

Thermodynamics of Resonant Scalars in AdS/CFT and implications for QCD

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EM, E.Ruiz Arriola, L.L.Salcedo, JHEP0601(2006), PRD80(2009),
PRD81(2010); EM, H.J.Pirner, K. Veschiogini, PRD83(2011),
PLB696(2011); EM, M.Valle, in preparation (2016).



Issues

1 Motivation

2 Thermodynamics of AdS/QCD

- Black Hole Thermodynamics
- The 5D Einstein-dilaton model at finite T
- Holographic Renormalization
- Conformal Anomaly

3 Analytical solution for the EoS

- High Temperature
- Low Temperature

4 Conclusions

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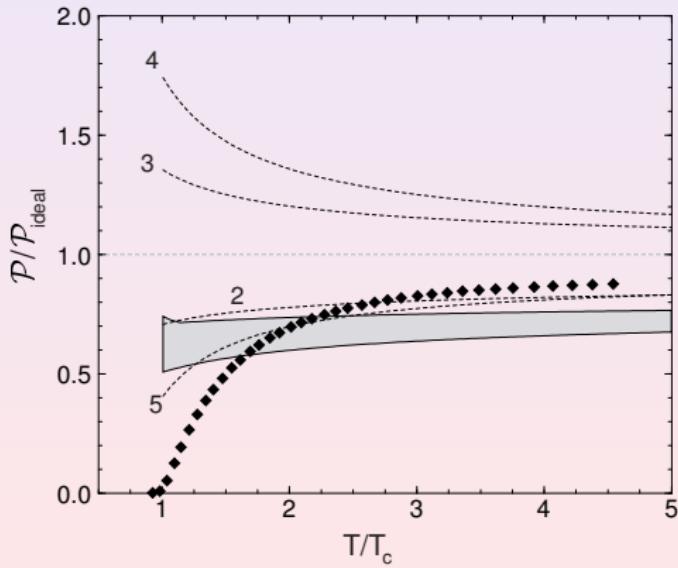
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Motivation

Pressure of Gluodynamics

Weak Coupling Expansion and Resummed Perturbation Theory
E. Braaten and A. Nieto (1996), J.O. Andersen et al (1999).

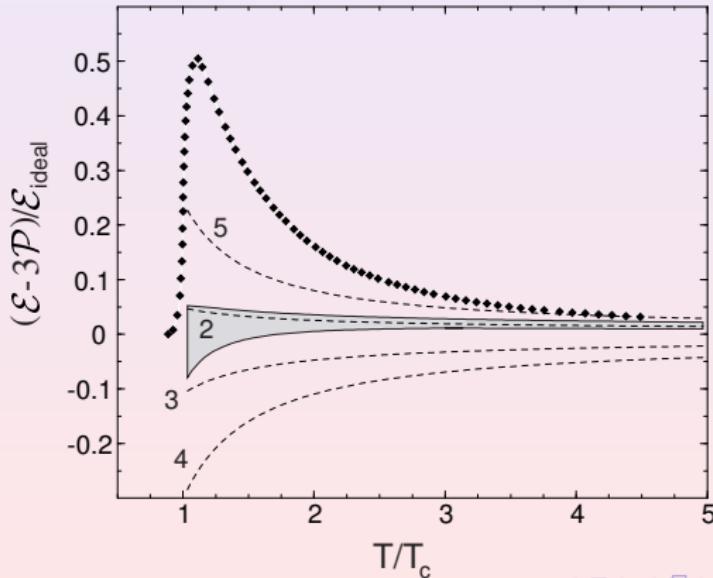


Motivation

Interaction Measure in Gluodynamics

Weak Coupling Expansion and Resummed Perturbation Theory

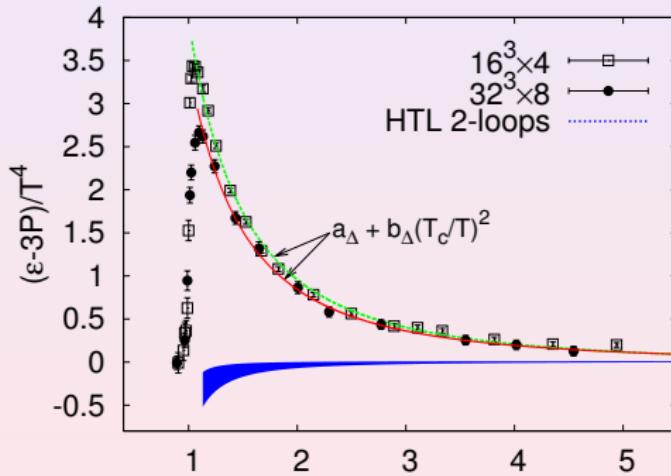
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Motivation

[EM, E.Ruiz Arriola, L.L.Salcedo, JHEP0601 '06, PRD80 '09, PRD81 '10]

Trace Anomaly $N_c = 3, N_f = 0$
G. Boyd et al., Nucl. Phys. B469, 419 (1996).

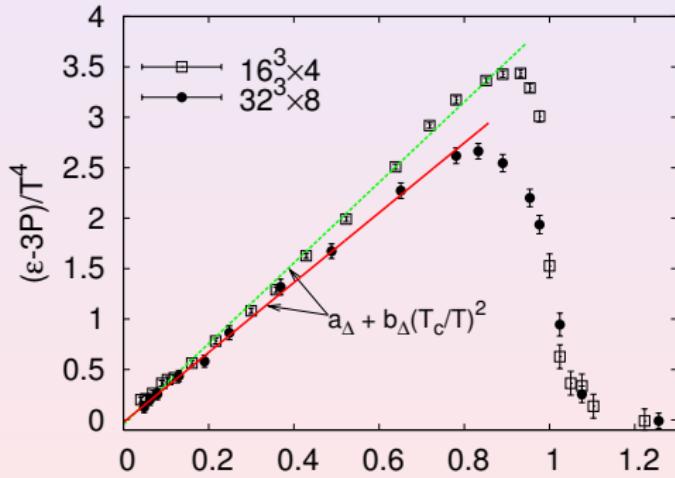


$$\frac{\epsilon - 3P}{T^4} = a_\Delta + \frac{b_\Delta}{T^2}, \quad b_\Delta = (3.46 \pm 0.13) T_c^2, \quad 1.13 T_c < T < 4.5 T_c.$$

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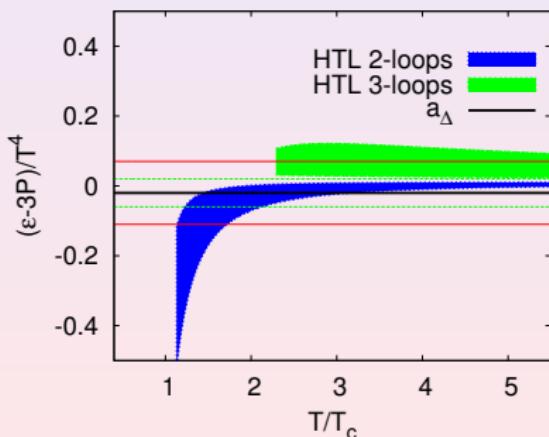
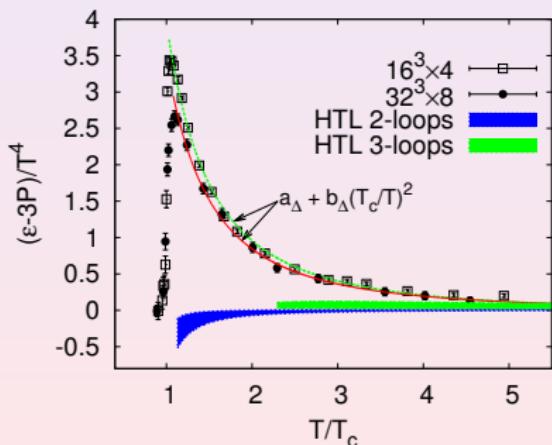


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Motivation

[EM, E.Ruiz Arriola, L.L.Salcedo, JHEP0601 '06, PRD80 '09, PRD81 '10]

$$\Delta \equiv \frac{\epsilon - 3P}{T^4} = \underbrace{\Delta_{HTL}(\mu_T)}_{\sim 1/\log T} + \frac{b_\Delta}{T^2}$$



Perturbation Theory and Hard Thermal Loops **only yield a_Δ !!.**

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Schwarzschild black hole

- General Relativity with no source \Rightarrow Einstein-Hilbert action

$$S_{EH} = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g} R, \quad R = g^{\mu\nu} R_{\mu\nu}$$

- Classical solution $\frac{\delta}{\delta g_{\mu\nu}}$ \Rightarrow Einstein equations

$$E_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \underset{\text{spherical}}{\implies} R_{\mu\nu} = 0$$

- Schwarzschild solution in spherical coordinates (1915):

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -f(r) dt^2 + \frac{1}{f(r)} dr^2 + r^2 d\Omega_2^2, \quad f(r) = 1 - \frac{r_h}{r}$$

- r_h is the horizon. Not physical singularity: $R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} = 12 \frac{r_h^2}{r^6}$.

Large distance limit: $g_{tt}(r) \underset{r \rightarrow \infty}{\sim} -(1 + 2V_{Newton}(r)) \Rightarrow r_h = 2G_4 M$.

Black Hole Thermodynamics

$$Z = \text{Tr} \left(e^{-\beta H} \right), \quad \beta = \frac{1}{T}$$

Periodicity in euclidean time ($\tau = it$): $\Phi(\tau + \beta) = \Phi(\tau)$

- **Regularity:** Expansion around the horizon $f(r_h) = 0$; $r = r_h(1+\rho^2)$:

$$ds_{\text{BH}}^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_2^2 \underset{\rho \rightarrow 0}{\sim} 4r_h^2 \left(d\rho^2 + \underbrace{\rho^2 \left(\frac{d\tau}{2r_h} \right)^2}_{d\theta^2} + \frac{1}{4}d\Omega_2^2 \right)$$

\implies Periodicity: $\frac{\tau}{2r_h} \rightarrow \frac{\tau}{2r_h} + 2\pi \implies \tau \rightarrow \tau + 4\pi r_h =: \tau + \beta$

$$T = \frac{1}{8\pi M G_4}$$

- **Thermodynamical interpretation of black holes:**

$$dM = TdS \implies S = \int \frac{dM}{T} = 4\pi G_4 M^2$$

$$\mathcal{A} = 4\pi r_h^2 = 16\pi(G_4 M)^2 \implies S_{\text{Black Hole}}(T) = \frac{\mathcal{A}(r_{\text{horizon}})}{4G_4} \quad \text{Bek-Hawking}$$

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The 5D Einstein-dilaton model at finite temperature

- AdS/CFT correspondence: Duality Gravity theory \leftrightarrow Field theory.
- Applications to strongly coupled plasmas:

Equation of state:

$$S_{\text{Black Hole}}(T) = \frac{\mathcal{A}(r_{\text{horizon}})}{4G_D} \longleftrightarrow S_{\text{QCD}}(T)$$

[Andreev et al. '06 '07 '09], [Thorlacius et al. '07], [Kiritsis et al. '08 '09], [Kajantie et al. '09],
 [EM, Pirner, Veschiini, PRD'11, PLB'11], [D.Li, M.Huang '13], [F.Zuo, JHEP1406 '14].

$$S_{5D} = \frac{1}{16\pi G_5} \int d^5x \sqrt{G} \left(R - \frac{4}{3} \partial_\mu \Phi \partial^\mu \Phi - V(\Phi) \right) - \frac{1}{8\pi G_5} \int_{\partial M} d^4x \sqrt{\gamma} K.$$

Finite temperature solutions [E. Kiritsis et al. JHEP (2009) 033]:

- Black hole solution (deconfined phase):

$$ds_{\text{BH}}^2 = b^2(z) \left[-f(z) dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)} \right]$$

The 5D Einstein-dilaton model at finite temperature

- Potential: $V(\Phi) = -\frac{6}{L^2} + \frac{1}{2}m^2\Phi^2$.
- Euclidean action:

$$S = \frac{1}{2\kappa^2} \int d\rho d^4x \sqrt{G} (-R + G^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + m^2\Phi^2) - \int_{\rho=\epsilon} d^4x \sqrt{\gamma} 2K + S_{\text{boundary}}$$

- Metric in Fefferman-Graham coordinates:

$$ds^2 = G_{\mu\nu}dx^\mu dx^\nu = \frac{L^2}{4\rho^2}d\rho^2 + \frac{L^2}{\rho}g_{\tau\tau}(\rho)d\tau^2 + \frac{L^2}{\rho}g_{xx}(\rho)d\vec{x}^2.$$

- Tachyonic mass: $m^2 L^2 = \Delta(\Delta - 4) < 0$,

with $\Delta \equiv \dim \mathcal{O}$, and \mathcal{O} is the operator dual of the scalar field Φ .

- From the *AdS/CFT dictionary*, this corresponds to a deformation:

$$\mathcal{L} = \mathcal{L}^{CFT} + \underbrace{\lambda \mathcal{O}}_{\delta \mathcal{L}: \lambda \equiv \text{source}, \mathcal{O} \equiv \text{operator}}, \quad \dim \mathcal{O} = \Delta \quad \text{and} \quad \dim \lambda = 4 - \Delta.$$

- We are interested in cases $\Delta = 1, 2, 3 \rightarrow$ Resonant Scalars.

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Holographic Renormalization

- Near boundary expansion:

- $\Delta \notin \mathbb{Z}$:

$$\Phi(\rho) = \underbrace{\phi_0}_{\text{source in } \Delta > 2} \cdot \rho^{(4-\Delta)/2} + \underbrace{\phi_{(2\Delta-4)}}_{\text{condensate in } \Delta > 2} \cdot \rho^{\Delta/2} + \dots \quad \rho \rightarrow 0.$$

- $\Delta = 1, 3$:

$$\Phi(\rho) = \underbrace{\chi}_{\text{source in } \Delta=3} \cdot \rho^{1/2} + \frac{\chi^3}{6} \rho^{3/2} \log \rho + \underbrace{\psi}_{\text{condensate in } \Delta=3} \cdot \rho^{3/2} + \dots,$$

$$g_{xx}(\rho) = 1 - \frac{\chi^2}{6} \rho + g_{(4)xx} \rho^2 - \frac{\chi^4}{24} \rho^2 \log \rho + \dots,$$

$$g_{\tau\tau}(\rho) = 1 - \frac{\chi^2}{6} \rho - 3g_{(4)xx} \rho^2 - \chi \psi \rho^2 + \frac{\chi^4}{9} \rho^2 - \frac{\chi^4}{24} \rho^2 \log \rho + \dots.$$

- Regularity at the horizon $\rho = \rho_h \rightarrow \psi$ and $g_{(4)xx} \equiv \text{fixed}$.
- We can choose $S_{\text{boundary}} = 0$. Counterterms:

$$S_{\text{ct}} = \frac{1}{\kappa^2 L} \int_{\rho=\epsilon} d^4x \sqrt{\gamma(\epsilon)} \left(3 + \frac{\Phi^2(\epsilon)}{2} + \frac{\Phi^4(\epsilon)}{12} \log(\epsilon \mu_0^2) \right).$$

- $\mu_0 \equiv \text{renormalization scale}$.

Condensate

- *Regularized action:*

$$\delta S_{\text{reg}} = -\frac{1}{\kappa^2 L} \int_{\rho=\epsilon} d^4x \sqrt{\gamma} 2\epsilon \Phi'(\epsilon) \delta\Phi(\epsilon) + \dots$$

- *Condensate for $\Delta = 3$:*

$$\begin{aligned} \langle \mathcal{O}_3 \rangle &= \frac{\delta(S_{\text{reg}} + S_{\text{ct}})}{\delta\chi} = \lim_{\epsilon \rightarrow 0} \left(\frac{1}{\epsilon^{-1/2}} \frac{\delta(S_{\text{reg}} + S_{\text{ct}})}{\delta\Phi(\epsilon)} \right) \\ &= \frac{L^3}{\kappa^2} \left(-2\psi - \frac{\chi^3}{3} + \frac{\chi^3}{3} \log \mu_0^2 \right). \end{aligned}$$

- For $\Delta = 1$ the computation is the same but the interpretation is different:

$$\dim \chi = 1 \implies \langle \mathcal{O}_1 \rangle = \chi$$

and $\frac{\delta(S_{\text{reg}} + S_{\text{ct}})}{\delta\chi}$ is the renormalized source of the operator \mathcal{O}_1 .

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Thermodynamics of $\int d^4x \chi \mathcal{O}_3$: Conformal anomaly

- *Energy density:*

$$\varepsilon = \langle T^{\tau\tau} \rangle = -2 \frac{\delta S_{\text{ren}}}{\delta g_{\tau\tau}(\epsilon)} = \frac{L^3}{\kappa^2} \left(6g_{(4)xx} + \chi\psi - \frac{5}{24}\chi^4 + \frac{\chi^4}{12} \log \mu_0^2 \right)$$

- *Pressure:*

$$p = \langle T^{xx} \rangle = -2 \frac{\delta S_{\text{ren}}}{\delta g_{xx}(\epsilon)} = \frac{L^3}{\kappa^2} \left(2g_{(4)xx} + \chi\psi - \frac{\chi^4}{72} - \frac{\chi^4}{12} \log \mu_0^2 \right)$$

- *Ward identity for the trace:*

$$-\varepsilon + 3p = \langle T_\mu^\mu \rangle = (\Delta - 4)\chi \langle \mathcal{O}_\Delta \rangle + \mathcal{A} \stackrel{(\Delta=3)}{=} -\chi \langle \mathcal{O}_3 \rangle + \mathcal{A}$$

where

$$\langle \mathcal{O}_3 \rangle = \frac{\delta(S_{\text{reg}} + S_{\text{ct}})}{\delta\chi} = \frac{L^3}{\kappa^2} \left(-2\psi - \frac{\chi^3}{3} + \frac{\chi^3}{3} \log \mu_0^2 \right) \quad (\text{Condensate})$$

$$\mathcal{A} = -\frac{L^3}{6\kappa^2}\chi^4 \quad (\text{Holographic conformal anomaly, only if } \Delta \in \mathbb{Z})$$

Thermodynamics of $\frac{\lambda}{2} \int d^4x \mathcal{O}_2^2$: Conformal anomaly

- $\Delta = 2 \implies$ near boundary expansion:

$$\Phi(\rho) = \phi_{0,0}\rho + \underbrace{\phi_{0,1}}_{\text{source}} \rho \log \rho + \dots, \quad \rho \rightarrow 0.$$

- Condensate for $\Delta = 2$:

$$\langle \mathcal{O}_2 \rangle = \frac{\delta(S_{\text{reg}} + S_{\text{ct}})}{\delta \phi_{0,1}} = \frac{2L^3}{\kappa^2} (\phi_{0,0} + \phi_{0,1} - \phi_{0,1} \log \mu_0^2)$$

- Ward identity for the trace:

$$-\varepsilon + 3p = \langle T_\mu^\mu \rangle = -2\phi_{0,1} \langle \mathcal{O}_2 \rangle + \mathcal{A}$$

with

$$\mathcal{A} = \frac{2L^3}{\kappa^2} \phi_{0,1}^2 \quad (\text{Holographic conformal anomaly, only if } \Delta \in \mathbb{Z})$$

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Thermodynamics of $W = \int d^4x \chi \mathcal{O}_3$

- *Temperature and entropy density:*

$$T = \frac{1}{2\pi} \sqrt{2\rho_h g''_{\tau\tau}(\rho_h)}, \quad s = \frac{2\pi}{\kappa^2} \left(\frac{L^2}{\rho_h} g_{xx}(\rho_h) \right)^{3/2}.$$

- General dependence:

$$s = T^3 \sigma \left(\frac{\chi}{T} \right) \implies p = T^4 H_1 \left(\frac{\chi}{T} \right) + \chi^4 H_2 \left(\frac{\chi}{\mu_0} \right).$$

$$\left. \begin{array}{l} -\varepsilon + 3p = \chi \frac{\partial p}{\partial \chi} - \frac{\chi^5}{\mu_0} H'_2 \left(\frac{\chi}{\mu_0} \right) \\ -\varepsilon + 3p = -\chi \langle \mathcal{O}_3 \rangle + \mathcal{A} \end{array} \right\} \implies \frac{\partial p}{\partial \chi} = -\langle \mathcal{O}_3 \rangle, \quad -\frac{\chi^5}{\mu_0} H'_2 \left(\frac{\chi}{\mu_0} \right) = \mathcal{A}.$$

- Analytical solution ($x \equiv \chi/T$)

$$H_1(x) = a_0 + a_2 x^2 + a_4 x^4 \log x + O(x^4), \quad H_2 \left(\frac{x}{\mu_0} \right) = \frac{L^3}{6\kappa^2} \log \frac{x}{\mu_0}.$$

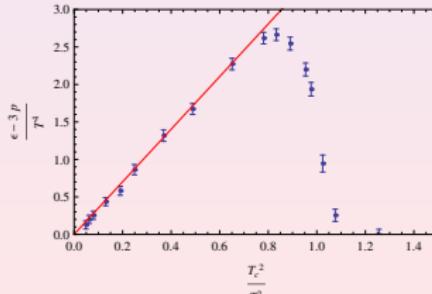
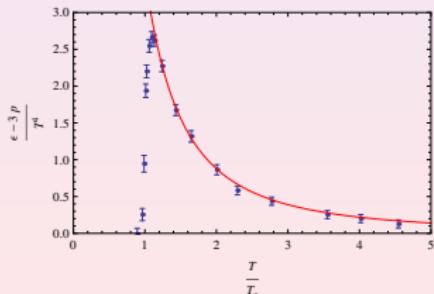
Thermodynamics of $W_\Delta = \int d^4x \phi_0 \mathcal{O}_\Delta$

- **Pressure:**

$$p = \frac{\pi^4 L^3 T^4}{2\kappa^2} \left(1 + 2^{3-\Delta} \left(\frac{\Lambda}{\pi T} \right)^{8-2\Delta} R(\Delta) + \dots \right), \quad \phi_0 = \Lambda^{4-\Delta},$$

$$R(\Delta) = -(\Delta - 2) \frac{\Gamma(\frac{3}{2} - \frac{\Delta}{4}) \Gamma(\frac{\Delta}{4})}{\Gamma(1 - \frac{\Delta}{4}) \Gamma(\frac{2+\Delta}{4})}.$$

- $\Delta = 3$: $p = \frac{\pi^4 L^3 T^4}{2\kappa^2} \left(1 + \left(\frac{\chi}{\pi T} \right)^2 R(3) \right) + \frac{L^3}{6\kappa^2} \chi^4 \log \frac{\chi}{\mu_0} + \dots$



→ Power corrections. See also [F. Zuo, JHEP1406 '14].

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Analytical solution at low temperature

- By defining $u(\phi) \equiv \rho\phi'(\rho)$, one gets

$$u''(\phi) = \frac{u'^2}{u} + \frac{3c_2\phi}{4u^2} u' + \frac{c_2^2\phi^2}{16u^3} - \frac{48 + 3c_2 + 4c_2\phi^2}{12u}, \quad c_2 \equiv \Delta(4-\Delta).$$

- Solutions with movable singularities near the horizon:*

$$u(\phi) = (\phi_h - \phi)^{1/2} \left(b_0 + \sum_{n=1}^{\infty} b_n (\phi_h - \phi)^n \right),$$

$$\text{with } b_0 = \frac{\sqrt{c_2\phi_h}}{2}, \quad b_1 = -\frac{96 + c_2(9 + 8\phi_h^2)}{48\sqrt{c_2\phi_h}}, \quad \dots.$$

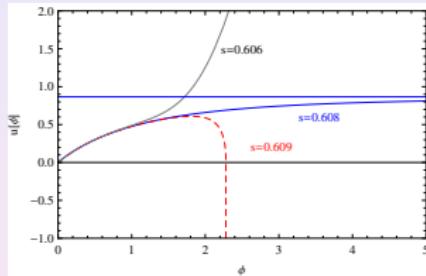
$\phi_h \equiv \phi(\rho_h)$ is an integration constant.

- Low Temperature $\iff \phi_h \gg 1$.
- Solution in the UV:*

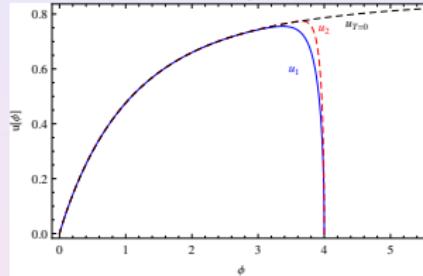
$$\phi \rightarrow 0 : \begin{cases} u(\phi) \simeq \phi \left(1 + \frac{1}{W_{-1}(-s\phi)} + \dots \right), & s \approx 0.608, \quad (\Delta = 2) \\ u(\phi) \simeq \frac{4-\Delta}{2}\phi + \dots, & (\Delta > 2) \end{cases}$$

Analytical solution at low temperature

$T = 0$



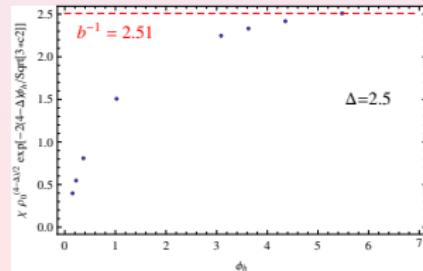
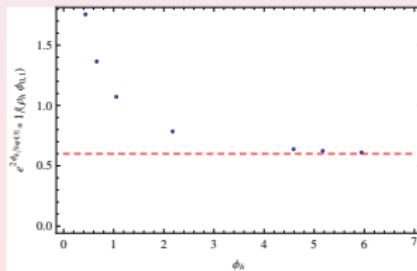
$T \neq 0$



- Relation between the source $(\phi_{0,1}, \chi)$ and ϕ_h :

$$\phi_{0,1} = -\frac{e^{\frac{2\phi_h}{\sqrt{3}}+R}}{s\rho_h} \text{ for } (\Delta = 2),$$

$$\frac{(b\chi)^{\frac{2}{4-\Delta}}}{T^2} = \frac{12\pi^2\phi_h^{-\frac{3}{2}+\frac{8}{\Delta(4-\Delta)}}}{\alpha\Delta(4-\Delta)} e^{\phi_h^2/3} \text{ for } (\Delta > 2).$$



Analytical solution at low temperature

- Near horizon solution:

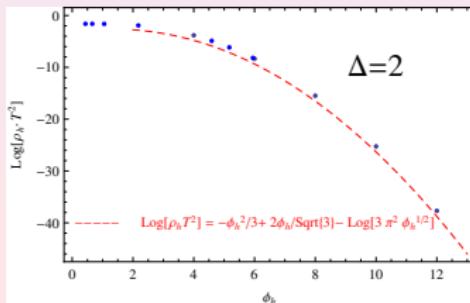
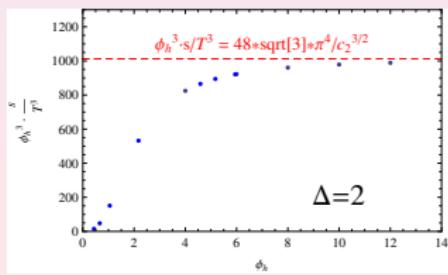
$$u(\phi) \simeq \sqrt{\frac{3c_2}{16} \left(1 - e^{-\frac{4}{3}\phi_h(\phi_h - \phi)}\right)},$$

$$g_{xx}(\phi) \simeq g_{xx}(\phi_h) e^{\frac{2}{3}\phi_h(\phi_h - \phi)}, \quad g_{\tau\tau}(\phi) \simeq -\frac{3g'_{\tau\tau}(\phi_h)}{4\phi_h} e^{\frac{2}{3}\phi_h(\phi_h - \phi)}.$$

→ Entropy and temperature (parametric dependence with ϕ_h):

$$\frac{s}{T^3} = \frac{L^3}{\kappa^2} \frac{48\sqrt{3}\pi^4}{c_2^{3/2} \phi_h^3},$$

$$\rho_h T^2 = \frac{\alpha c_2}{12\pi^2} \phi_h^{\frac{3}{2} - \frac{8}{c_2}} e^{-\frac{\phi_h^2}{3} + \frac{4\phi_h}{\sqrt{3}c_2}}.$$



Conclusions:

- AdS/CFT serves as a powerful tool to study the non-perturbative (NP) regime of Gluodynamics/QCD at zero and finite temperature.
- We have studied a model of conformal symmetry breaking in 5D based on dilatons.
- Thermodynamics depends on the dimension of the condensate associated to the scalar field: $\dim(\mathcal{O}) = \Delta$.
- The NP behaviour of QCD near and above T_c is characterized by power corrections in T^2 . These power corrections can be conveniently described with a deformation $\dim(\mathcal{O}) = 3$, so that this dimension is preferred to describe the EoS of QCD.
- i) Numerical description of the EoS in the whole regime of temperatures.
ii) Analytical results of the EoS at low and high temperatures.
- We have studied the role of the holographic conformal anomaly \mathcal{A} when $\Delta \in \mathbb{Z}$.

Thank You!