QCD@work 2016
Martina Franca, 2016/06/29

Chiral Phase Transition in Electric and Magnetic Fields

Critical temperature and effects of a chiral density background

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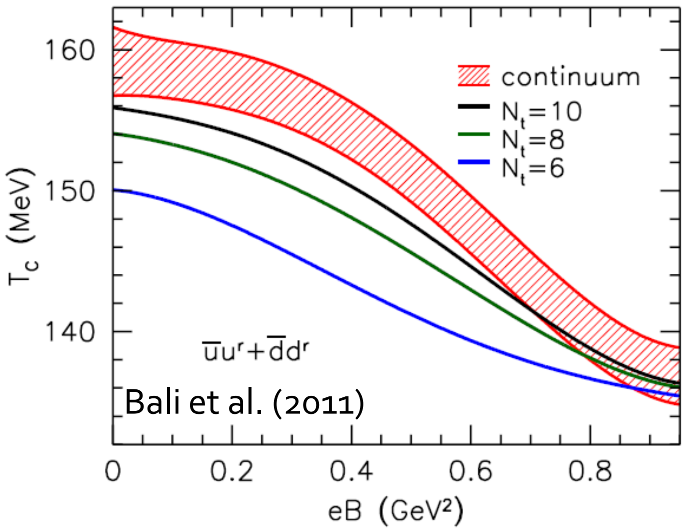
Plan of the talk

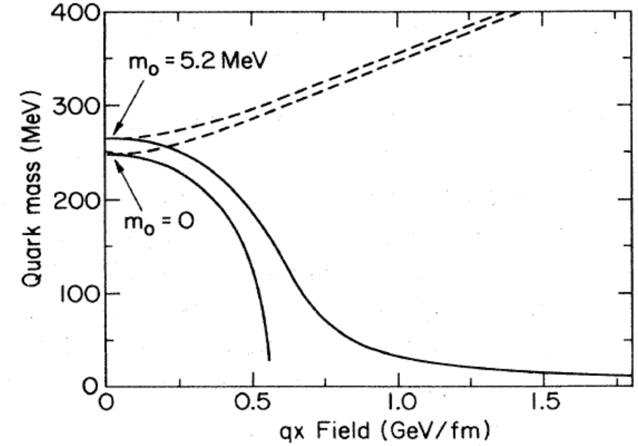
- Introduction
- NJL model in external electric and magnetic fields
- ***Results***
  - Critical temperature for chiral phase transition
  - Chiral density effect on critical temperature
- Conclusions

QCD in external fields

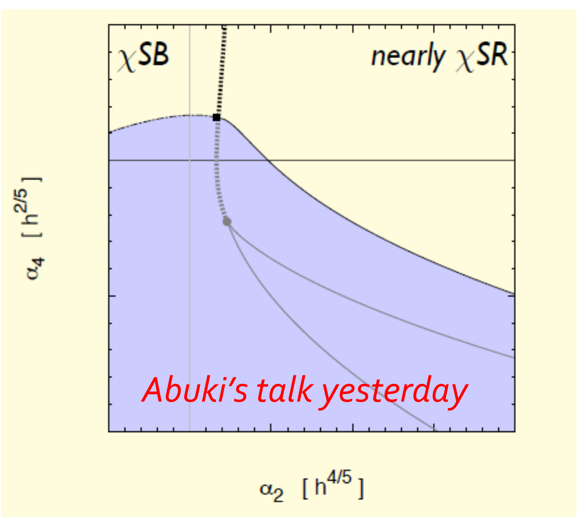
Critical temperature of QCD in magnetic field

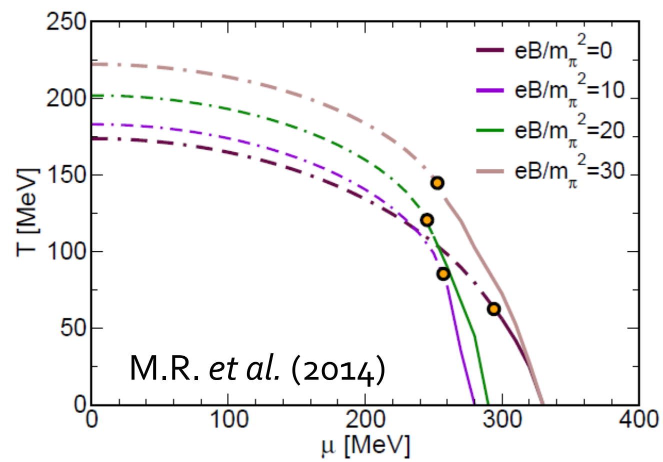


Effects of fields on chiral symmetry breaking



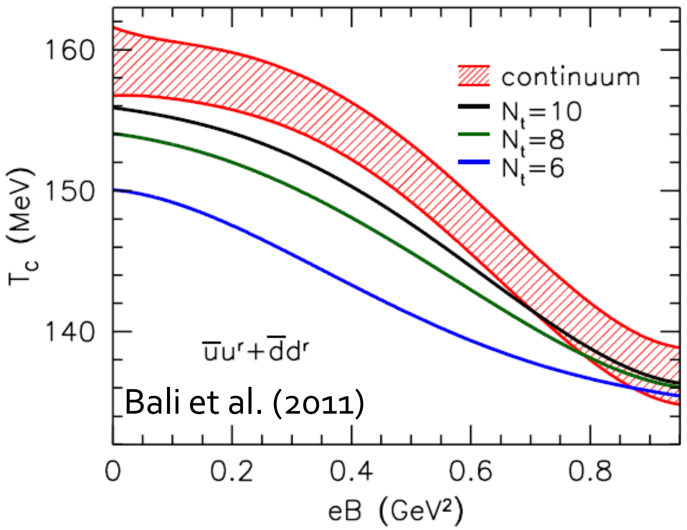
Klevansky and Lemmer (1989)





QCD in external fields

Critical temperature of QCD in magnetic field



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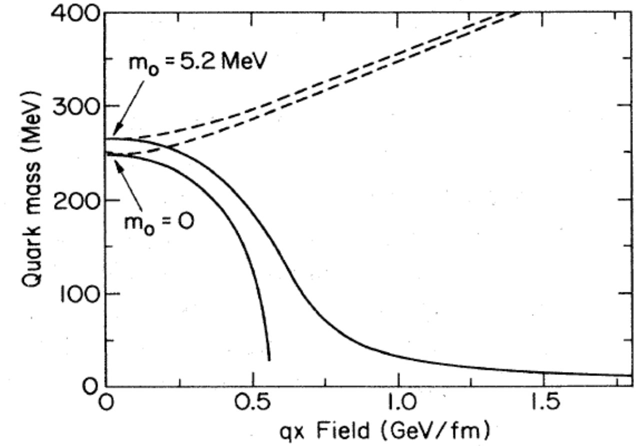
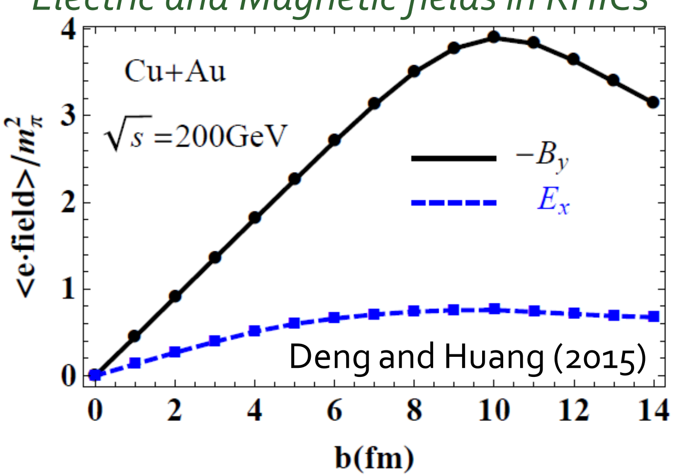


Klevansky and Lemmer (1989)

Electric and Magnetic fields in RHICs



Chiral Magnetic Effect in RHICs

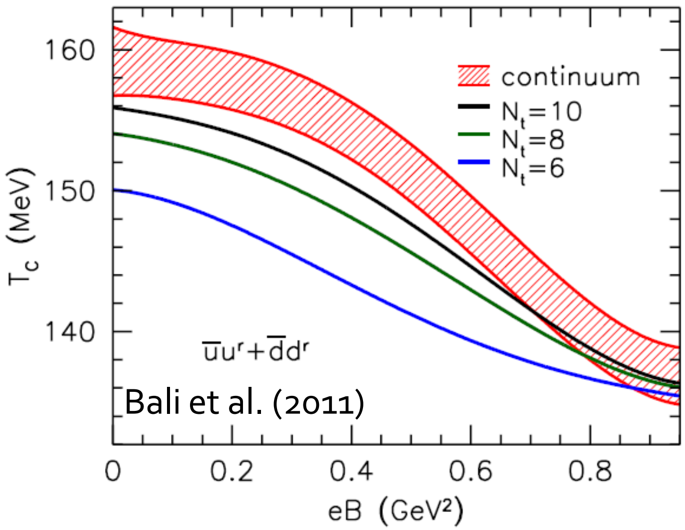
Charge separation due to the interplay of:

- Magnetic fields
- QCD topological configurations and ABJ anomaly

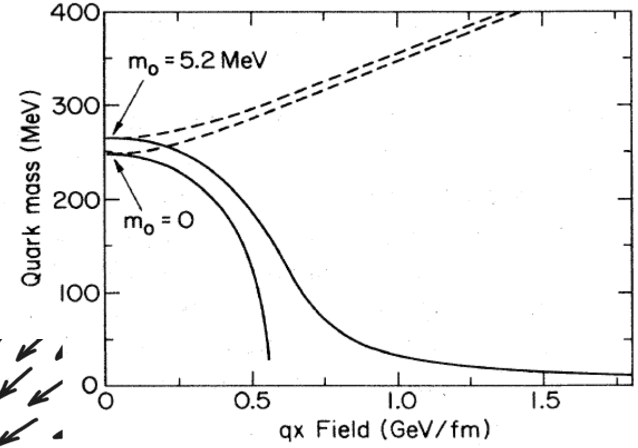
Kharzeev, McLerran and Warringa (2008)
Fukushima, Kharzeev and Warringa (2008)
STAR collaboration (2015)
ALICE collaboration (2014)
Kharzeev *et al.* (2014)

Focus

Critical temperature of QCD in magnetic field



Effects of fields on chiral symmetry breaking



Klevansky and Lemmer (1989)

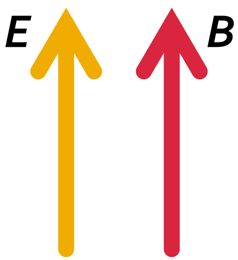
Effects of external electric ***E*** and magnetic ***B*** fields on the QCD chiral phase transition:

- Simultaneous ***E*** and ***B***, ***E***||***B***, interesting for chiral density n_5 dynamical production

Use of an effective model rather than full QCD.

NJL model in external fields

We consider quark matter in the background of *parallel* ***electric (E)*** and ***magnetic (B)*** fields:



$$\mathcal{L} = \bar{\psi}\left(i\not{D} - m_0\right)\psi + G\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi\right)^2\right]$$

NJL model in external fields

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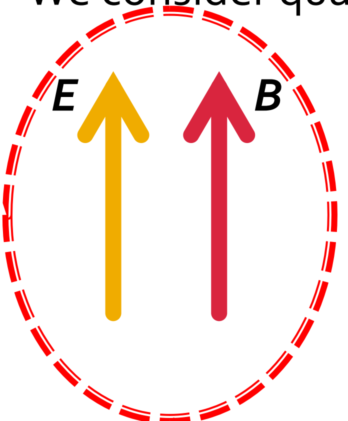

$$\mathcal{L} = \bar{\psi}\left(i\not{D} - m_0\right)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \boldsymbol{\tau}\psi)^2\right]$$

Important for:

.) ***Dynamical production of chiral density***

.) *Model for a QCD sphaleron*

.) *Condensed matter experiments*

.) *Simplified model of Glasma*

NJL model in external fields

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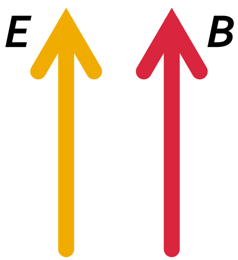

$$\mathcal{L} = \bar{\psi}\left(i\not{D} - m_0\right)\psi + \boxed{G\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi\right)^2\right]}$$

Responsible of *spontaneous chiral symmetry breaking*
and of
Interaction with collective modes

NJL model in external fields

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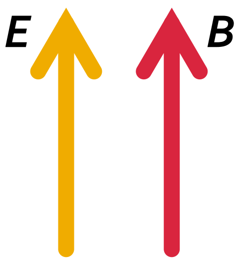

$$\mathcal{L} = \boxed{\bar{\psi}\left(i\not{D} - m_0\right)\psi} + \boxed{G\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi\right)^2\right]}$$

Responsible of *spontaneous chiral symmetry breaking*
and of
Interaction with collective modes

Kinetic term
and
Interaction with external fields via QED covariant derivative

NJL model in external fields

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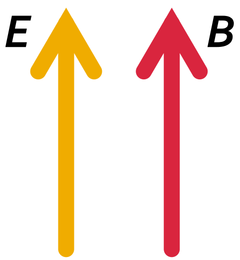


$$\mathcal{L} = \bar{\psi}\left(i\not{D} - m_0\right)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)^2\right]$$

Mean field thermodynamic potential at finite temperature:

$$\Omega = \frac{(M_q - m_0)^2}{4G} - \frac{1}{\beta V}\mathrm{Tr}\log\beta(i\not{D} - M_q)$$

$$\boxed{M_q = m_0 - 2G\langle\bar{\psi}\psi\rangle}$$

M_q computed by solving the ***gap equation***:

$$\boxed{\partial\Omega/\partial M_q = 0} \quad \Longleftrightarrow \quad \frac{M_q - m_0}{2G} - \frac{1}{\beta V}\mathrm{Tr}\mathcal{S}(x,x') = 0$$

$$\frac{M_q - m_0}{2G} = M_q\frac{N_c}{4\pi^2}\sum_f\int_0^\infty\frac{ds}{s^2}e^{-M_q^2 s}\mathcal{F}(s) + M_q\frac{N_c N_f}{4\pi^2}\int_{1/\Lambda^2}^\infty\frac{ds}{s^2}e^{-M_q^2 s},$$

$$\mathcal{F}(s) = \theta_3\left(\frac{\pi}{2}, e^{-|\mathcal{A}|}\right)\frac{q_f eBs}{\tanh(q_f eBs)}\frac{q_f eEs}{\tan(q_f eEs)} - 1$$

See also:
Cao and Huang (2015), Klevansky (1989), Schwinger (1951)

M.R. and Peng (2016)
M.R., Lu and Peng (in preparation)

Chiral density equilibration

$$n_5 \equiv n_R - n_L$$

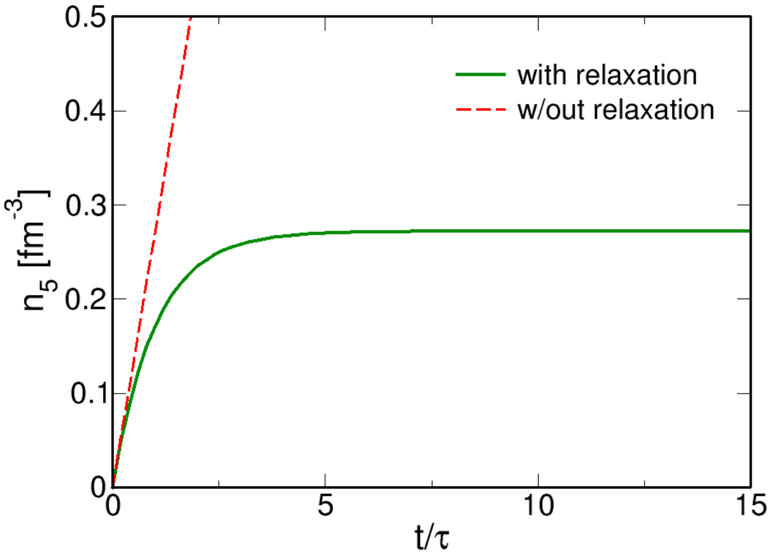
ABJ anomaly $\boldsymbol{E} \parallel \boldsymbol{B}$
[Adler (1969), Bell and Jackiw (1969), Warringa (2012)]

$$\frac{dn_5}{dt} = \frac{q_f^2 (eE)(eB)}{2\pi^2} e^{-\frac{\pi M^2}{|q_f eE|}}$$

Ethernal production of n_5
Related to Schwinger effect

However, chirality changing processes occurr in the thermal bath on a time scale τ_M

$$\frac{dn_5}{dt} = \frac{q_f^2 (eE)(eB)}{2\pi^2} e^{-\frac{\pi M^2}{|q_f eE|}} - \frac{n_5}{\tau_M}$$



Equilibration of n_5

$$n_5^{\mathrm{eq}} = \frac{q_f^2 (eE)(eB)}{2\pi^2} e^{-\frac{\pi M^2}{|q_f eE|}} \tau_M$$

Introduce the *chiral chemical potential*:

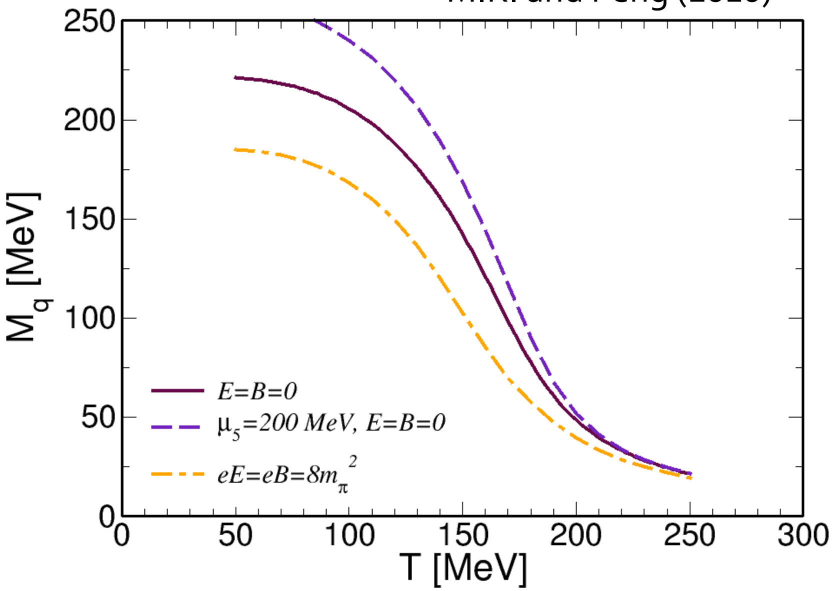
$$n_5^{\mathrm{eq}} = -\frac{\partial \Omega}{\partial \mu_5},$$

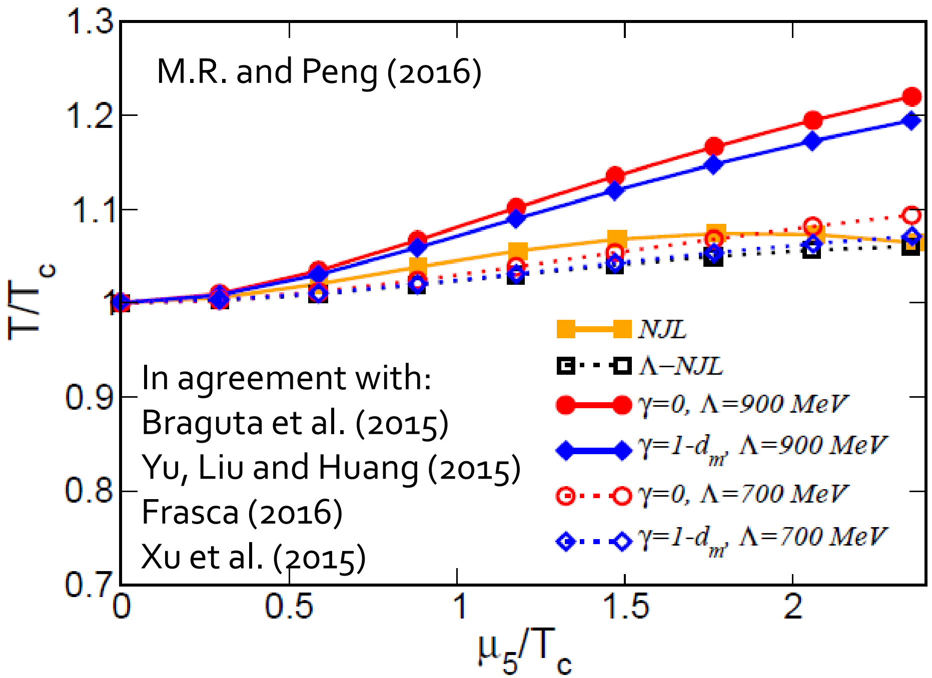
Number Equation

Fields create a medium because of anomaly.

Expected effect of $\mu 5$ on T_c

Chiral chemical potential *increases* the critical temperature of chiral restoration.





Self-consistent computation of μ_5 is necessary to give a firm conclusion about the net effect of the fields on chiral symmetry restoration.

Problem of chiral condensate in a medium made of a chiral imbalanced background, rather than a more common baryon density background.

Relaxation time of chiral density

$$n_5 \equiv n_R - n_L$$

By definition:

$$\frac{dn_5}{dt} = N_c N_f \int \frac{d^3p}{(2\pi)^3}\left(\frac{df_R}{dt} - \frac{df_L}{dt}\right)$$

$f_{L/R}$: distribution functions of L/R quarks

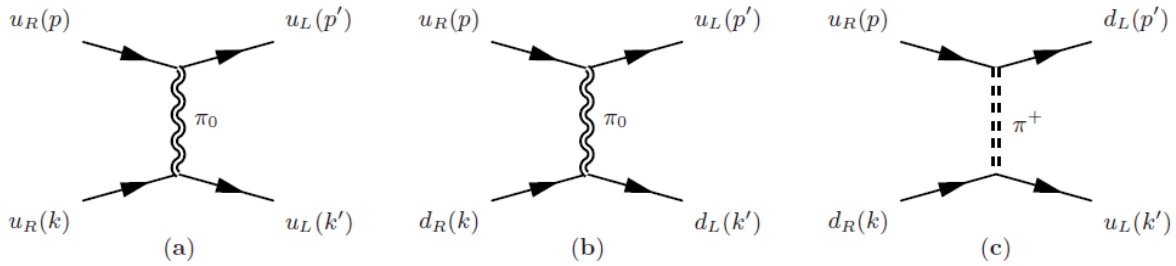
$f_{L/R}$ satisfy the kinetic equation:

$$\frac{df_R(p)}{dt} = \int d\Pi \frac{(2\pi)^4\delta^4(p+k-p'-k')}{2E_p}|\mathcal{M}|^2 F$$

$$d\Pi = \frac{d^3k}{(2\pi)^3 2E_k}\frac{d^3k'}{(2\pi)^3 2E'_k}\frac{d^3p'}{(2\pi)^3 2E'_p}$$

$$\begin{aligned} F(p,k,p',k') \;=\; & f_L(p')f_L(k')[1-f_R(p)][1-f_R(k)] \\ & - f_R(p)f_R(k)[1-f_L(p')][1-f_L(k')] \end{aligned}$$

$$|\mathcal{M}|^2$$

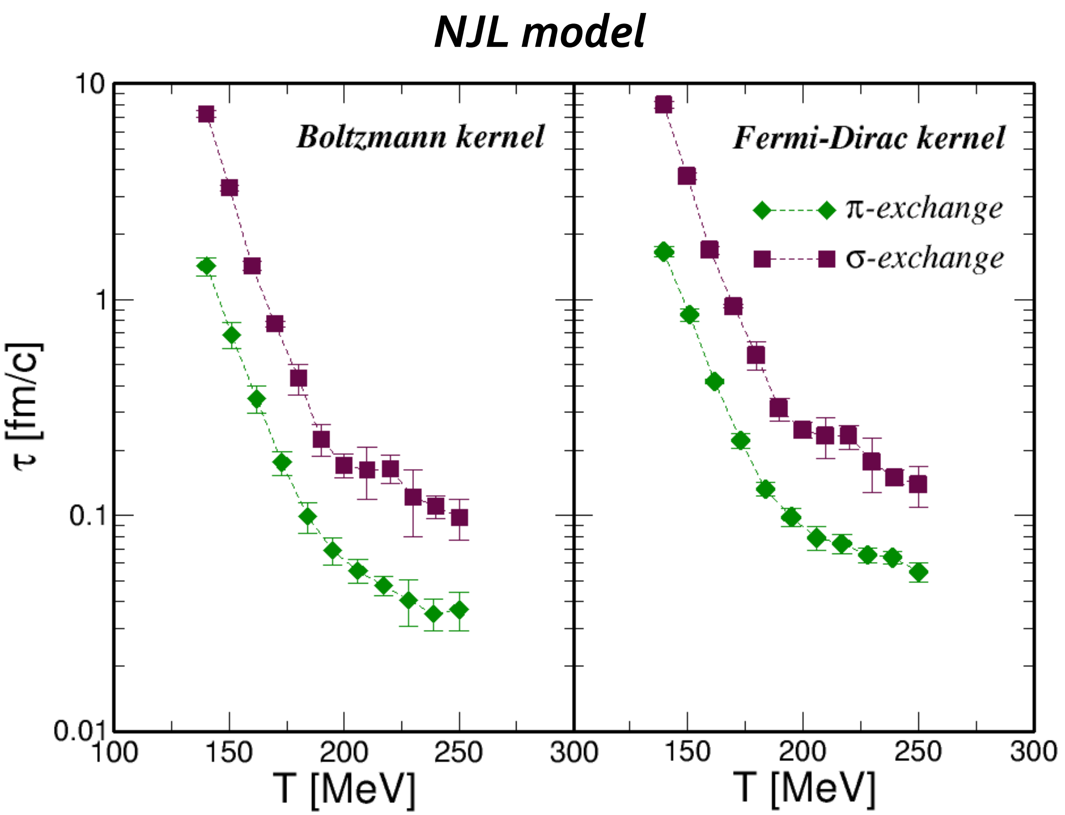


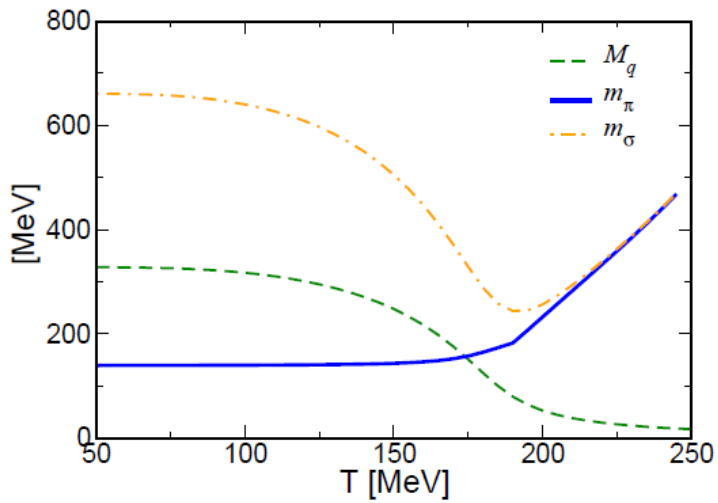
Rate: $\Gamma = -\frac{1}{n_5}\frac{dn_5}{dt}$

Relaxation time: $\tau = 1/\Gamma$

M.R., Chernodub and Peng (2016)

Relaxation time of chiral density





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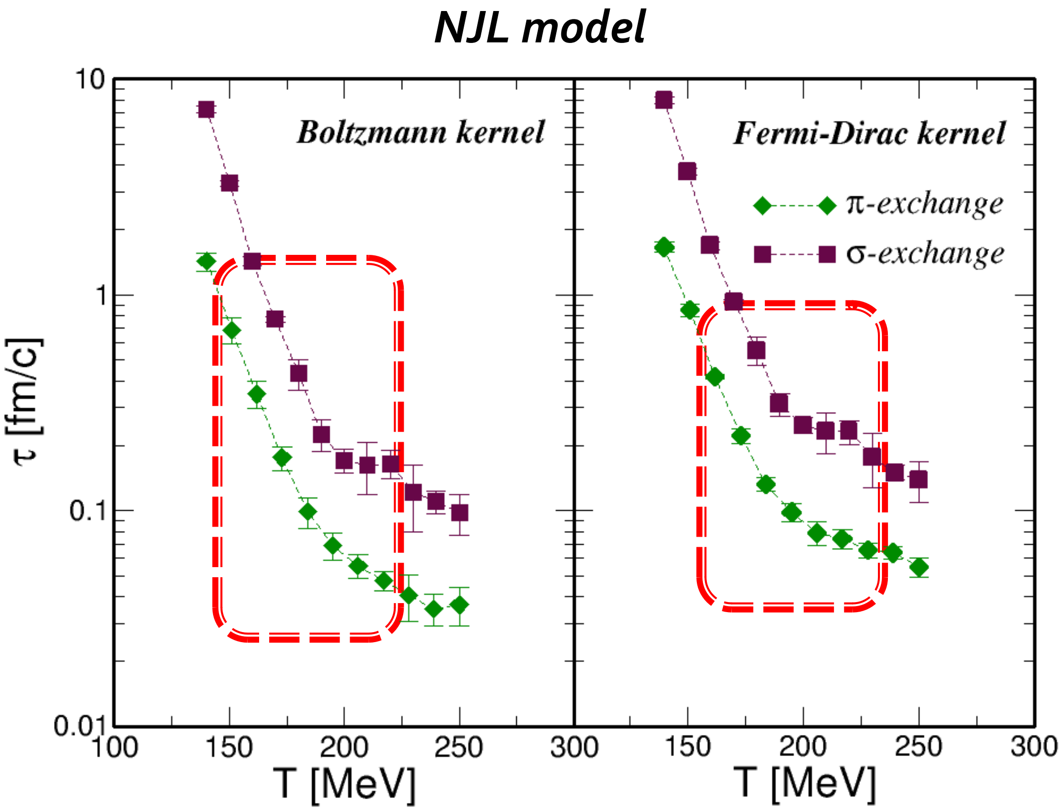
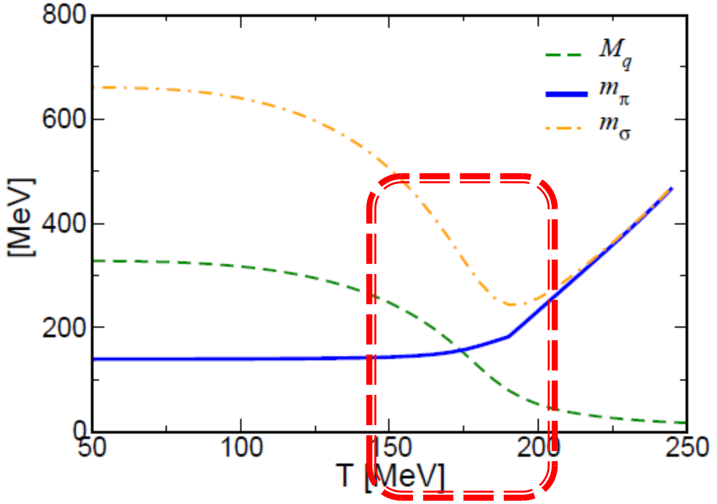
Relaxation time of chiral density





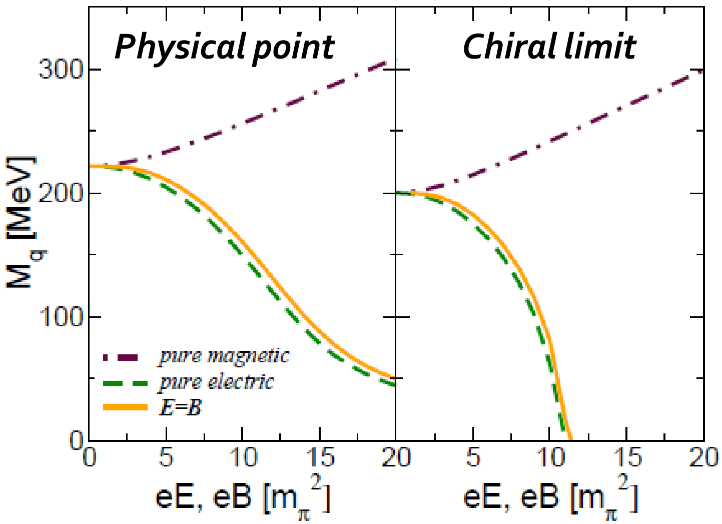
In the chiral crossover region:

$$\boxed{\tau \simeq 0.1 \div 2\ \mathrm{fm/c}}$$

RESULTS

M.R. and Peng (2016)

Inverse catalysis, T=0



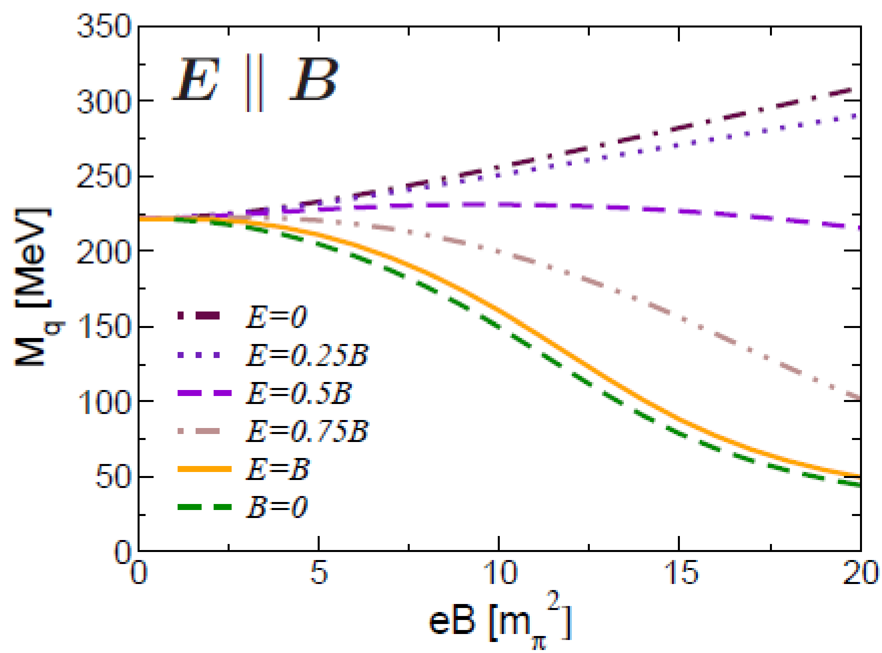
Competition of electric and magnetic fields

$$\boxed{\delta m = \frac{1}{2N_f|E_i(-M_0^2/\Lambda^2)|}(\Upsilon_1 + \Upsilon_2)}$$

$$\Upsilon_1 = \frac{q_u^2 + q_d^2}{3M_0^3}\mathcal{I}_1,$$

$$\Upsilon_2 = -\frac{q_u^4 + q_d^4}{45M_0^7}(\mathcal{I}_1^2 + 7\mathcal{I}_2^2),$$

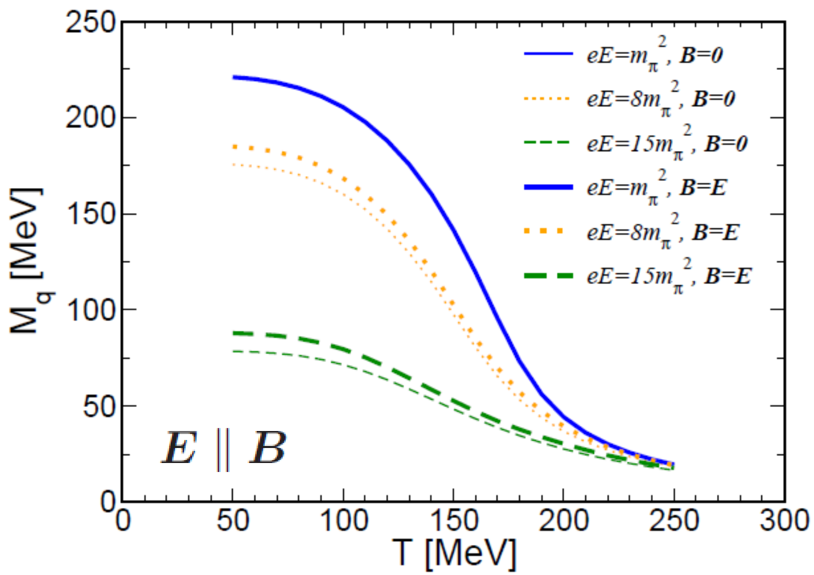
$$\mathcal{I}_1 \equiv (eB)^2 - (eE)^2$$
$$\mathcal{I}_2 \equiv (eE)(eB)$$



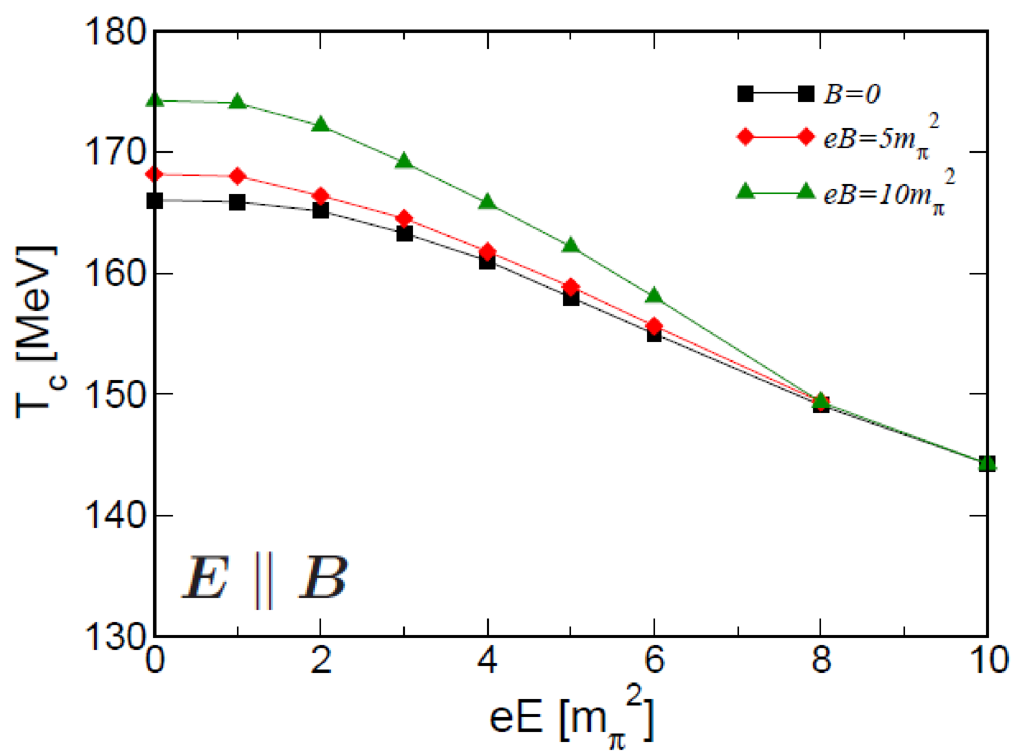
In agreement with:
Cao and Huang (2015), Klevansky (1989), Gorbar et al. (1998)

M.R. and Peng (2016)

The suggested phase diagram

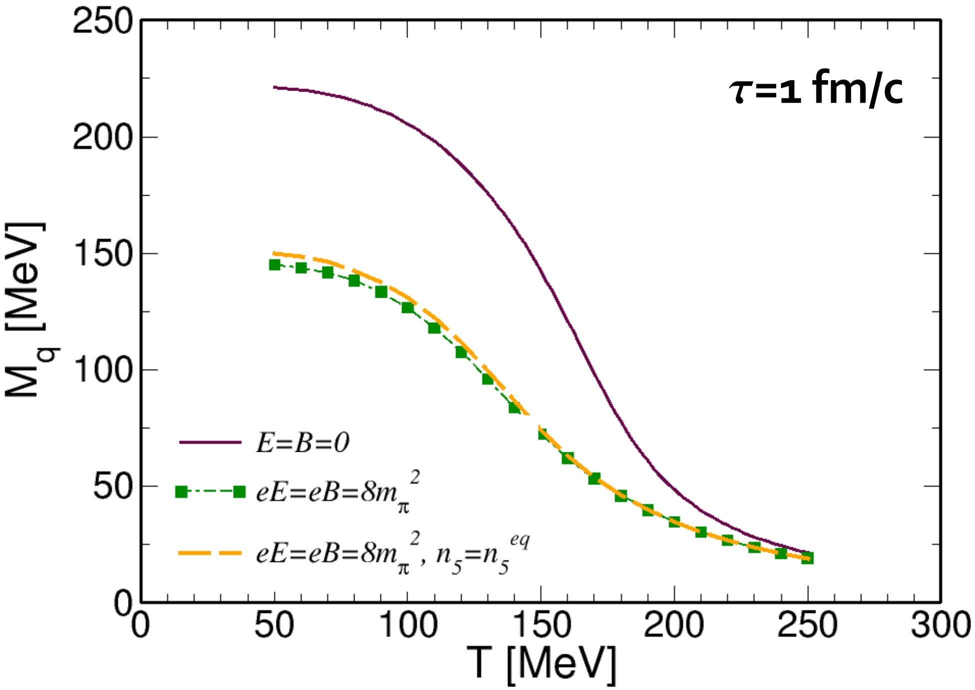


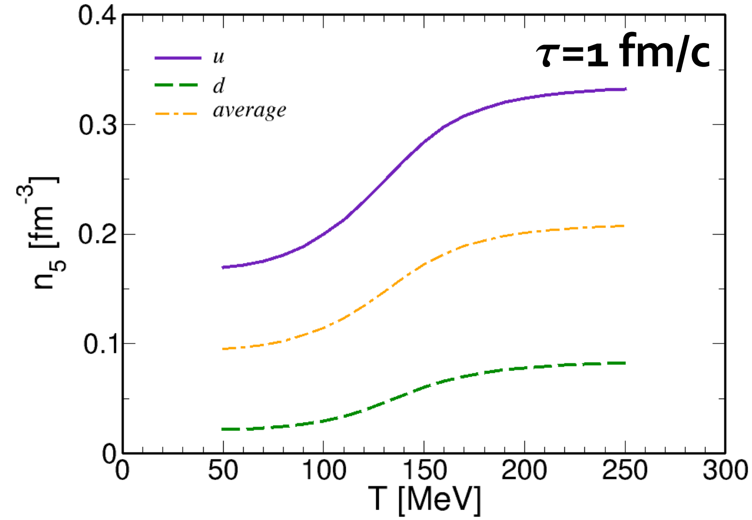
*Simultaneous **E** and **B** with **E**||**B** induce inverse catalysis of chiral symmetry breaking.*

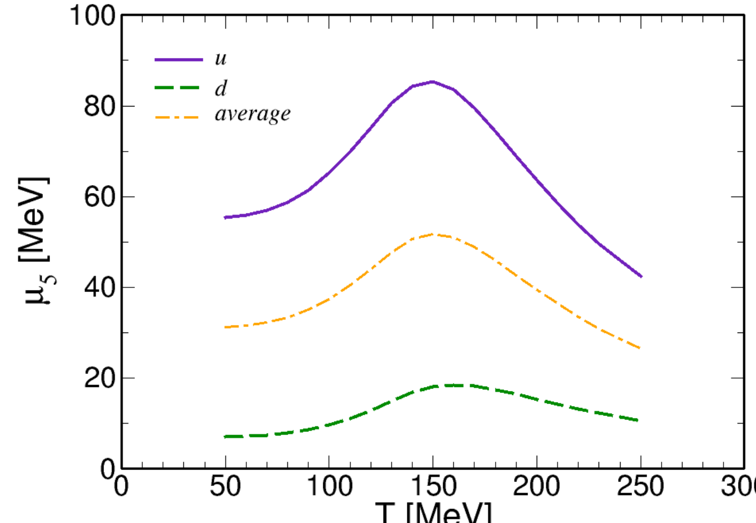


*Even with a large magnetic background, a modest electric field induces **inverse catalysis of chiral symmetry breaking**.*

Chiral density at phase transition







- *Chiral densities of u and d quarks differ because of different electric charges:*

$$\frac{n_{5u}^{\mathrm{eq}}}{n_{5d}^{\mathrm{eq}}} = \frac{q_u^2}{q_d^2} e^{-\frac{\pi M^2}{|eE|}\left(\frac{1}{q_u} - \frac{1}{|q_d|}\right)}$$

- *Increase of T_c due to μ_5 is not enough to spoil the inverse catalysis induced by the electric field.*

Conclusions

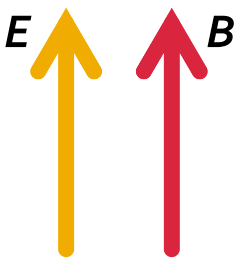
- Electric field acts as an inhibitor of chiral symmetry breaking.
- ***E***||***B*** inhibites chiral symmetry and leads to a lowering of T_c.
- Chiral density is produced dynamically by ***E***||***B*** and equilibrates within few fm/c.
- Chiral density n_5, dynamically produced, does not change drastically the phase diagram.

Thanks for your attention.

APPENDIX

NJL model in external fields

We consider quark matter in the background of *parallel* ***electric (E)*** and ***magnetic (B)*** fields:



$$\mathcal{L} = \bar{\psi}\left(i\not{D} - m_0\right)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)^2\right]$$

Mean field thermodynamic potential at finite temperature:

$$\Omega = \frac{(M_q - m_0)^2}{4G} - \frac{1}{\beta V}\mathrm{Tr}\log\beta(i\not{D} - M_q)$$

$$\boxed{M_q = m_0 - 2G\langle\bar{\psi}\psi\rangle}$$

M_q computed by solving the ***gap equation***:

$$\boxed{\partial\Omega/\partial M_q = 0} \iff \frac{M_q - m_0}{2G} - \frac{1}{\beta V}\mathrm{Tr}\mathcal{S}(x, x') = 0$$

$$\frac{M_q - m_0}{2G} = M_q\frac{N_c}{4\pi^2}\sum_f\int_0^\infty\frac{ds}{s^2}e^{-M_q^2 s}\mathcal{F}(s) + M_q\frac{N_c N_f}{4\pi^2}\int_{1/\Lambda^2}^\infty\frac{ds}{s^2}e^{-M_q^2 s},$$

$$\mathcal{F}(s) = \theta_3\left(\frac{\pi}{2}, e^{-|\mathcal{A}|}\right)\frac{q_f eBs}{\tanh(q_f eBs)}\boxed{\frac{q_f eEs}{\tan(q_f eEs)}} - 1$$

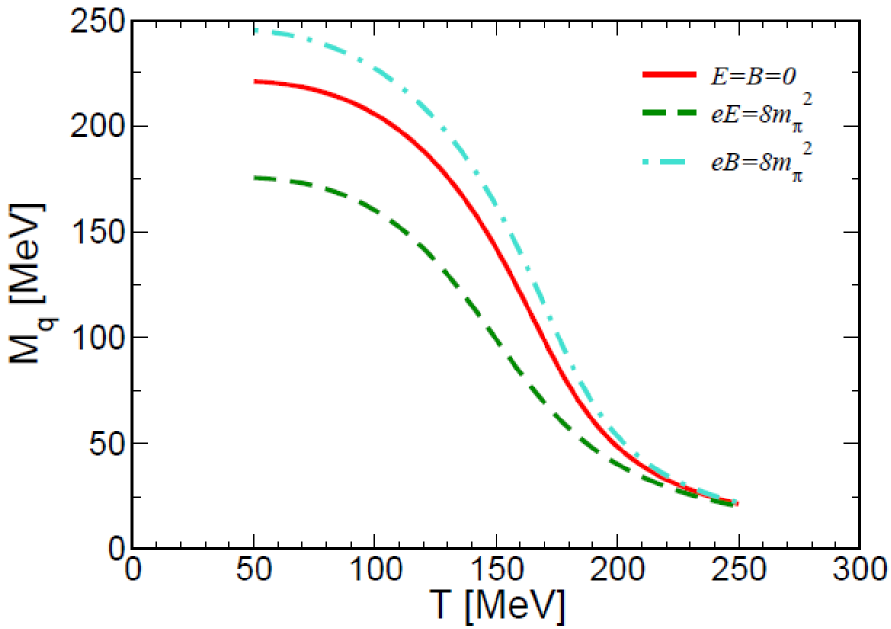
Simple poles: $|q_f eEs| = n\pi, \quad n = 1, 2, \ldots$

Vacuum instability (Schwinger effect)

See also:
Cao and Huang (2015), Klevansky (1989), Schwinger (1951)

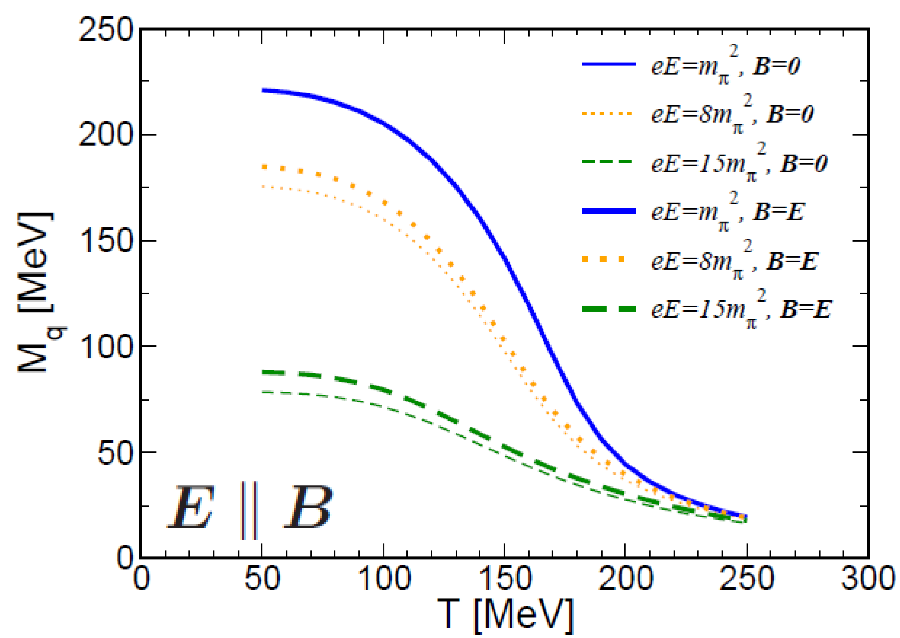
M.R. and Peng (2016)

Inverse catalysis at finite T



Pure electric and magnetic effects

Combined effect of fields

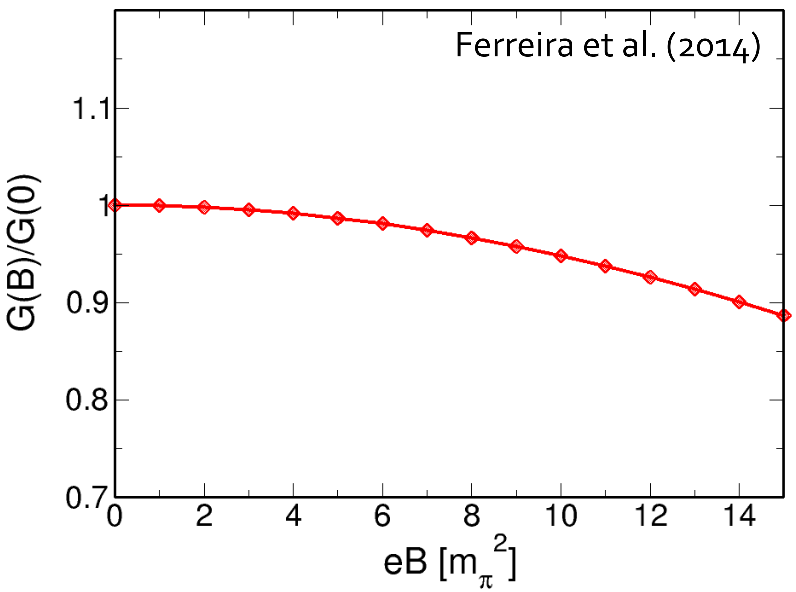


$$\Omega = \frac{\alpha_2}{2} M_q^2 + O(M_q^4),$$

$$\begin{aligned}\alpha_{2,2} &= -\sum_f q_f^2 \frac{N_c}{4\pi^2} \int ds\ \Theta_3(T,s) \frac{(eB)^2 - (eE)^2}{3} \\ &\quad -\sum_f q_f^2 \frac{N_c}{48\pi^2 T^2} \int \frac{ds}{s} e^{-\frac{1}{4T^2 s}} \Delta_3(T,s)(eE)^2\end{aligned}$$

M.R., Chernodub and Peng (in preparation)

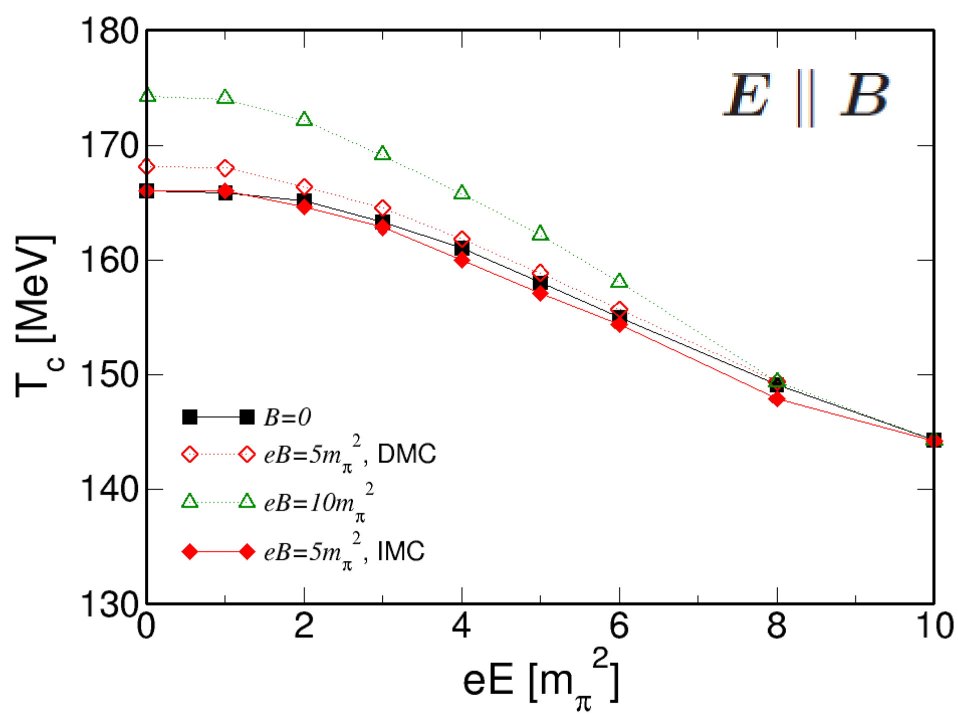
Phase diagram with IMC



Inverse magnetic catalysis (IMC) in the NJL model:

$$G_s(\zeta) = G_s^0 \left(\frac{1 + a\,\zeta^2 + b\,\zeta^3}{1 + c\,\zeta^2 + d\,\zeta^4} \right) \qquad \zeta = eB/\Lambda_{QCD}^2$$

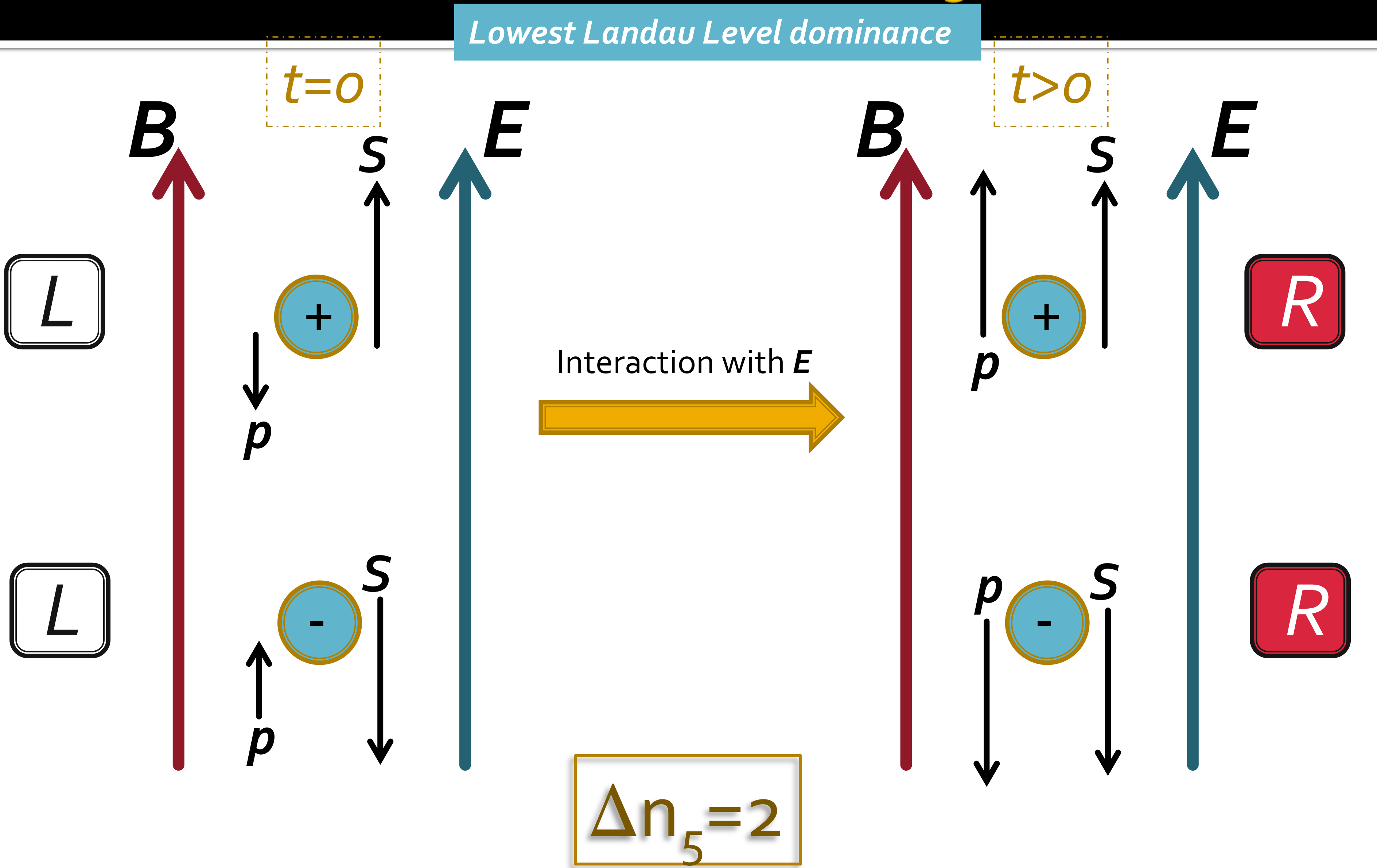
Parameters fixed to reproduce Lattice T_c(B). Bali et al. (2011)



Qualitative picture is almost unchanged:

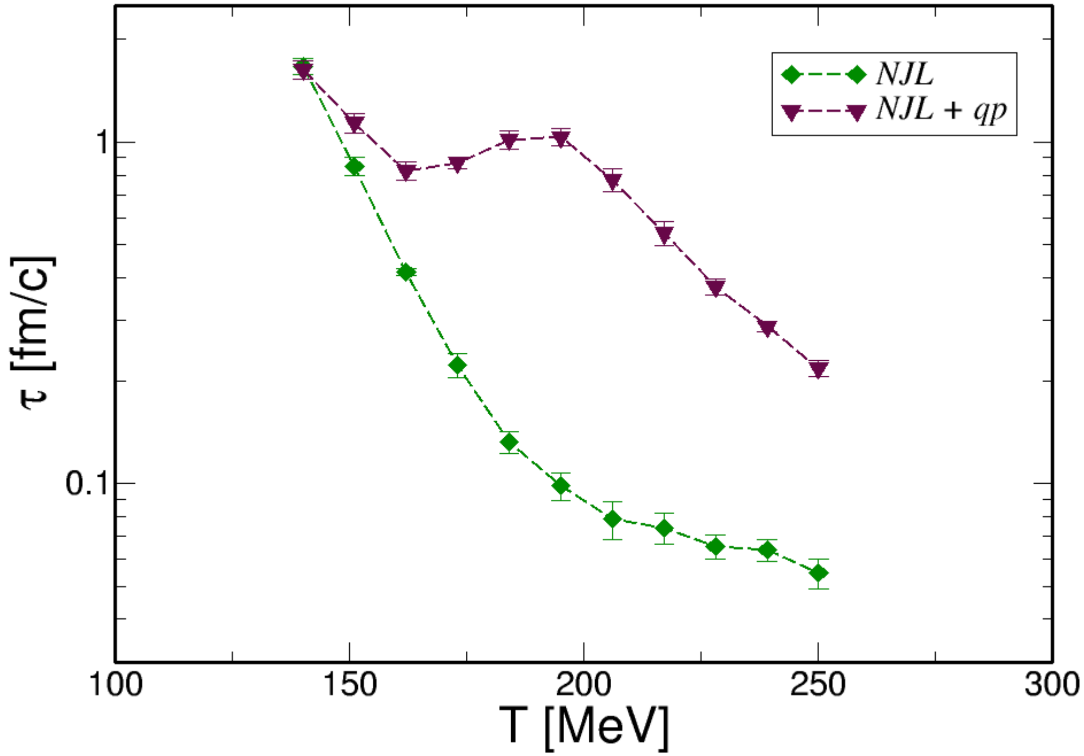
- Both E and B induce inverse catalysis;
- E effect is larger than the B one.

Schwinger effect and n_5

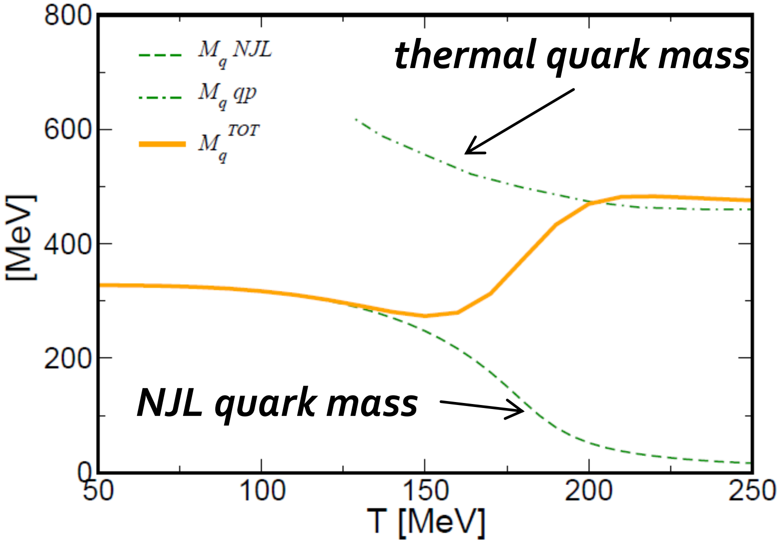


M.R., Chernodub and Peng (in preparation)

Relaxation time of chiral density



Thermal mass adapted from Plumari et al. (2011)



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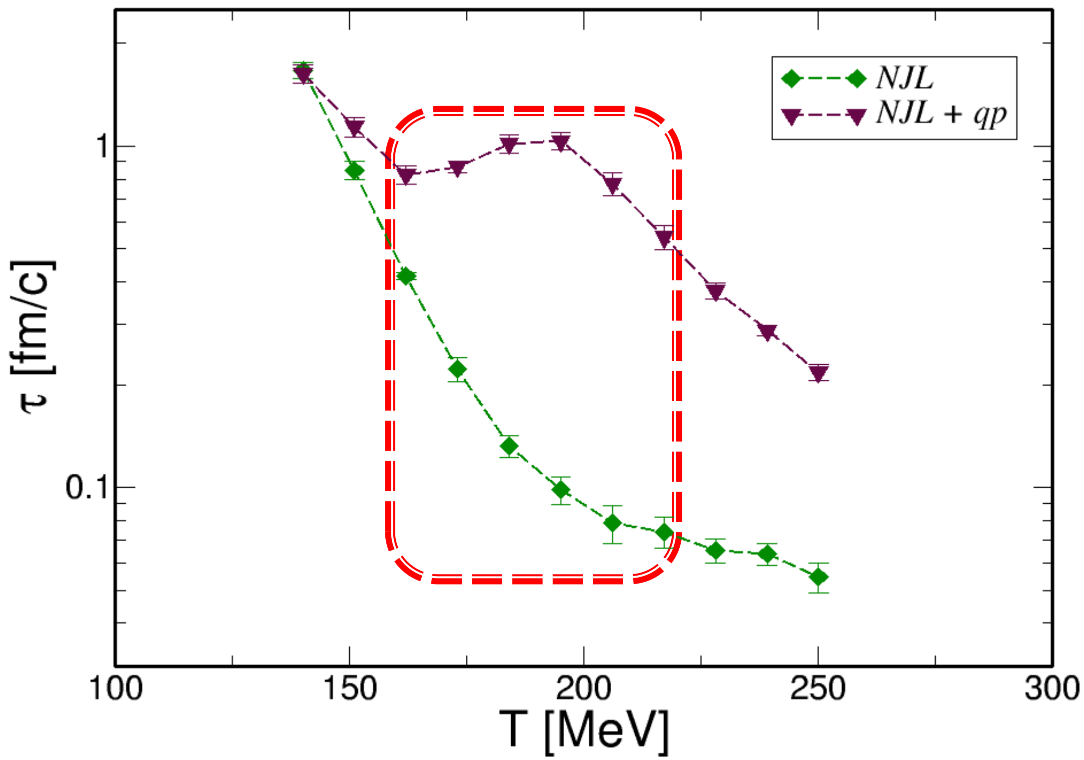

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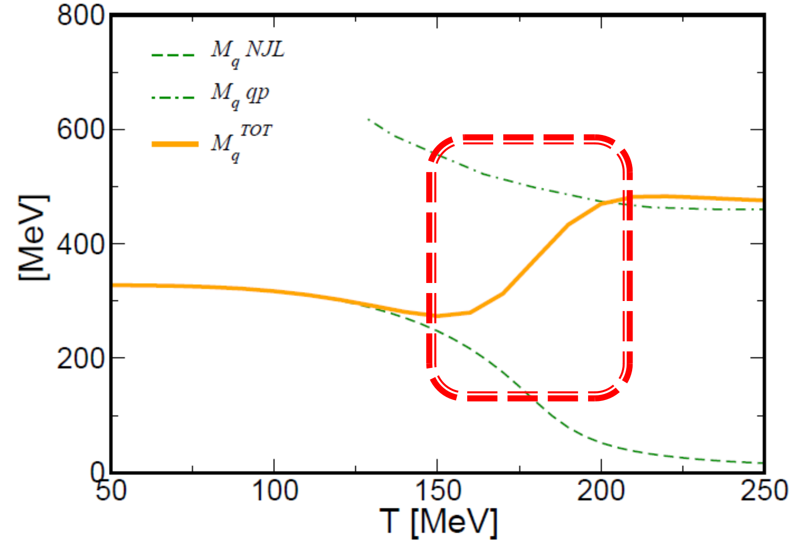

Increase of relaxation time due to increase of quark mass which leads to a partial lowering of the available phase space for collisions.

In the chiral crossover region taking into account also σ:

$$\boxed{\tau \simeq 0.1 \div 2 \ \mathrm{fm/c}}$$