A perturbative approach to the confinement-deconfinement transition

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Based on collaborations with:

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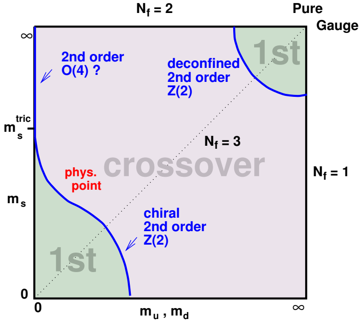
Motivation



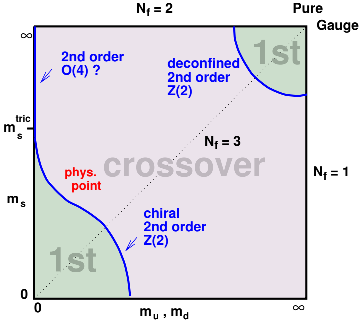
Aim:

Show that various non-trivial aspects of the QCD phase structure can be accessed from perturbative methods.

Here:

Heavy quark limit.

Motivation



Relevant order parameter: Polyakov loop(s)

$$\ell \equiv \frac{1}{3}\left\langle \mathrm{tr}\,\mathcal{P}\, e^{ig\int_0^\beta d\tau\, A_0} \right\rangle \propto e^{-\beta F_{\text{quark}}}$$

$$\bar{\ell} \equiv \frac{1}{3}\left\langle \mathrm{tr}\left(\mathcal{P}\, e^{ig\int_0^\beta d\tau\, A_0}\right)^\dagger \right\rangle \propto e^{-\beta F_{\text{antiquark}}}$$

Relevant symmetry: center-symmetry

if unbroken $\Rightarrow$ $\ell = e^{\pm i2\pi/3}\ell$ $\Rightarrow$ $\ell = 0$

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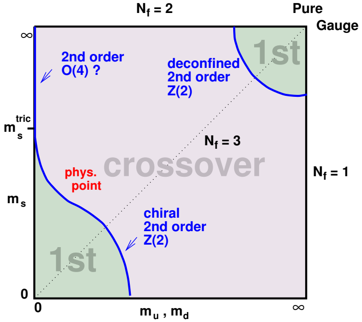

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Relevant symmetry: center-symmetry

if unbroken $\Rightarrow$ $\ell = e^{\pm i2\pi/3}\ell$ $\Rightarrow$ $\ell = 0$

$\Rightarrow$ Study the phase diagram from the Polyakov loop effective potential $V(\ell, \bar{\ell})$.

$\Rightarrow$ Work in a gauge that does not break center-symmetry from the start.

Choice of gauge and gauge-fixing completion

One possibility is to consider the Landau-DeWitt gauge: [Abbot (1981); Braun, Pawlowski, Gies (2010)]

$$S_{\bar{A}}[A,h,c,\bar{c}] = \int_x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{D}_\mu \bar{c}^a (D_\mu c)^a + i h^a \bar{D}_\mu (A^a_\mu - \bar{A}^a_\mu) \right\}$$

where $\bar{D}_\mu \varphi^a \equiv \partial_\mu \varphi^a + g f^{abc} \bar{A}^b \varphi^c$. Does not break center-symmetry!

However, not a complete gauge-fixing due to the presence of Gribov copies.
Not relevant in the UV, but could become important in the IR:

$\rightarrow$ try to model the effect of Gribov copies with the hope that, once a good (and simple) model is found the rest is a perturbative expansion.

Various models on the market:

- Gribov-Zwanziger and refined Gribov-Zwanziger actions;
- Here: massive extensions of Faddeev-Popov actions.

Massive extension of the Landau-DeWitt gauge

We model the effect of Gribov copies by adding a phenomenological mass term:

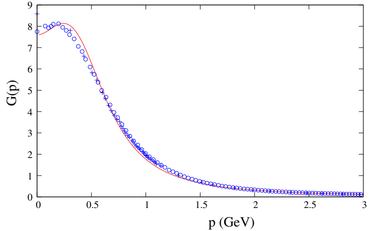
$$\int_x \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{D}_\mu \bar{c}^a (D_\mu c)^a + i h^a \bar{D}_\mu (A^a_\mu - \bar{A}^a_\mu) + \frac{1}{2} m^2 (A^a_\mu - \bar{A}^a_\mu)(A^a_\mu - \bar{A}^a_\mu) \right\}$$

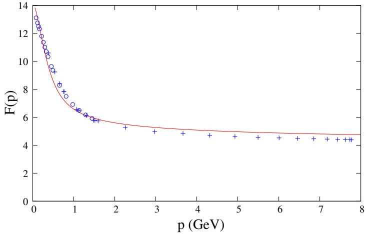
→ Minimal extension, only one additional parameter.

→ It is renormalizable.

→ No IR Landau pole!

Another source of motivation lies on how good the lattice $T = 0$ correlators are reproduced. The fit of the lattice results gives $m \simeq 500\,\mathrm{MeV}$ in the SU(3) case.





[M. Tissier, N. Wschebor, PRD84 (2011); M. Peláez, M. Tissier, N. Wschebor PRD88 (2013)]

One-loop Polyakov-loop potential: expression

$$\left.\begin{aligned} & V_{1\text{loop}}(r_3, r_8)\,,\ \ r_a = g\beta\bar{A}_a^0 \\ & \ell = \frac{e^{-i\frac{r_8}{\sqrt{3}}} + 2e^{i\frac{r_8}{2\sqrt{3}}}\cos(r_3/2)}{3} \\ & \bar{\ell} = \frac{e^{i\frac{r_8}{\sqrt{3}}} + 2e^{-i\frac{r_8}{2\sqrt{3}}}\cos(r_3/2)}{3} \end{aligned}\right\} \quad \Rightarrow \quad V_{1\text{loop}}(\ell, \bar{\ell}) = {\color{green}V_{\text{matter}}(\ell, \bar{\ell})} + {\color{red}V_{\text{glue}}(\ell, \bar{\ell})}$$

One-loop Polyakov-loop potential: expression

$$V_{\rm matter}(\ell,\bar\ell) = -\frac{T}{\pi^2}\int_0^\infty dq\, q^2 \left(\ln\left[1+3\ell\, e^{-\beta(\tilde\varepsilon_q-\mu)}+3\bar\ell\, e^{-2\beta(\tilde\varepsilon_q-\mu)}+e^{-3\beta(\tilde\varepsilon_q-\mu)}\right]\right.$$
$$\left.+\ln\left[1+3\bar\ell\, e^{-\beta(\tilde\varepsilon_q+\mu)}+3\ell\, e^{-2\beta(\tilde\varepsilon_q+\mu)}+e^{-3\beta(\tilde\varepsilon_q+\mu)}\right]\right)$$

$$V_{\rm glue}(\ell,\bar\ell) = \frac{3}{2}\mathcal{W}_m(\ell,\bar\ell) - \frac{1}{2}\mathcal{W}_0(\ell,\bar\ell)$$

$$\boxed{T \gg m,\ V_{\rm glue} \approx \mathcal{W}_0 \quad \text{vs} \quad T \ll m,\ V_{\rm glue} \approx -\frac{1}{2}\mathcal{W}_0}$$

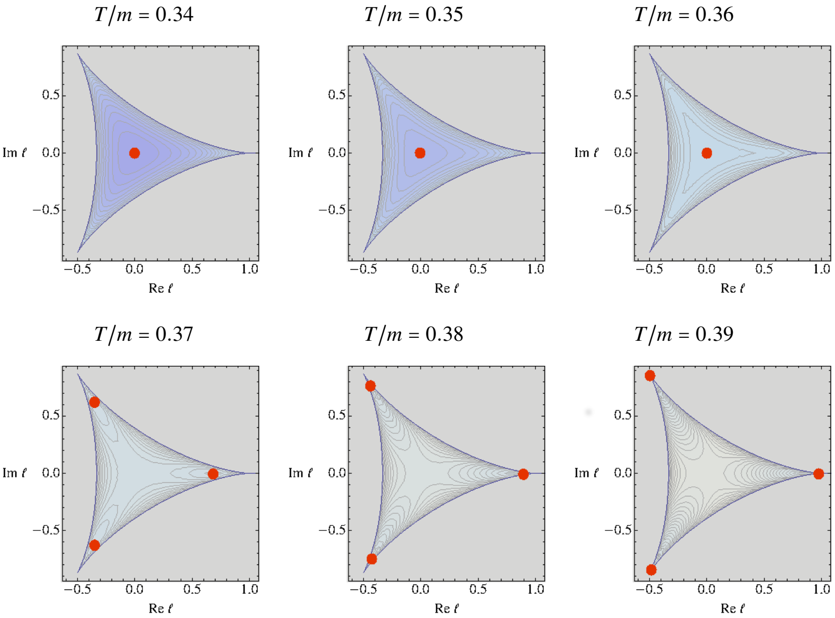
[confinement scenario à la Braun, Gies & Pawlowski]

$$\mathcal{W}_m(\ell,\bar\ell) = \frac{T}{\pi^2}\int_0^\infty dq\, q^2 \ln\Big[1+e^{-8\beta\varepsilon_q}-(9\ell\bar\ell-1)(e^{-\beta\varepsilon_q}+e^{-7\beta\varepsilon_q})$$
$$+(27\ell^3+27\bar\ell^3-27\ell\bar\ell+1)(e^{-2\beta\varepsilon_q}+e^{-6\beta\varepsilon_q})$$
$$-(81\ell^2\bar\ell^2-27\ell\bar\ell+2)(e^{-3\beta\varepsilon_q}+e^{-5\beta\varepsilon_q})$$
$$+(162\ell^2\bar\ell^2-54\ell^3-54\bar\ell^3+18\ell\bar\ell-2)e^{-4\beta\varepsilon_q}\Big]$$

$$\boxed{\varepsilon_q = \sqrt{q^2+m^2}}$$

[UR, J. Serreau, M. Tissier, N. Wschebor PLB 742 (2015); UR, J. Serreau, M. Tissier, PRD92 (2015)]

Pure glue case: spontanous breaking of center-symmetry



Pure glue case: order and temperature of the transition

order	lattice	fRG	model at 1-loop	model at 2-loop
SU(2)	2nd	2nd	2nd	2nd
SU(3)	1st	1st	1st	1st
SU(4)	1st	1st	1st	1st
Sp(2)	1st	1st	1st	1st

$T_c\,(MeV)$	lattice	fRG$^{(*)}$	model at 1-loop$^{(**)}$	model at 2-loop$^{(***)}$
SU(2)	295	230	238	284
SU(3)	270	275	185	254

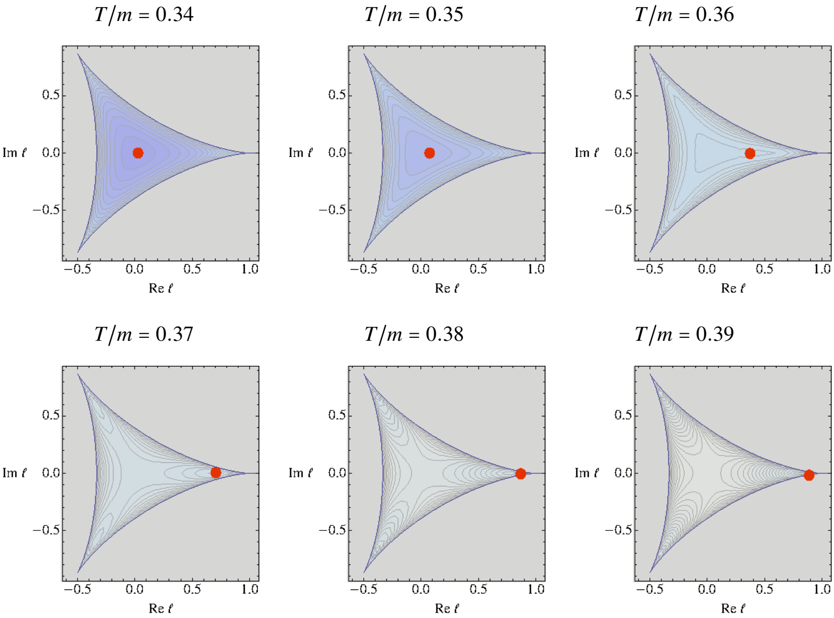
$^{(*)}$ L. Fister and J. M. Pawlowski, Phys.Rev. D88 (2013) 045010 .

$^{(**)}$ SU(2) and SU(3): UR, J. Serreau, M. Tissier and N. Wschebor, PLB742 (2015).

$^{(***)}$ SU(2): UR, J. Serreau, M. Tissier and N. Wschebor, Phys.Rev. D91 (2015) 045035.

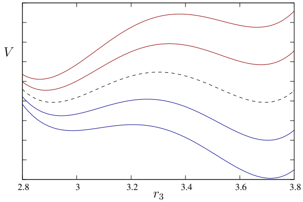
$^{(***)}$ SU(3) and beyond: UR, J. Serreau, M. Tissier and N. Wschebor, in preparation.

Heavy quarks, $\mu = 0$: transition



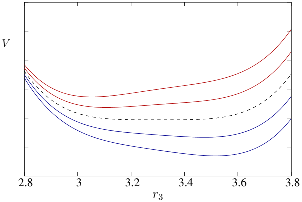
Heavy quarks, $\mu = 0$: mass dependence of the transition

$M > M_c$:



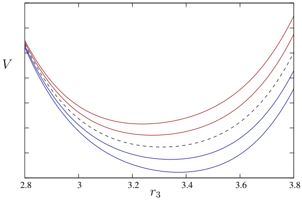
first order

$M = M_c$:



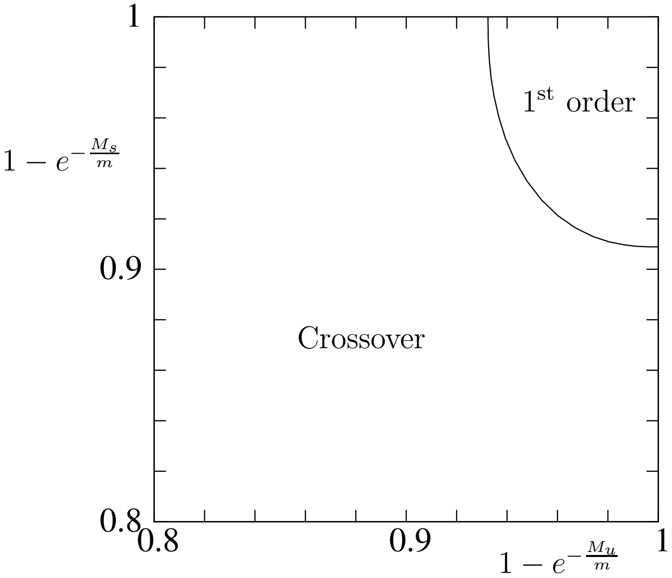
second order

$M < M_c$:



crossover

Heavy quarks, $\mu = 0$: Columbia plot



Heavy quarks, $\mu = 0$: comparison to other approaches

N_f	$(M_c/T_c)^{\text{our model (*)}}$	$(M_c/T_c)^{\text{lattice (**)}}$	$(M_c/T_c)^{\text{matrix (***)}}$	$(M_c/T_c)^{\text{SD (****)}}$
1	6.74	7.22	8.04	1.42
2	7.59	7.91	8.85	1.83
3	8.07	8.32	9.33	2.04

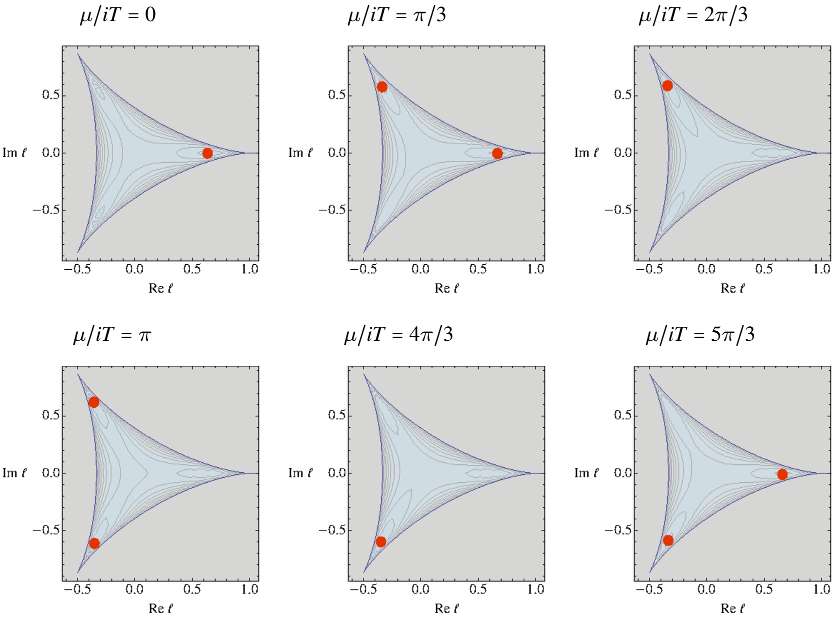
(*) UR, J. Serreau and M. Tissier, PRD92 (2015).

(**) M. Fromm, J. Langelage, S. Lottini and O. Philipsen, JHEP 1201 (2012) 042.

(***) K. Kashiwa, R. D. Pisarski and V. V. Skokov, Phys.Rev. D85 (2012) 114029.

(****) C. S. Fischer, J. Luecker and J. M. Pawlowski, Phys.Rev. D91 (2015) 1, 014024.

Heavy quarks, μ imaginary: Roberge-Weiss transition



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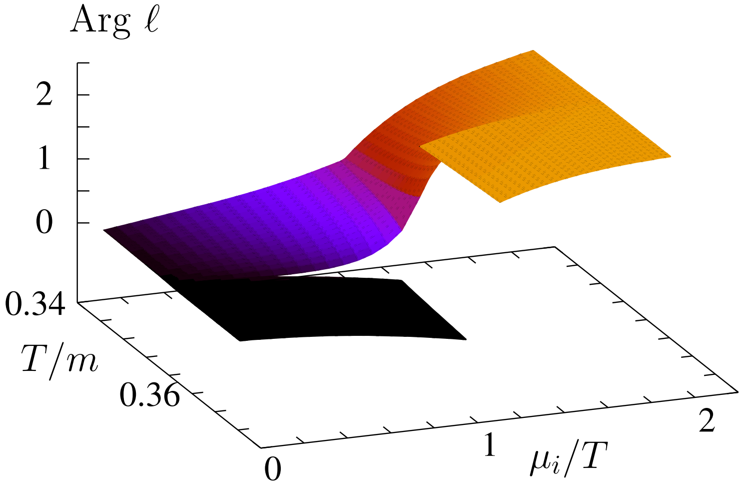
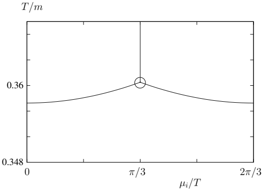
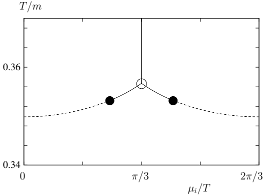


Heavy quarks, μ imaginary: mass dependence of the transition

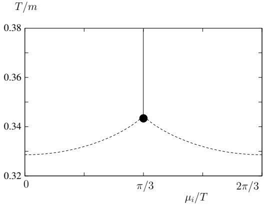
$M > M_c(0)$:



$M \in [M_c(i\pi/3), M_c(0)]$:

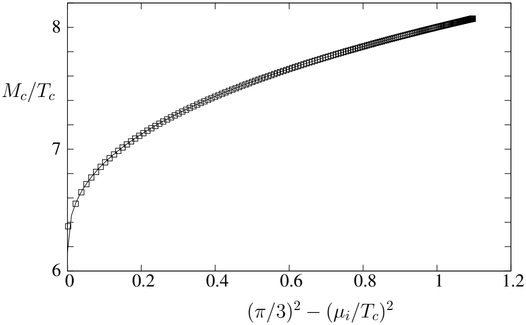


$M = M_c(i\pi/3)$:



Similar structure as in the lattice study of [P. de Forcrand, O. Philipsen, Phys.Rev.Lett. 105 (2010)]

Heavy quarks, μ imaginary: comparison to other approaches



$$\frac{M_c}{T_c} = \frac{M_{\text{tric.}}}{T_{\text{tric.}}} + K\left[\left(\frac{\pi}{3}\right)^2 + \left(\frac{\mu}{T}\right)^2\right]^{2/5}$$

	our model(*)	lattice(**)	SD(***)
K	1.85	1.55	0.98
$\frac{M_{\text{tric.}}}{T_{\text{tric.}}}$	6.15	6.66	0.41

(*) UR, J. Serreau and M. Tissier, arXiv:1504.02916.

(**) Fromm et.al., JHEP 1201 (2012) 042.

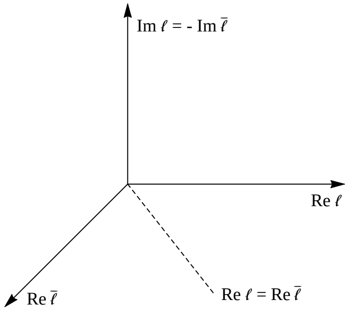
(***) Fischer et.al., Phys.Rev. D91 (2015) 1, 014024.

Heavy quarks, μ real: "small" sign problem

$\ell \equiv \langle \mathrm{tr}\, L \rangle$, $\bar{\ell} \equiv \langle \mathrm{tr}\, L^\dagger \rangle$ with $\mathrm{tr}\, L^\dagger = (\mathrm{tr}\, L)^*$.
But we not always have $\bar{\ell} = \ell^*$:

$\underline{\mu \in i\mathbb{R}}$: the action is real
$\Rightarrow \bar{\ell} = \ell^*$.

$\underline{\mu \in \mathbb{R}}$: the action is complex.
One shows that $\ell, \bar{\ell} \in \mathbb{R}$.



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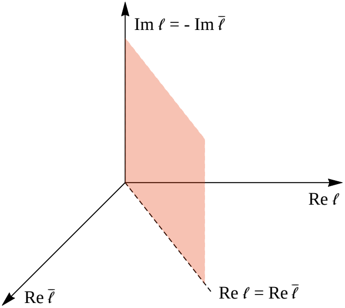

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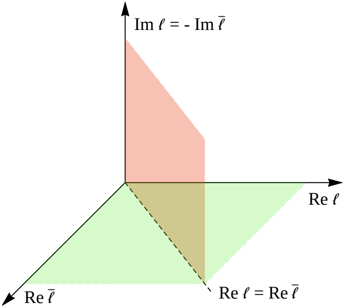

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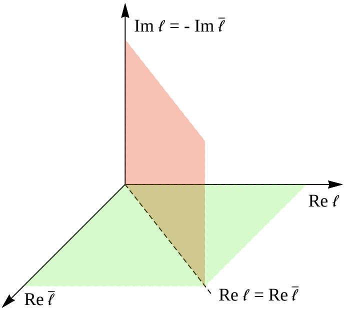


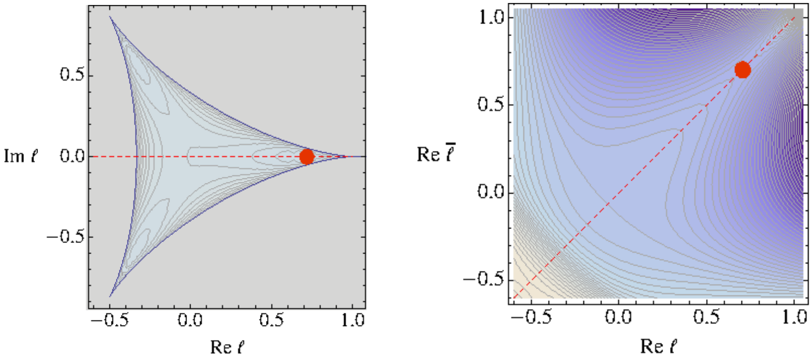
How to extract the physics from $V(\ell, \bar{\ell})$?

$\mu \in i\mathbb{R}$: a real action implies that the physical point is the absolute minimum of $V(\ell, \ell^*)$ for $\ell \in \mathbb{C}$.

$\mu \in \mathbb{R}$: with a complex action it is not clear which extremum to choose.

Heavy quarks, μ real: our recipe

At $\mu = 0$, it is possible to study $V(\ell, \bar{\ell})$ both for $(\ell, \bar{\ell}) \in \{(z, z^*) | z \in \mathbb{C}\}$ and for $(\ell, \bar{\ell}) \in \mathbb{R} \times \mathbb{R}$.

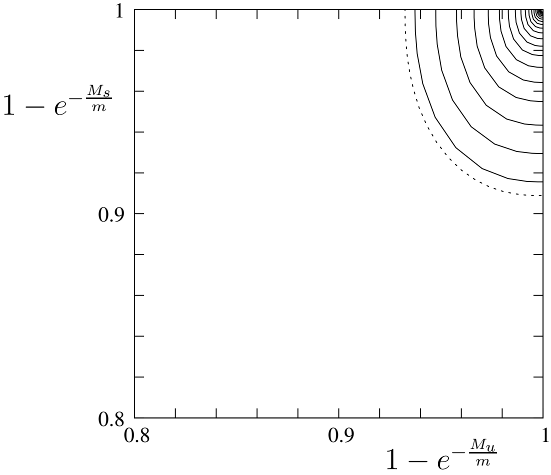


The minimum in the plane $(\mathrm{Re}\,\ell, \mathrm{Im}\,\ell)$ appears as the deepest saddle point in the plane $(\mathrm{Re}\,\ell, \mathrm{Re}\,\bar{\ell})$.

For $\mu > 0$, we keep on choosing the deepest saddle point.

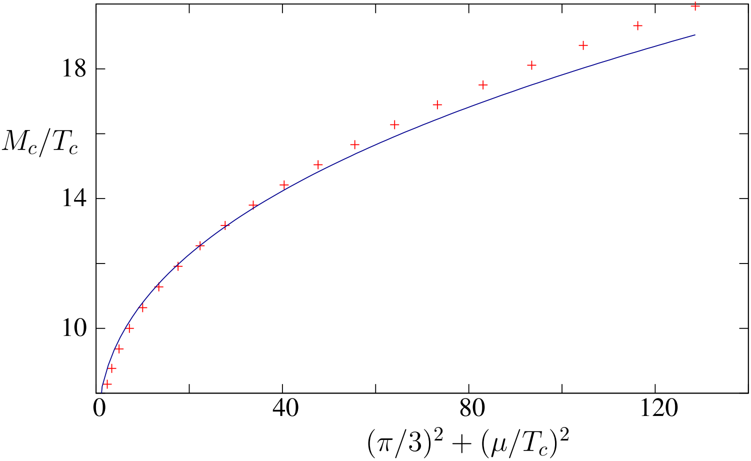
Heavy quarks, μ real: Columbia plot

As observed on the lattice, the critical line moves towards larger masses as μ is increased:



Heavy quarks, μ real: tricritical scaling

As observed on the lattice, the tricritical scaling survives deep in the $\mu^2 > 0$ region:



Conclusions

Simple one-loop calculations in a model aimed at fixing the Gribov ambiguity account for qualitative and quantitative features of the QCD phase diagram in the heavy quark limit:

- Correct account of the order parameter in the quenched limit;
- Critical line of the Columbia plot at $\mu = 0$;
- Roberge-Weiss phase diagram and its mass dependence.

Certain aspects require the inclusion of two-loop corrections:

- value of T_c;
- consistent thermodynamics.

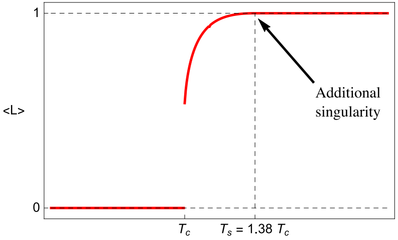
TODO: [In progress]

- Lower left corner of the Columbia plot? Chiral phase transition?
- Propagators in the Landau-DeWitt gauge [today on the arXiv]. Comparison to Lattice results?
- Our approach is not completely void of problems: how to define the physical space?

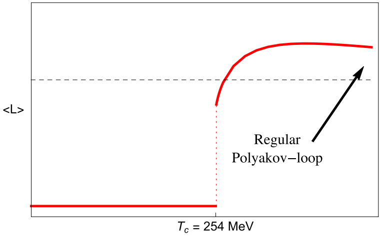
Backup

Polyakov loop: 1-loop vs 2-loop

One-loop artefact:

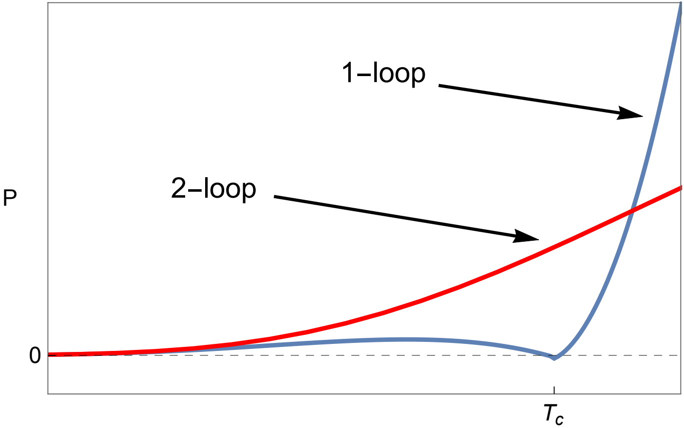


Disappears at two-loop order:



Thermodynamics

One-loop thermodynamics is inconsistent around T_c but two-loop thermodynamics is consistent:



However there remain non-exponentially suppressed T^4 contributions to the pressure as $T \to 0$.

The same problem appears in many other approaches (GZ, presumably functional RG, . . .).

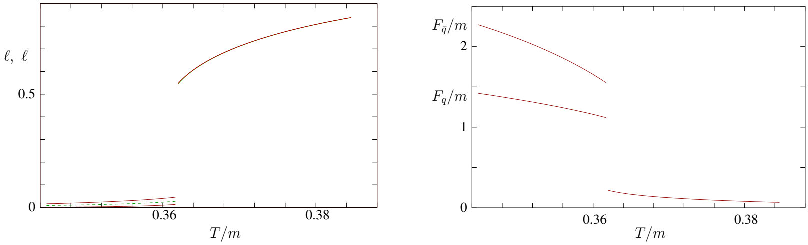
Heavy quarks, μ real: quark and anti-quark free-energies

Having both ℓ and $\bar{\ell}$ real corresponds to an imaginary background $r_8 \equiv \beta g \bar{A}_8^0 \equiv i\tilde{r}_8$!

$$\ell = \frac{e^{\frac{\tilde{r}_8}{\sqrt{3}}} + 2e^{-\frac{\tilde{r}_8}{2\sqrt{3}}}\cos(r_3/2)}{3} \in \mathbb{R} \qquad \bar{\ell} = \frac{e^{-\frac{\tilde{r}_8}{\sqrt{3}}} + 2e^{\frac{\tilde{r}_8}{2\sqrt{3}}}\cos(r_3/2)}{3} \in \mathbb{R}$$

In line with [H. Nishimura, M. C. Ogilvie, K. Pangeni, Phys.Rev. D90, 045039 (2014)] (saddle-point approximation)

We obtain not only real Polyakov loops, in line with $\ell = e^{-F_{\text{quark}}}$ and $\bar{\ell} = e^{-F_{\text{antiquark}}}$ ...
... but also $\ell \neq \bar{\ell}$ and $F_{\text{quark}} \neq F_{\text{antiquark}}$, in line with the breaking of C by $\mu \neq 0$:



In other approaches the choice $r_8 = 0$ is made, which leads to $\ell = \bar{\ell}$, so $F_{\text{quark}} = F_{\text{antiquark}}$.