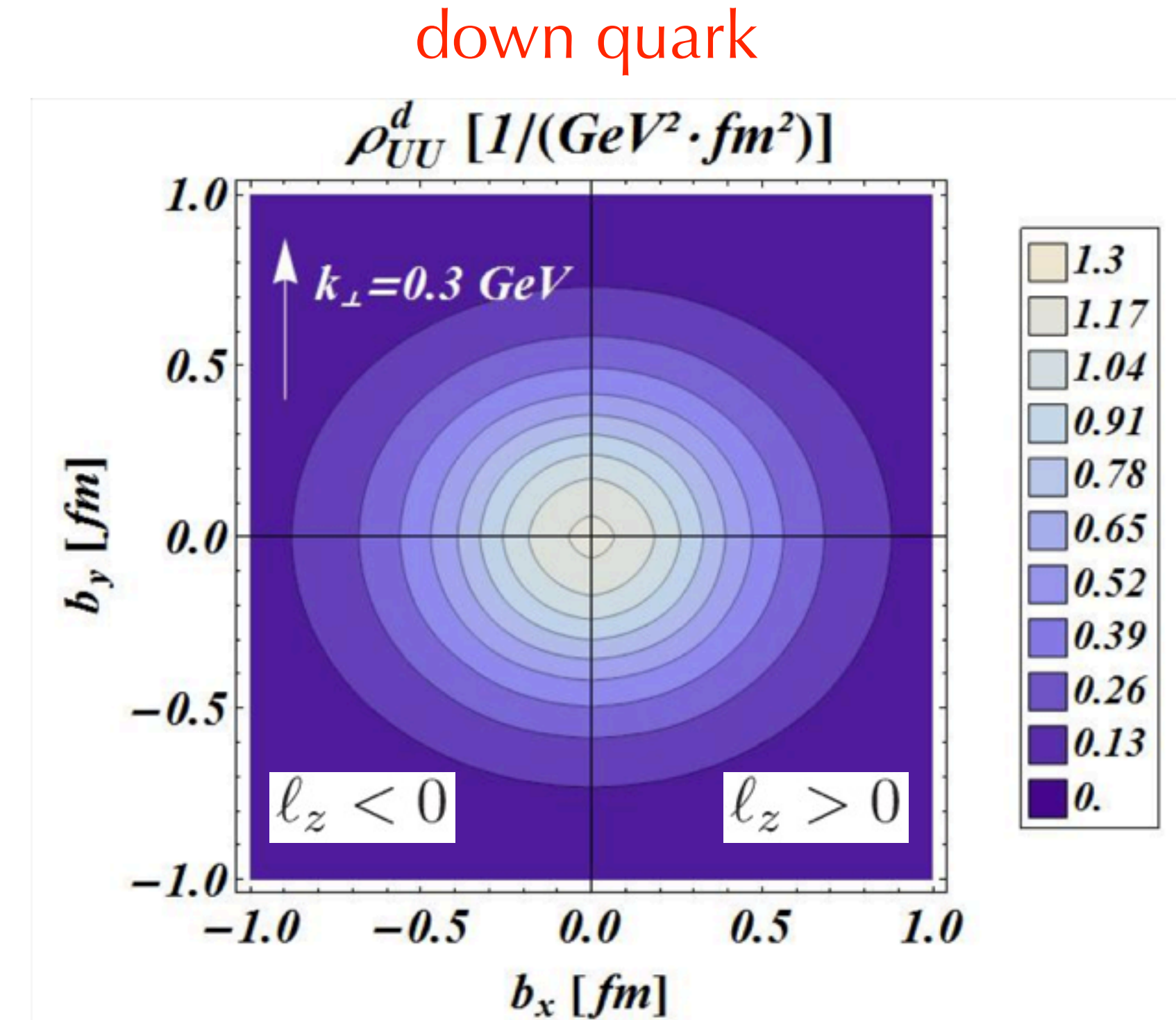
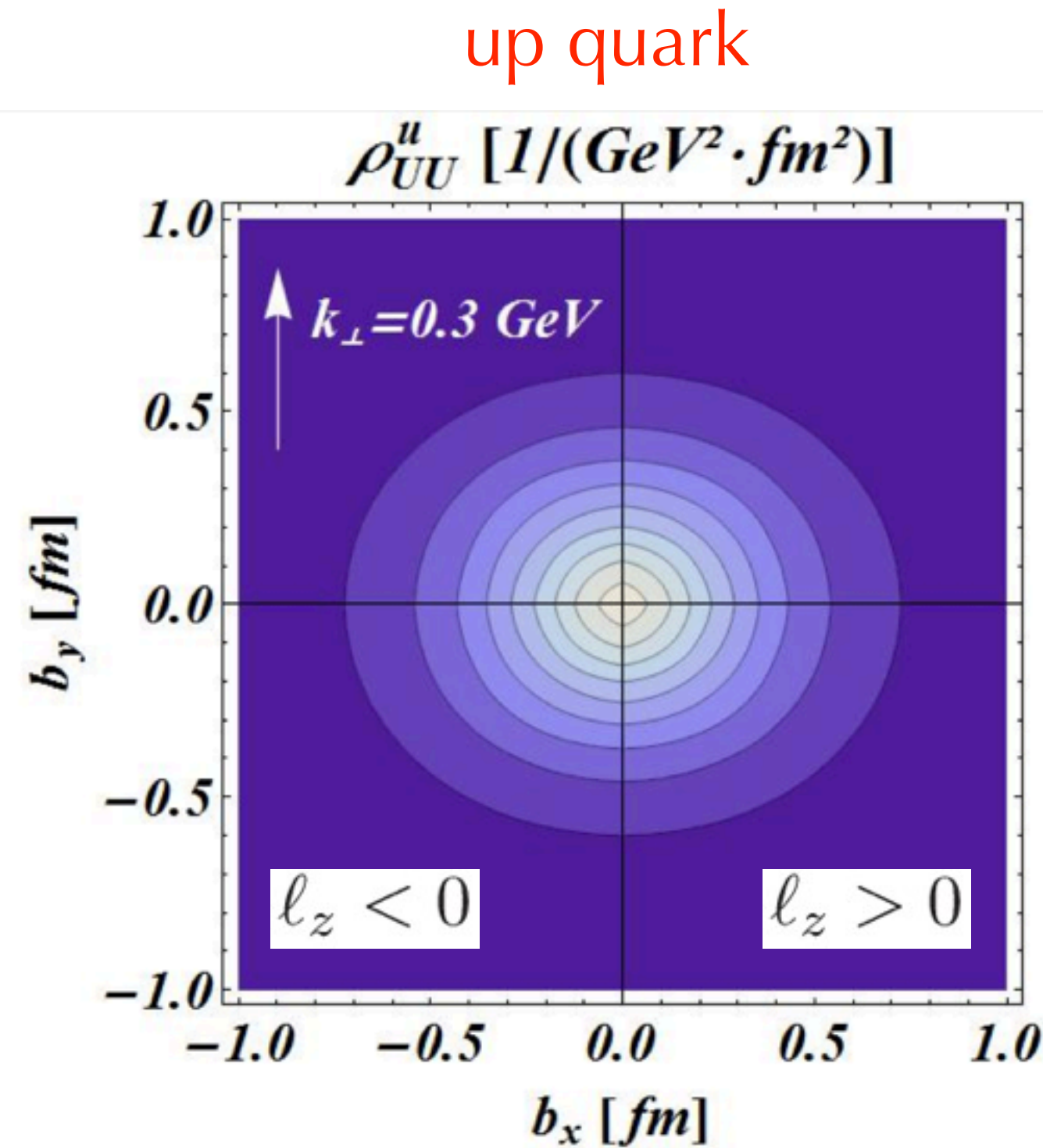
- Gauge-variant decomposition
- Missing observables for the OAM (Δg and $\Delta \Sigma$ measured by COMPASS, HERMES, RHIC)

Improvements:

- OAM accessible via Wigner distributions and it can be calculated on the lattice

Unpolarized quarks in unpolarized proton

fixed $\vec{k}_\perp \uparrow$
integrated over x



Heisenberg uncertainty principle \longrightarrow not probabilistic interpretation

Distortion due to correlations between \vec{k}_\perp and \vec{b}_\perp

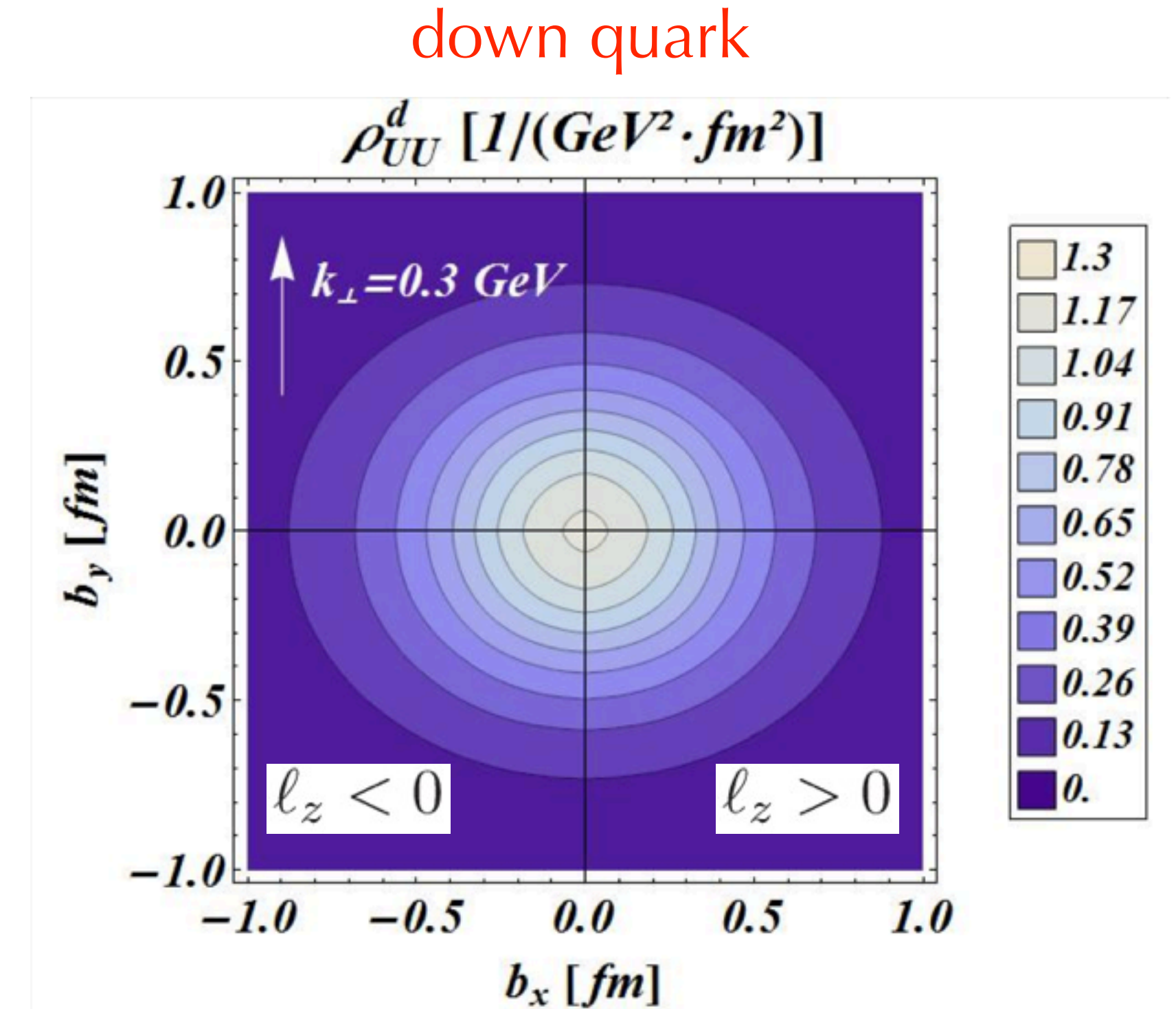
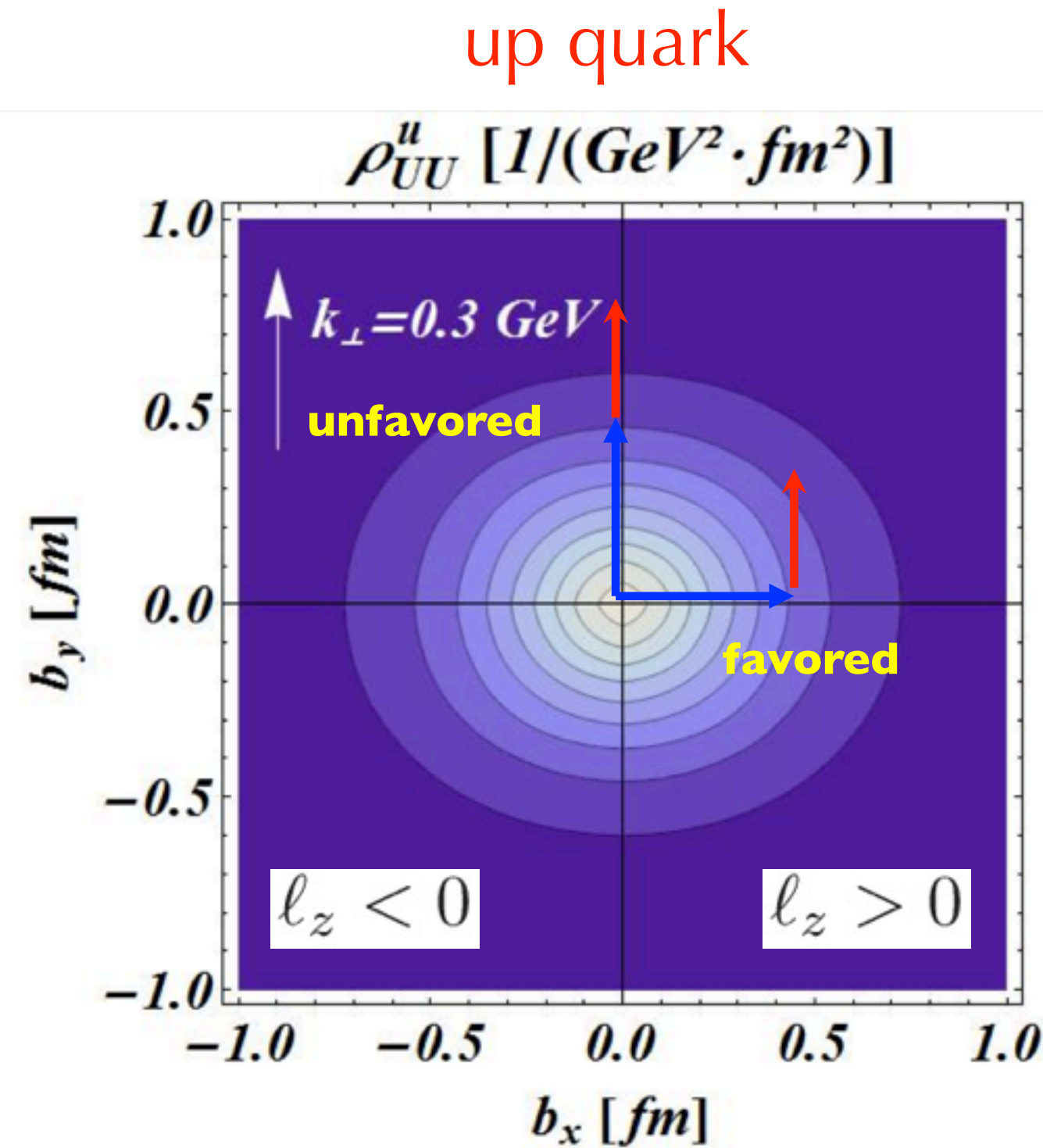
\searrow absent in **GPD** and **TMD** !

Left-right symmetry \longrightarrow no net quark OAM

Lorcé, Pasquini (2011)

Unpolarized quarks in unpolarized proton

fixed $\vec{k}_\perp \uparrow$
integrated over x



Heisenberg uncertainty principle \longrightarrow not probabilistic interpretation

Distortion due to correlations between \vec{k}_\perp and \vec{b}_\perp

\searrow absent in **GPD** and **TMD** !

Left-right symmetry \longrightarrow no net quark OAM

Lorcé, Pasquini (2011)

Quark Orbital Angular Momentum

$$\ell_z^q = \int dx d^2\vec{k}_\perp d^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp) \rho_{LU}^q(\vec{b}_\perp, \vec{k}_\perp, x)$$



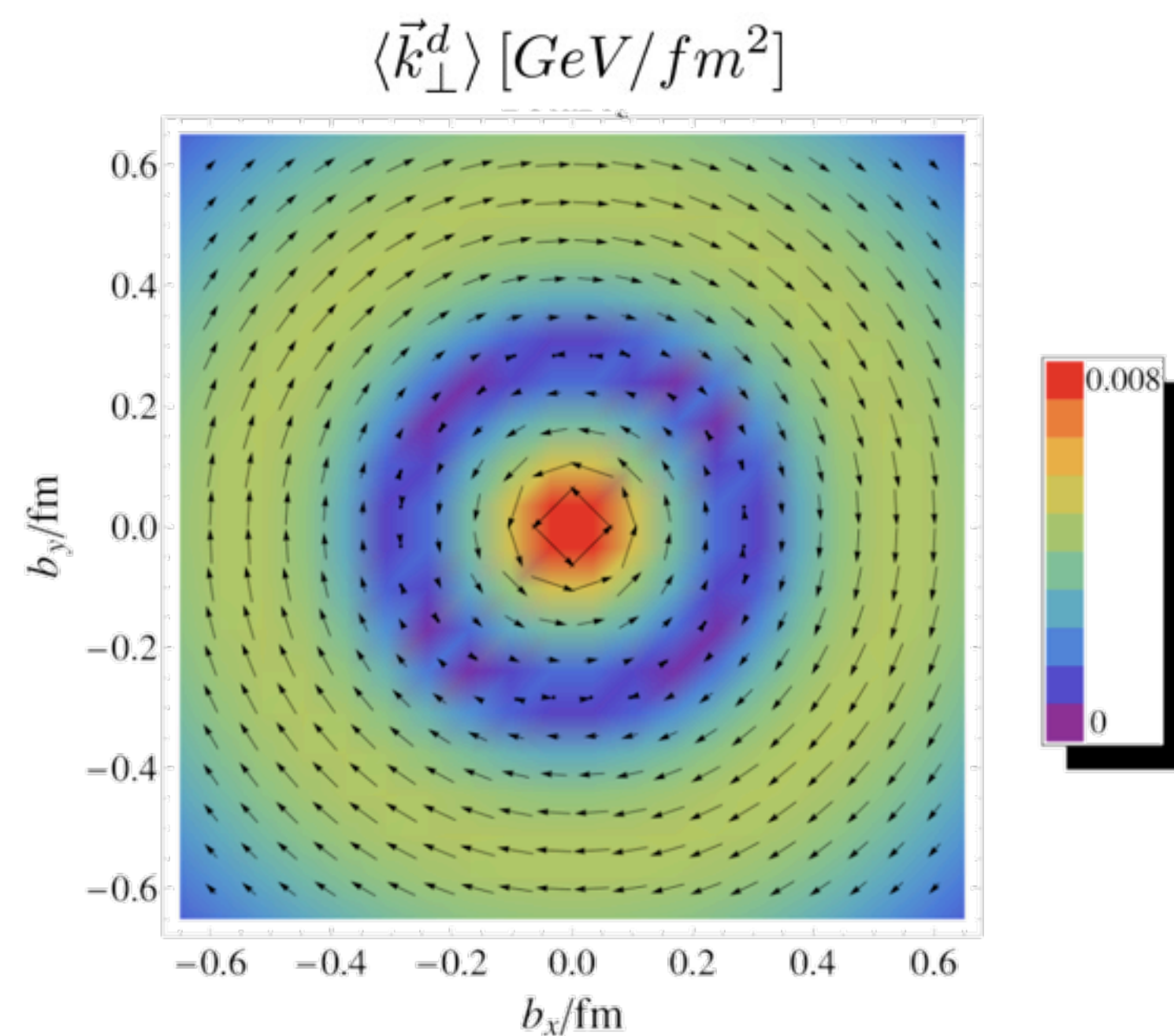
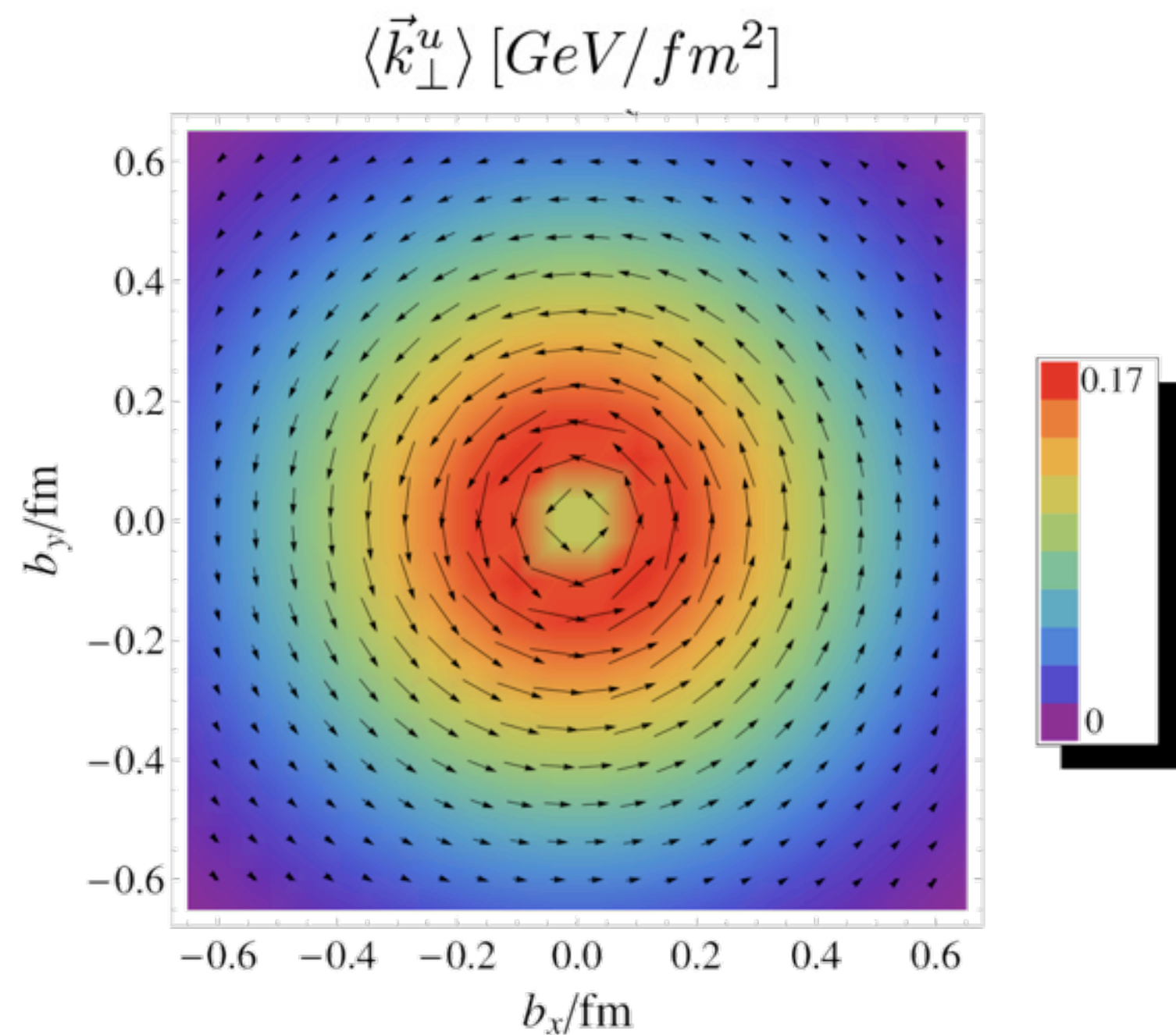
Wigner distribution for
Unpolarized quark in a Longitudinally pol. nucleon

Quark Orbital Angular Momentum

$$\begin{aligned}\ell_z^q &= \int dx \, d^2\vec{k}_\perp d^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp) \rho_{LU}^q(\vec{b}_\perp, \vec{k}_\perp, x) \\ &= \int d^2\vec{b}_\perp \vec{b}_\perp \times \langle \vec{k}_\perp^q \rangle \longrightarrow \langle \vec{k}_\perp^q \rangle = \int dx \, d^2\vec{k}_\perp \vec{k}_\perp \rho_{LU}^q(\vec{b}_\perp, \vec{k}_\perp, x)\end{aligned}$$

Quark Orbital Angular Momentum

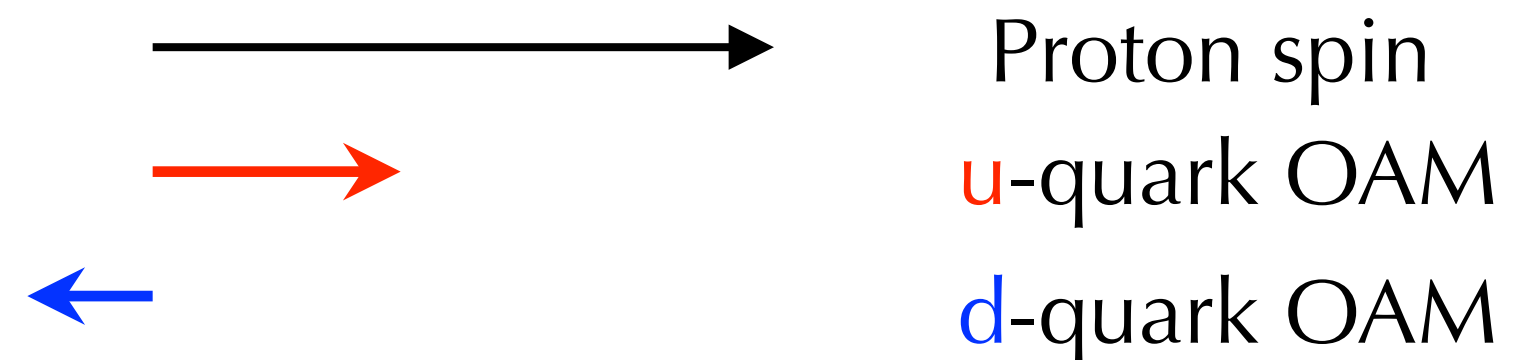
$$\begin{aligned}\ell_z^q &= \int dx d^2\vec{k}_\perp d^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp) \rho_{LU}^q(\vec{b}_\perp, \vec{k}_\perp, x) \\ &= \int d^2\vec{b}_\perp \vec{b}_\perp \times \langle \vec{k}_\perp^q \rangle \longrightarrow \langle \vec{k}_\perp^q \rangle = \int dx d^2\vec{k}_\perp \vec{k}_\perp \rho_{LU}^q(\vec{b}_\perp, \vec{k}_\perp, x)\end{aligned}$$



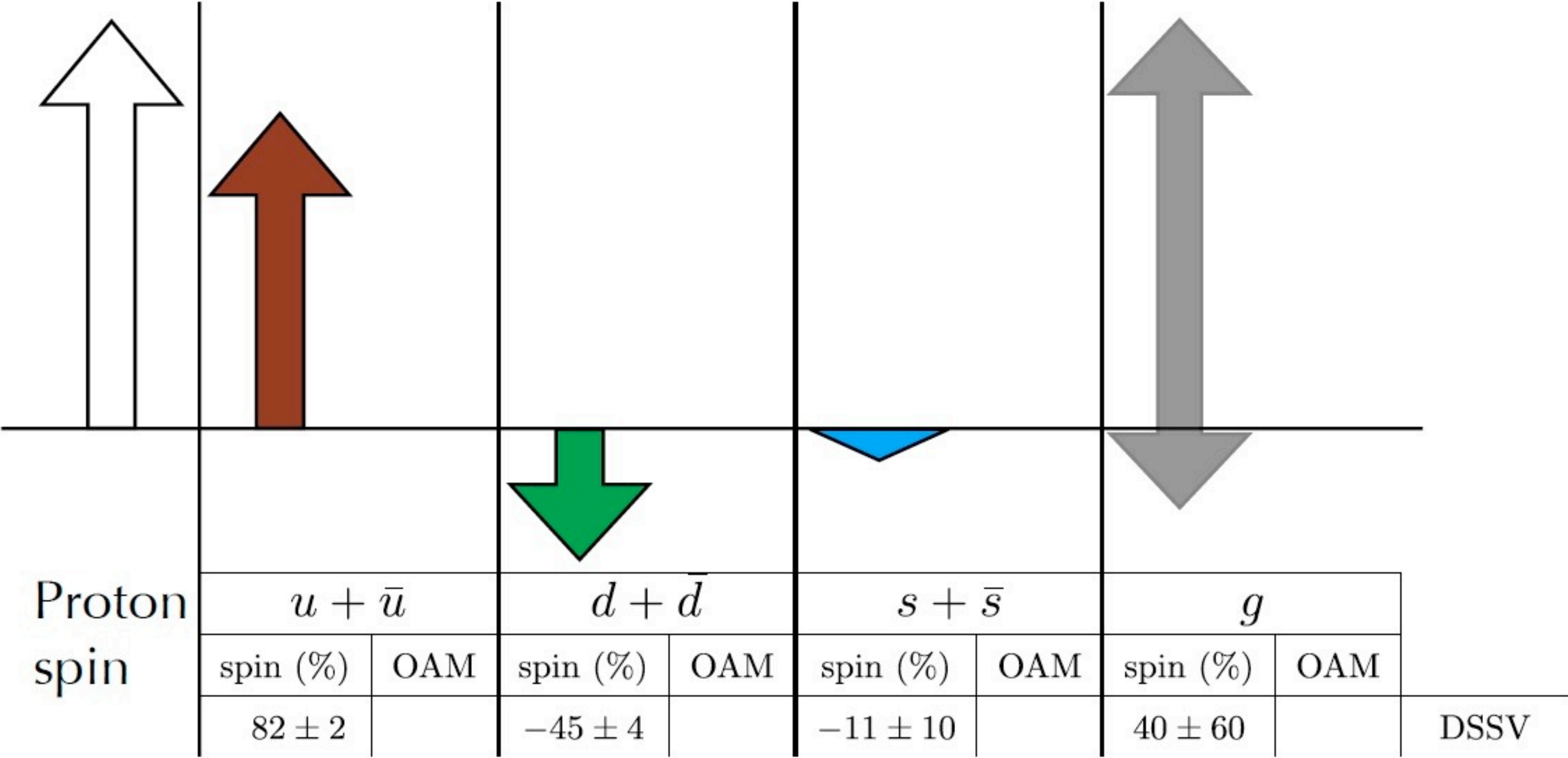
Results in a light-front constituent quark model:

Lorcé, BP, PRD **84** (2011) 014015

Lorcé, BP, Xiong, Yuan, PRD **85** (2012) 114006

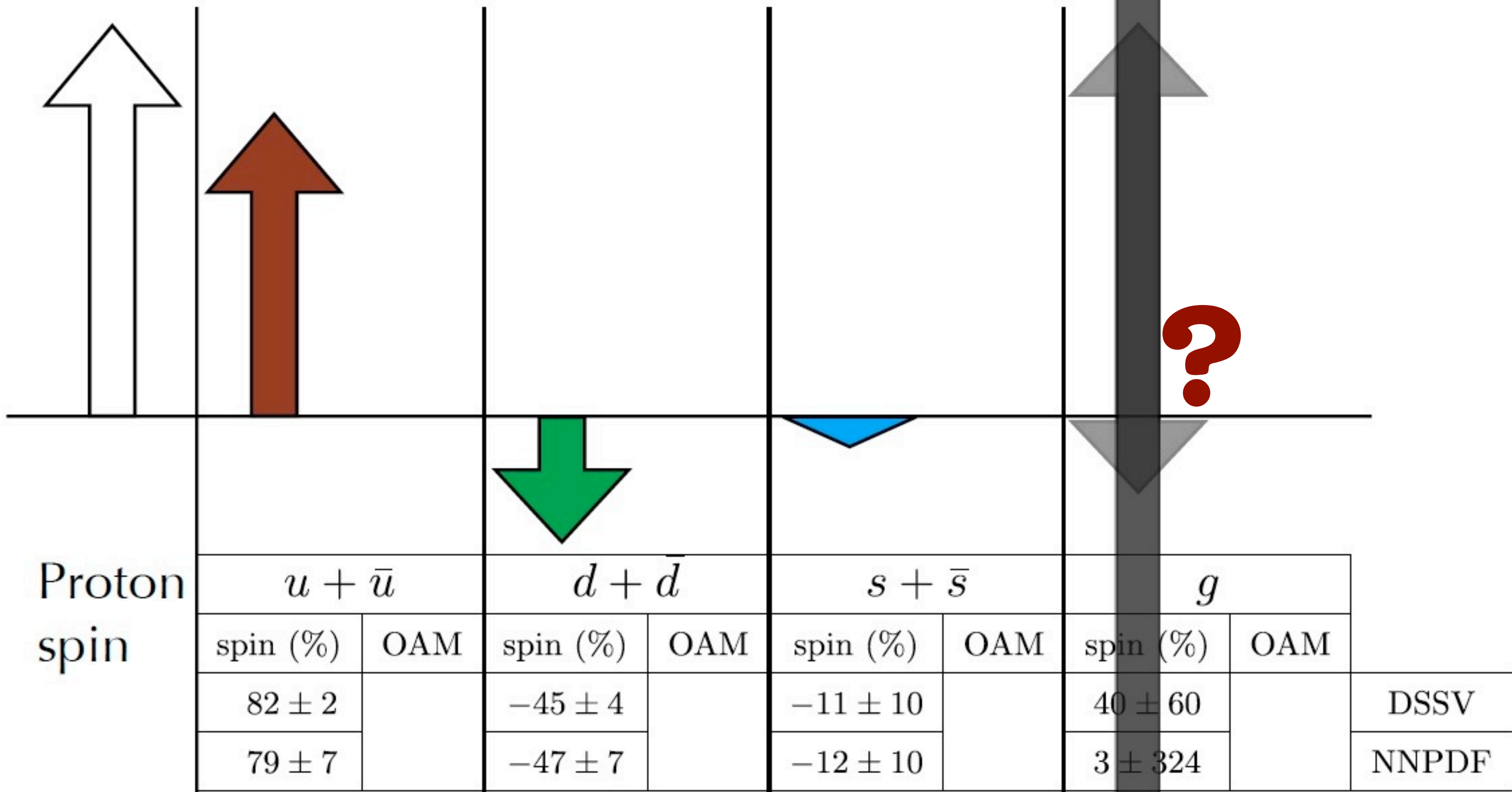


Status of spin sum rule



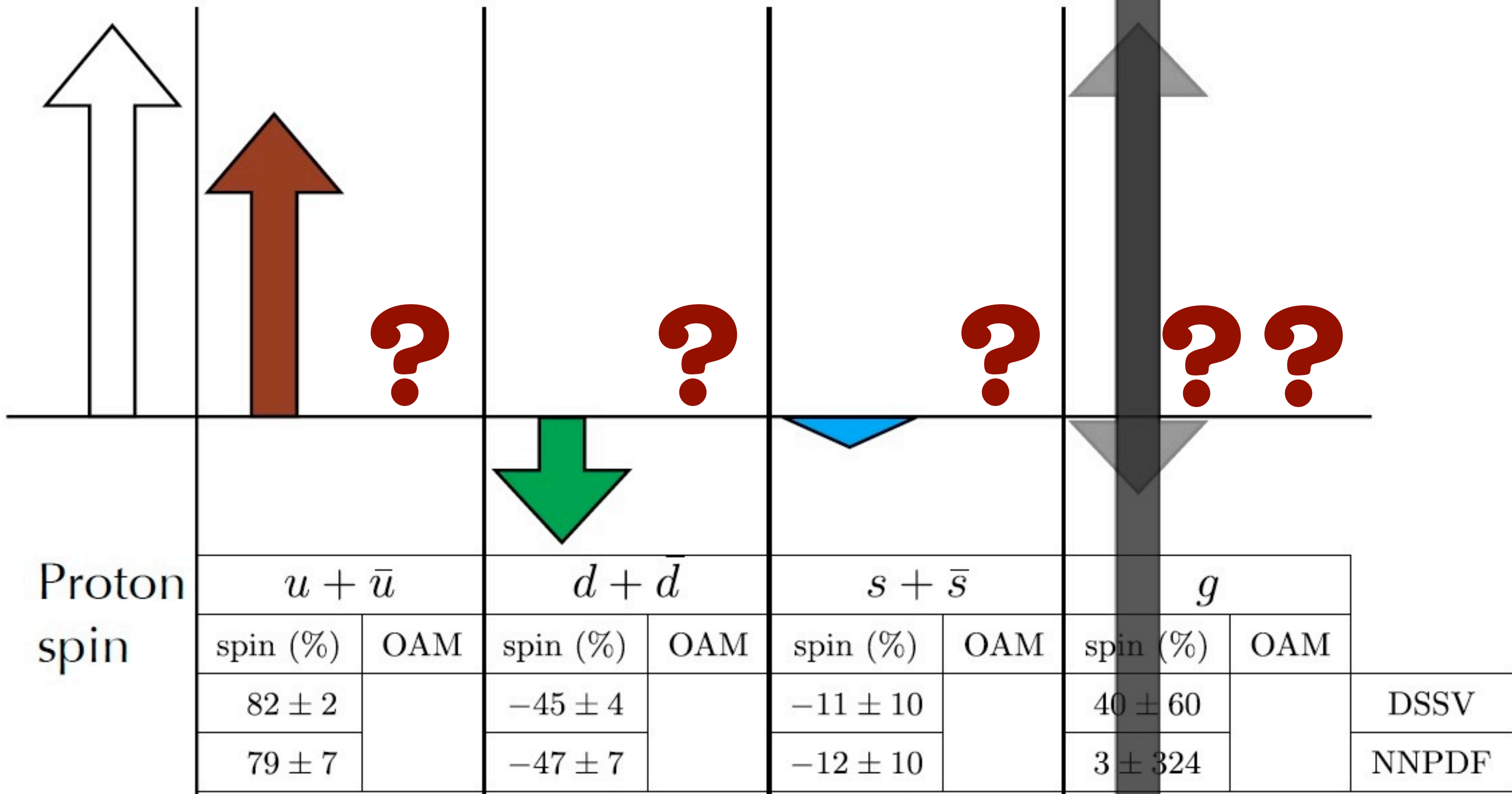
de Florian, Sassot, Stratmann, Vogelsang, PRL 113 (14)
NNPDF, Ball... Nocera... NPB 887 (14), Tab. 12, 13

Status of spin sum rule



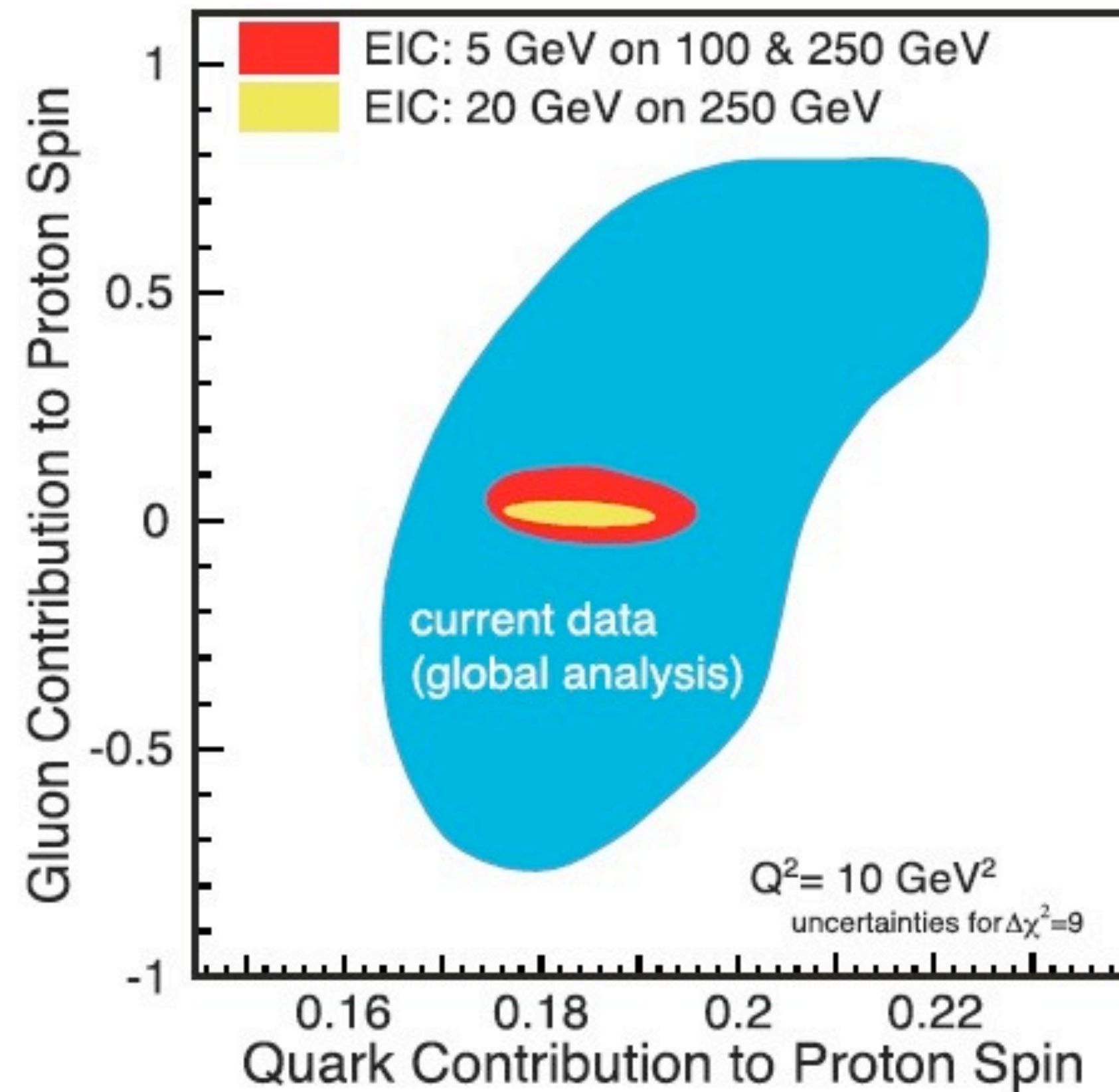
de Florian, Sassot, Stratmann, Vogelsang, PRL 113 (14)
NNPDF, Ball... Nocera... NPB 887 (14), Tab. 12, 13

Status of spin sum rule



de Florian, Sassot, Stratmann, Vogelsang, PRL 113 (14)
NNPDF, Ball... Nocera... NPB 887 (14), Tab. 12, 13

Impact of EIC on proton spin



Aschenauer, Stratmann, Sassot, PRD86 (2012)

Geesaman, et al., Reaching for the horizon: The 2015 long range plan for nuclear science (2015)

Conclusions

- TMDs and GPDs extend the concept of standard PDFs and provide a 3D description of the partonic structure of the nucleon
- TMDs and GPDs provide complementary information and allow us to investigate aspects of nucleon structure that are not accessible to standard collinear PDFs
- A lot of data is already available, but we expect more from e^+e^- , SIDIS at higher energies, Drell-Yan, DVCS,
- Some parametrizations of TMDs and GPDs are available, but we are a long way from anything similar to PDF global fits