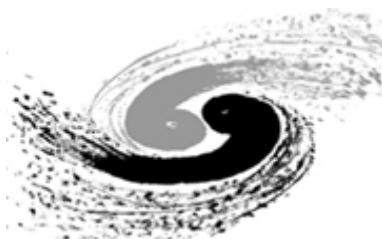


# **Strongly interacting matter from holographic QCD model**

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**QCD@Work, June 27-30, 2016, Matina Franca, Italy**

# Content

I. Dynamical hQCD model

II. Hadron spectra

Glueball, light-flavor meson spectra

III. sQGP

Equation of state, transport properties

IV. Chiral phase transition

V. Conclusion and discussion

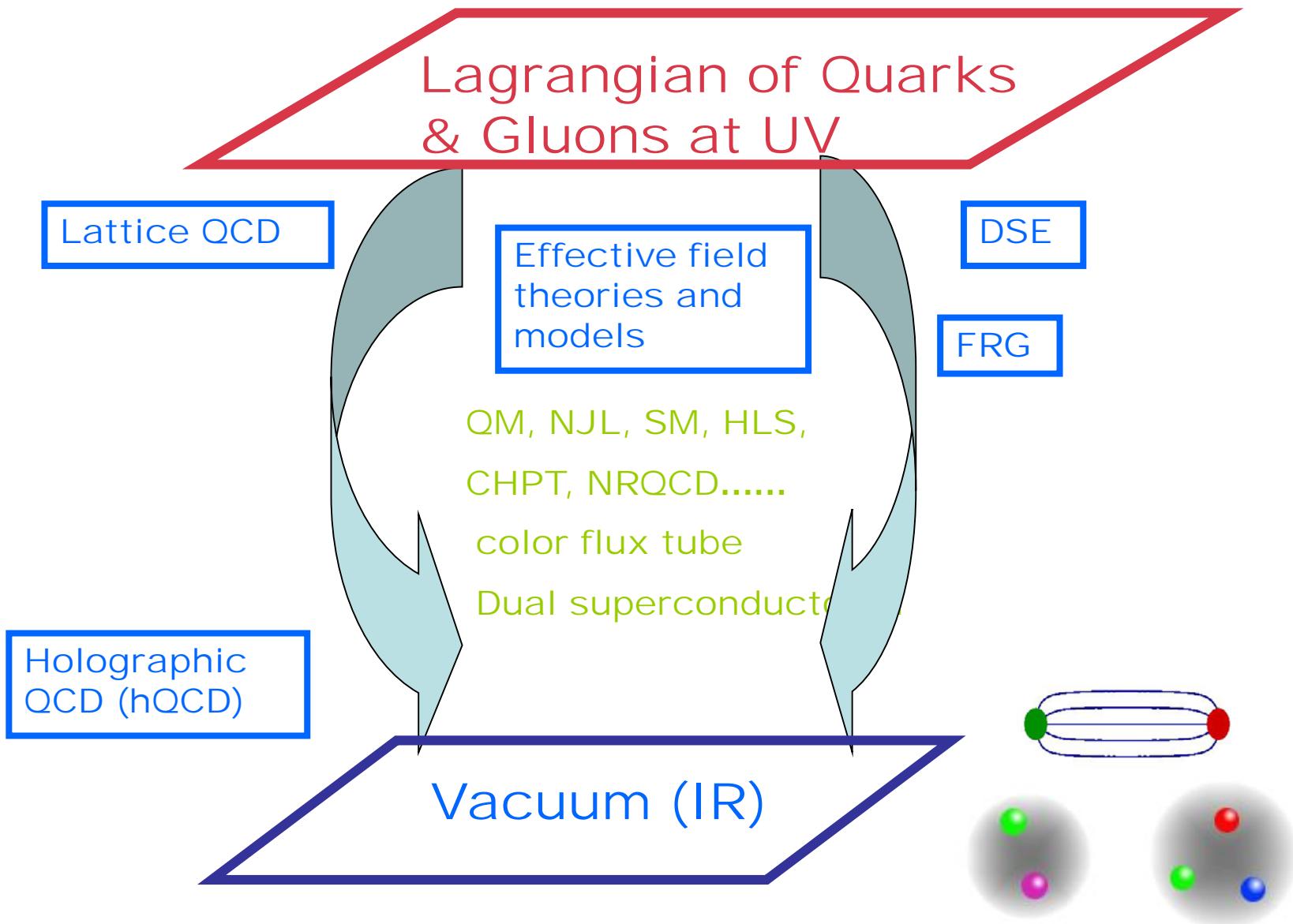
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Collaborators: Danning Li, Song He, Yidian Chen

# I. Dynamical hQCD model

## ----- 5D effective QCD model

# Strong QCD



# Holographic Duality: Gravity/QFT

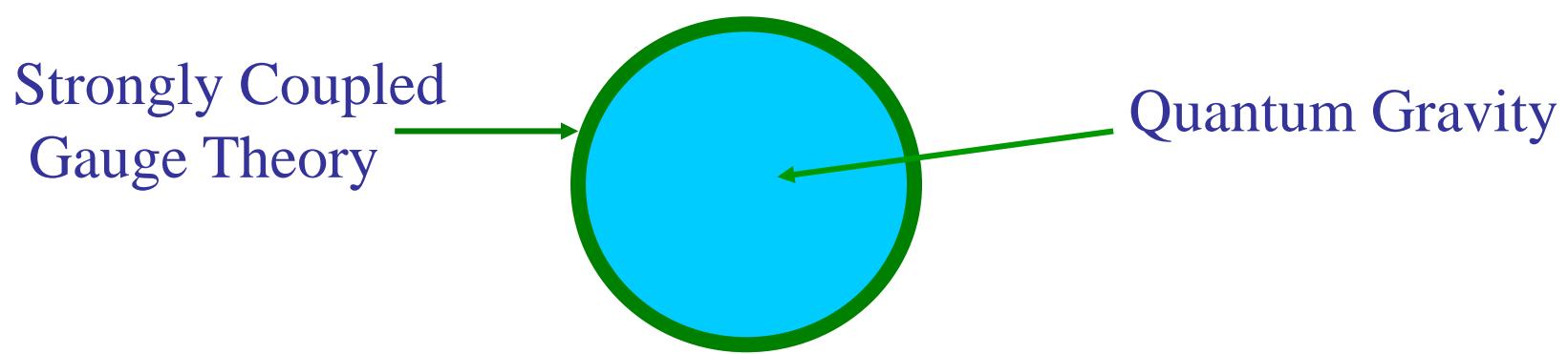
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AdS/CFT :Original discovery of duality

J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998)

**Supersymmetry and conformality are required for AdS/CFT.**

Holographic Duality:  $(d+1)$ -Gravity/  $(d)$ -QFT

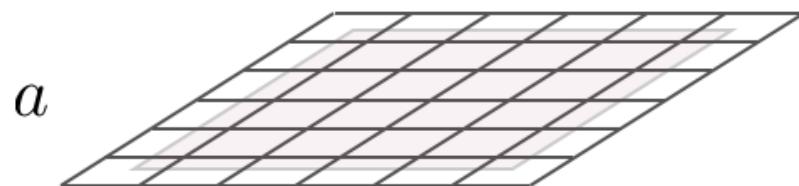


# Holographic Duality & RG flow

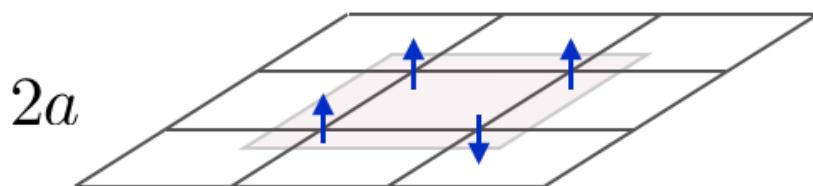
## Coarse graining spins on a lattice: Kadanoff and Wilson

$$H = \sum_{x,i} J_i(x) \mathcal{O}^i(x)$$

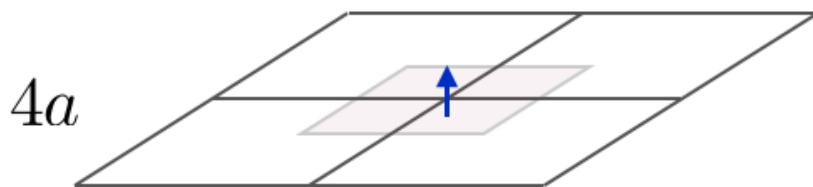
J(x): coupling constant or source for the operator



$$H = \sum_i J_i(x, a) \mathcal{O}^i(x)$$



$$H = \sum_i J_i(x, 2a) \mathcal{O}^i(x)$$



$$H = \sum_i J_i(x, 4a) \mathcal{O}^i(x)$$

$$u \frac{\partial}{\partial u} J_i(x, u) = \beta_i(J_j(x, u), u)$$

arXiv:1205.5180

# Holographic Duality & RG flow

**QFT on lattice equivalent to GR problem from Gravity**

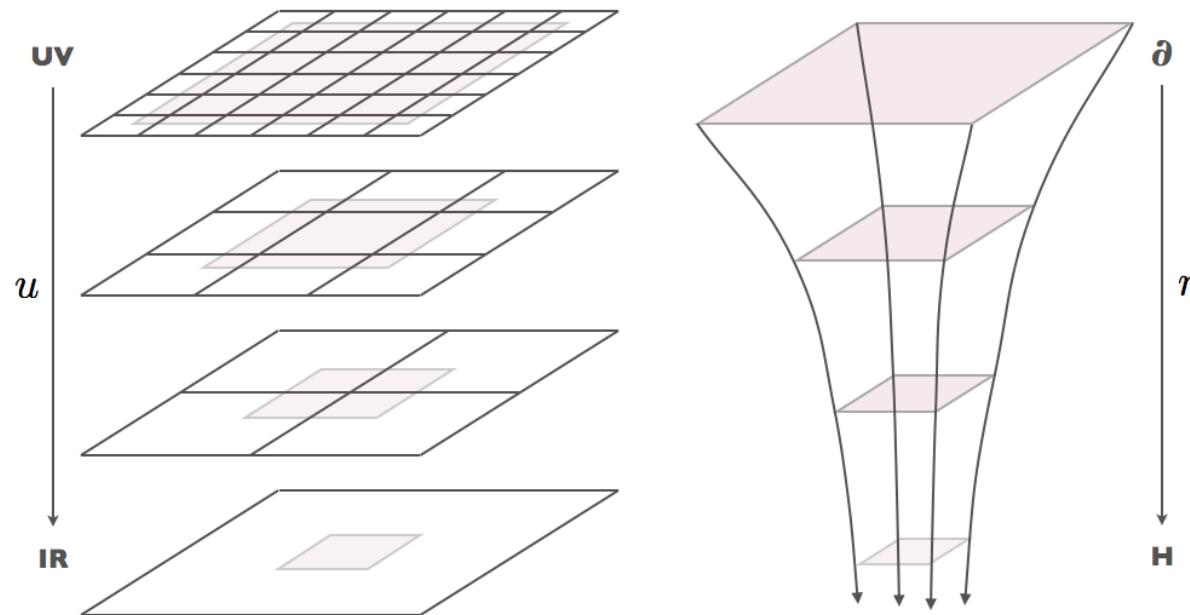
**RG scale -> an extra spatial dimension**

**Coupling constant -> dynamical field**

**arXiv:1205.5180**

$$J_i|_{UV} =$$

$$\Phi_i|_{\partial}$$



**The extra dimension plays the role of energy scale in QFT, with motion along the extra dimension representing a change of scale, or renormalization group (RG) flow.**

# A systematic framework: Graviton-dilaton system

$$S_G = \frac{1}{16\pi G_5} \int d^5x \sqrt{g_s} e^{-2\Phi} (R_s + 4\partial_M \Phi \partial^M \Phi - V_G^s(\Phi))$$

N=4 Super YM  
conformal

AdS<sub>5</sub>

$$ds^2 = \frac{L^2}{z^2} (dt^2 + d\vec{x}^2 + dz^2)$$

$$V_E(\phi) = -\frac{12}{L^2}$$

QCD  
nonconformal

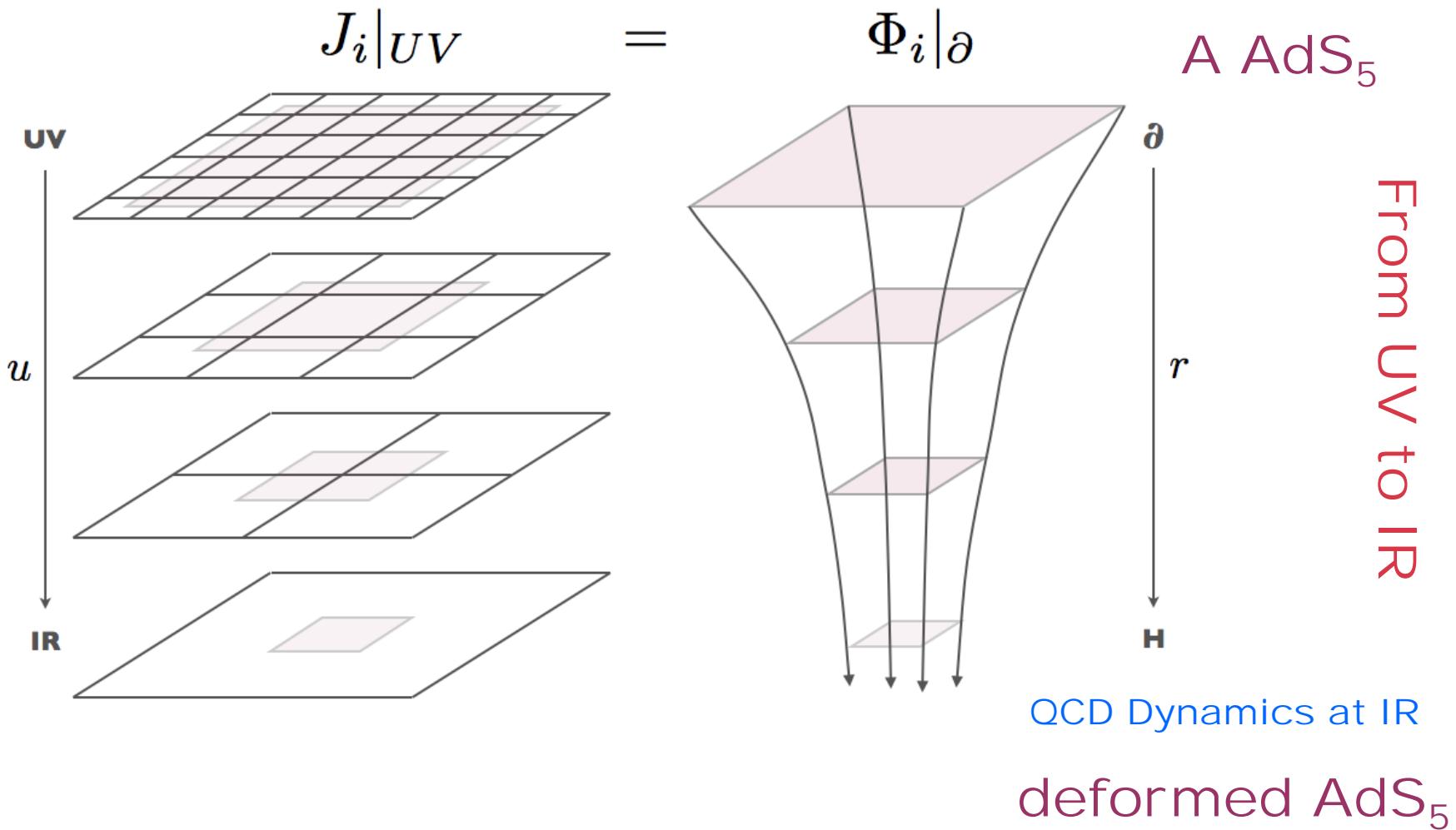
deformed AdS<sub>5</sub>

$$ds^2 = \frac{h(z)L^2}{z^2} (dt^2 + d\vec{x}^2 + dz^2)$$

**Dilaton field breaks conformal symmetry**

**Input: QCD dynamics at IR**  
**Solve: Metric structure, dilaton potential**

# Dynamical hQCD & RG



The goal is to describe

Hadron spectra  
chiral symmetry breaking  
& linear confinement

Phase transitions  
equation of state

Transport properties

in one systematic framework

## III. Hadron spectra:

Glueball spectra  
Light-flavor meson spectra

D.N. Li, M.H., JHEP2013, arXiv:1303.6929

Yidian Chen, M.H., arXiv: 1511.07018

# Pure gluon system: Gluonic background

D.N. Li, M.H., JHEP2013, arXiv:1303.6929

$$\mathcal{L}_G = -\frac{1}{4} G_{\mu\nu}^a(x) G^{\mu\nu,a}(x),$$

IR: Gluon condensate  $\text{Tr}\langle G^2 \rangle$   
Effective gluon mass  $\langle g^2 A^2 \rangle$

---

## 5D action: graviton-dilaton

$$S_G = \frac{1}{16\pi G_5} \int d^5x \sqrt{g_s} e^{-2\Phi} (R_s + 4\partial_M \Phi \partial^M \Phi - V_G^s(\Phi))$$

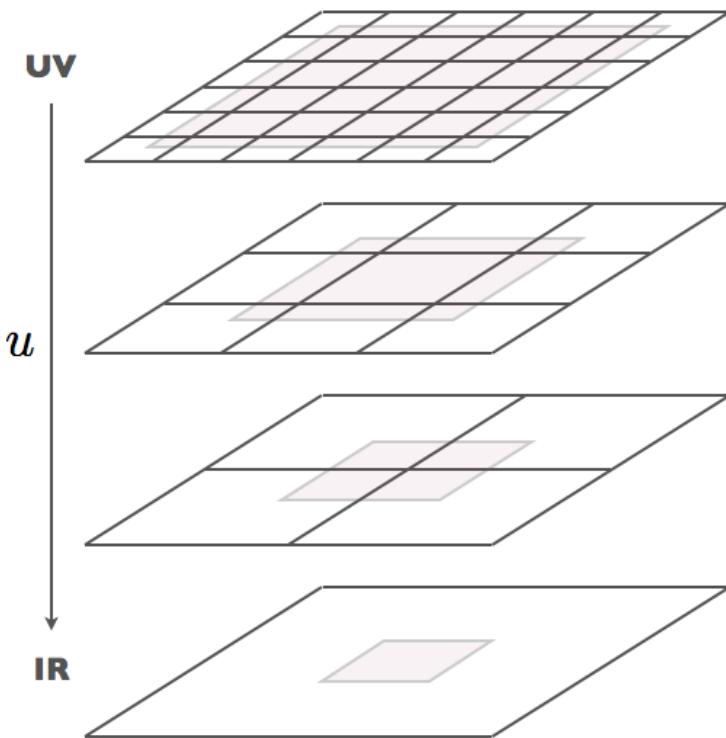
$\text{Tr}\langle G^2 \rangle$   $\langle g^2 A^2 \rangle$  dual to  $\Phi(z)$

$$\Phi(z) = \mu_G^2 z^2 \tanh(\mu_{G^2}^4 z^2 / \mu_G^2)$$

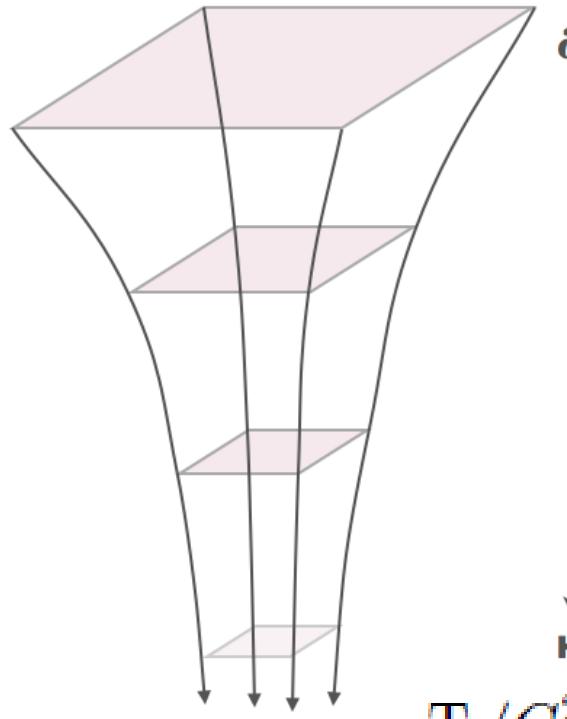
$$\Phi(z) \xrightarrow{z \rightarrow 0} \mu_{G^2}^4 z^4, \quad \Phi(z) \xrightarrow{z \rightarrow \infty} \mu_G^2 z^2.$$

# Graviton-dilaton system

$$J_i|_{UV} =$$



$$\Phi_i|_{\partial}$$



A AdS<sub>5</sub>

From UV to IR

$\text{Tr}\langle G^2 \rangle \langle g^2 A^2 \rangle$   
deformed AdS<sub>5</sub>

$$g_{MN}^s = b_s^2(z)(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu), \quad b_s(z) \equiv e^{A_s(z)}$$

# Holographic Duality: Dictionary

Boundary QFT

Bulk Gravity

Local operator  $\mathcal{O}_i(x)$

Bulk field  $\Phi_i(x, r)$

$$\Delta(d - \Delta) = m^2 L^2$$

-----

Strongly coupled

Semi-classical

$$Z_{\text{QFT}}[J_i] = Z_{\text{QG}}[\Phi[J_i]]$$

$$Z_{\text{QFT}}[J] \simeq e^{-I_{\text{GR}}[\Phi[J]]}$$

$$\langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle = \frac{\delta^n I_{\text{GR}}[\Phi[J_i]]}{\delta J_1(x_1) \dots \delta J_n(x_n)} \Big|_{J_i=0}$$

# Two-gluon and tri-gluon Glueball spectra:

Yidian Chen, M.H., arXiv: 1511.07018

$$M_5^2 = (\Delta - f)(\Delta + f - 4)$$

$J^{PC}$	$4D : \mathcal{O}(x)$	$\Delta$	$f$	$M_5^2$
$0^{++}$	$Tr(G^2)$	4	0	0
$0^{--}$	$Tr(\tilde{G}\{D_{\mu_1}D_{\mu_2}G, G\})$	8	0	32
$0^{-+}$	$Tr(G\tilde{G})$	4	0	0
$1^{\pm-}$	$Tr(G\{G, G\})$	6	1	15
$2^{++}$	$Tr(G_{\mu\alpha}G_{\alpha\nu} - \frac{1}{4}\delta_{\mu\nu}G^2)$	4	2	4
$2^{++}$	$E_i^a E_j^a - B_i^a B_j^a - trace$	4	2	4
$2^{-+}$	$E_i^a B_j^a + B_i^a E_j^a - trace$	4	2	4
$2^{\pm-}$	$Tr(G\{G, G\})$	6	2	16

tri-gluon

tri-gluon

tri-gluon  
15

# Two-gluon and tri-gluon Glueball spectra:

C. -F. Qiao and L. Tang, “Finding the  $0^{--}$  Glueball,” Phys. Rev. Lett. **113**, 221601 (2014).

C. F. Qiao and L. Tang, arXiv:1509.00305 [hep-ph].

## Tri-gluon glueball

$$j_{0^{--}}^A \sim d^{abc} [g_{\alpha\beta}^t \tilde{G}_{\mu\nu}^a] [\partial_\alpha \partial_\beta G_{\nu\rho}^b] [G_{\rho\mu}^c],$$

$$j_{0^{--}}^B \sim d^{abc} [g_{\alpha\beta}^t G_{\mu\nu}^a] [\partial_\alpha \partial_\beta \tilde{G}_{\nu\rho}^b] [G_{\rho\mu}^c],$$

$$j_{0^{--}}^C \sim d^{abc} [g_{\alpha\beta}^t G_{\mu\nu}^a] [\partial_\alpha \partial_\beta G_{\nu\rho}^b] [\tilde{G}_{\rho\mu}^c],$$

$$j_{0^{--}}^D \sim d^{abc} [g_{\alpha\beta}^t \tilde{G}_{\mu\nu}^a] [\partial_\alpha \partial_\beta \tilde{G}_{\nu\rho}^b] [\tilde{G}_{\rho\mu}^c],$$

$$j_{\mu\alpha}^{2+-}, A(x) = g_s^3 d^{abc} [G_{\mu\nu}^a(x)] [G_{\nu\rho}^b(x)] [G_{\rho\alpha}^c(x)],$$

$$j_{\mu\alpha}^{2+-}, B(x) = g_s^3 d^{abc} [G_{\mu\nu}^a(x)] [\tilde{G}_{\nu\rho}^b(x)] [\tilde{G}_{\rho\alpha}^c(x)],$$

$$j_{\mu\alpha}^{2+-}, C(x) = g_s^3 d^{abc} [\tilde{G}_{\mu\nu}^a(x)] [G_{\nu\rho}^b(x)] [\tilde{G}_{\rho\alpha}^c(x)],$$

$$j_{\mu\alpha}^{2+-}, D(x) = g_s^3 d^{abc} [\tilde{G}_{\mu\nu}^a(x)] [\tilde{G}_{\nu\rho}^b(x)] [G_{\rho\alpha}^c(x)].$$

## Excitations from gluonic background

$$S_{\mathcal{G}} = -\frac{1}{2} \int d^5x \sqrt{g_s} e^{-p\Phi} (\partial_M \mathcal{G} \partial^M \mathcal{G} + M_{\mathcal{G},5}^2(z) \mathcal{G}^2),$$

$$S_V = -\frac{1}{2} \int d^5x \sqrt{g_s} e^{-p\Phi} (\frac{1}{2} F^{MN} F_{MN} + M_{V,5}^2(z) V^2),$$

$$\begin{aligned} S_T = & -\frac{1}{2} \int d^5x \sqrt{g_s} e^{-p\Phi} (-\nabla_L h_{MN} \nabla^L h^{MN} - 2\nabla_L h^{LM} \nabla^N h_{NM} + 2\nabla_M h^{MN} \nabla_N h \\ & - \nabla_M h \nabla^M h + M_{h,5}^2(z) (h^{MN} h_{MN} - h^2)) \end{aligned}$$

$M_5^2(z) = M_5^2 e^{-2\Phi/3}$ ,  $p = 1$  for even parity and  $p = -1$  for odd parity.

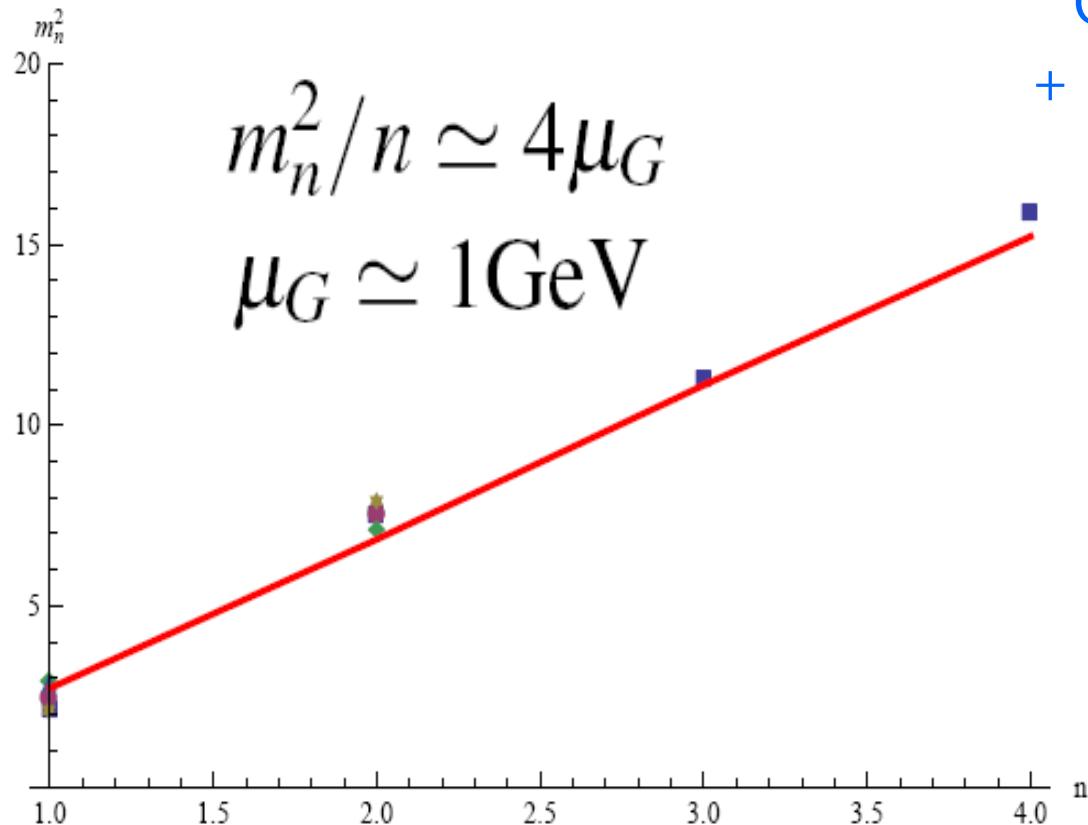
EOM:

$$-\mathcal{A}_n'' + V_{\mathcal{A}} \mathcal{A}_n = m_{\mathcal{A},n}^2 \mathcal{A}_n,$$

$$V_{\mathcal{A}} = \frac{cA_s'' - p\Phi''}{2} + \frac{(cA_s' - p\Phi')^2}{4} + e^{2A_s - \frac{2}{3}\Phi} M_{\mathcal{A},5}^2,$$

Only one parameter determined from the Regge slope of the scalar glueball spectra:

$$\mu_G = 1 \text{ GeV}$$

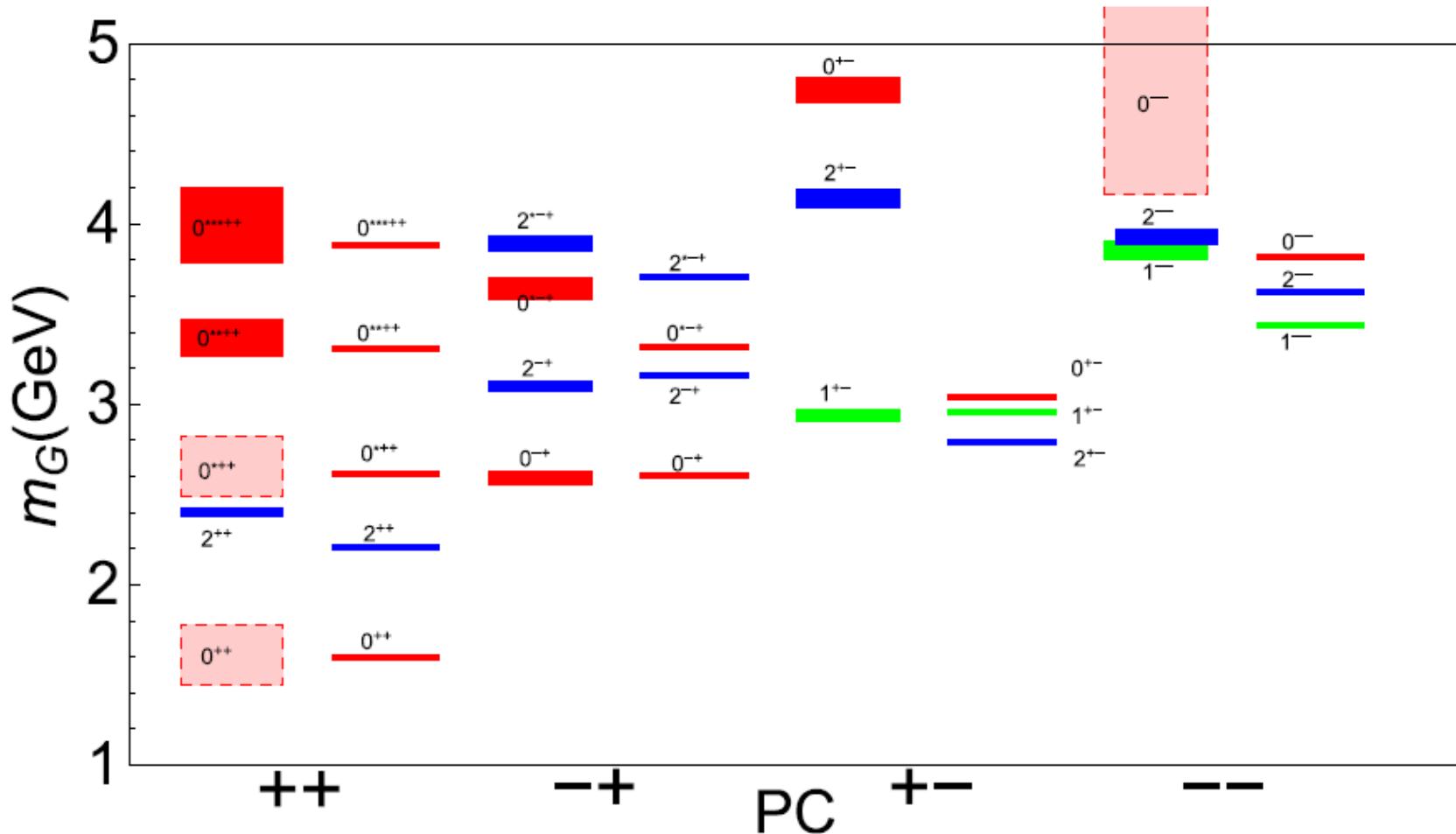


Ground state  
+ Regge slope !

hep-lat/0508002.  
[hep-lat/0510074].  
[hep-lat/0103027].  
[hep-lat/9901004]

# Glueball spectra:

Yidian Chen, M.H., arXiv: 1511.07018



Agree well with lattice result except  
three triluon glueball  $0^{--}$ ,  $0^{+-}$  and  $2^{+-}$

# Glueball spectra:

Yidian Chen, M.H., arXiv: 1511.07018

$J^{PC}$	LQCD	Flux tube model	QCDSR	MDSM
$0^{++}$	1.475-1.73	1.52	1.5	1.593
$0^{*++}$	2.67-2.83	2.75	—	2.618
$0^{**++}$	3.37	—	—	3.311
$0^{***++}$	3.99	—	—	3.877
$0^{-+}$	2.59	2.79	2.05	2.606
$0^{*-+}$	3.64	—	—	3.317
$0^{--}$	5.166	2.79	3.81	3.817
$0^{+-}$	4.74	2.79	4.57	3.04
$0^{++\S}$	—	—	3.1	2.667
$1^{+-}$	2.94	2.25	—	2.954
$1^{--}$	3.85	—	—	3.44
$2^{++}$	2.4	2.84	2	2.203
$2^{-+}$	3.1	2.84	—	3.161
$2^{*-+}$	3.89	—	—	3.703
$2^{+-}$	4.14	2.84	6.06	2.786
$2^{--}$	3.93	2.84	—	3.619

**All two-gluon and tri-gluon glueball spectra agree well with lattice result except three trigluon glueballs  
 $0^{--}$  ,  $0^{+-}$  and  $2^{+-}$**

**These three trigluon glueballs  
 $0^{--}$  ,  $0^{+-}$  and  $2^{+-}$   
are dominated by three-gluon condensate.**

**Our model only considered two-gluon condensate.**

# Add flavor dynamics

---

D.N. Li, M.H., JHEP2013, arXiv:1303.6929

Action for pure gluon system: Graviton-dilaton coupling

$$S_G = \frac{1}{16\pi G_5} \int d^5x \sqrt{g_s} e^{-2\Phi} (R + 4\partial_M \Phi \partial^M \Phi - V_G(\Phi))$$

**Gluonic background**

Action for light hadrons: KKSS model

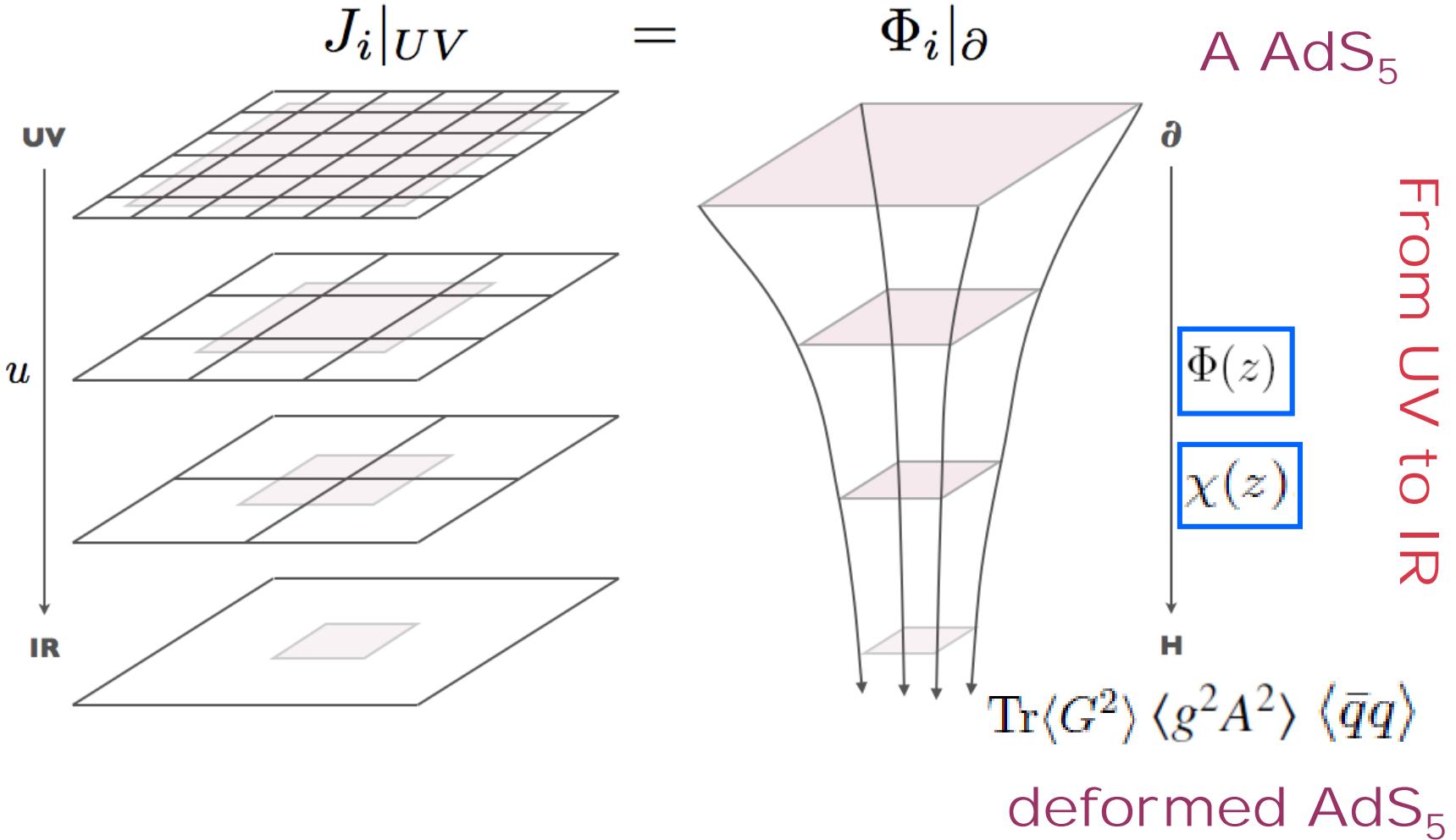
$$S_{KKSS} = - \int d^5x \sqrt{g_s} e^{-\Phi} Tr(|DX|^2 + V_X(X^+ X, \Phi) + \frac{1}{4g_5^2}(F_L^2 + F_R^2)).$$

**5D linear sigma model**

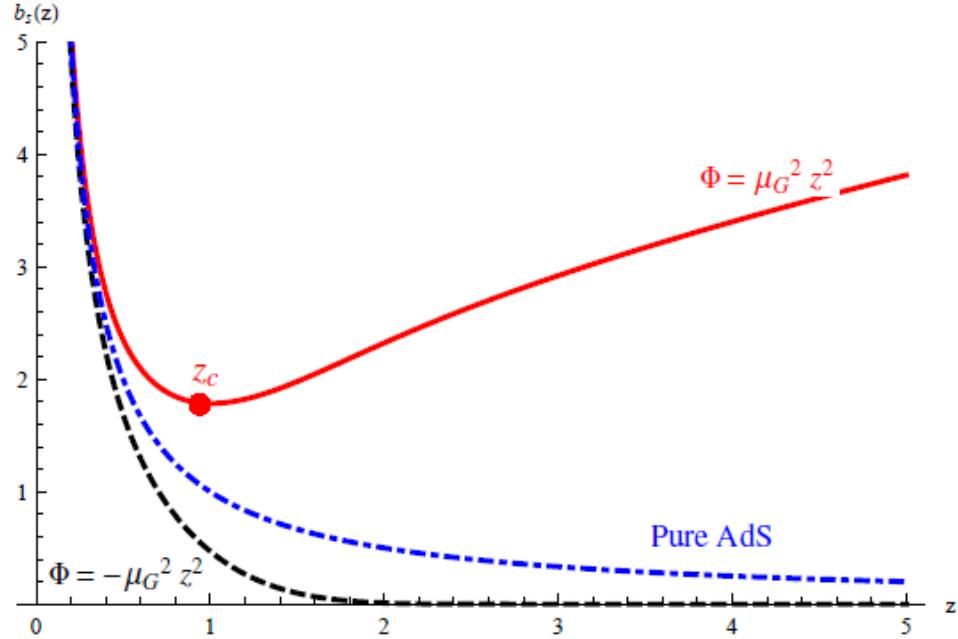
Total action:

$$S = S_G + \frac{N_f}{N_c} S_{KKSS}.$$

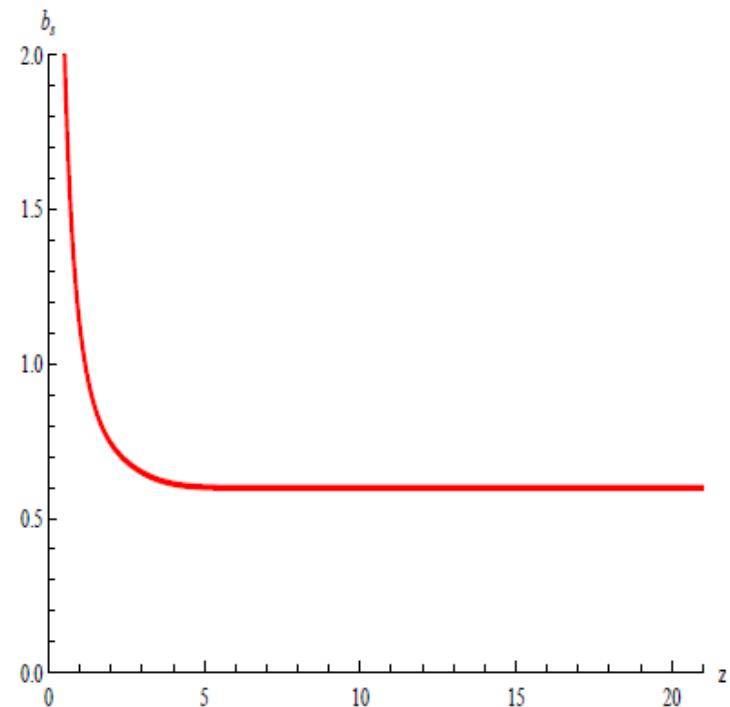
# Graviton-dilaton-scalar system



# Quenched background



# Unquenched background



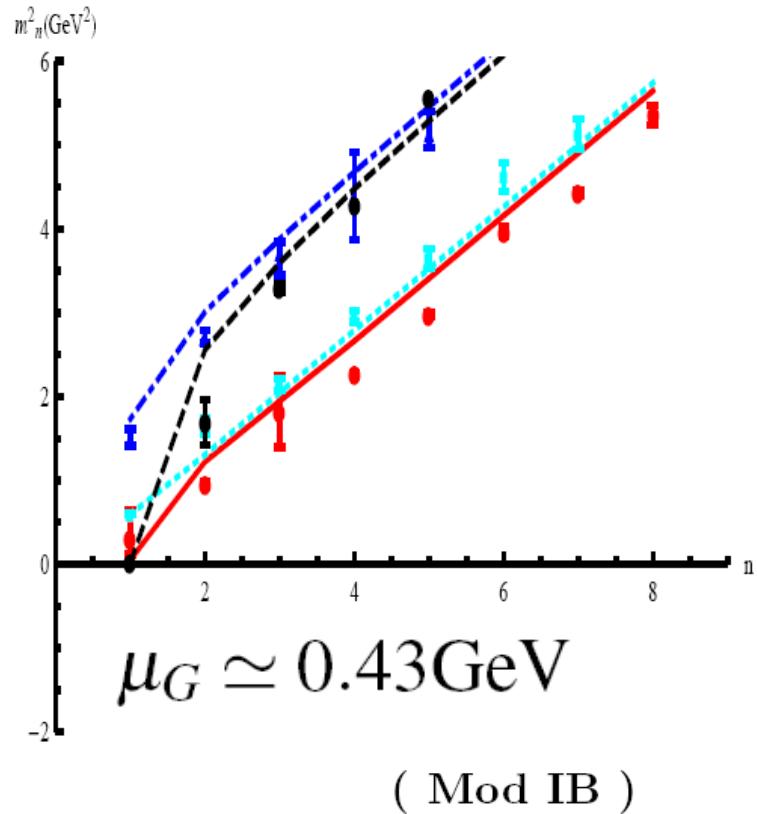
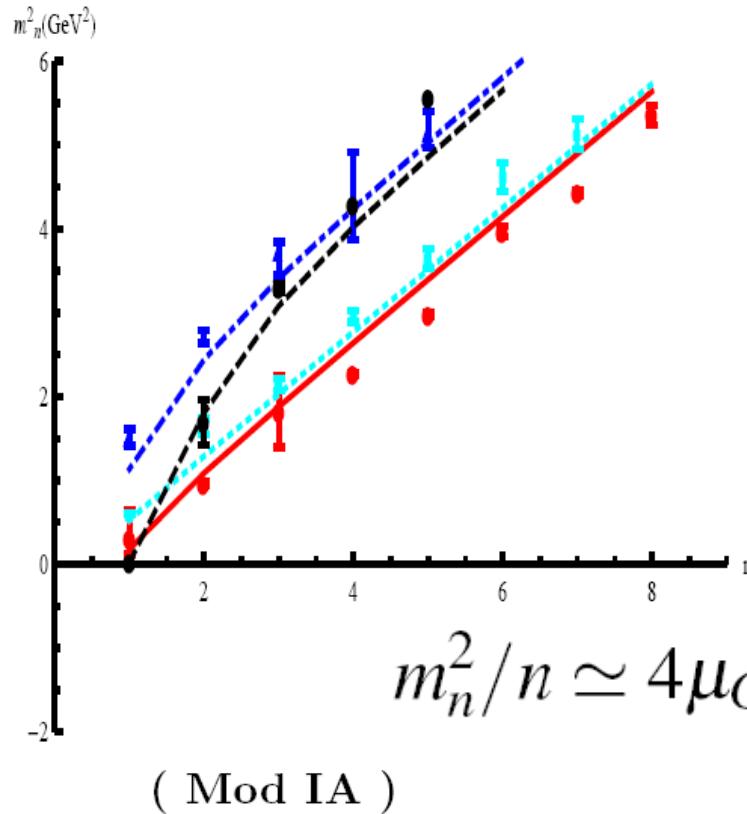
$$-A_s'' + A_s'^2 + \frac{2}{3}\Phi'' - \frac{4}{3}A_s'\Phi' - \frac{\lambda}{6}e^\Phi\chi'^2 = 0,$$

$$\Phi'' + (3A_s' - 2\Phi')\Phi' - \frac{3\lambda}{16}e^\Phi\chi'^2 - \frac{3}{8}e^{2A_s - \frac{4}{3}\Phi}\partial_\Phi \left( V_G(\Phi) + \lambda e^{\frac{7}{3}\Phi}V_C(\chi, \Phi) \right) = 0,$$

$$\chi'' + (3A_s' - \Phi')\chi' - e^{2A_s}V_{C,\chi}(\chi, \Phi) = 0.$$

# Produced hadron spectra compared with data

D.N. Li, M.H., JHEP2013, arXiv:1303.6929



Ground states: chiral symmetry breaking  
Excitation states: linear confinement

## III. sQGP

### Equation of state & transport properties

D.N. Li, S. He, M.H., Q. S. Yan, arXiv:1103.5389, JHEP2011

Danning Li, Jinfeng Liao, M.H. arXiv:1401.2035, PRD2014

Danning Li, Song He, M.H. arXiv:1411.5332, JHEP2015

# Phase transition and EOS

5D graviton action:

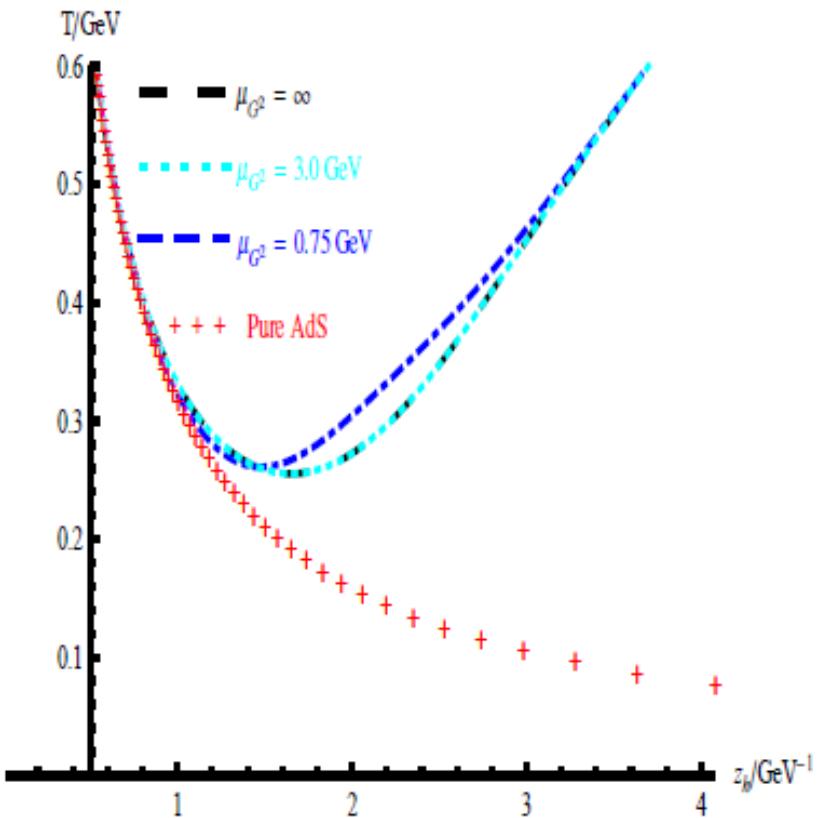
$$S_{5D} = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g^E} \left( R - \frac{4}{3} \partial_\mu \phi \partial^\mu \phi - V_E(\phi) \right)$$

$$ds_S^2 = \frac{L^2 e^{2A_s}}{z^2} \left( -f(z) dt^2 + \frac{dz^2}{f(z)} + dx^i dx^i \right),$$

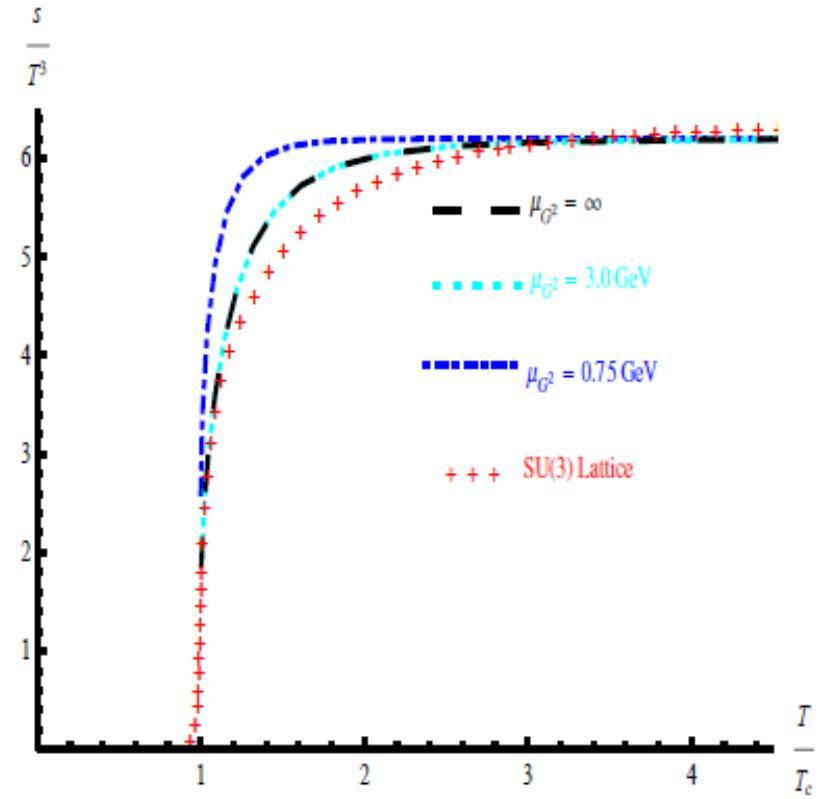
**Metric structure, blackhole, Dilaton field and  
Dilaton potential should be solved self-  
consistently from the Einstein equations.**

$$T = \frac{|f'(z_h)|}{4\pi}.$$

$$s = \frac{A_{area}}{4G_5 V_3} = \frac{L^3}{4G_5} \left( \frac{e^{A_s - \frac{2}{3}\phi}}{z} \right)^3.$$

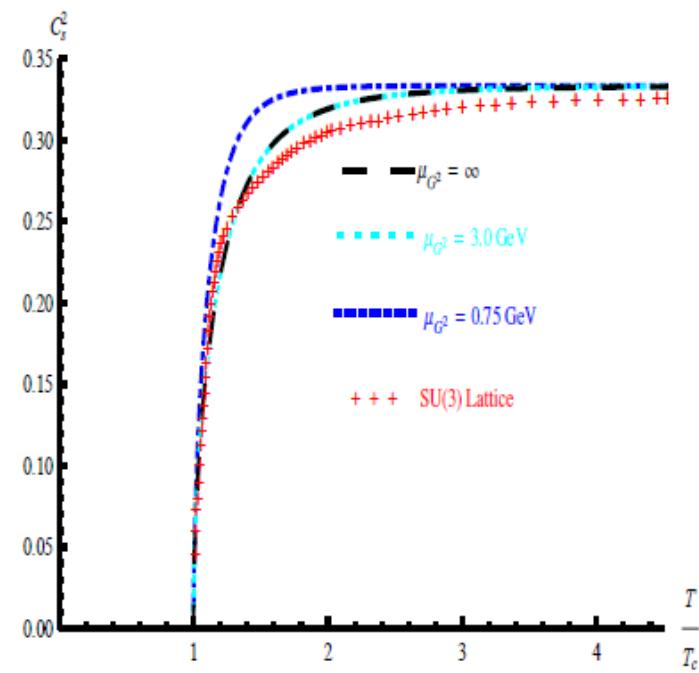
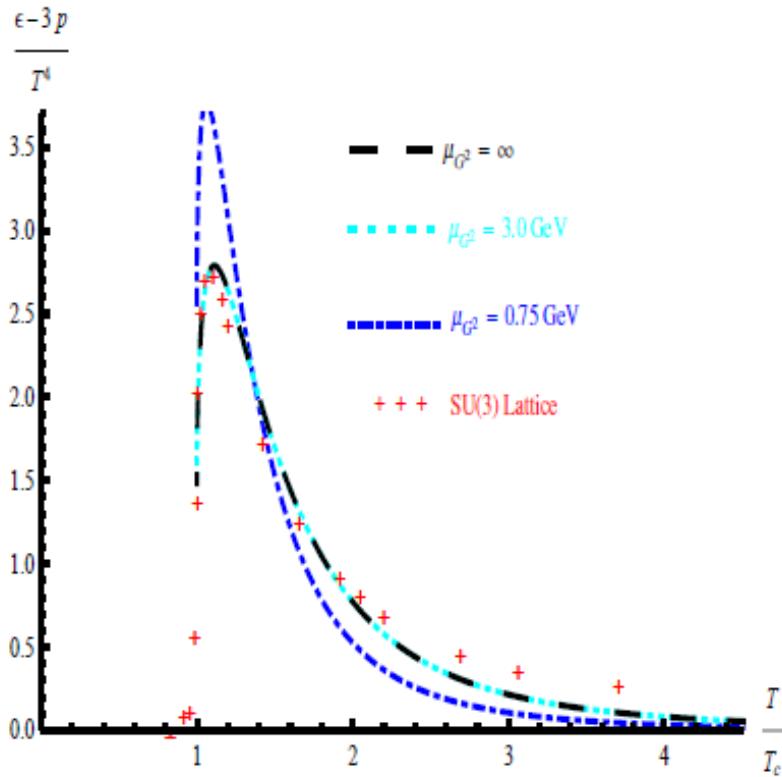


Hawking-Page  
phase transition



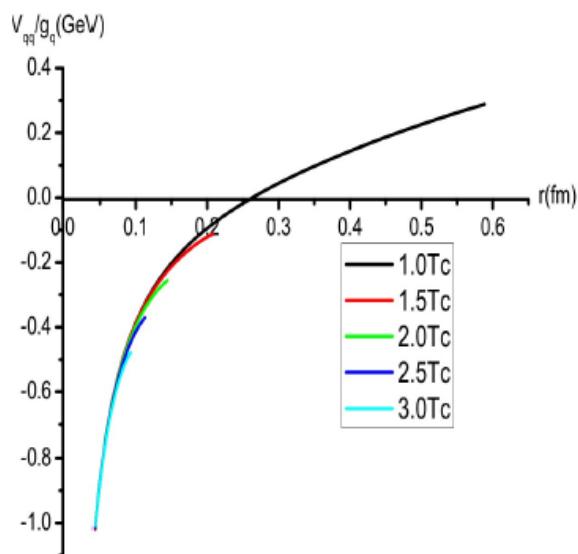
# Trace anomaly

$$c_s^2 = \frac{d \log T}{d \log s} = \frac{s}{T ds/dT},$$

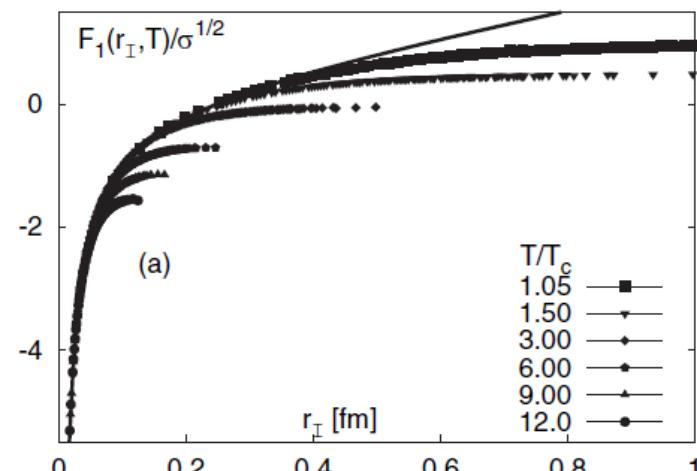


Danning Li, Jinfeng Liao, M.H. arXiv:1401.2035, PRD2014

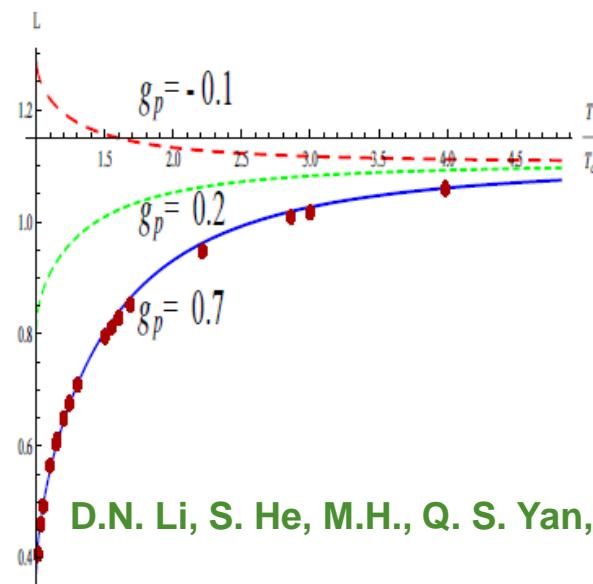
# Electric screening



# Heavy quark potential

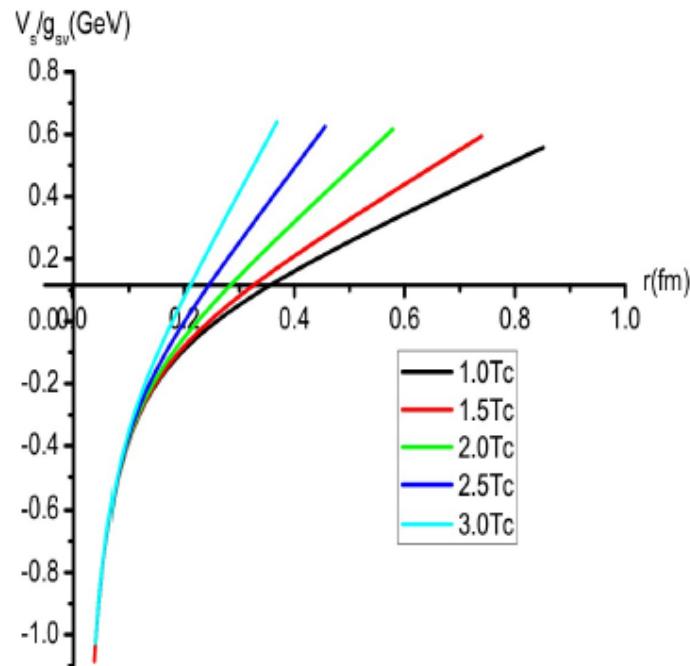


Polyakov loop:  
color electric  
deconfinement

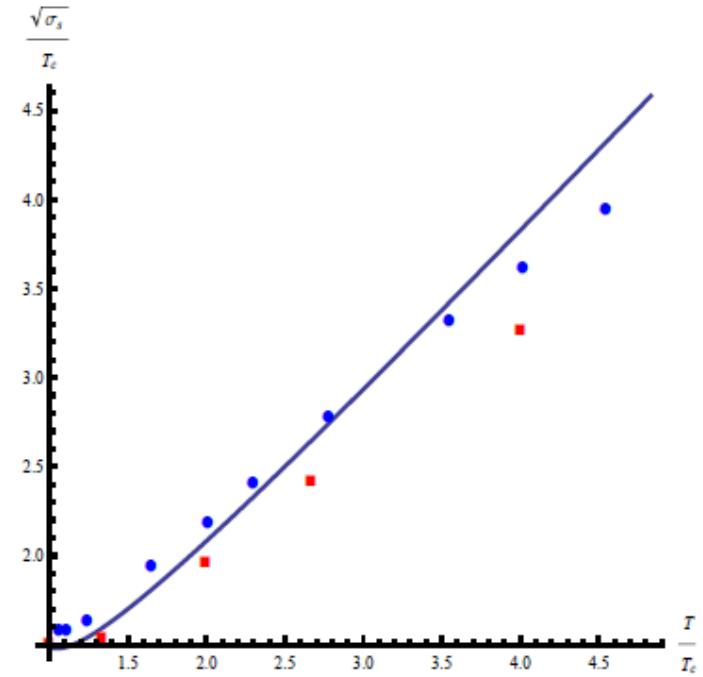


D.N. Li, S. He, M.H., Q. S. Yan, arXiv:1103.5389, JHEP2011

# Magnetic screening and magnetic confinement



spatial Wilson loop



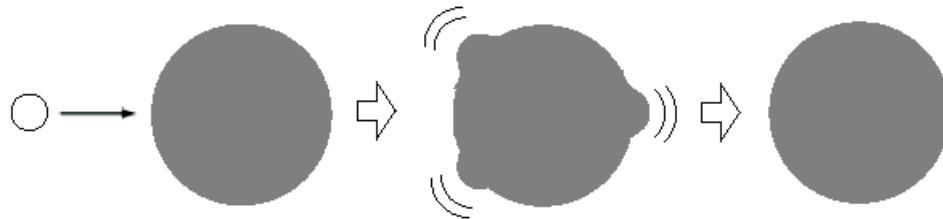
spatial string tension

D.N. Li, S. He, M.H., Q. S. Yan, arXiv:1103.5389, JHEP2011

# **Temperature dependent transport properties reflect phase transitions?**

**shear/bulk viscosity,  
Jet quenching parameter  
Electric conductivity**

# Shear viscosity from AdS/CFT



shear viscosity  $\Leftrightarrow$  absorption cross section of graviton

$$\eta = \pi N^2 T^3 / 8$$

entropy  $\Leftrightarrow$  horizon area

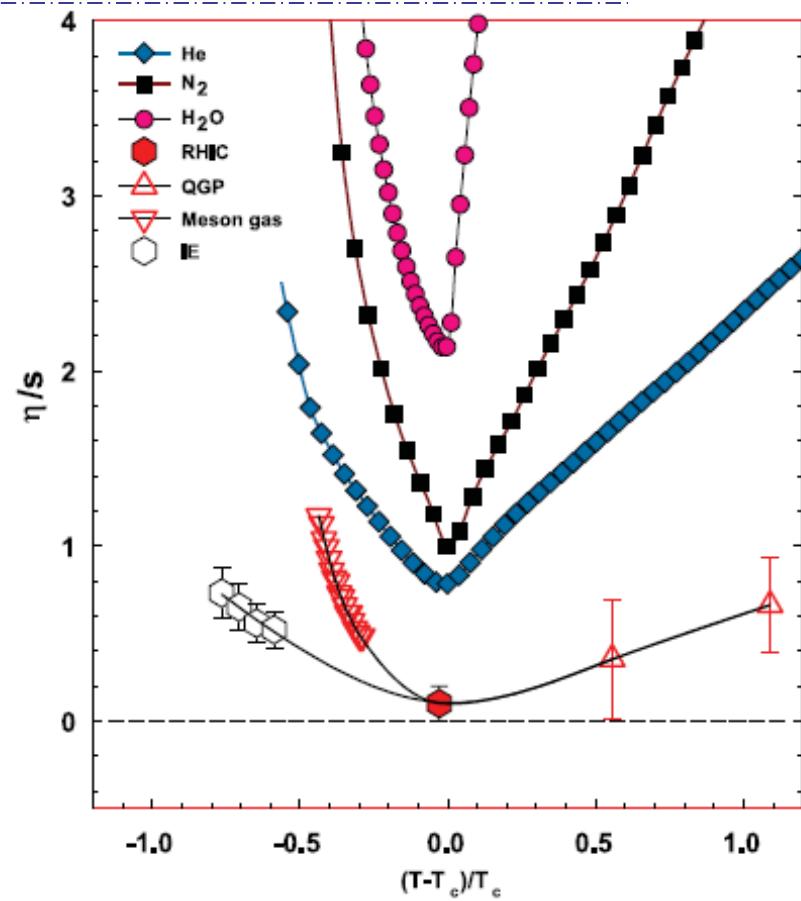
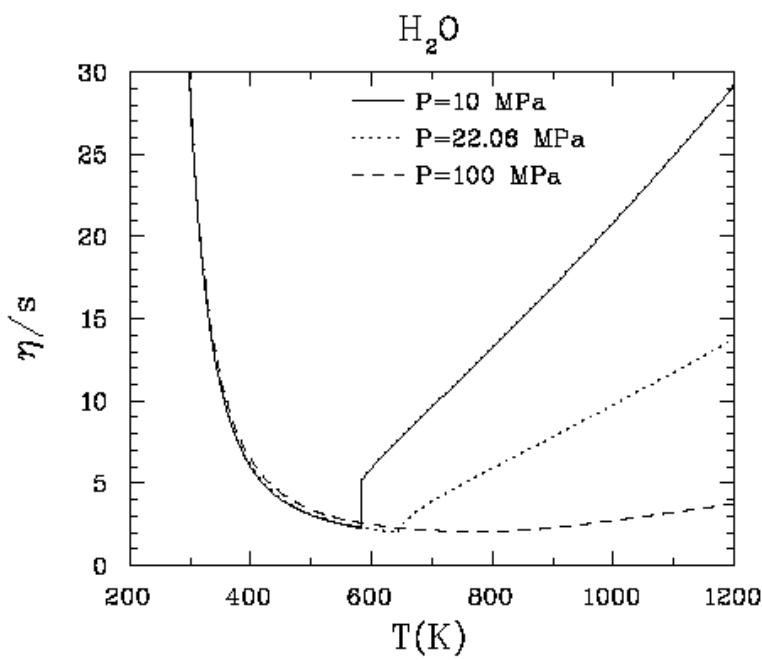
$$s = \pi^2 \dot{N}^2 T^3 / 2$$

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Kovtun - Son - Starinets (2004)

Minimum bound

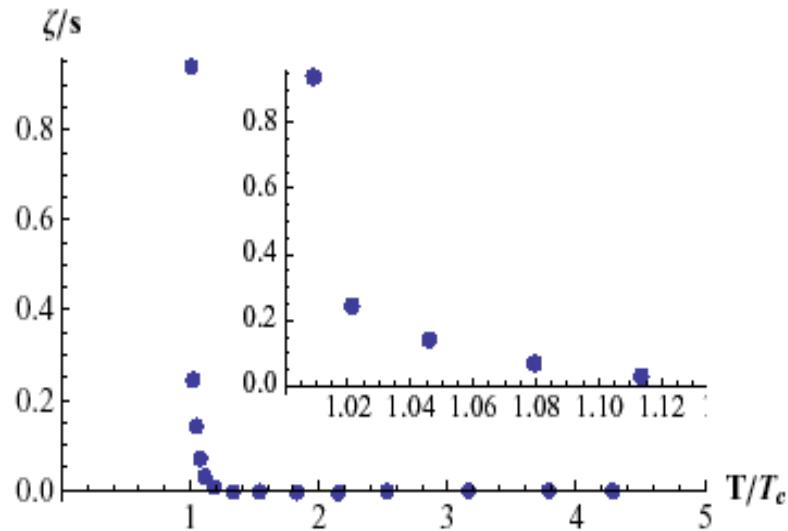
# Shear viscosity over entropy density: minimum near phase transition



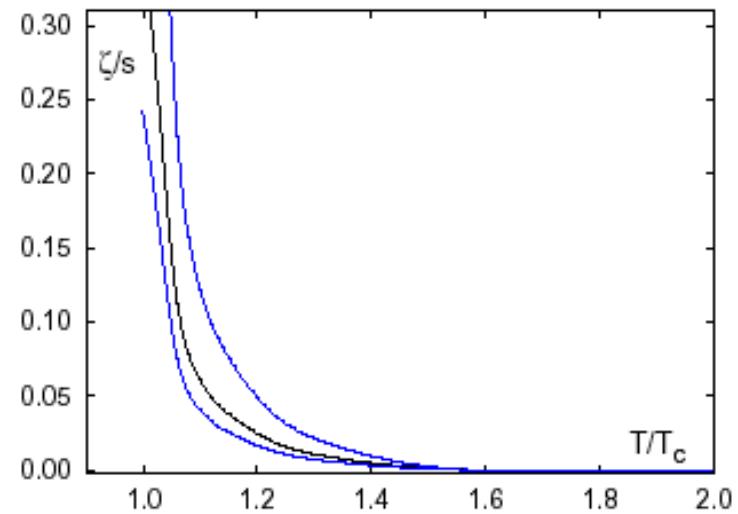
Csernai et.al. Phys.Rev.Lett.97:152303,2006

Lacey et al., PRL 98:092301,2007

# Bulk viscosity over entropy density: LQCD sharply rising near phase transition



Pure gluodynamics



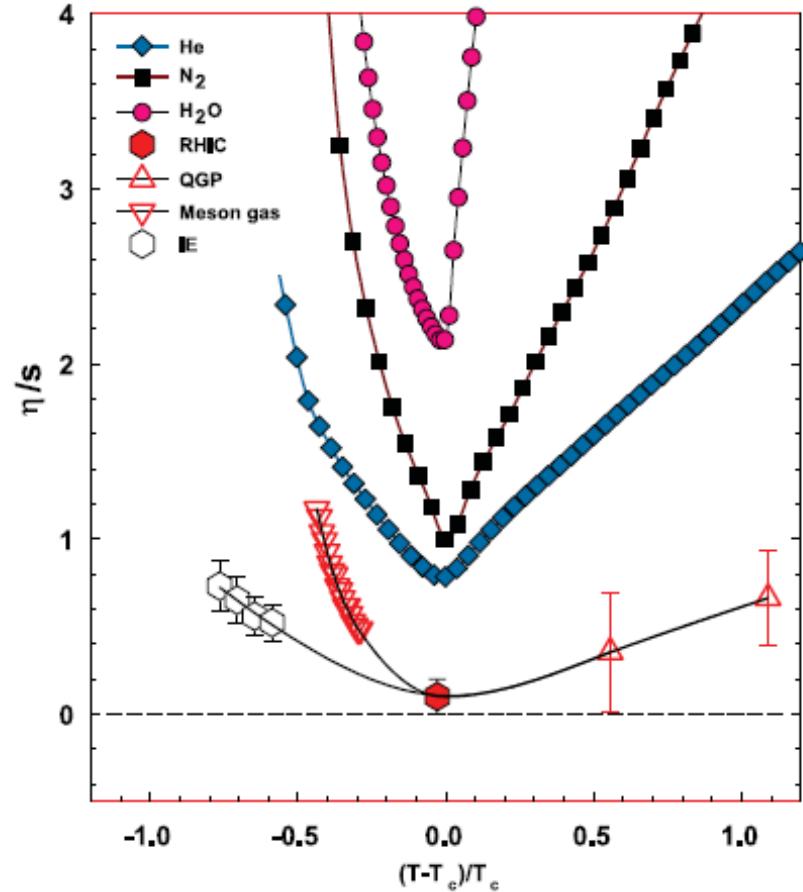
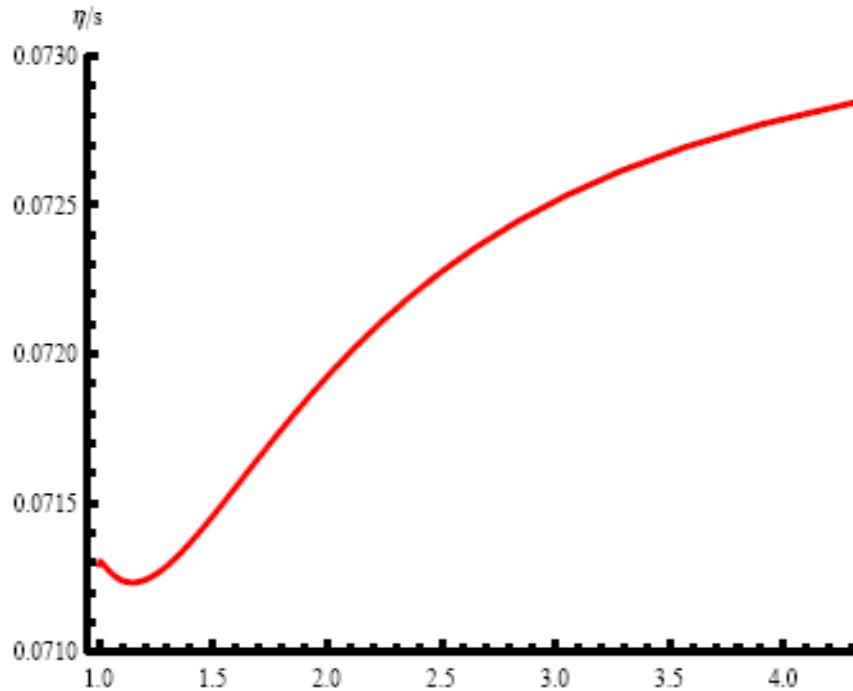
2-flavor case

$$\zeta = \frac{1}{9\omega_0} \left\{ T^5 \frac{\partial}{\partial T} \frac{(\epsilon_T - 3p_T)}{T^4} + 16|\epsilon_v| \right\}$$

Dmitri Kharzeev, Kirill Tuchin arXiv:0705.4280 [hep-ph],  
F.Karsch, Dmitri Kharzeev, Kirill Tuchin arXiv:0711.0914 [hep-ph],  
Harvey Meyer arXiv:0710.3717 [hep-ph],

# Shear viscosity from dynamical hQCD

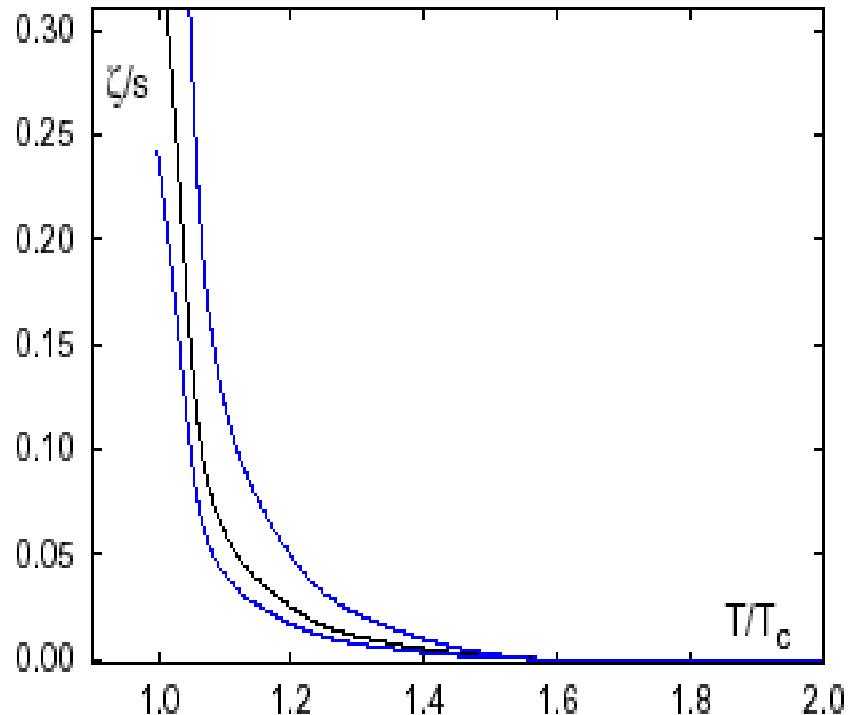
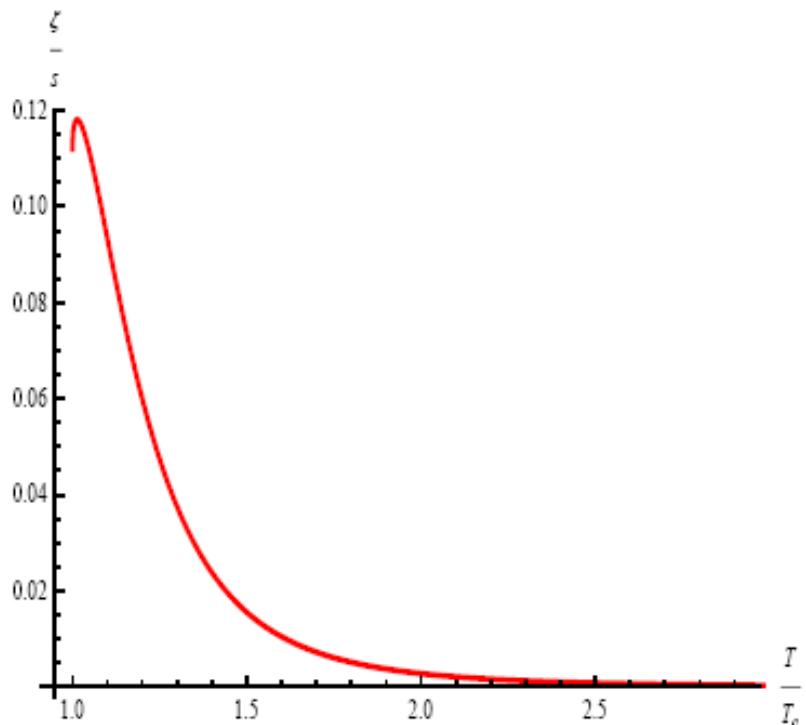
$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[ R - \frac{4}{3} (\nabla\phi)^2 + V(\phi) + \ell^2 \beta e^{\sqrt{\frac{2}{3}}\gamma\phi} R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho} \right]$$



Danning Li, Song He, M.H. JHEP2015

Lacey et al., PRL 98:092301,2007

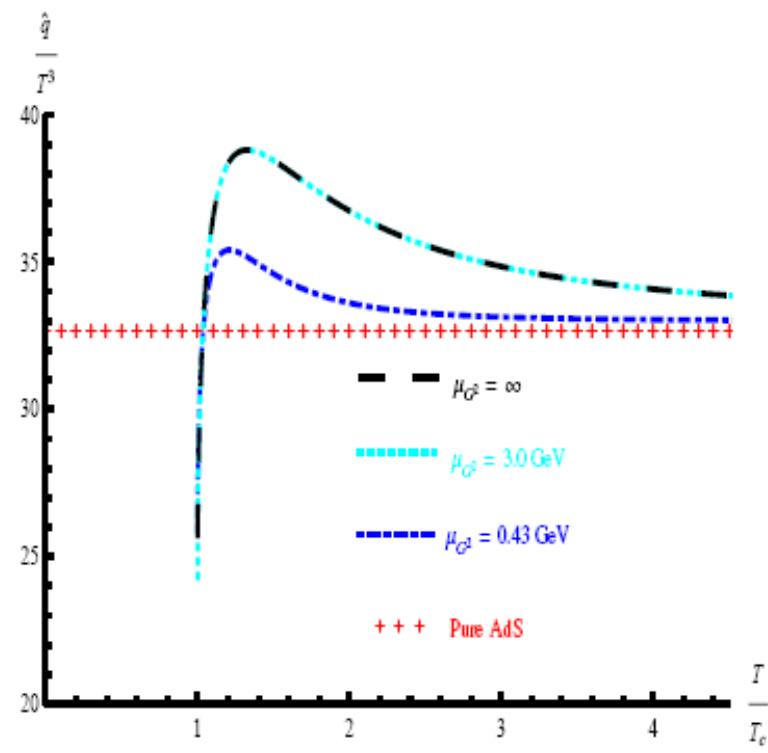
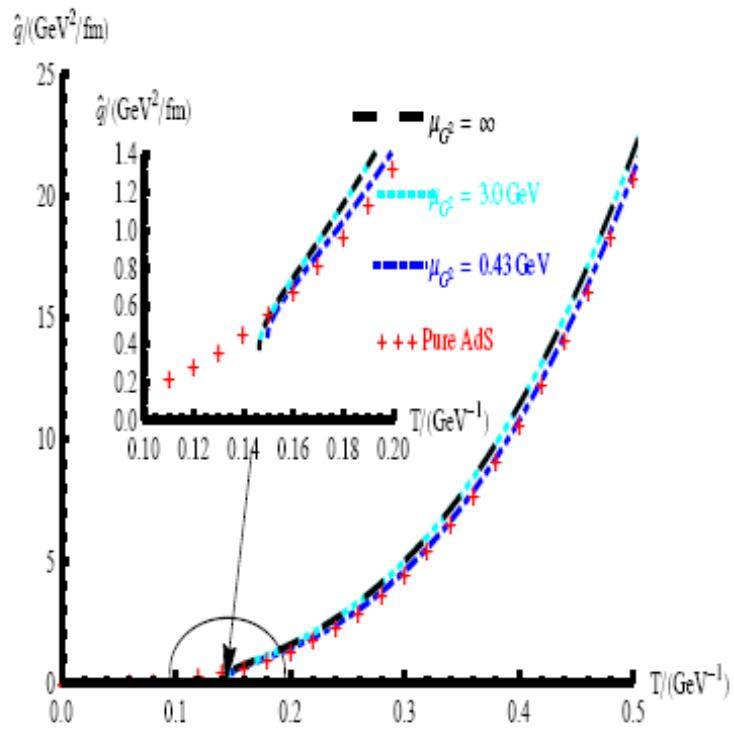
# Bulk viscosity from dynamical hQCD



Danning Li, Song He, M.H. JHEP2015 Dmitri Kharzeev, Kirill Tuchin arXiv:0705.4280,

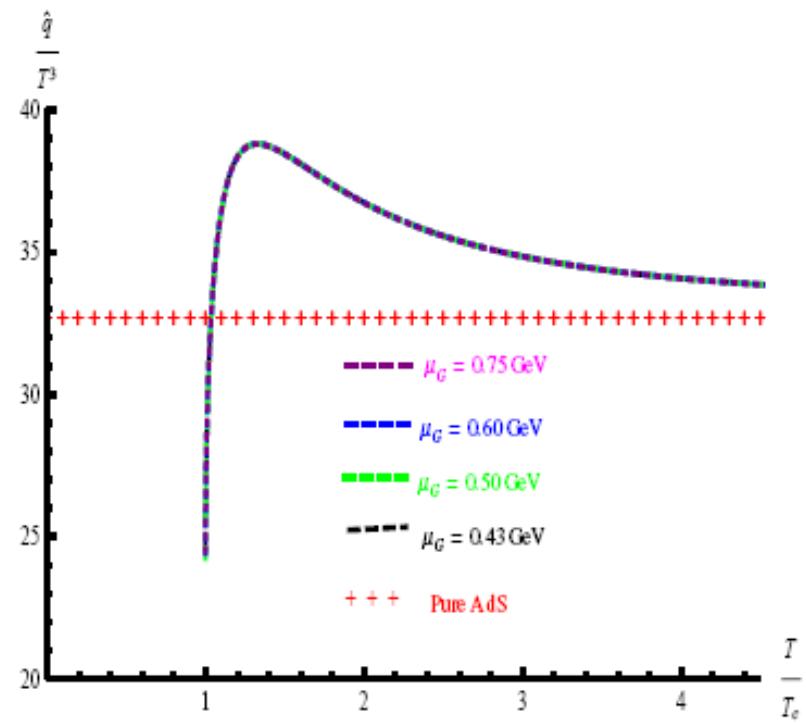
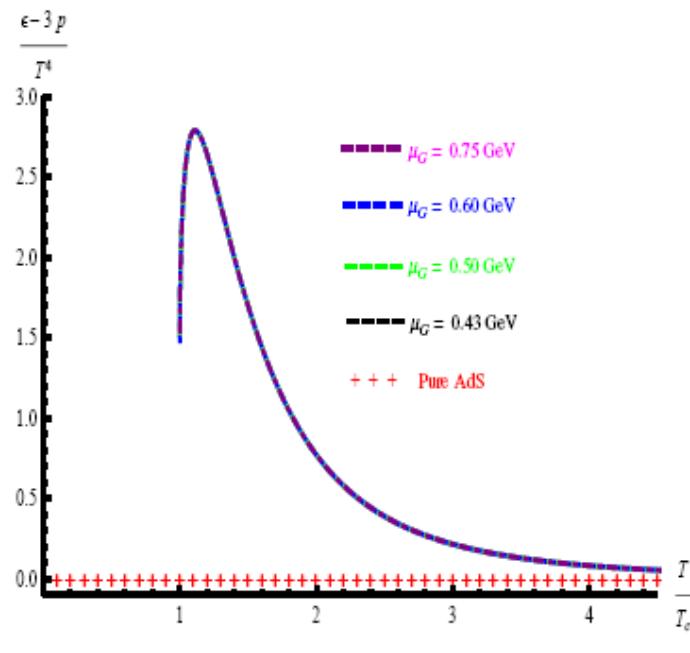
# Jet quenching from dynamical hQCD

Danning Li, Jinfeng Liao, M.H. PRD2014

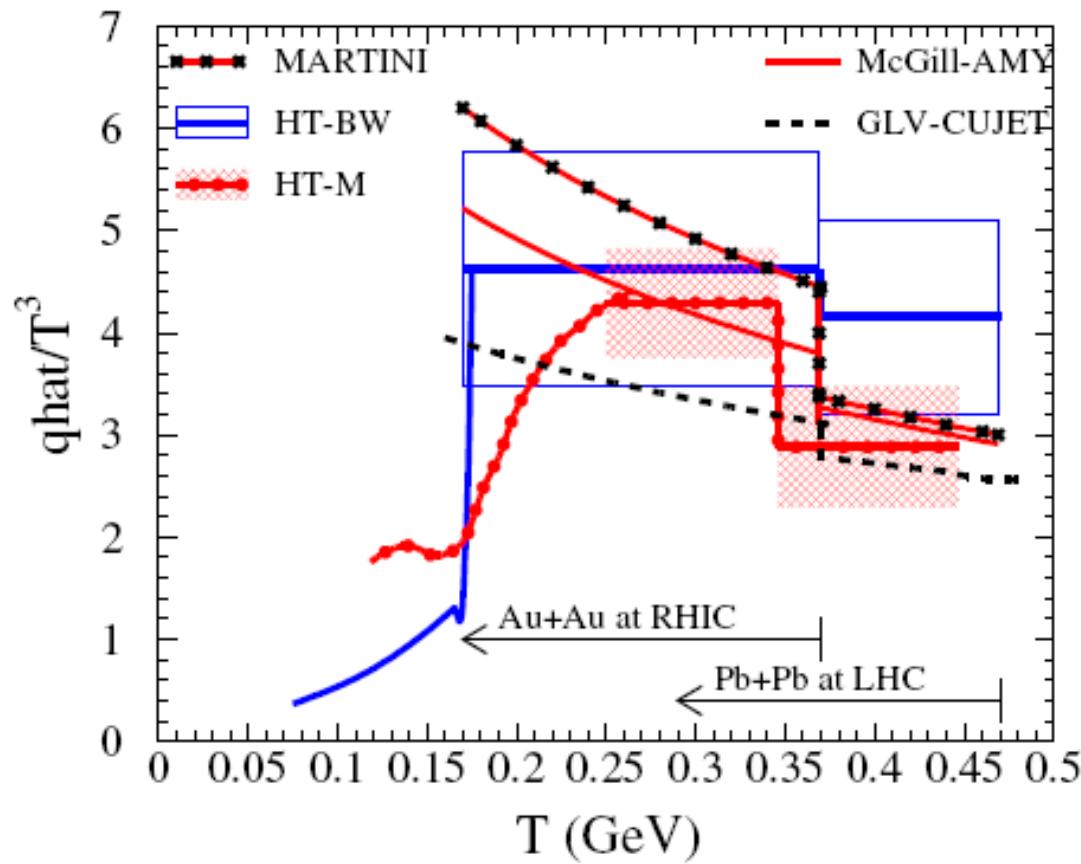


# Jet quenching characterizing phase transition!

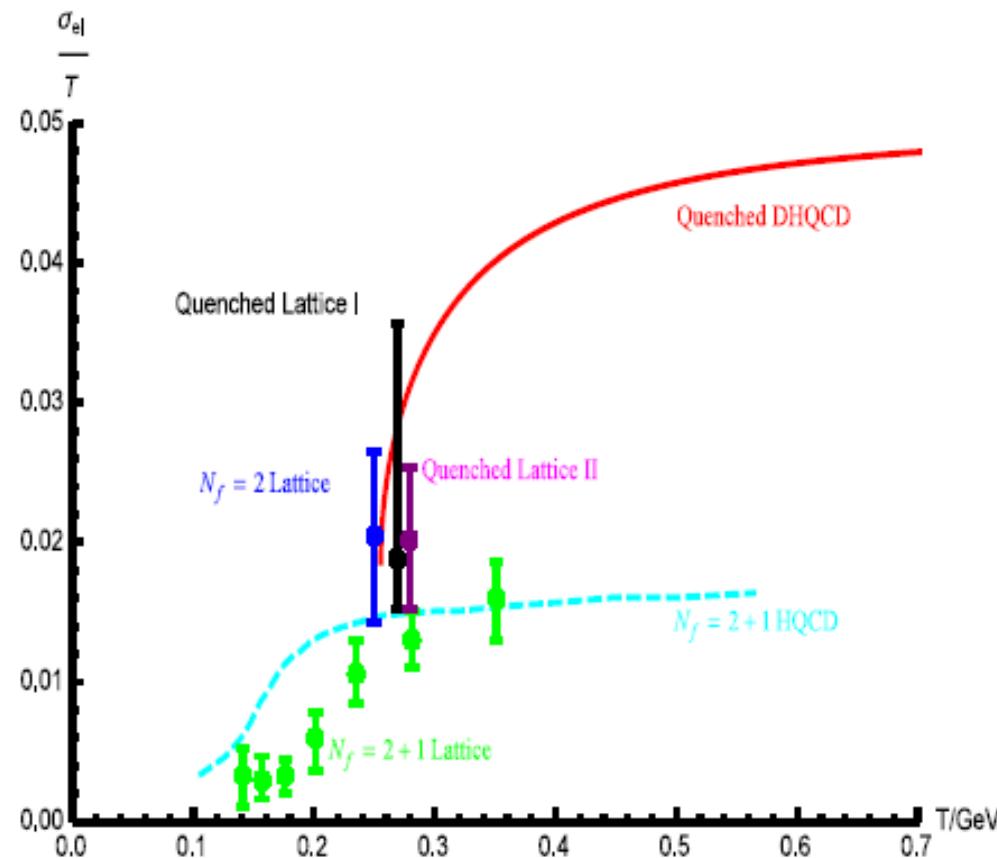
Danning Li, Jinfeng Liao, M.H. PRD2014



## Temperature dependence of jet quenching parameter [Jet Collaboration] arXiv:1312.5003



## Electric conductivity



**Temperature dependent transport properties  
reflect phase transitions!**

**shear/bulk viscosity,  
Jet quenching parameter  
Electric conductivity**

## IV. Realization of chiral symmetry

breaking & restoration

First realization of Landau phase transition  
in holographic QCD model

K. Chelabi, Z.Fang, M.Huang, D.N.Li, Y.L.Wu,  
arXiv:1511.02721, PRD2016  
arXiv:1512.06493, JHEP2016

Only focus on the scalar sector:

$$SU(N_f)_L \times SU(N_f)_R$$

$$S = - \int d^5x \sqrt{-g} e^{-\Phi} Tr(D_m X^+ D^m X + V_X(|X|)).$$

$$ds^2 = e^{2A_s(z)}(-f(z)dt^2 + \frac{1}{f(z)}dz^2 + dx_i dx^i),$$

$$A_s(z) = -\log(z),$$

$$f(z) = 1 - \frac{z^4}{z_h^4}.$$

$$S_\chi = - \int d^5x \sqrt{-g} e^{-\Phi} \left( \frac{1}{2} g^{zz} \chi'^2 + V(\chi) \right),$$

$$X_0 = \frac{\chi(z)}{\sqrt{2N_f}} I_{N_f}$$

Profile of the scalar potential

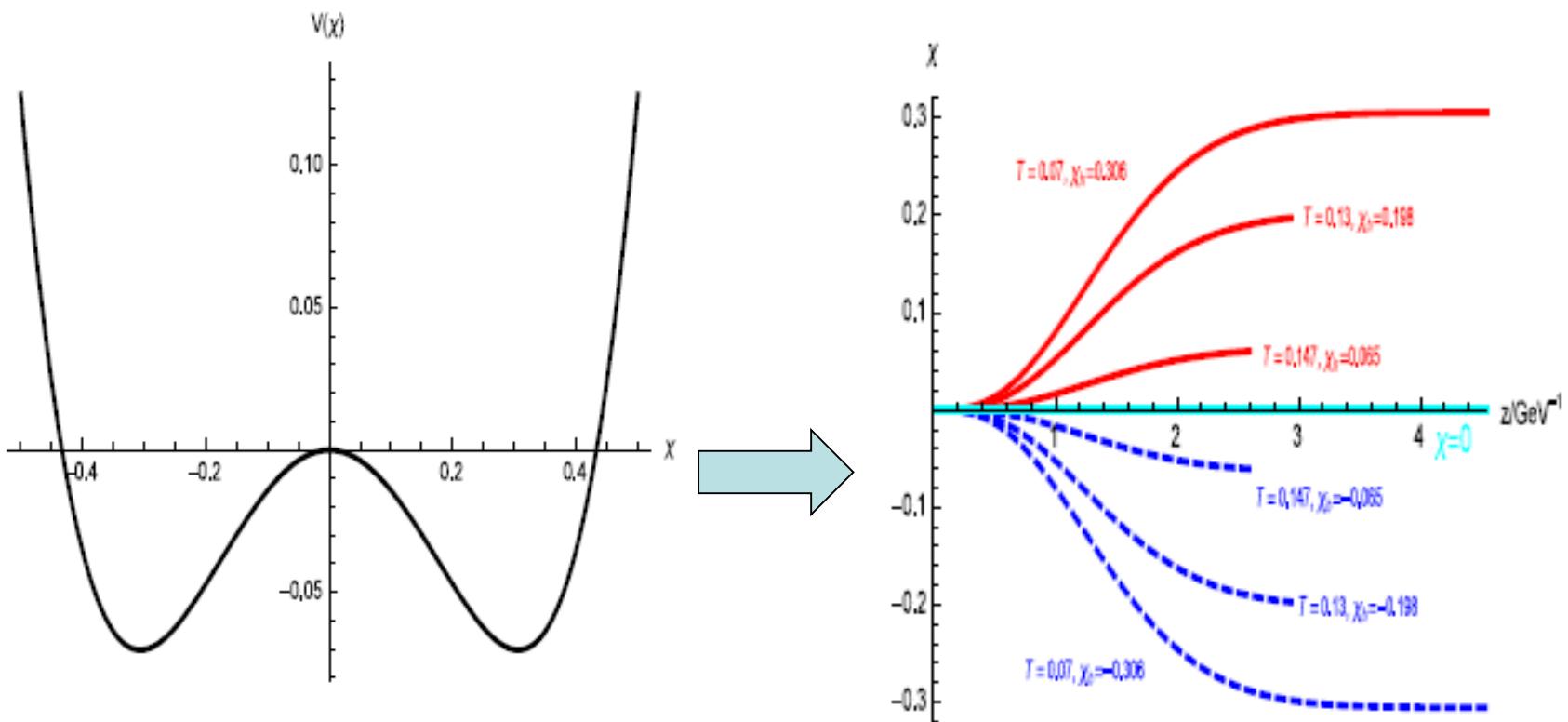
$$V(\chi) \equiv Tr(V_X(|X|)) = -\frac{3}{2}\chi^2 + v_3\chi^3 + v_4\chi^4.$$

  
 Only for three-flavor scalar

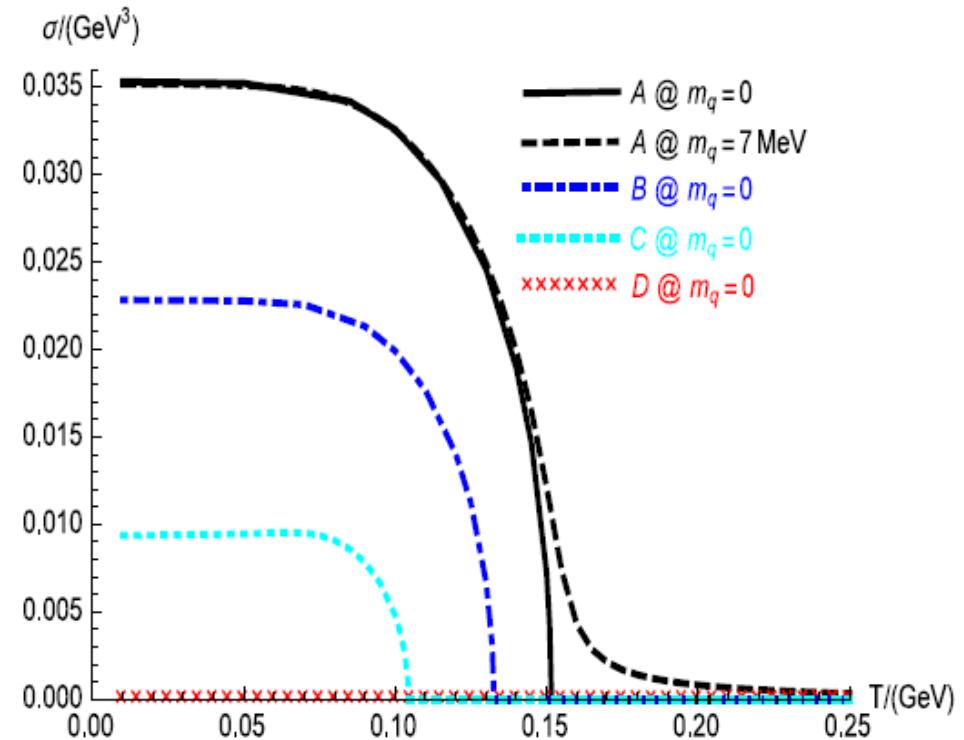
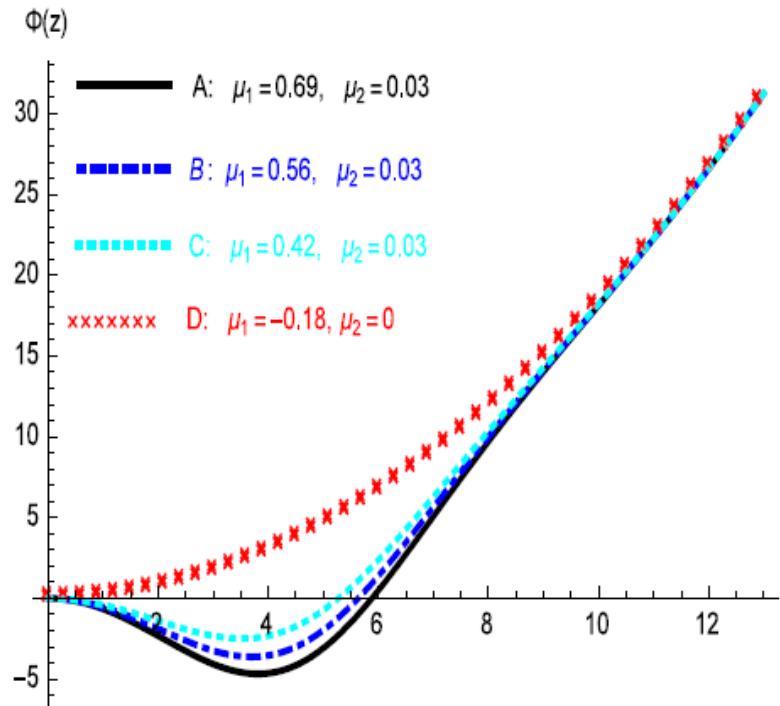
Profile of the dilaton field

$$\Phi(z) = -\mu_1 z^2 + (\mu_1 + \mu_0) z^2 \tanh(\mu_2 z^2),$$

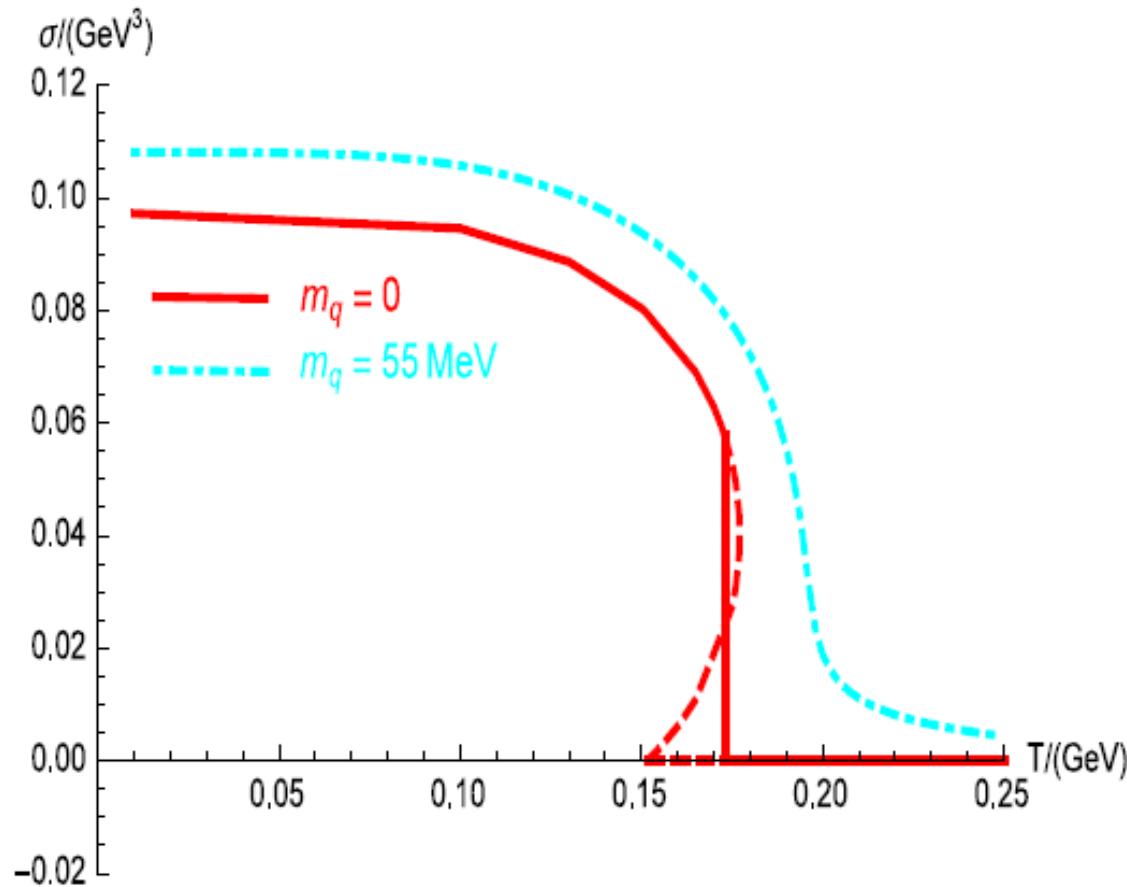
# Profile of the scalar potential determines the possible solution of the chiral condensate



Profile of the dilaton field represents the gluodynamics, and it determines the real solution of the chiral condensate

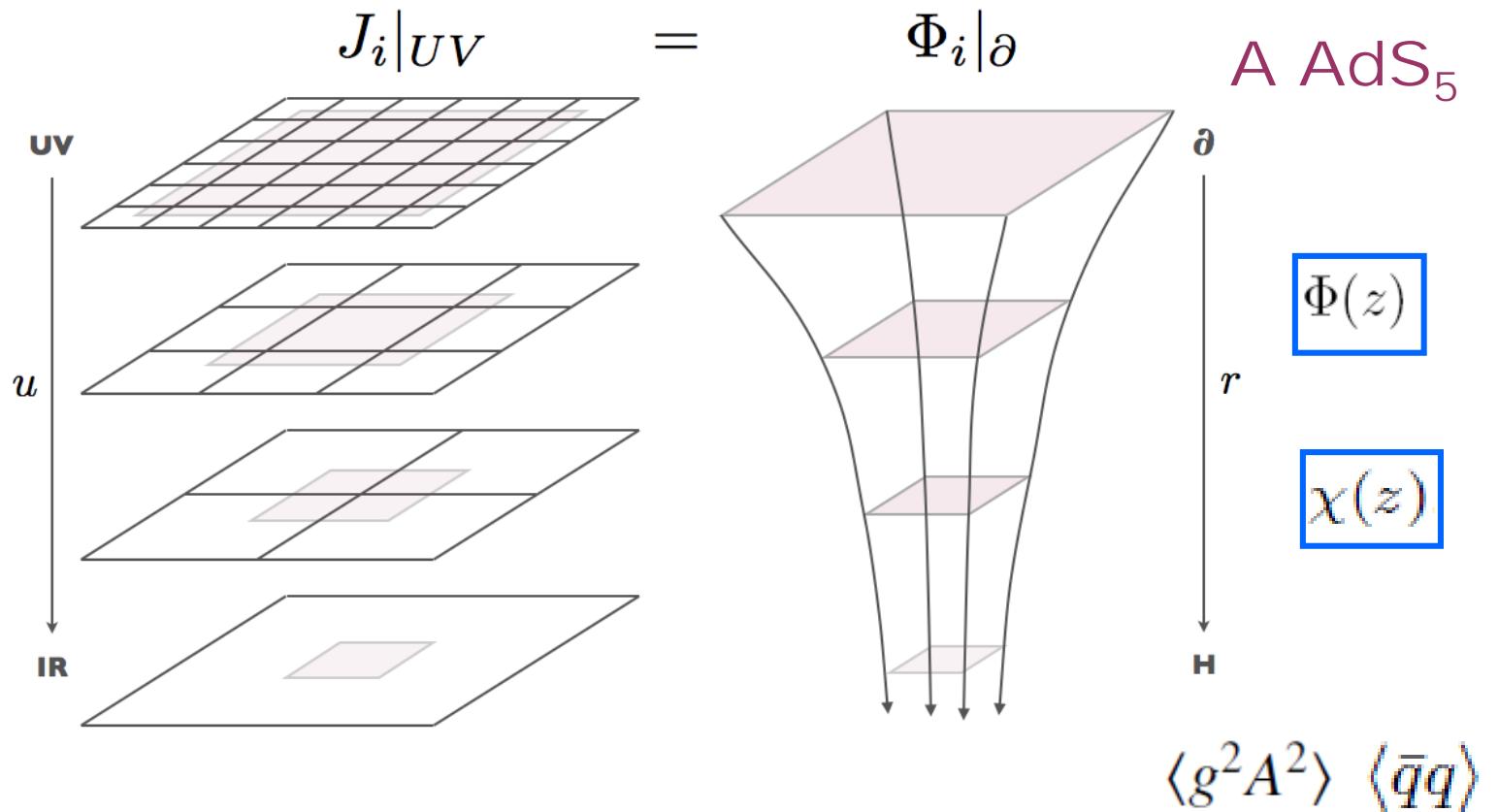


Two-flavor case:  
chiral limit, 2<sup>nd</sup> order phase transition  
nonzero current quark mass, cross-over



Three-flavor case:  
chiral limit, 1<sup>st</sup> order phase transition  
finite current quark mass, cross-over

# V. Conclusion and discussion



Correct gluon dynamics and chiral dynamics running from UV to IR gives correct physics! 50

In the DhQCD model, we have achieved:

1, QCD vacuum properties

glueball spectra, light-flavor meson spectra,

chiral symmetry breaking and linear confinement

2, QCD phase transitions

deconfinement phase transition(HP) & chiral phase transition(Landau)

3, Equation of state for QCD matter agree with lattice result.

4, Temperature dependent transport properties reflect phase  
transitions of QCD.

5D effective QCD model is more powerful than

4D effective QCD model!

# **Thanks for your attention!**