QCD@Work 2016

Martina Franca, 27 – 30 June 2016

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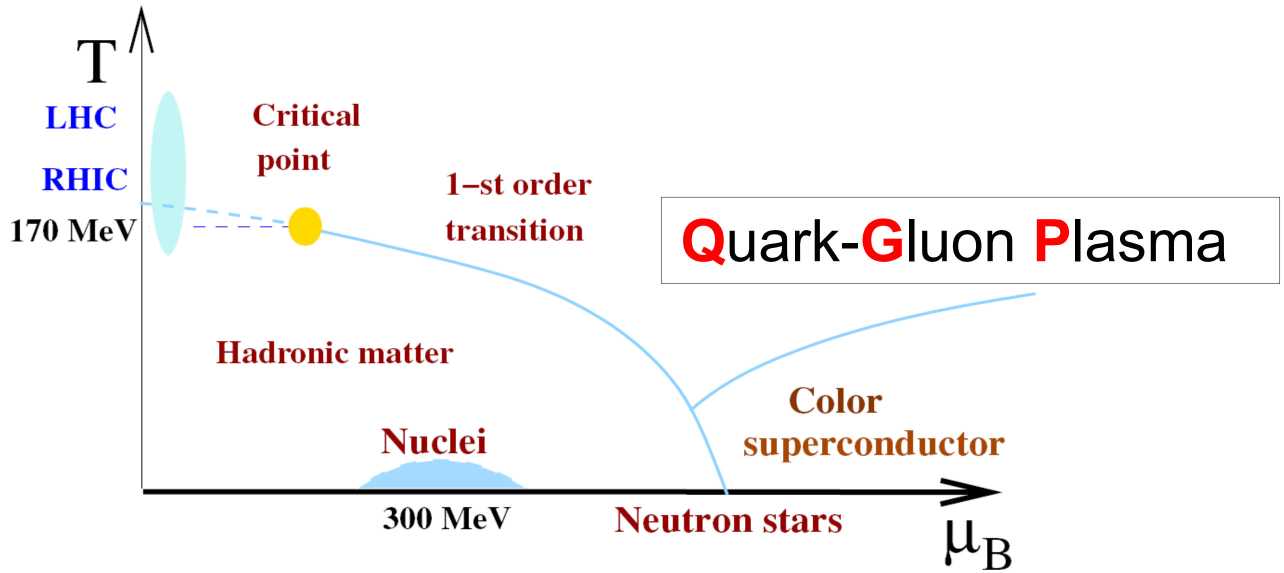
Thermalization of a strongly interacting plasma

JHEP **1507** (2015) 053

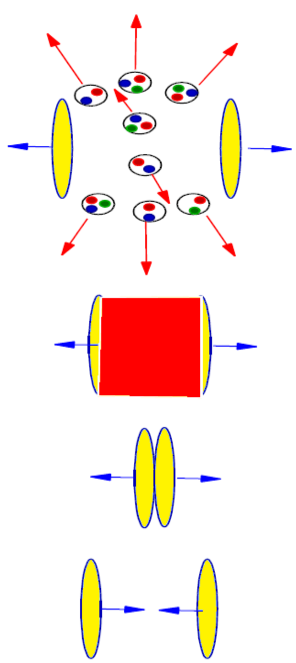
arXiv:1603.08849, to appear in PRD

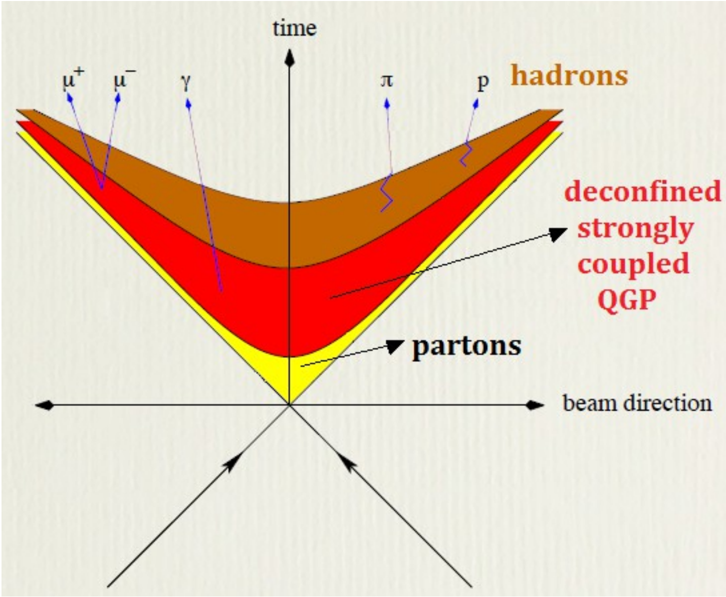
In collaboration with: P. Colangelo, F. De Fazio, F. Giannuzzi, S. Nicotri

Relaxation of a far-from-equilibrium QGP



Physical picture of **QGP** formation in Heavy Ion Collisions (LHC, RHIC)





OUR FOCUS:

evolution of the QGP from a pre-equilibrium state and estimate of physical observables (effective temperature, entropy density, energy density, pressure, non-local probes)

QGP as a strongly coupled fluid

Evidence from the RHIC and LHC experiments:

onset of the hydrodynamic regime for time scales **t $\gtrsim$1 fm/c** after the collision

PRE-EQUILIBRIUM EVOLUTION:

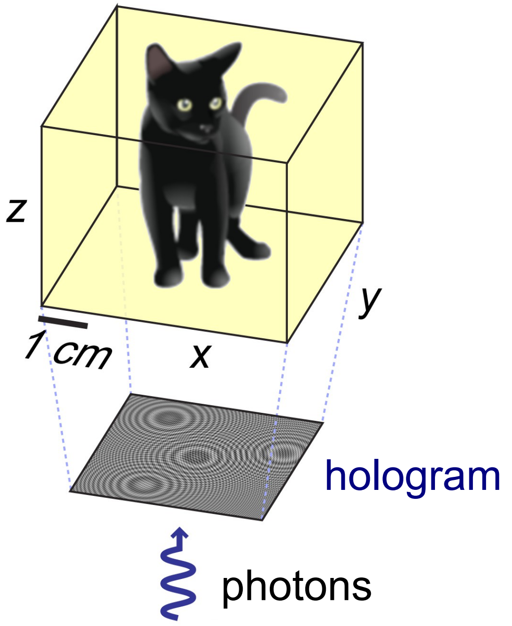
QGP behaves as a strongly-coupled fluid

⇨ Perturbative and lattice QCD methods are inapplicable

Possible solution: a holographic QCD model

- Analogy with holograms produced via optical techniques
- Application of the holographic principle ('t Hooft, Susskind)

Information related to a system is projected on the boundary enclosing its volume



The AdS/CFT correspondence

Strongly-coupled

Conformal **F**ield **T**heory

on the Minkowski space

$\mathcal{M}_4$

A conformal compactification gives

the *Boundary* of

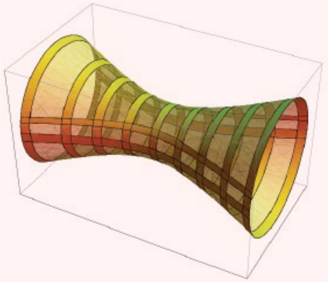
duality

Wealky-coupled gravitational string theory on

$AdS_5 \times S^5$

Anti-**de** **S**itter hyperboloid

Bulk



Quantum field theory on $\mathcal{M}_4$

at finite stationary T

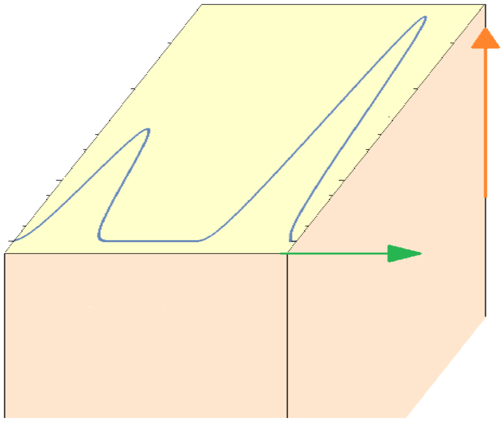
duality

AdS_5 / BH metric

Black **H**ole: horizon $\longrightarrow$ T

QGP formation and relaxation in holography

BOUNDARY SOURCING:



a time-dependent deformation pulse (quench) is introduced to the metric on the boundary in order to mimic the effects of heavy ion collisions.

QGP evolution towards equilibrium is computed in the 5-dimensional dual space from Einstein equations.

1. $\mathcal{M}_4$ + quench deformation pulse

→ duality →

2. 5d curved metric driven out of equilibrium

→ Einstein equations →

3. Intermediate and late time solution

→

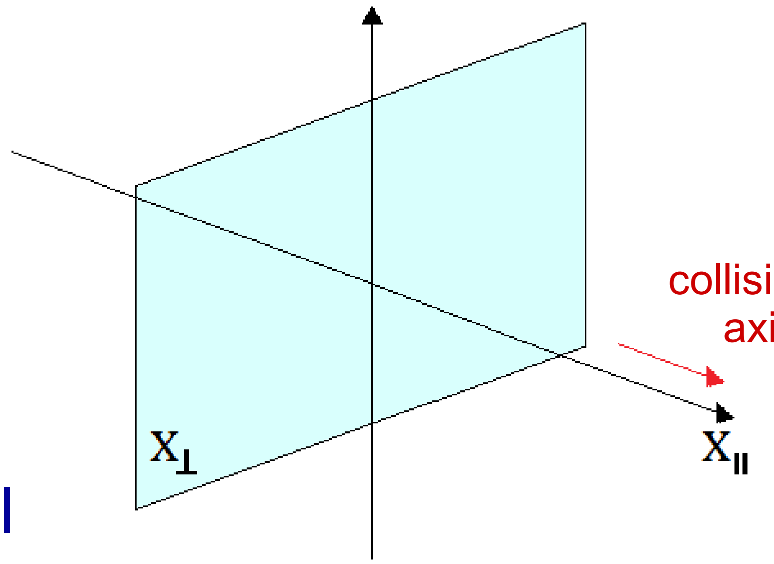
4. Relaxation and hydrodynamic regime: thermalization and isotropization of QGP

Simplification

Space-time symmetries

- Translation and rotation invariance in the $x_\perp$ plane
- Boost invariance along the $x_\parallel$ direction

Approximately realized at the central part of the QGP



Local thermal equilibrium : expansion is much slower than relaxation

All the portions of the fluid share the same (time dependent) temperature

Boundary sourcing and QGP evolution

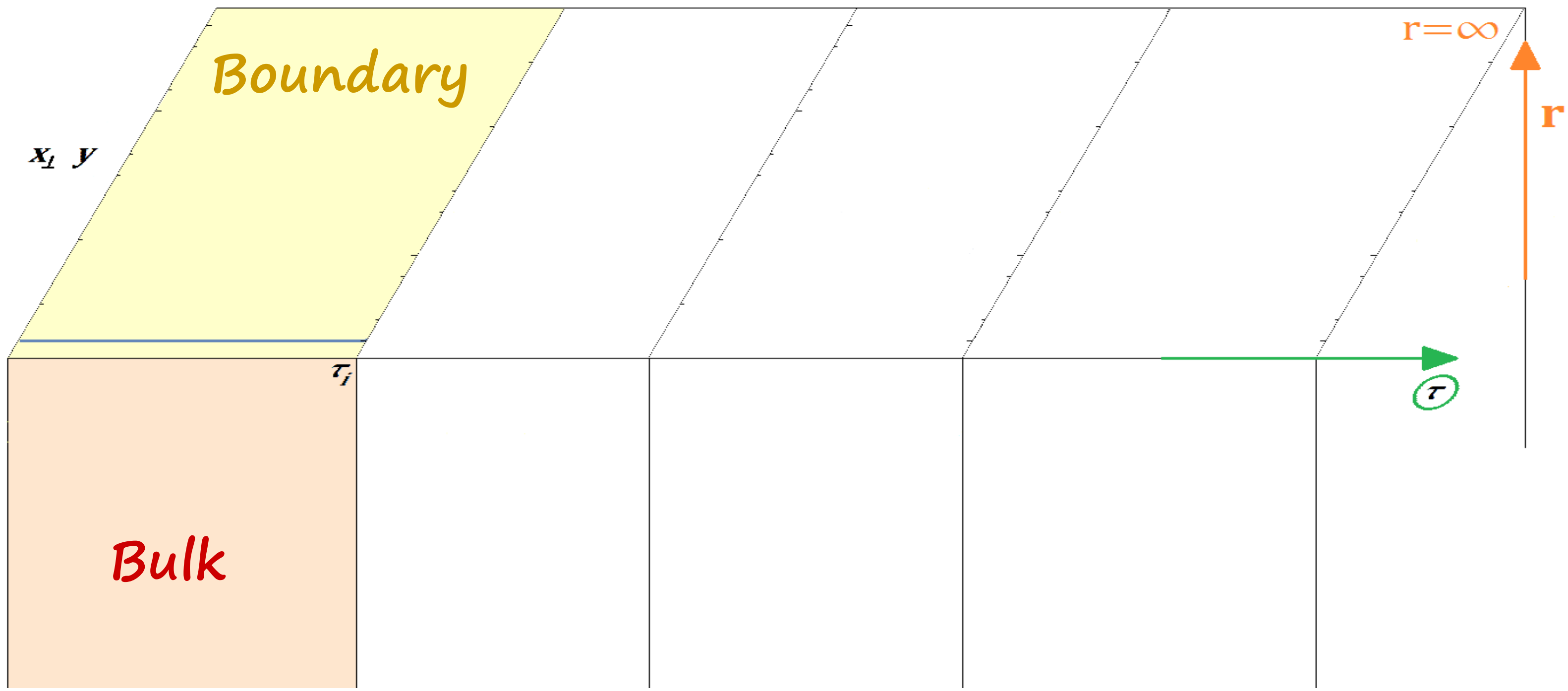
Coordinates: $(\tau, \boldsymbol{x}_\perp, y)$

y → rapidity

τ → proper time

$$\begin{cases} x^0 = \tau \cosh y \\ x_\parallel = \tau \sinh y \end{cases}$$

Fifth holographic coordinate: bulk radius r



GROUND STATE

$$\mathcal{M}_4: \quad ds^2 = -d\tau^2 + d\boldsymbol{x}_\perp^2 + \tau^2 dy^2$$

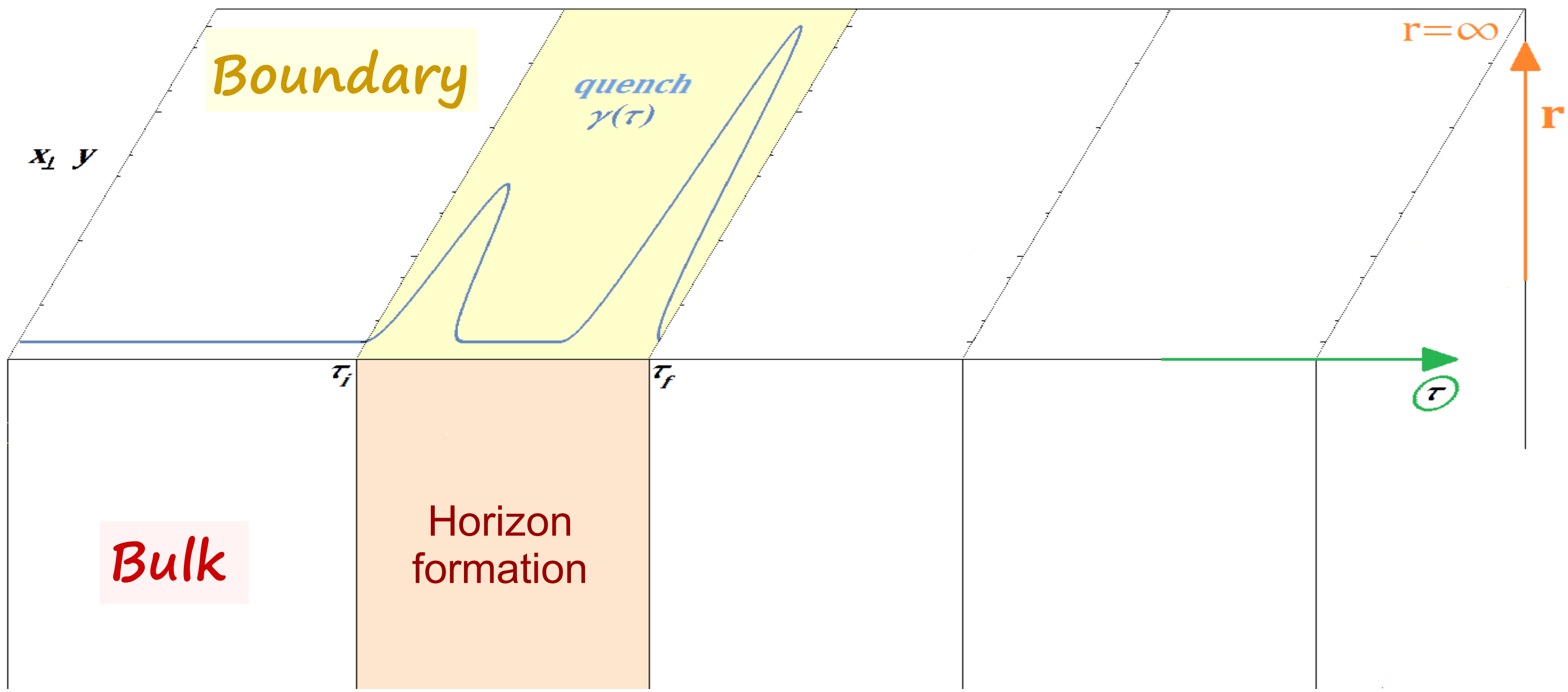
$$AdS_5: \quad ds^2 = r^2\left[-d\tau^2 + d\boldsymbol{x}_\perp^2 + \left(\tau + \tfrac{1}{r}\right)^2 dy^2\right] + 2\,dr\,d\tau$$

Boundary sourcing and QGP evolution

Coordinates: $(\tau, \mathbf{x}_\perp, y)$ — y: rapidity; τ: proper time

$$\begin{cases} x^0 = \tau \cosh y \\ x_\parallel = \tau \sinh y \end{cases}$$

Fifth holographic coordinate: bulk radius r



BOOST-INVARIANT DEFORMATION

4d : $ds^2 = -d\tau^2 + e^{\gamma(\tau)} d\mathbf{x}_\perp^2 + \tau^2 e^{-2\gamma(\tau)} dy^2$

5d : $ds^2 = -A(r,\tau)\,d\tau^2 + \Sigma^2(r,\tau)\left[e^{B(r,\tau)} d\mathbf{x}_\perp^2 + e^{-2B(r,\tau)} dy^2\right] + 2\,dr\,d\tau$

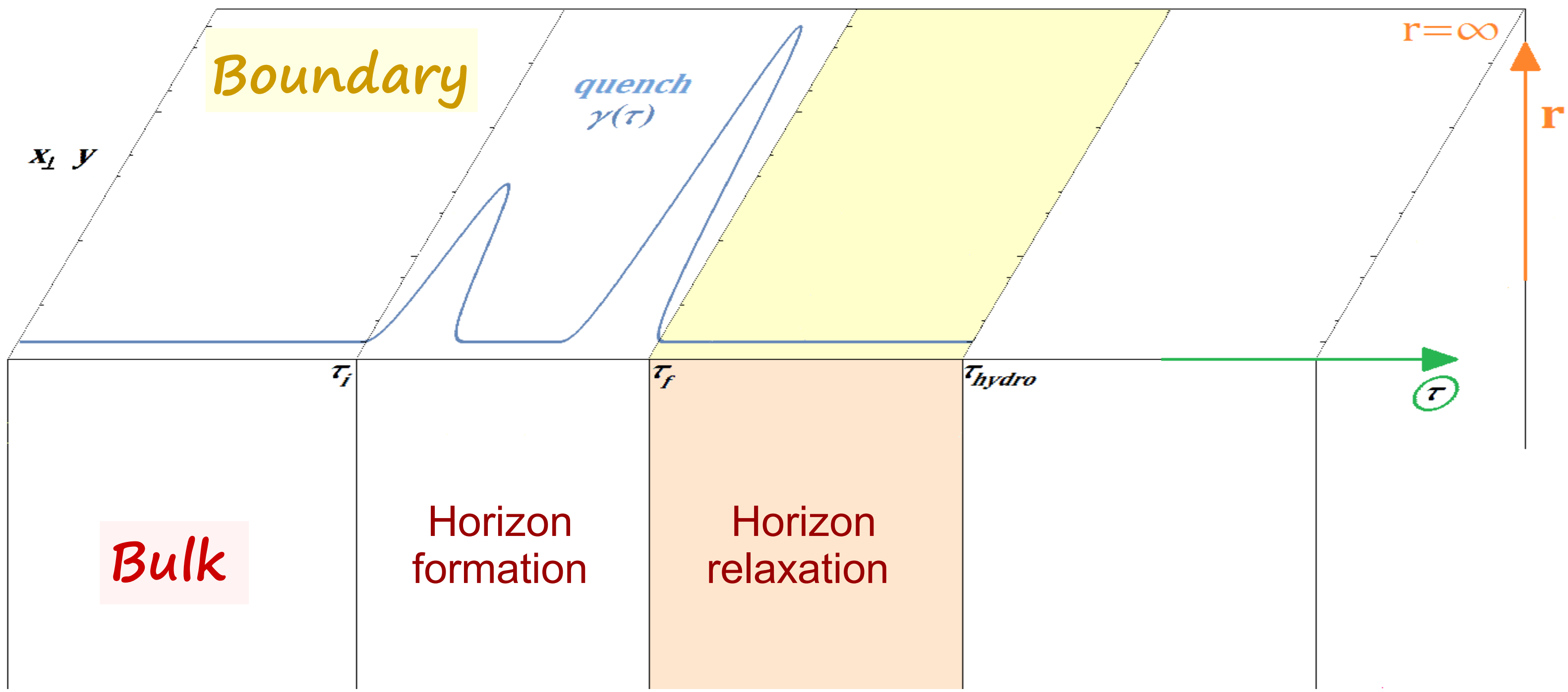
Einstein's equations

Boundary sourcing and QGP evolution

Coordinates: $(\tau, \boldsymbol{x}_\perp, y)$ — y: rapidity; τ: proper time

$$\begin{cases} x^0 = \tau \cosh y \\ x_\parallel = \tau \sinh y \end{cases}$$

Fifth holographic coordinate: bulk radius r



$$\tau_f \le \tau \le \tau_{\text{hydro}}$$

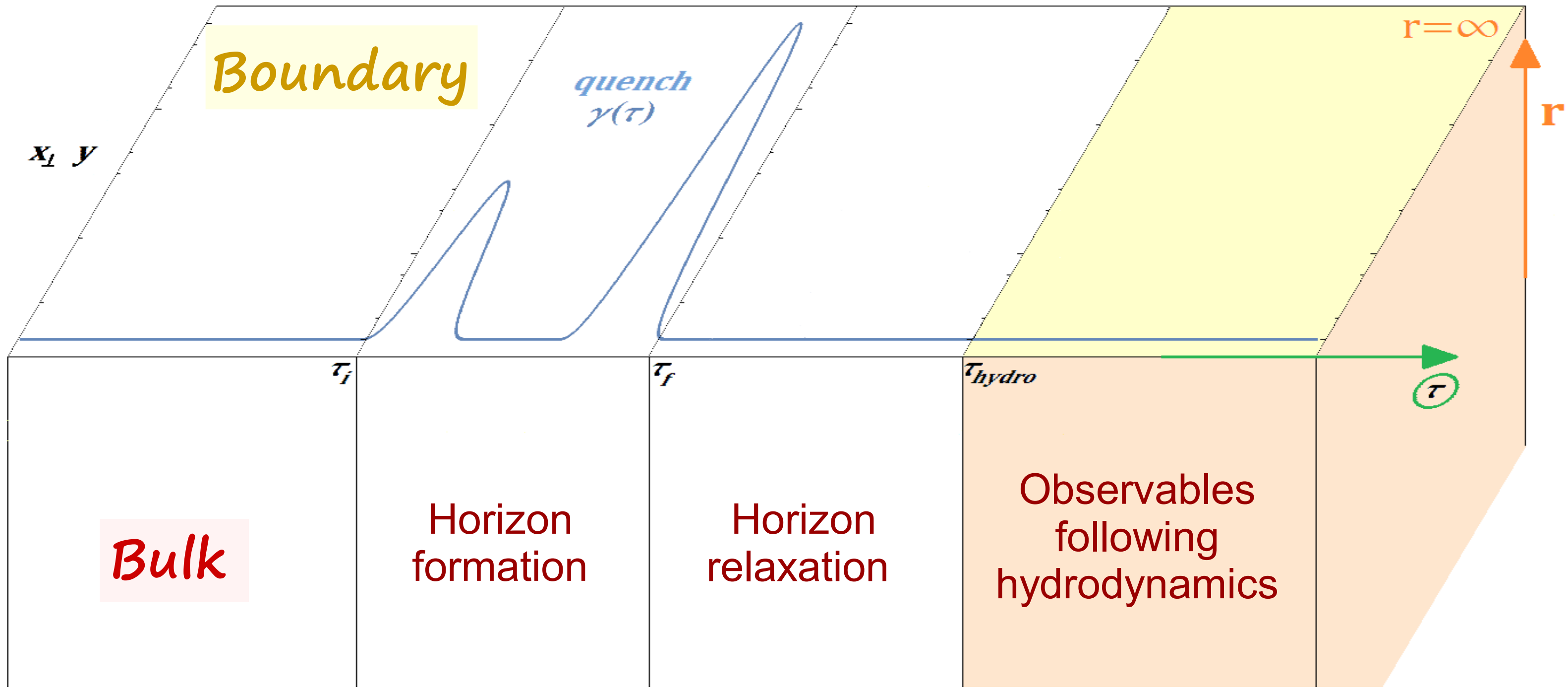
TERMALIZATION and **ISOTROPIZATION** of the system after the quench

Boundary sourcing and QGP evolution

Coordinates: $(\tau, \boldsymbol{x}_{\perp}, y)$ → y: rapidity, τ: proper time

$$\begin{cases} x^0 = \tau \cosh y \\ x_{\parallel} = \tau \sinh y \end{cases}$$

Fifth holographic coordinate: bulk radius r



$\tau \geq \tau_{\text{hydro}}$ **HYDRODYNAMIC REGIME:** both temperature and stress-energy tensor follow hydrodynamics

$$T_{\text{eff}}(\tau) \propto \tau^{-1/3}, \qquad \mathcal{E}(\tau), \mathcal{P}_{\perp}(\tau), \mathcal{P}_{\parallel}(\tau) \propto \tau^{-4/3}$$

Observables computed

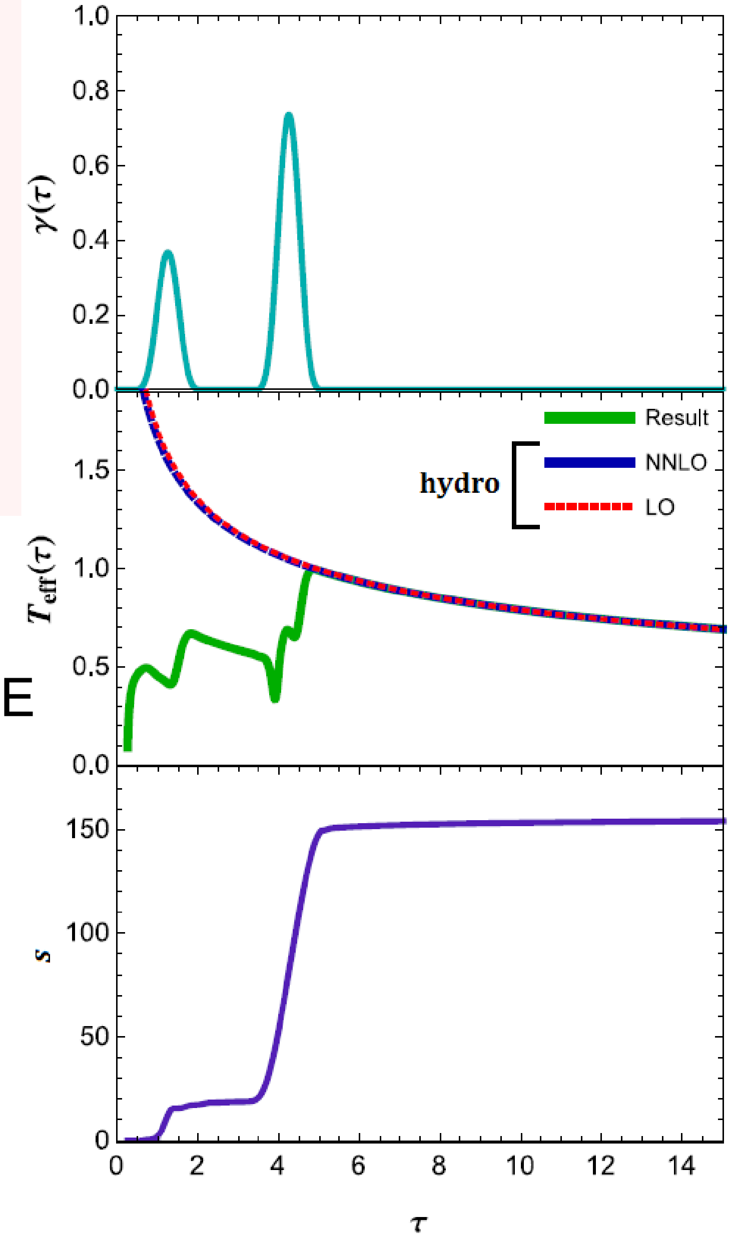
- Temperature
- Entropy density → Multiplicity of particles produced in heavy ion collisions
- Energy density
- Pressure
- Non-local probes → Two-point correlation function and expectation values of rectangular and circular Wilson loops

Temperature and entropy density

At late times the *Bulk* geometry evolves towards the AdS_5 / BH form

The computed horizon $r_h(\tau)$:

- follows the distortion profile
- asymptotically relaxes as $r_h(\tau) \propto \tau^{-1/3}$



BH THERMODYNAMICS + HOLOGRAPHIC PRINCIPLE allow to define:

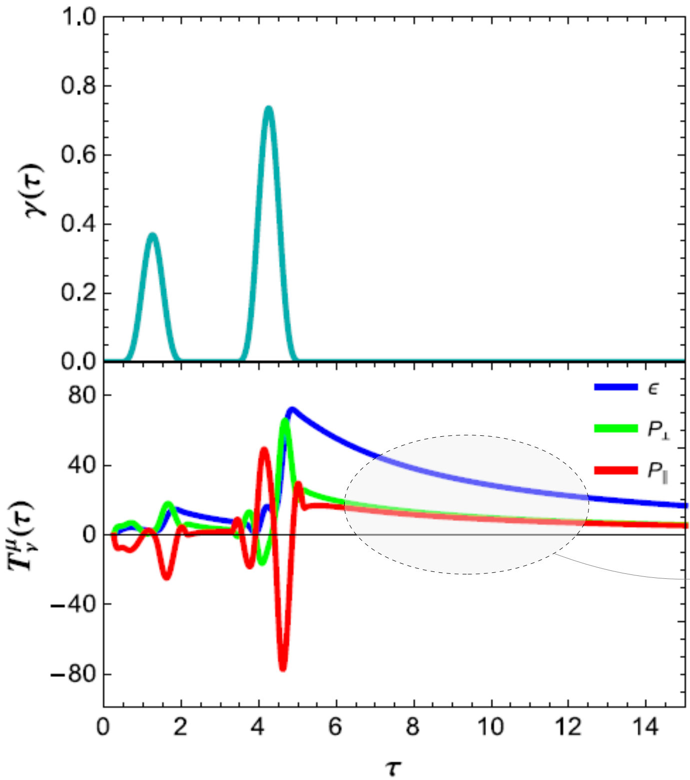
- Effective temperature from horizon position

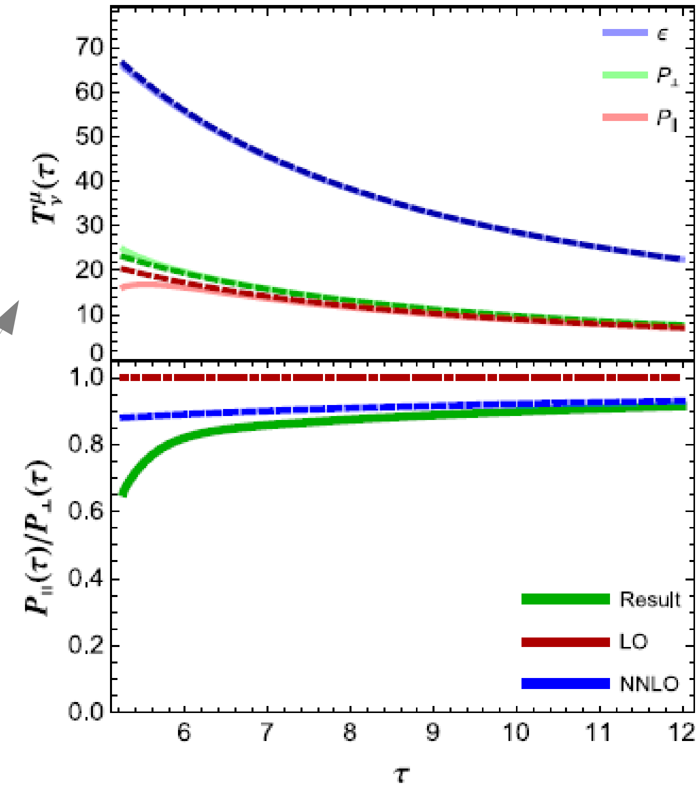
$$T_{\text{eff}}(\tau) = \frac{r_h(\tau)}{\pi}$$

- Entropy from horizon area (Bekenstein-Hawking definition)

Energy density and pressure

Boundary Stress-Energy Tensor $T_{\mu\nu} = \frac{N_c^2}{2\pi^2}\,\mathrm{diag}\left(\mathcal{E}, \mathcal{P}_\perp, \mathcal{P}_\perp, \mathcal{P}_\parallel\right)$





Energy density $\mathcal{E}(\tau) = \frac{3}{4}\pi^4 T_{\text{eff}}(\tau)^4$ starts to follow hydrodynamics as soon as the quench is switched off $(\tau = \tau_f)$

Setting the scale $T_{\text{eff}}(\tau_f) = 500\ \text{MeV}$, pressure isotropy is reached after a time $\boxed{\tau_{\text{hydro}} - \tau_f \simeq 0.6\ \text{fm}/c}$

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Non-local probes of thermalization

NON-LOCAL PROBES

EQUAL-TIME TWO-POINT CORRELATION FUNCTION of boundary operators with large conformal dimension

EXPECTATION VALUES OF SPACELIKE WILSON LOOPS defined on the boundary

- Deeper investigation of the bulk spacetime
- Sensitivity to a wide range of energy scales in the boundary field theory
- Possible thermalization mechanisms:

BOTTOM-UP *hard* quanta of gauge theory equilibrate radiating *softer* quanta

TOP-DOWN energetic gauge field modes equilibrate first, soft modes last

Non-local probes of thermalization

Equal-time correlation function (CF)

$$\left\langle \mathcal{O}\left(t_0, -\frac{\ell}{2}, x_2, y\right) \mathcal{O}\left(t_0, \frac{\ell}{2}, x_2, y\right)\right\rangle \simeq e^{-\mathcal{L}(t_0, \ell)\Delta}$$

Length of the spacelike geodesics extending in the bulk and connecting the boundary points

QCD boundary operator of large conformal dimension Δ

geodesics approximation (AdS/CFT)

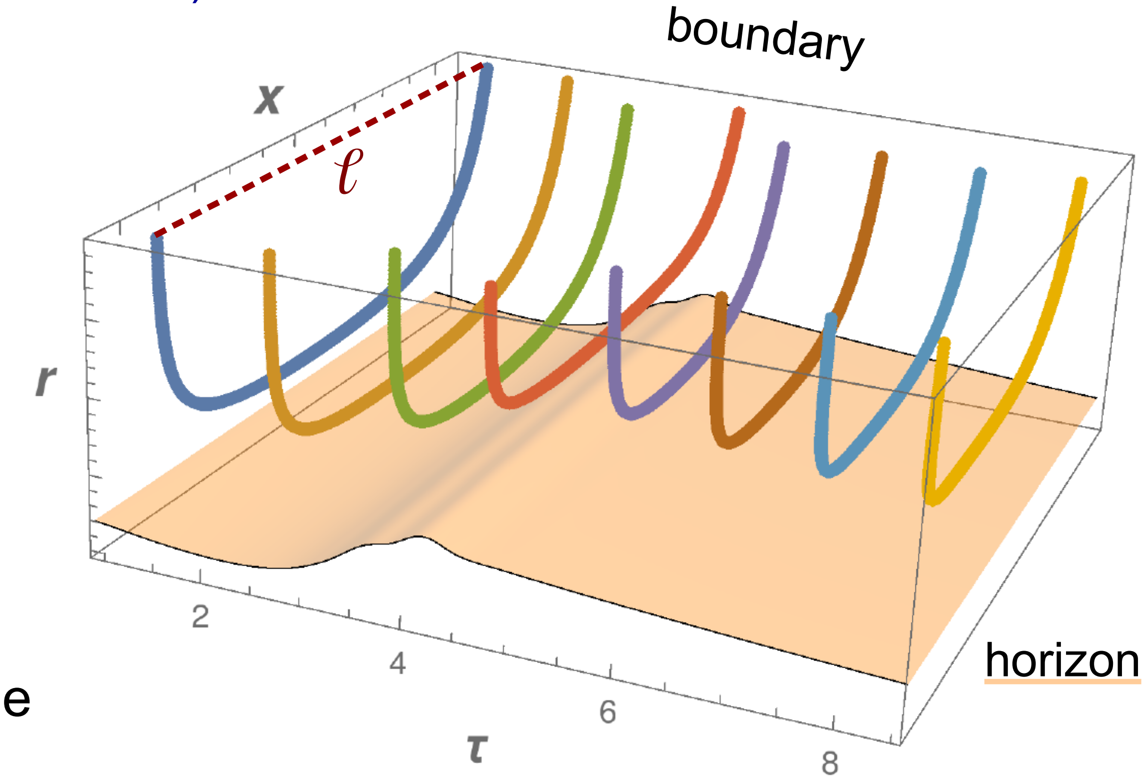
Geodesics parametrized by $x_1 \equiv x$:

$$(r(x), \tau(x))$$

(x_2, y) fixed

ℓ distance between the points in the CF

$\tau\left(\pm\frac{\ell}{2}\right) = t_0$ boundary field theory time



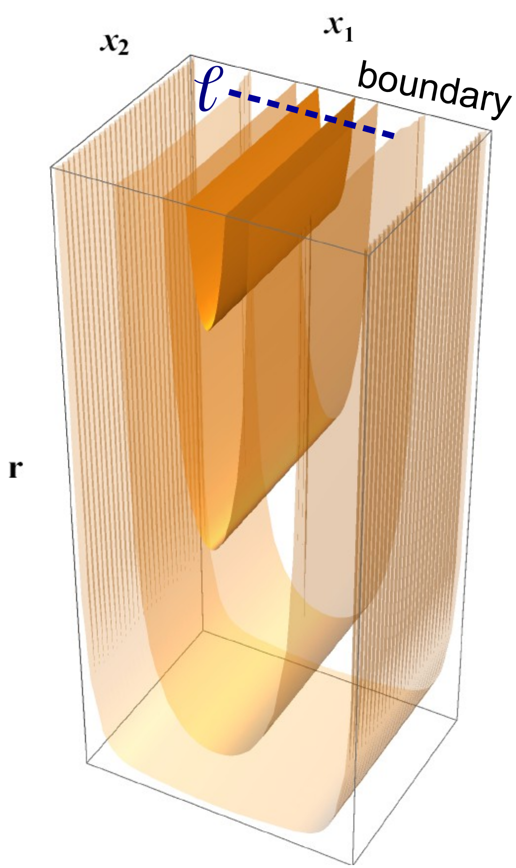
Non-local probes of thermalization

Expectation values of spacelike Wilson loops (WL)

$$\left\langle \mathcal{W}_C \right\rangle \simeq e^{-\mathcal{A}(t_0,\ell)}$$

Area of the minimal string surface extending in the bulk, bounded by the path C on the boundary

Infinite rectangular WL



Parametrization (x_1, x_2) with translation invariance along x_2

$$\left(x_2 \in [0,q], \quad q \gg 1\right)$$

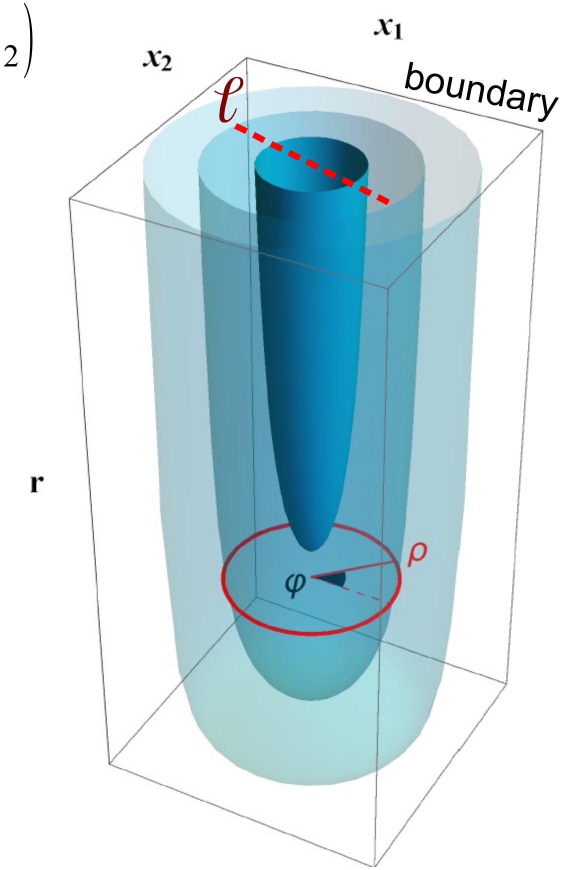
$$\left(r(x_1), \tau(x_1)\right)$$

y fixed

ℓ finite side of the rectangular path

$$\tau\left(\pm\frac{\ell}{2}\right) = t_0$$

Circular WL



Parametrization (ρ, φ)

$$\left(r(\rho), \tau(\rho)\right)$$

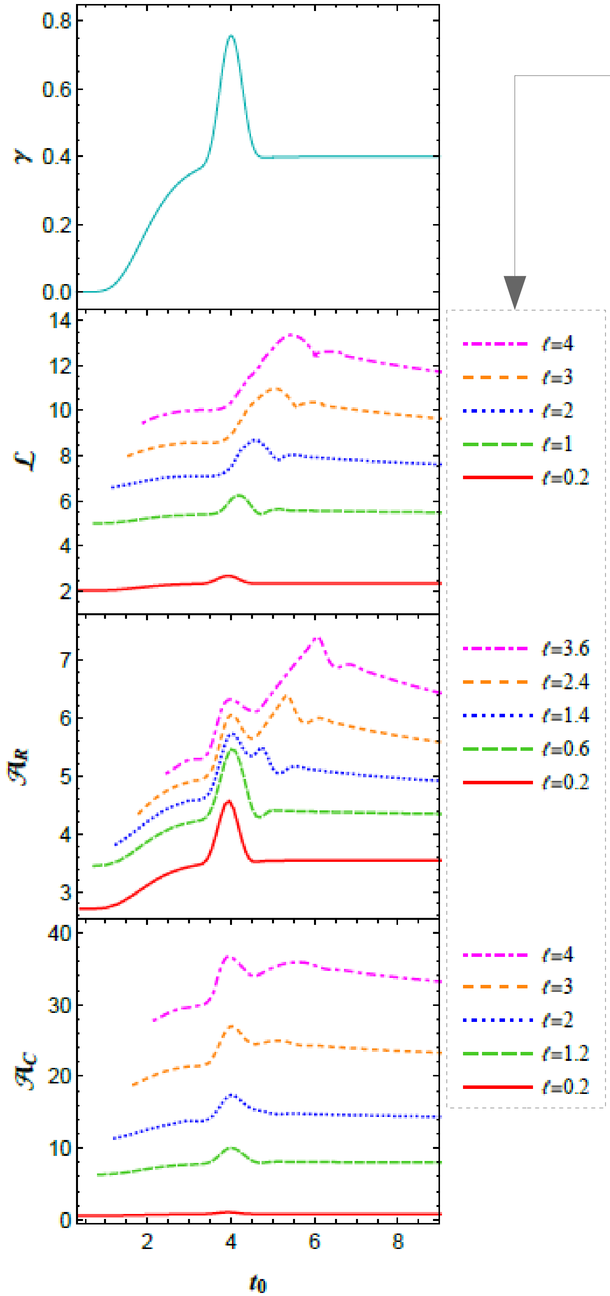
$$(x_1, x_2) = \rho(\cos\varphi, \sin\varphi)$$

y fixed

ℓ diameter of the circular path

$$\tau\left(\frac{\ell}{2}\right) = t_0$$

Geometrical invariants

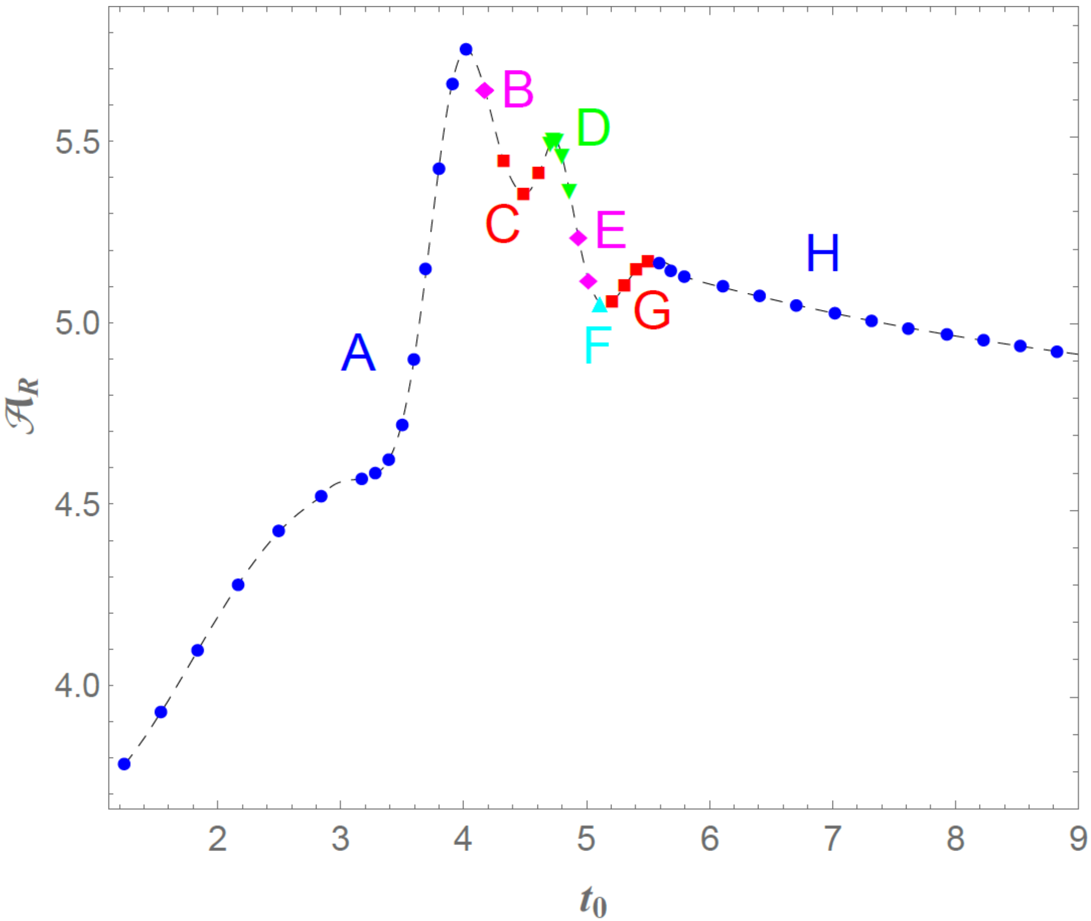


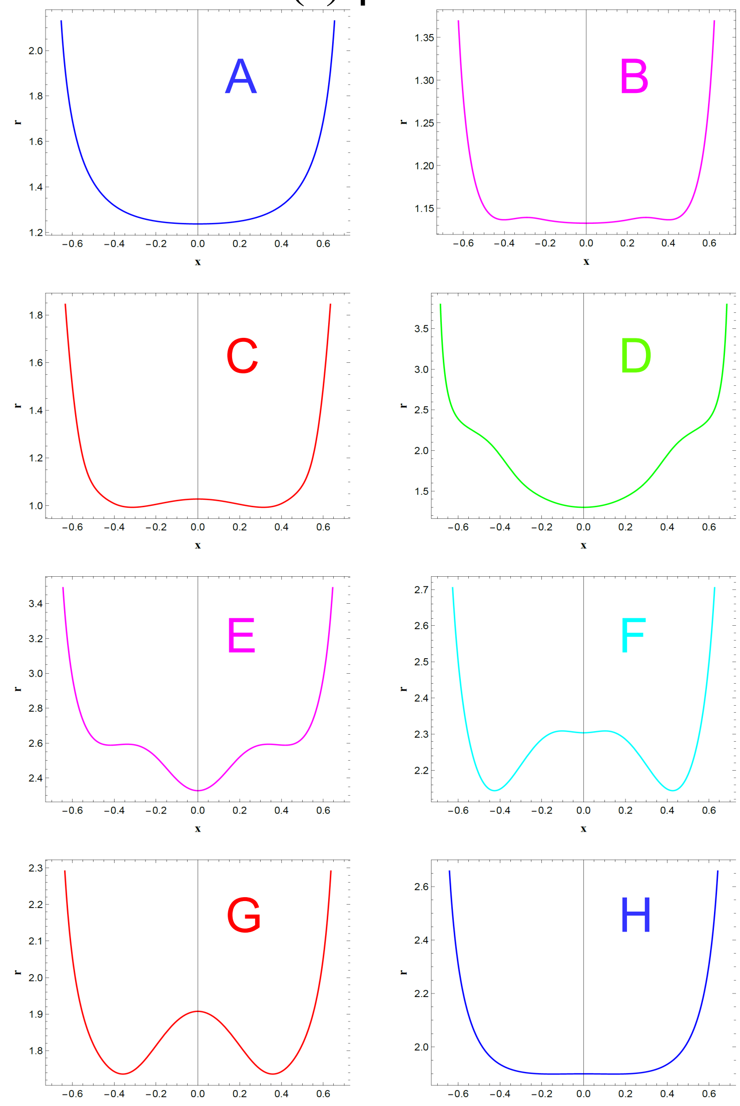
ℓ boundary separation
(size of the probe)

- The three observables follow the quench profile
- Additional structures (local minima) are found

Topologies of the $r(x)$ profiles

The structures in $\mathcal{A}_R\,(t_0, \ell\!=\!1.4)$ are related to the topologies of extremal surfaces



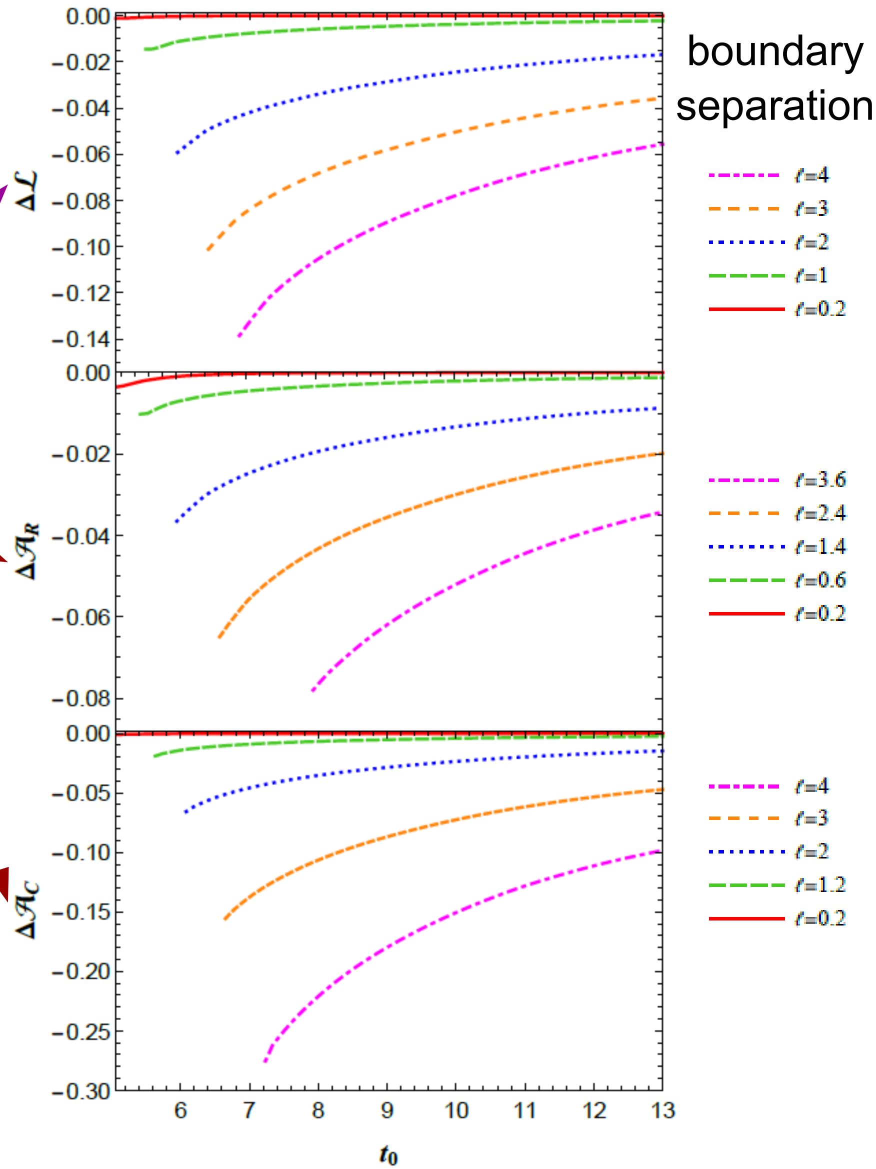


For some points in branches A (upper part), B, C, D, G, corresponding to the largest time variations of the geometry, $r\,(x)$ crosses the event horizon

Δ-probes of relaxation to hydrodynamics

$\Delta\mathcal{L} = \mathcal{L} - \mathcal{L}_{Hydro}$

$\Delta\mathcal{A} = \mathcal{A} - \mathcal{A}_{Hydro}$



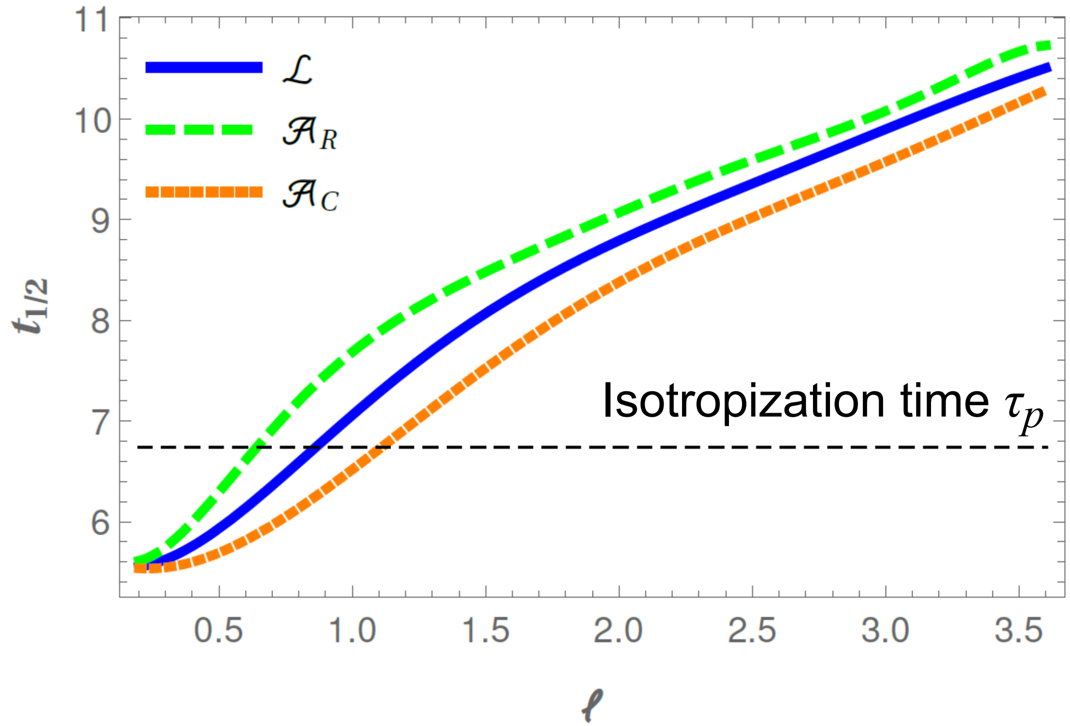
Comparison with the results for viscous hydrodynamics

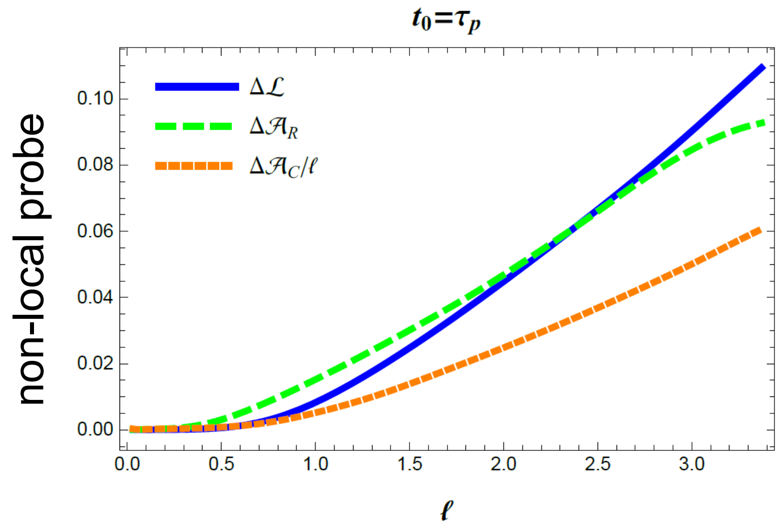
⇓

Hydrodynamic regime reached faster for smaller ℓ

Thermalization times: local vs non-local probes

At the boundary times $t_0 = t_{1/2}(\ell)$, the probes $\Delta\mathcal{L}$, $\Delta\mathcal{A}_R$, $\Delta\mathcal{A}_C$ are reduced by 1/2 with respect to their values at the end of the quench.





The rectangular Wilson loop takes more time to thermalize

HIERARCHY OF THERMALIZATION TIMES

$$\tau_\varepsilon \simeq \tau_f < t_{1/2}(\ell) < \tau_p \qquad (\text{small } \ell)$$

$$\tau_\varepsilon \simeq \tau_f < \tau_p < t_{1/2}(\ell) \sim \ell \qquad (\text{large } \ell)$$

Top-down thermalization

Conclusions and perspectives

- The evolution of a boost-invariant non-Abelian plasma has been analyzed in the holographic picture. The plasma is driven out-of-equilibrium by introducing a perturbation (quench) to the Minkowski boundary.

- The temperature and the energy density acquire the hydrodynamical form as soon as the quench is switched off, while pressure isotropy is restored with a time delay of O(fm/c) [scale $T_{\mathrm{eff}}(\tau_f)=500\,\mathrm{MeV}$].

- The thermalization time of non-local observables (correlation function, expectation values of rectangular and circular Wilson loops) increases with the boundary separation.

- A hierarchy is found among the thermalization times of the energy density, pressures and large probes, indicating that relaxation is faster at the UV scales (top-down thermalization). The result is independent on the quench profile.

- Local and non-local probes for less symmetric systems, and in presence of a confinement/deconfinement phase transition.

Thank you!

Bonus material

Evolution in the 5-dimensional bulk

A, B, Σ from Einstein's equations

$$\begin{cases} \Sigma(\dot{\Sigma})' + 2\Sigma'\dot{\Sigma} - 2\Sigma^2 = 0 \\ \Sigma(\dot{B})' + \frac{3}{2}(\Sigma'\dot{B} + B'\dot{\Sigma}) = 0 \\ A'' + 3B'\dot{B} - 12\frac{\Sigma'\dot{\Sigma}}{\Sigma^2} + 4 = 0 \\ \ddot{\Sigma} + \frac{1}{2}(\dot{B}^2\Sigma - A'\dot{\Sigma}) = 0 \\ \Sigma'' + \frac{1}{2}B'^2\Sigma = 0 \end{cases} \Rightarrow$$

- **Event horizon:** the critical geodesics $r_h(\tau)$ such that
  $$\lim_{\tau\to\infty} A(r_h(\tau), \tau) = 0$$
- **Apparent horizon:** from
  $$\dot{\Sigma}(r_h(\tau), \tau) = 0$$

  Effective temperature and entropy density

Directional derivatives :

$f' \equiv \partial_r f$ along infalling radial null geodesics

$\dot{f} \equiv \partial_\tau f + \frac{1}{2} A \partial_r f$ along outgoing radial null geodesics

Testing the numerical algorithm

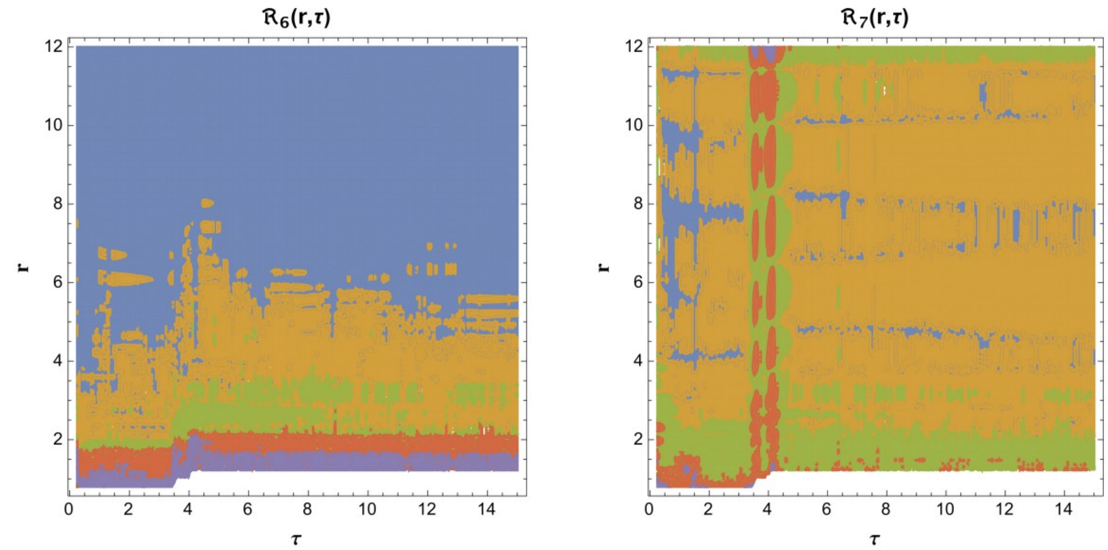
$$\mathcal{R}_6 = \frac{\Sigma(\dot\Sigma)' + 2\Sigma'\dot\Sigma - 2\Sigma^2}{|\Sigma(\dot\Sigma)'| + |2\Sigma'\dot\Sigma| + |2\Sigma^2|}$$

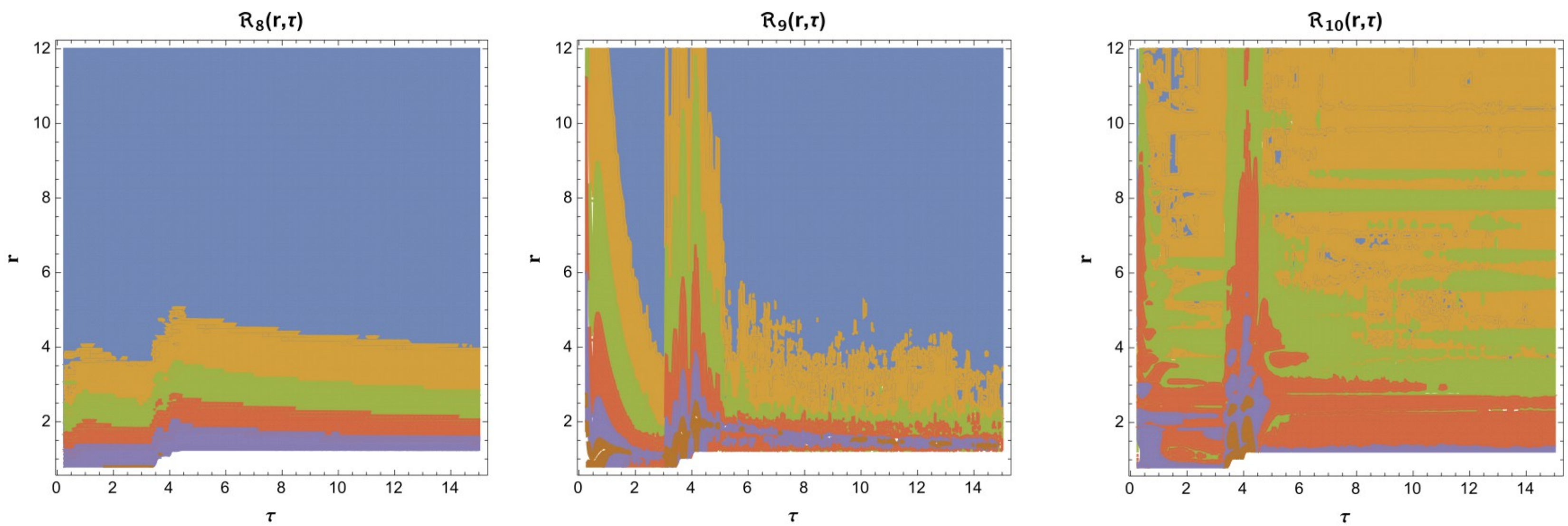
$$\mathcal{R}_7 = \frac{\Sigma(\dot B)' + (3/2)(\Sigma'\dot B + B'\dot\Sigma)}{|\Sigma(\dot B)'| + (3/2)(|\Sigma'\dot B| + |B'\dot\Sigma|)}$$

$$\mathcal{R}_8 = \frac{A'' + 3B'\dot B - 12\Sigma'\dot\Sigma/\Sigma^2 + 4}{|A''| + 3|B'\dot B| + 12\,|\Sigma'\dot\Sigma|/|\Sigma^2| + 4}$$

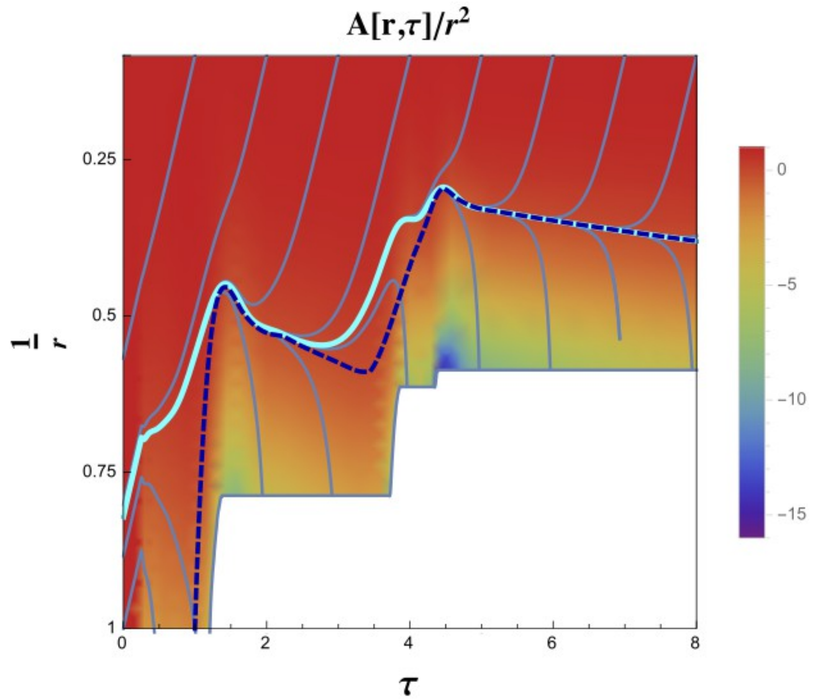
$$\mathcal{R}_9 = \frac{\ddot\Sigma + (1/2)(\dot B^2\Sigma - A'\dot\Sigma)}{|\ddot\Sigma| + (1/2)(|\dot B^2\Sigma| + |A'\dot\Sigma|)}$$

$$\mathcal{R}_{10} = \frac{\Sigma'' + (1/2)B'^2\Sigma}{|\Sigma''| + (1/2)\,|B'^2\Sigma|}$$





Apparent and event horizon



- The gray lines are radial null outgoing geodesics $\dfrac{d\,r}{d\,\tau} = \dfrac{A(r,\tau)}{2}$;
- The dashed dark blue line is the apparent horizon from $\dot{\Sigma}(r_h(\tau),\tau)=0$;
- The continuos cyan line is the event horizon obtained as the critical geodesics $r_h(\tau)$ such that $\lim_{\tau\to\infty} A(r_h(\tau),\tau)=0$.