Factorization, resummation and sum rules for heavy-to-light form factors

Yu-Ming Wang

University of Vienna

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Why heavy-to-light form factors?

- Interesting to understand the strong interaction dynamics of heavy quark decays.
  - Factorization properties of exclusive B-meson decay amplitudes.
  - Renormalization and asymptotic properties of B-meson DAs.
  - Interplay of different QCD techniques based upon the HQE.

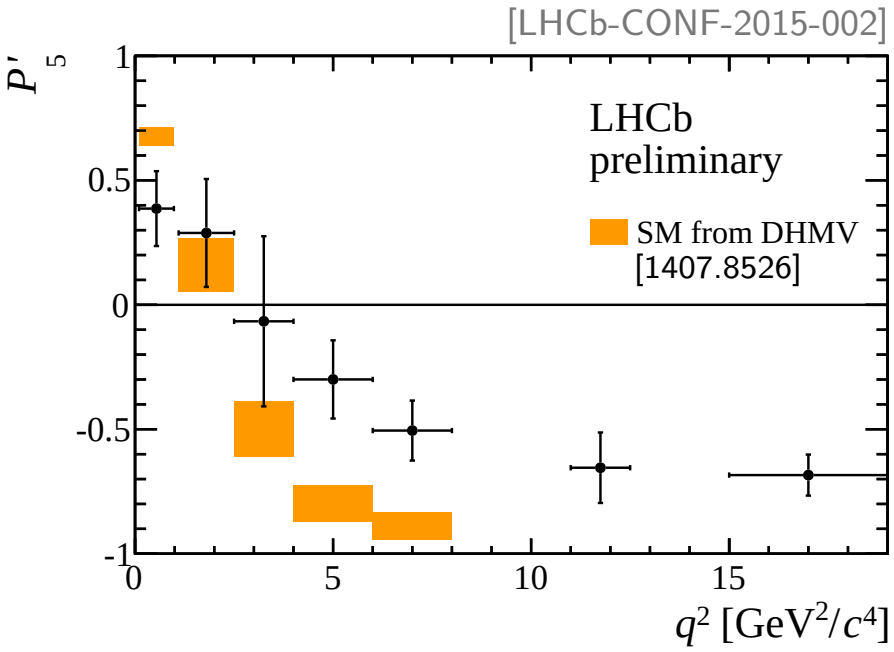
- Precision determination of the CKM matrix element $|V_{ub}|$.
  $B \to \pi \ell \nu$, $B \to \rho \ell \nu$, $\Lambda_b \to p \ell \nu$.

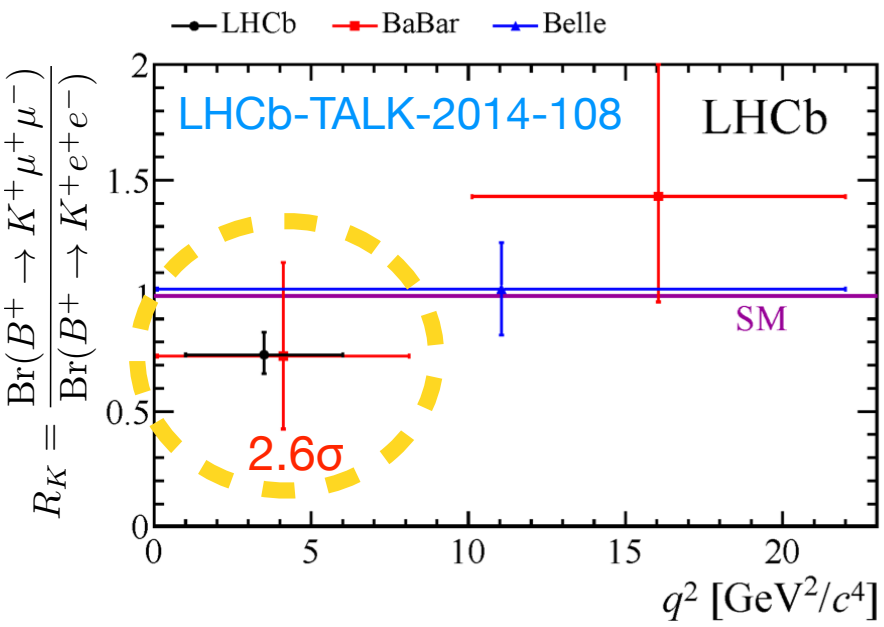
- Fundamental inputs for QCD descriptions of FCNC decays and hadronic decays.
  $B \to K^* \ell\ell$, $\Lambda_b \to \Lambda \ell\ell$, $B \to \pi\pi$, $\Lambda_b \to p\pi$.
  - Sensitive to the BSM physics.
  - CP violating asymmetries and the CKM angles.
  - More complicated than FFs.

- Crucial to understand the flavour puzzles.
  - Important source of theory uncertainties.
  - Systematical treatment of sub-leading power/twist contributions.

Anomalies in FCNC processes

A few “anomalies" exist in $B \to K^{(*)}\ell^+\ell^-$.

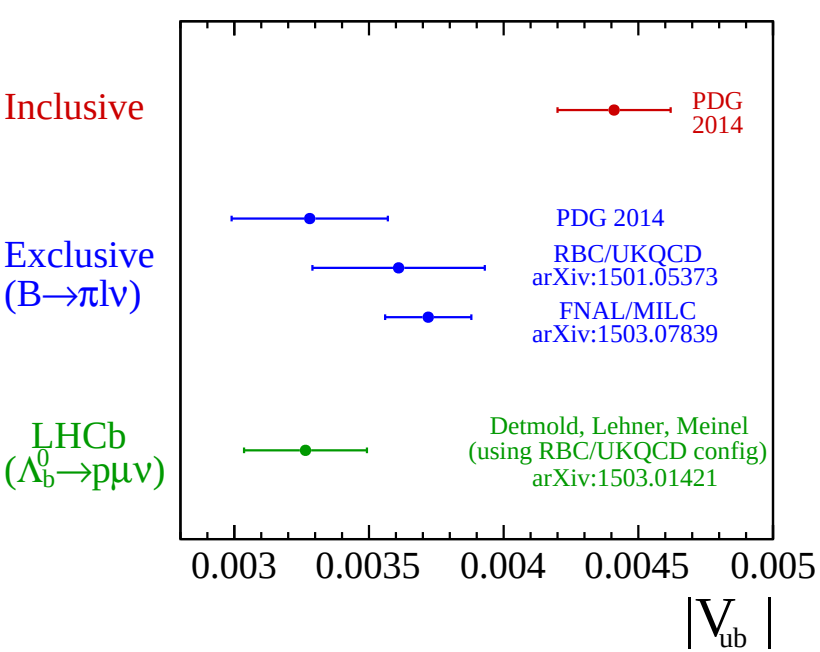


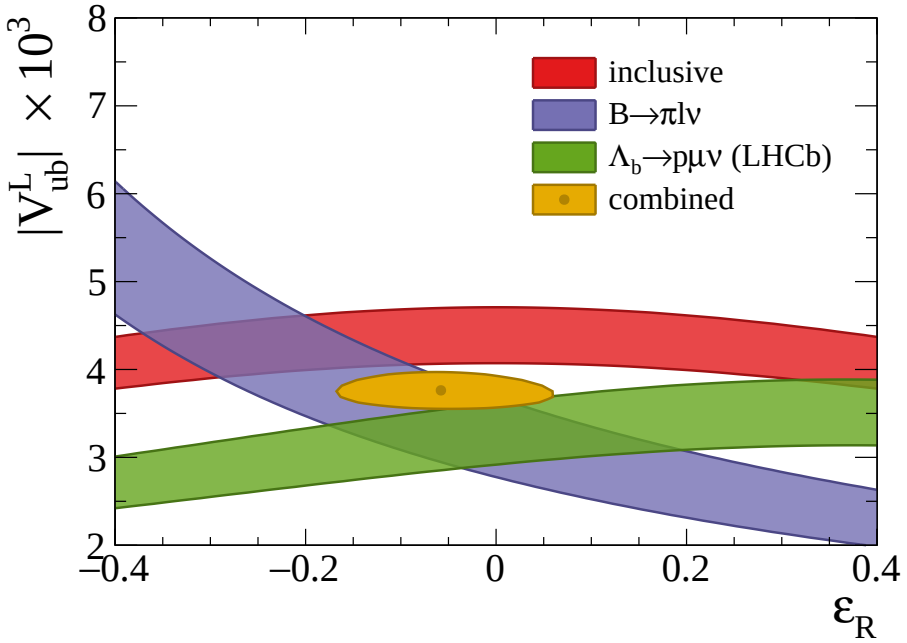


- Indication of BSM physics or ignorance of QCD dynamics?
- P'_5 anomaly below $6\,\mathrm{GeV}^2$ more serious [power corrections].
- Violation of lepton flavor universality [QED corrections].
- Need more data and theoretical efforts.

$|V_{ub}|$ puzzle

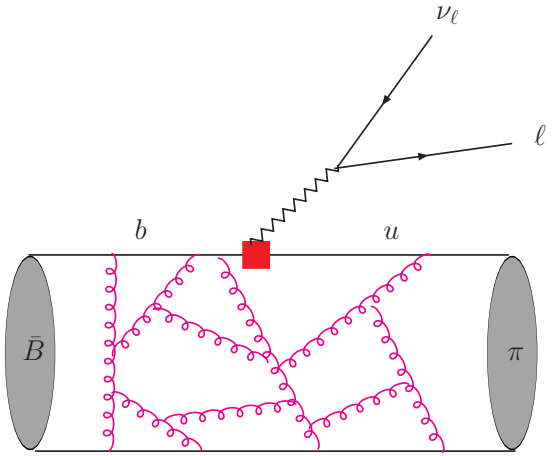
$3\,\sigma$ tension between exclusive and inclusive $|V_{ub}|$ [arXiv:1504.01568].





right handed current, underestimate of QCD uncertainties?

Semileptonic $B \to \pi \ell \nu$ decays



Hadronic matrix element:

$$\langle \pi(p)|\bar{u}\gamma_\mu b|\bar{B}(p+q)\rangle = f^+_{B\pi}(q^2)\left[p_B + p - \frac{m_B^2 - m_\pi^2}{q^2} q\right]_\mu + f^0_{B\pi}(q^2)\,\frac{m_B^2 - m_\pi^2}{q^2} q_\mu \,.$$

- Lepton spectrum:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3 q^4 m_B^2}(q^2 - m_l^2)^2 |\vec{p}_\pi| \times \left[\left(1 + \frac{m_l^2}{q^2}\right) m_B^2 |\vec{p}_\pi|^2 |f^+_{B\pi}(q^2)|^2 + \frac{3 m_l^2}{8q^2}(m_B^2 - m_\pi^2)^2 |f^0_{B\pi}(q^2)|^2\right] .$$

- Still the best way to determine $|V_{ub}|$ exclusively in the continuum approach!

- $\Lambda_b \to p\ell\nu$ decays also become important now [LHCb, arXiv:1504.01568].

$$|V_{ub}| = \left(3.27 \pm 0.23\right) \times 10^{-3} .$$

$B \to \pi$ form factors in QCD factorization

- QCD factorization of $B \to \pi$ form factors [Beneke, Feldmann, 2001]:

$$\boxed{F_i(q^2) = C_i(E)\,\xi_P(E) + \Phi_B(\omega) \otimes T_i(E; \ln\omega, v) \otimes \Phi_\pi(v)\,.}$$

- QCD correction to $B \to \pi$ form factors:

$$\begin{aligned}
\xi_P(E) &\equiv f_+(q^2)\ \text{[factorization scheme]}\,, \\
f_0 &= \frac{2E}{M}\,\xi_P\left(1 + \frac{\alpha_s\,C_F}{4\pi}\,[2-2L]\right) + \frac{\alpha_s\,C_F}{4\pi}\,\Delta f_0, \\
\Delta f_0 &= \frac{M-2E}{2E}\,\frac{8\pi^2 f_B f_P}{N_C M}\int dl_+\,\frac{\phi_+^B(l_+)}{l_+}\int du\,\frac{\phi(u)}{\bar{u}}\,.
\end{aligned}$$

- SCET factorization of $B \to P$ form factors [Beneke, Feldmann, 2003]:

$$\begin{aligned}
F_i(q^2) &= C_i(E)\,\underbrace{\xi_P(E)}_{\langle P(p)|(\bar{\xi}\,W_c)\,h_v|\bar{B}_v\rangle} + C_i^{(B1)}(E,\tau) \otimes \underbrace{\Xi_P(\tau,E)}_{\langle P(p)|(\bar{\xi}\,W_c)\left(W_c^\dagger\,i\not{D}_{c\perp}\,W_c\right)(rn)\,h_v|\bar{B}_v\rangle}, \\
\Xi_P(\tau,E) &= J_P(\tau; v, \omega) \otimes \Phi_B(\omega) \otimes \Phi_P(v)\,.
\end{aligned}$$

  - $\xi_P(E)$ defined in SCET_I.
  - Three-particle DAs contribute to $\xi_P(E)$ at LP.

- Factorization of $\xi_P(E)$ in SCET_II: missing field modes?
  SCET_II operators describing the endpoint region and overlapping with the pion DAs.

Alternative approaches to $B \to \pi$ form factors

- Traditional QCD light-cone sum rules [Braun et al; Khodjamirian et al]:
  - Replace the B-meson by a space-like interpolating current.
  - QCD factorization of the correlation function at leading twist.

$$\text{correlation function} \sim \sum_n T^{(n)} \otimes \phi_\pi^{(n)} \,.$$

    Twist-3 factorization only in the asymptotic limit.
  - No separation of hard and hard-collinear scales.
    $\Rightarrow$ No resummation of large logarithms.

- QCD light-cone sum rules with B-meson DAs:
  - Replace the pion by a space-like hard-collinear current.
  - QCD factorization for the vacuum-to-B-meson correlation function.

$$\Pi_i(n\cdot p, \bar{n}\cdot p) \quad \sim \quad \sum_{k=\pm} \underbrace{C_i(k)(n\cdot p,\mu)} \int \frac{d\omega}{\omega - \bar{n}\cdot p}\, J_i^{(k)}\left(\frac{\mu^2}{n\cdot p\,\omega}, \frac{\omega}{\bar{n}\cdot p}\right) \phi_B^{(k)}(\omega,\mu)\,.$$

Hard matching coefficients of the QCD weak currents.

  - Reproduce the structure of QCD factorization for $B \to \pi$ form factors.
  - Can be formulated in SCET [De Fazio, Feldmann and Hurth, 2005, 2008].

$B \to \pi$ form factors from LCSRs with B-meson DAs

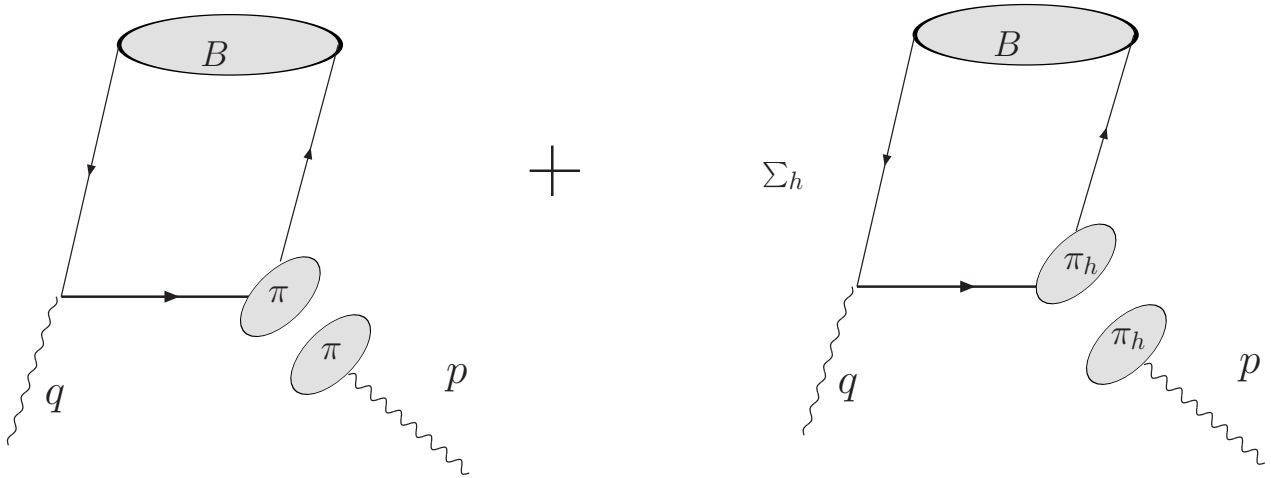
- Starting point: correlation function [Y.M.W and Y.L. Shen, 2015]

$$\begin{aligned}
\Pi_\mu(p,q) &= \int d^4x\, e^{ip\cdot x} \langle 0|T\left\{\bar{d}(x)\not{n}\gamma_5 u(x), \bar{u}(0)\gamma_\mu b(0)\right\}|\bar{B}(p+q)\rangle \\
&= \Pi(n\cdot p, \bar{n}\cdot p) n_\mu + \widetilde{\Pi}(n\cdot p, \bar{n}\cdot p)\bar{n}_\mu\,, \\
n\cdot p &= \frac{m_B^2 + m_\pi^2 - q^2}{m_B}, \qquad \bar{n}\cdot p \sim O(\Lambda)\,, \qquad p+q \equiv m_B v = \frac{m_B}{2}(n+\bar{n})\,.
\end{aligned}$$

Similar to $B \to \gamma \ell \nu$ decay: replacing the pion current by the e.m. current.

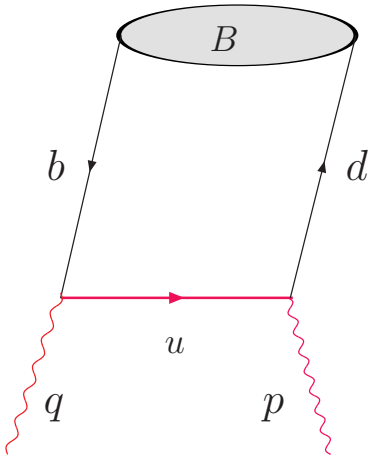
- Inserting complete set of pion states $\Rightarrow$ hadronic sum:

$\widetilde{\Pi}(n\cdot p, \bar{n}\cdot p) =$



$$\frac{f_\pi(n\cdot p)\, m_B}{2(m_\pi^2 - p^2)} \underbrace{\left[\frac{n\cdot p}{m_B} f_{B\pi}^{+}(n\cdot p) + f_{B\pi}^{0}(n\cdot p)\right]}_{\text{relative sign changes for } \Pi(n\cdot p, \bar{n}\cdot p)} \qquad \int_{\omega_s}^{+\infty} d\omega' \frac{\widetilde{\rho}^h(n\cdot p, \omega')}{\omega' - \bar{n}\cdot p}$$

OPE calculation of the correlation function



Factorization at tree level:

$$\widetilde{\Pi}^{(0)}(n\cdot p,\bar{n}\cdot p) = \tilde{f}_B m_B \int_0^{+\infty} d\omega' \frac{\phi_B^-(\omega')}{\omega' - \bar{n}\cdot p - i0},$$

$$\Pi^{(0)}(n\cdot p,\bar{n}\cdot p) = 0,$$

$$\Rightarrow f_{B\pi}^0(n\cdot p) = \frac{n\cdot p}{m_B} f_{B\pi}^+(n\cdot p) + O(\alpha_s).$$

- Light-cone DAs of the B-meson [Grozin and Neubert, 1996]:

$$i m_B \tilde{f}_B \phi_B^+(\omega) = \frac{1}{2\pi}\int dt\, e^{i\omega t} \langle 0|(\bar{q}_s Y_s)(t\bar{n})\, \not{\bar{n}}\gamma_5\, (Y_s^\dagger b_v)(0)|\bar{B}(v)\rangle.$$

  - One-loop renormalization of $\phi_B^+(\omega,\mu)$ [Lange and Neubert, 2003].
  - Renormalization of $[\bar{q}_s(t\bar{n})\Gamma b_v(0)]$ does not commute with the shot-distance expansion [Braun, Ivanov and Korchemsky, 2004].

$$[(\bar{q}_s Y_s)(t\bar{n})\not{\bar{n}}\Gamma(Y_s^\dagger b_v)(0)]_R = \sum_{p=0} \frac{t^p}{p!}\left[\bar{q}_s(0)\,(n\cdot \overleftarrow{D})^p\, \not{\bar{n}}\Gamma b_v(0)\right]_R.$$

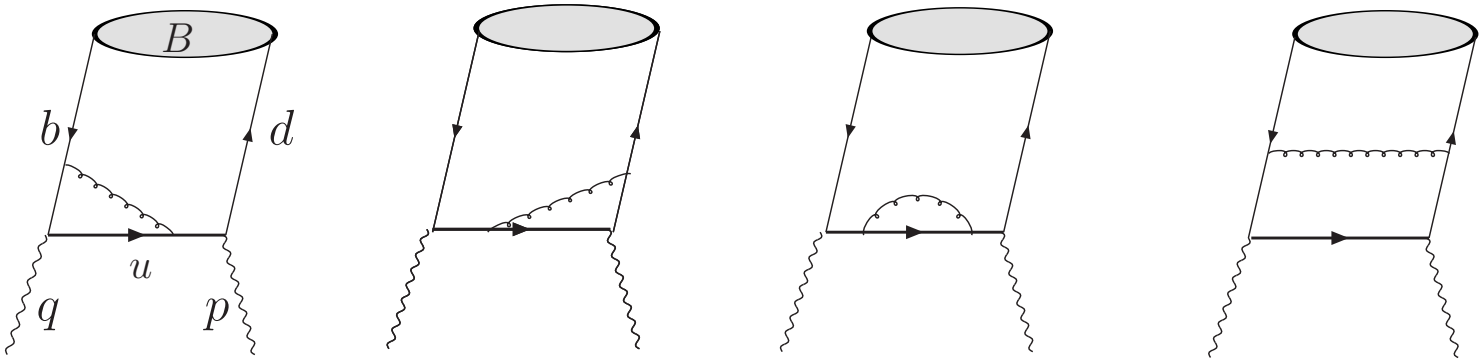
  - Eigenfunctions of the Lange-Neubert kernel [Bell, Feldmann, YMW and Yip, 2013].
  - $\phi_B^-(\omega)$ defined in a similar way, renormalization kernel available [Bell and Feldmann, 2008].

- QCD correction involving $\phi_B^+(\omega')$ at NLO must be IR finite.

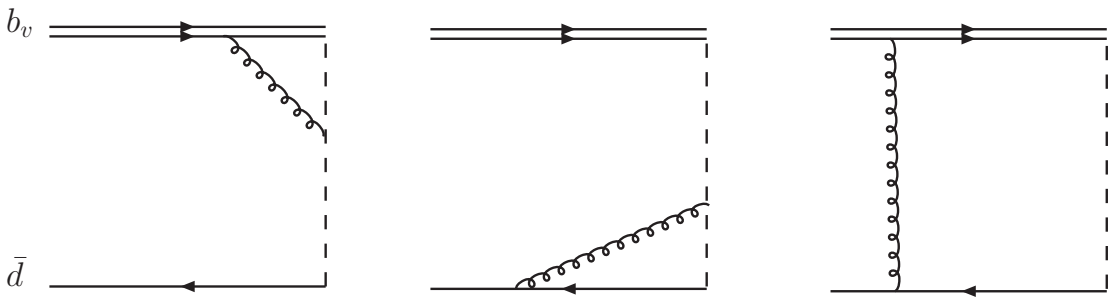
- Symmetry breaking of the form-factor relations at NLO must be IR finite.

Factorization of the correlation function

- Light-cone OPE: $|\bar{n}\cdot p| \sim O(\Lambda_{\rm QCD})$.



- Cancellation of the soft divergences.



- Diagrammatic factorization:

$$\begin{aligned}\Pi_\mu &= \Pi_\mu^{(0)} + \Pi_\mu^{(1)} + \ldots = \Phi_B \otimes T \\ &= \Phi_B^{(0)} \otimes T^{(0)} + \left[\Phi_B^{(0)} \otimes T^{(1)} + \Phi_B^{(1)} \otimes T^{(0)}\right] + \ldots .\end{aligned}$$

$$\Downarrow$$

$$\boxed{\Phi_B^{(0)} \otimes T^{(1)} = \Pi_\mu^{(1)} - \Phi_B^{(1)} \otimes T^{(0)} .}$$

Sample calculation: the weak vertex diagram

- Strategy:
  (i) Identify the leading regions of the QCD amplitudes.
  (ii) Evaluate the leading contributions with the method of regions [Beneke and Smirnov, 1997].
  (iii) Perform the soft subtraction [the same as the QCD amplitude in the soft region].

- QCD amplitude:

$$\Pi^{(1)}_{\mu,weak} = \frac{g_s^2 C_F}{2(\bar{n}\cdot p-\omega)} \int \frac{d^D l}{(2\pi)^D} \frac{1}{[(p-k+l)^2+i0][(m_b v+l)^2-m_b^2+i0][l^2+i0]}$$
$$\bar{d}(k)\,\not{n}\,\gamma_5\,\not{\bar{n}}\,\underbrace{\gamma_\rho\,(\not{p}-\not{k}+\not{l})\,\gamma_\mu\,(m_b\not{v}+\not{l}+m_b)\,\gamma^\rho}_{\substack{\text{soft}\Downarrow\text{region} \\ 2\,n\cdot p\,m_b\,\gamma_\mu}}\,b(p_b)\,,$$

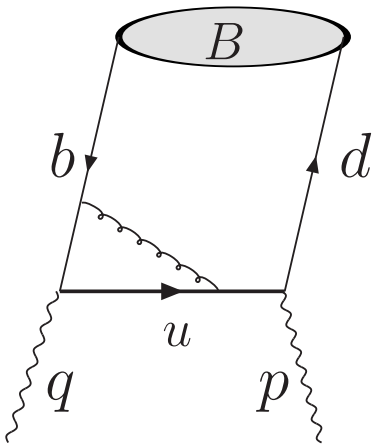
  - Leading contributions from the hard, hard-collinear and soft regions.
  - Important that the collinear region absent at leading power.

- Soft subtraction [Wilson-line Feynman rules]:

$$\Phi^{(1)}_{B,weak}\otimes T^{(0)} = \frac{g_s^2 C_F}{2(\bar{n}\cdot p-\omega)} \int \frac{d^D l}{(2\pi)^D} \frac{1}{[\bar{n}\cdot(p-k+l)+i0][v\cdot l+i0][l^2+i0]}$$
$$\bar{d}(k)\,\not{n}\,\gamma_5\,\not{\bar{n}}\,\gamma_\mu\,b(p_b)\,.$$

Precise cancellation of the soft contribution.

Sample calculation: the weak vertex diagram



Hard contribution:

$$\Pi^{(1),h}_{\mu,weak} = \frac{\alpha_s C_F}{4\pi}\left\{\bar{n}_\mu\left[\frac{1}{\varepsilon^2}+\frac{1}{\varepsilon}\left(2\ln\frac{\mu}{n\cdot p}+1\right)+2\ln^2\frac{\mu}{n\cdot p}\right.\right.$$
$$\left.+2\ln\frac{\mu}{m_b}-\ln^2 r-2\,\mathrm{Li}_2\left(-\frac{\bar{r}}{r}\right)+\frac{2-r}{r-1}\ln r+\frac{\pi^2}{12}+3\right]$$
$$\left.+n_\mu\left[\frac{1}{r-1}\left(1+\frac{r}{\bar{r}}\ln r\right)\right]\right\}\widetilde{\Pi}^{(0)}(n\cdot p,\bar{n}\cdot p)\,.$$

Hard function only from the weak vertex diagram and renormalization of the external b-quark field.

- Hard-collinear contribution:

$$\Pi^{(1),hc}_{\mu,weak} = \frac{g_s^2 C_F}{2(\bar{n}\cdot p-\omega)}\int\frac{d^D l}{(2\pi)^D}\frac{2m_b\, n\cdot(p+l)}{[n\cdot(p+l)\,\bar{n}\cdot(p-k+l)+l_\perp^2+i0][m_b\, n\cdot l+i0][l^2+i0]}$$
$$\bar{d}(k)\,\not{n}\,\gamma_5\,\not{\bar{n}}\,\gamma_\mu\, b(p_b)\,.$$

Can be also obtained from the hard-collinear contribution in $B\to\gamma\ell\nu$.

- Compute the hard-collinear contribution with the light-cone projector [Beneke, Feldmann, 2001]:

$$\mathscr{M}^B_{\beta\,\alpha} = -\frac{i\tilde{f}_B\, m_B}{4}\left\{\frac{1+\not{v}}{2}\left[\phi_B^+(\omega)\not{n}+\phi_B^-(\omega)\not{\bar{n}}-\frac{2}{D-2}\,\omega\,\phi_B^-(\omega)\,\gamma_\perp^\mu\frac{\partial}{\partial k_\perp^\mu}\right]\right\}_{\alpha\beta}\,.$$

Factorization of the correlation function

- Factorization of the correlation function.

$$\widetilde{\Pi}(n\cdot p,\bar{n}\cdot p) \quad = \quad \tilde{f}_B\, m_B \sum_{k=\pm} \widetilde{C}^{(k)}(n\cdot p,\mu) \int \frac{d\omega}{\omega-\bar{n}\cdot p}\, \widetilde{J}^{(k)}\left(\frac{\mu^2}{n\cdot p\,\omega},\frac{\omega}{\bar{n}\cdot p}\right) \phi_B^{(k)}(\omega,\mu)\,.$$

  Similar factorization formula for $\Pi(n\cdot p,\bar{n}\cdot p)$.

- Hard functions:

$$C^{(+)}(n\cdot p,\mu) = \tilde{C}^{(+)}(n\cdot p,\mu) = 1, \qquad C^{(-)}(n\cdot p,\mu) = \frac{\alpha_s\, C_F}{4\,\pi}\,\frac{1}{\bar{r}}\left[\frac{r}{\bar{r}}\ln r + 1\right], \qquad r = \frac{n\cdot p}{m_b}\,,$$

$$\tilde{C}^{(-)}(n\cdot p,\mu) = 1 - \frac{\alpha_s\, C_F}{4\,\pi}\left[2\ln^2\frac{\mu}{n\cdot p} + 5\ln\frac{\mu}{n\cdot p} - \ln^2 r - 2\mathrm{Li}_2\left(\frac{r-1}{r}\right) + \frac{2-r}{r-1}\ln r + \frac{\pi^2}{12} + 5\right].$$

- Hard matching coefficient of the QCD weak current [Bauer et al, 2001; Beneke et al, 2004]:

$$\bar{q}\,\gamma_\mu\, b \to \left[C_4\,\bar{n}_\mu + C_5\, v_\mu\right]\bar{\xi}_{\bar{n}}\, b_v + \ldots.$$

  Perturbative matching coefficients independent of the external states $\Rightarrow$

$$\boxed{C^{(-)} = \frac{1}{2}\,C_5, \qquad \tilde{C}^{(-)} = C_4 + \frac{1}{2}\,C_5\,.}$$

Factorization of the correlation function

- Jet functions [Y.M.W and Y.L. Shen, 2015]:

$$
\begin{aligned}
J^{(+)}(\bar{n}\cdot p,\omega,\mu) &= \frac{1}{r}\tilde{J}^{(+)}(\bar{n}\cdot p,\omega,\mu) = \frac{\alpha_s C_F}{4\pi}\left(1-\frac{\bar{n}\cdot p}{\omega}\right)\ln\left(1-\frac{\omega}{\bar{n}\cdot p}\right),\\
J^{(-)}(\bar{n}\cdot p,\omega,\mu) &= 1,\\
\tilde{J}^{(-)}(\bar{n}\cdot p,\omega,\mu) &= 1+\frac{\alpha_s C_F}{4\pi}\left[\ln^2\frac{\mu^2}{n\cdot p(\omega-\bar{n}\cdot p)}-2\ln\frac{\bar{n}\cdot p-\omega}{\bar{n}\cdot p}\ln\frac{\mu^2}{n\cdot p(\omega-\bar{n}\cdot p)}\right.\\
&\quad\left.-\ln^2\frac{\bar{n}\cdot p-\omega}{\bar{n}\cdot p}-\left(1+\frac{2\bar{n}\cdot p}{\omega}\right)\ln\frac{\bar{n}\cdot p-\omega}{\bar{n}\cdot p}-\frac{\pi^2}{6}-1\right].
\end{aligned}
$$

In agreement with the jet functions computed in SCET [De Fazio, Feldmann and Hurth, 2008].

- Cancellation of the factorization-scale dependence:

$$
\begin{aligned}
\frac{d}{d\ln\mu}\tilde{C}^{(-)}(n\cdot p,\mu) &= -\frac{\alpha_s C_F}{4\pi}\left[4\ln\frac{\mu}{n\cdot p}+5\right]\tilde{C}^{(-)}(n\cdot p,\mu),\\
\frac{d}{d\ln\mu}\tilde{J}^{(-)}(\bar{n}\cdot p,\omega,\mu) &= \frac{\alpha_s C_F}{4\pi}\left[4\ln\frac{\mu^2}{n\cdot p\,\omega}\right]\tilde{J}^{(-)}(\bar{n}\cdot p,\omega,\mu)\\
&\quad+\frac{\alpha_s C_F}{4\pi}\int_0^\infty d\omega'\,\omega\,\Gamma^{(1)}(\omega,\omega',\mu)\,\tilde{J}^{(-)}(\bar{n}\cdot p,\omega',\mu),\\
\frac{d}{d\ln\mu}\left[\tilde{f}_B\,\phi_B^-(\omega,\mu)\right] &= -\frac{\alpha_s C_F}{4\pi}\left[4\ln\frac{\mu}{\omega}-5\right]\left[\tilde{f}_B\,\phi_B^-(\omega,\mu)\right]\\
&\quad-\frac{\alpha_s C_F}{4\pi}\int_0^\infty d\omega'\,\omega\,\Gamma^{(1)}(\omega,\omega',\mu)\,\left[\tilde{f}_B\,\phi_B^-(\omega',\mu)\right],
\end{aligned}
$$

NLL resummation for $B \to \pi$ form factors

- No common scale μ to avoid the large logarithms in the hard functions, the jet functions, $\tilde{f}_B(\mu)$ and the B-meson DAs.
- Resummation for the hard functions [see also, Beneke and Rohrwild, 2011]:

$$\begin{aligned} \tilde{C}^{(-)}(n\cdot p,\mu) &= U_1(n\cdot p,\mu_{h1},\mu)\,\tilde{C}^{(-)}(n\cdot p,\mu_{h1})\,, \\ \tilde{f}_B(\mu) &= U_2(\mu_{h2},\mu)\tilde{f}_B(\mu_{h2})\,. \end{aligned}$$

RG evolutions at NLL:

$$\frac{d}{d\ln\mu}\,U_1(n\cdot p,\mu_{h1},\mu) = \left[-\underbrace{\Gamma_{\rm cusp}(\alpha_s)}_{\text{at } \mathcal{O}(\alpha_s^3)}\ln\frac{\mu}{n\cdot p}+\underbrace{\gamma(\alpha_s)}_{\text{at } \mathcal{O}(\alpha_s^2)\ [\text{Asatrian et al, 2008; Bell, 2008}]}\right]U_1(n\cdot p,\mu_{h1},\mu)\,,$$

$$\frac{d}{d\ln\mu}\,U_2(\mu_{h2},\mu) = \underbrace{\tilde{\gamma}(\alpha_s)}_{\text{at } \mathcal{O}(\alpha_s^2)[\text{Ji and Musolf, 1991; Broadhurst and Grozin 1991}]}\,U_2(\mu_{h2},\mu)\,.$$

- Resummation of parametrically large logarithms in the B-meson DAs ignored.

$$\frac{d\phi_B^-(\omega,\mu)}{d\ln\mu} = -\left[\Gamma_{\rm cusp}(\alpha_s)\ln\frac{\mu}{\omega}+\underbrace{\gamma_-(\alpha_s)}_{\text{at } \mathcal{O}(\alpha_s^2)}\right]\phi_B^-(\omega,\mu)-\omega\int_0^\infty d\eta\,\underbrace{\Gamma(\omega,\eta,\alpha_s)}_{\text{at } \mathcal{O}(\alpha_s^2)}\,\phi_B^-(\eta,\mu)\,.$$

  - Unclear whether the structure of the renormalization kernel holds at $\mathcal{O}(\alpha_s^2)$.
  - Whether Bessel functions are still the eigenfunctions of the evolution kernel at $\mathcal{O}(\alpha_s^2)$?

$B \to \pi$ form factors from the B-meson LCSR

- B-meson LCSR @ NLL:

$$
\begin{aligned}
& f_\pi \; e^{-m_\pi^2/(n\cdot p\,\omega_M)} \left\{ \frac{n\cdot p}{m_B} f_{B\pi}^{+}(n\cdot p)\,,\; f_{B\pi}^{0}(n\cdot p) \right\} \\
& = \left[U_2(\mu_{h2},\mu)\tilde{f}_B(\mu_{h2})\right] \int_0^{\omega_s} d\omega'\; e^{-\omega'/\omega_M} \Bigg[r\,\widetilde{C}^{(+)}(n\cdot p,\mu)\,\phi_{B,\mathrm{eff}}^{+}(\omega',\mu) \\
& \quad + \left[U_1(n\cdot p,\mu_{h1},\mu)\,\widetilde{C}^{(-)}(n\cdot p,\mu_{h1})\right] \phi_{B,\mathrm{eff}}^{-}(\omega',\mu) \\
& \quad \pm \frac{n\cdot p - m_B}{m_B} \left(C^{(+)}(n\cdot p,\mu)\, \underbrace{\phi_{B,\mathrm{eff}}^{+}(\omega',\mu)}_{\text{“hc” correction}} + \underbrace{C^{(-)}(n\cdot p,\mu)}_{\text{hard correction}}\, \phi_B^{-}(\omega',\mu) \right) \Bigg] .
\end{aligned}
$$

- Effective DAs:

$$
\phi_{B,\mathrm{eff}}^{+}(\omega',\mu) = 0 + \frac{\alpha_s C_F}{4\pi} \int_{\omega'}^{\infty} \frac{d\omega}{\omega}\, \phi_B^{+}(\omega,\mu)\,,
$$

$$
\begin{aligned}
\phi_{B,\mathrm{eff}}^{-}(\omega',\mu) = \phi_B^{-}(\omega',\mu) + \frac{\alpha_s C_F}{4\pi} \Bigg\{ & \int_0^{\omega'} d\omega \left[\frac{1}{\omega-\omega'} \left(2\ln\frac{\mu^2}{n\cdot p\,\omega} - 4\ln\frac{\omega'-\omega}{\omega'} \right) \right]_+ \phi_B^{-}(\omega,\mu) \\
& - \int_{\omega'}^{\infty} d\omega \left[\ln^2\frac{\mu^2}{n\cdot p\,\omega} - \left(2\ln\frac{\mu^2}{n\cdot p\,\omega} + 3 \right) \ln\frac{\omega-\omega'}{\omega'} + 2\ln\frac{\omega}{\omega'} + \frac{\pi^2}{6} - 1 \right] \frac{d\phi_B^{-}(\omega,\mu)}{d\omega} \Bigg\} .
\end{aligned}
$$

- Power counting: $\omega \sim \Lambda\,, \qquad \omega_s \sim \omega_M \sim O(\Lambda^2/m_b) \Rightarrow \omega' \sim O(\Lambda^2/m_b)\,,$

  $\Rightarrow \ln\left((\omega-\omega')/\omega'\right) \sim \ln\left(\omega/\omega'\right) \sim \ln\left(m_b/\Lambda\right).$

The B-meson LCDAs

- Light-cone distribution amplitudes of the B meson:

$$\phi^+_{B,\mathrm{I}}(\omega,\mu_0) = \frac{\omega}{\omega_0^2}\, e^{-\omega/\omega_0}\,, \qquad \text{[Grozin and Neubert, 1997]}$$

$$\phi^+_{B,\mathrm{II}}(\omega,\mu_0) = \frac{1}{4\pi\,\omega_0}\,\frac{k}{k^2+1}\left[\frac{1}{k^2+1} - \frac{2(\sigma_B-1)}{\pi^2}\ln k\right],\ k = \frac{\omega}{1\ \mathrm{GeV}}\,, \qquad \text{[Braun et al, 2004]}$$

$$\phi^+_{B,\mathrm{III}}(\omega,\mu_0) = \frac{2\omega^2}{\omega_0\,\omega_1^2}\, e^{-(\omega/\omega_1)^2}\,,\ \omega_1 = \frac{2\omega_0}{2\sqrt{\pi}}\,, \qquad \text{[De Fazio, Feldmann, Hurth, 2008]}$$

$$\phi^+_{B,\mathrm{IV}}(\omega,\mu_0) = \frac{\omega}{\omega_0\,\omega_2}\,\frac{\omega_2-\omega}{\sqrt{\omega(2\omega_2-\omega)}}\,,\ \omega_2 = \frac{4\omega_0}{4-\pi}\,, \qquad \text{[De Fazio, Feldmann, Hurth, 2008]}$$

Perturbative constraints on the B-meson DAs at large ω [Feldmann, Lange and Y.M.W, 2014].

- The shape of $f^+_{B\pi}(q^2)$ less model dependent.

  blue curve from pion LCSR, solid, dotted, dashed and dot-dashed curves from Model-I, II, III and IV.

  fitting $f^+_{B\pi}(q^2=0) = 0.28 \pm 0.03$ from pion LCSR $\Rightarrow$

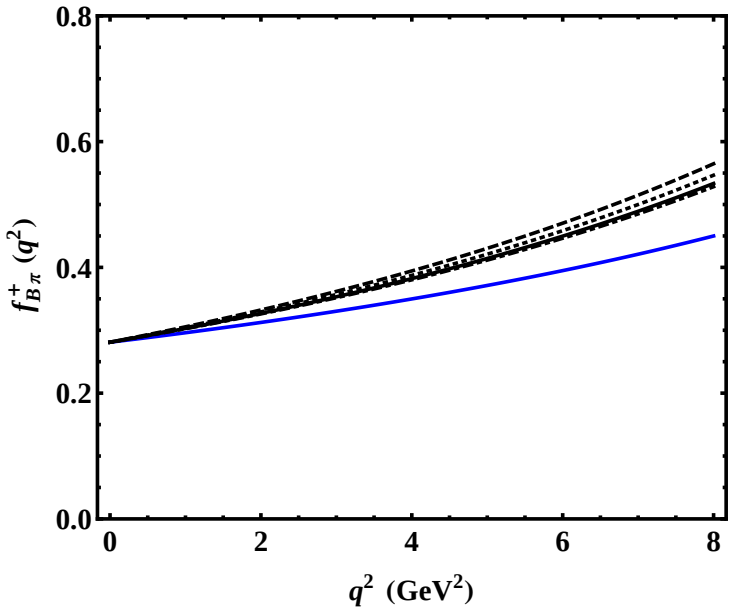
  Model-I: $\omega_0 = 360^{+40}_{-30}\,\mathrm{MeV}$,

  Model-II: $\omega_0 = 375^{+40}_{-35}\,\mathrm{MeV}$,

  Model-III: $\omega_0 = 395^{+35}_{-30}\,\mathrm{MeV}$,

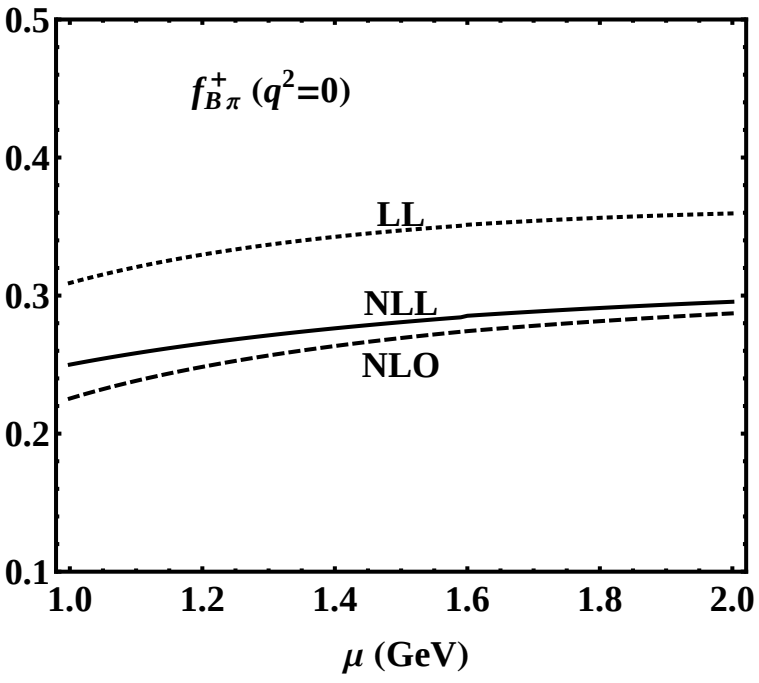
  Model-IV: $\omega_0 = 310^{+40}_{-30}\,\mathrm{MeV}$.

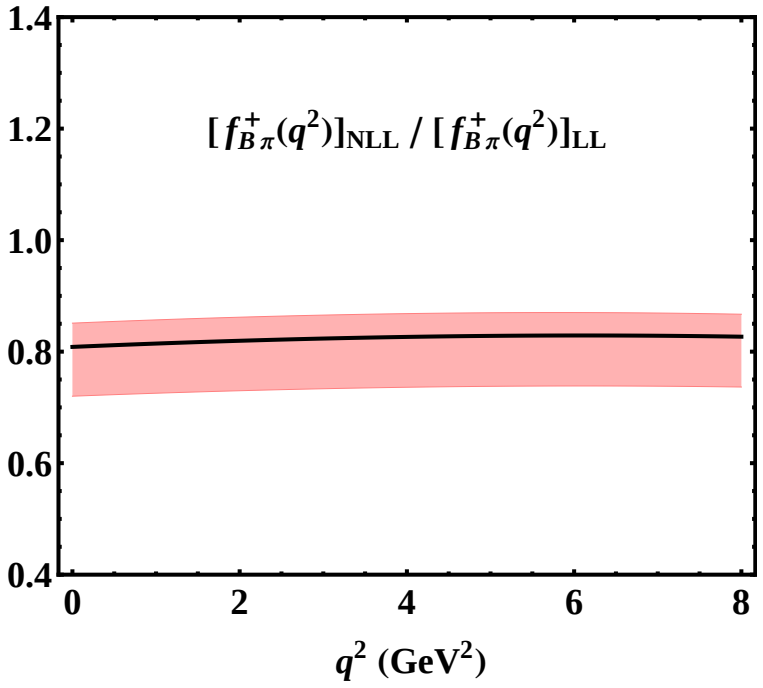
  Determination of ω_0 from $B \to \gamma\ell\nu$.



$B \to \pi$ form factors from the B-meson LCSR

- Factorization scale dependence and radiative correction:





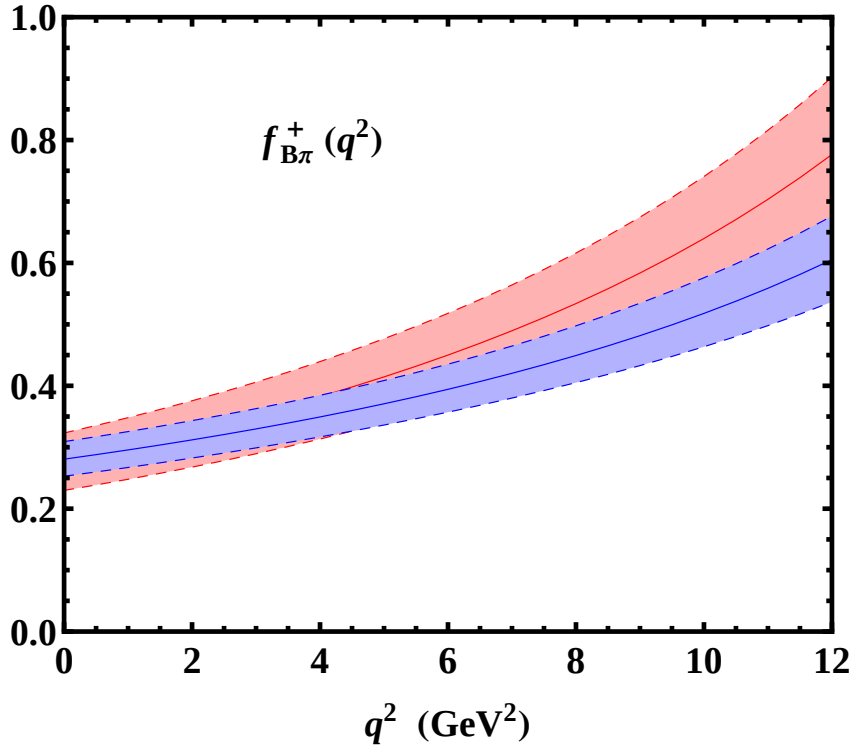
  - Dominant radiative effect from the NLO QCD correction instead of the QCD resummation.
  - Resummation improvement does stabilize the factorization scale dependence.
  - Radiative effect can induce 20 % reduction of the form factor.

$B \to \pi$ form factors from the B-meson LCSR

- The predicted form factor $f^+_{B\pi}(q^2)$:

Pink band: B-meson LCSR @ NLO,
Blue band: pion LCSR @ NLO.

Rapidly increasing $f^+_{B\pi}(q^2)$ from B-meson LCSR:
(i) Different pattern of higher power/twist contributions?
(ii) Different quark-hadron quality ansatz?



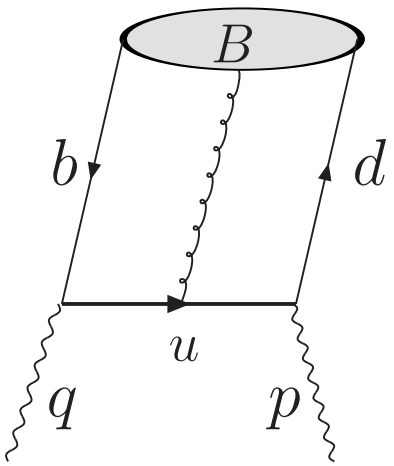
- Exclusive $|V_{ub}|$ from B-meson LCSR @ NLO [Y.M.W and Y.L. Shen, 2015]:

$$|V_{ub}| = \left(3.05^{+0.54}_{-0.38}\Big|_{\text{th.}} \pm 0.09\Big|_{\text{exp.}}\right) \times 10^{-3}\,.$$

- Exclusive $|V_{ub}|$ from $B \to \tau\nu$ [Belle, combined two tagging methods, arXiv: 1503.05613]:

$$|V_{ub}| = \left(3.28^{+0.37}_{-0.42}\right) \times 10^{-3}\,.$$

Three-particle B-meson DA's contributions



- Quark propagator in the background gluon field [Balitsky and Braun, 1988]:

$$\langle 0|T\{q(x)\,,\bar{q}(0)\}|0\rangle|_G$$
$$\supset -\frac{i}{16\pi^2}\frac{1}{x^2}\int_0^1 du\,\left[\not{x}\,\sigma_{\alpha\beta}-4\,i\,u\,x_\alpha\,\gamma_\beta\right]$$
$$\times\underbrace{G^{\alpha\beta}(ux)}_{\equiv g_s\,T^a\,G^a_{\mu\nu}}\,.$$

- Three-particle B-meson DA's contributions [Khodjamirian, Mannel and Offen, 2007]:

$$\langle 0|\bar{u}_\alpha(x)\,G_{\lambda\rho}(ux)\,b_v(0)|B^-(v)\rangle\big|_{x^2=0}$$
$$=\frac{F_{\rm stat}(\mu)}{4}\int_0^\infty d\omega\int_0^\infty d\xi\,e^{-i(\omega+u\,\xi)\,v\cdot x}\left[(1+\not{v})\left\{(v_\lambda\,\gamma_\rho-v_\rho\,\gamma_\lambda)\,[\Psi_A(\omega,\xi)-\Psi_V(\omega,\xi)]\right.\right.$$
$$\left.\left.-i\,\sigma_{\lambda\rho}\,\Psi_V(\omega,\xi)-\frac{x_\lambda\,v_\rho-x_\rho\,v_\lambda}{v\cdot x}\,X_A(\omega,\xi)+\frac{x_\lambda\,\gamma_\rho-x_\rho\,\gamma_\lambda}{v\cdot x}\,Y_A(\omega,\xi)\right\}\gamma_5\right].$$

See also [Kawamura, Kodaira, Qiao and Tanaka, 2001; Geye and Witzel, 2013].

- Work in the coordinate space, compute the $\int d^4x\,e^{ip\cdot x}$ integral, and do the power counting.

Three-particle B-meson DA [Braun, Manashov and Offen, 2015]

- One-loop renormalization of the three-particle DA $\tilde{\Psi}_3(z_1,z_2)$:

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g_s)\frac{\partial}{\partial g_s}+\frac{\alpha_s}{2\pi}\mathscr{H}\right]F_{\rm stat}(\mu)\,\tilde{\Psi}_3(z_1,z_2,\mu)=0\,,$$

$$\tilde{\Psi}_3(z_1,z_2)\equiv\tilde{\Psi}_A(z_1,z_2)-\tilde{\Psi}_V(z_1,z_2)\,,\qquad \mathscr{H}=N_c\,H_0+N_c^{-1}\,\delta H\,.$$

An additional "hidden" symmetry for H_0: $[\hat{Q}_1,\hat{Q}_2]=[\hat{Q}_1,H_0]=[\hat{Q}_2,H_0]=0$.

- Eigenfunctions:

$$H_0\,Y_{s,x}(z_1,z_2)=E(s,x)\,Y_{s,x}(z_1,z_2)\,,\qquad Y_{s,i/2}(z_1,z_2)=\frac{i\,s^2}{z_1^2\,z_2^3}\int_0^1 du\,u\,\bar{u}\,e^{is(u/z_1+\bar{u}/z_2)}\,,$$

$$\Delta E=\underbrace{E(s,0)}_{\text{continuous spectrum}}-\underbrace{E(s,i/2)}_{\text{ground state}}=2\,\psi(3/2)-\psi(2)-\psi(1)\,.$$

- Expansion, "asymptotics" and RGE of ϕ_B^-:

$$\tilde{\Psi}_3(z_1,z_2,\mu)=\int_0^\infty ds\left[\underbrace{\eta_0(s,\mu)\,Y_{s,i/2}(z_1,z_2)}_{\text{"asymptotical" behaviour}}+\frac{1}{2}\int_{-\infty}^{+\infty}dx\,\eta(s,x,\mu)\,Y_{s,x}(z_1,z_2)\right].$$

$$\Psi_3^{\rm asy}(\omega_1,\omega_2,\mu)=\frac{\omega_1\,\omega_2}{\omega_1+\omega_2}\left[f_1(\omega_1+\omega_2)-f_0(\omega_1+\omega_2)\right]+\omega_1\left[f_1(\omega_1+\omega_2)-f_1(\omega_1)\right].$$

$$\phi_B^-(\omega,\mu)=\int_0^\infty ds\left[\hat{\phi}_B^+(s,\mu)+\underbrace{\eta_0(s,\mu)}_{\text{continuous spectrum of }\tilde{\Psi}_3(z_1,z_2,\mu)\text{ irrelevant}}\right]J_0(2\sqrt{\omega\,s})\,.$$

Concluding Remarks

- Heavy-to-light form factors as fundamental inputs of describing heavy hadron decays.
  - Factorization properties not fully understood in QCD.
  - Can reproduce the factorization structure in QCD light-cone sum rules.
  - Diagrammatic factorization of the correlation functions with the method of regions.
  - Different $B \to \pi$ form factor shapes from different sum rules at leading twist.

- Further developments:
  - Higher Fock-state contributions to $B \to \pi$ form factors.
    $\Rightarrow$ Renormalization properties of three-particle B-meson DAs.
  - Power suppressed contributions from the B-meson LCSR.
  - Factorization of $\xi_P(E)$ in $\mathrm{SCET_{II}}$.
  - Factorization of $B \to D^{(*)}\ell\nu$ and $\Lambda_b \to \Lambda\ell\ell$ in QCD.