

Pion polarizabilities: Theory vs. Experiment

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Martina Franca, Italy

Contents

Introduction

Experiment

Theory: overview

Chiral Perturbation Theory (ChPT)

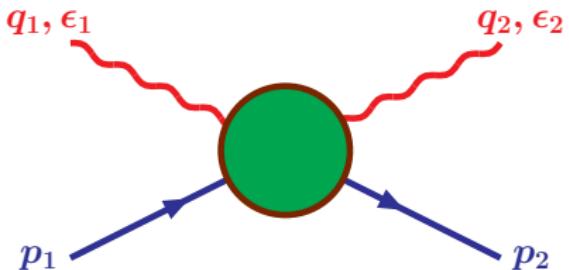
Numerics

Primakoff reaction

Summary

Introduction

Pion polarizabilities: **definition**



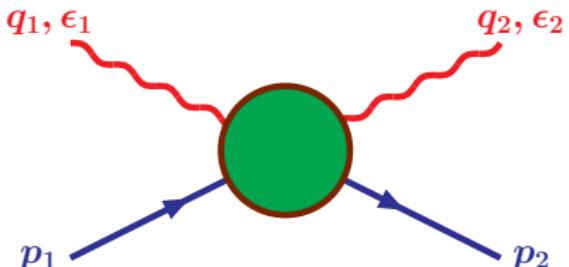
Transverse gauge $\epsilon_i = (0, \vec{\epsilon}_i)$

LAB frame $p_1 = (M_\pi, \vec{0})$

Expansion in $q_i = (\omega_i, \vec{q}_i)$:

Introduction

Pion polarizabilities: **definition**



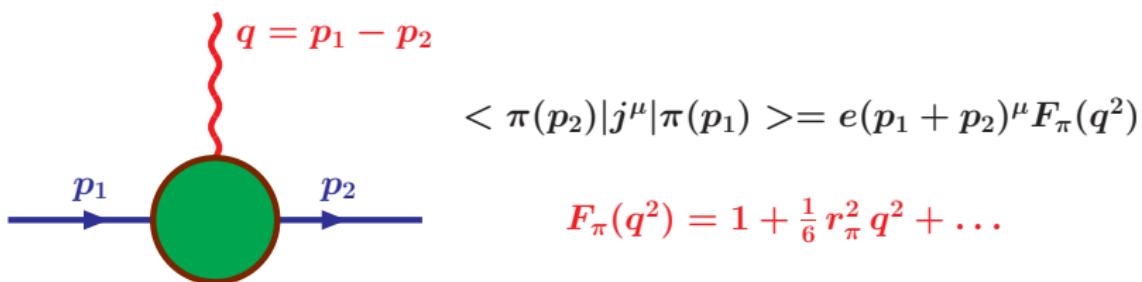
Transverse gauge $\epsilon_i = (0, \vec{\epsilon}_i)$

LAB frame $p_1 = (M_\pi, \vec{0})$

Expansion in $q_i = (\omega_i, \vec{q}_i)$:

$$\begin{aligned}
 T_{\gamma\pi^+ \rightarrow \gamma\pi^+} &= \underbrace{-2 e^2 \vec{\epsilon}_1 \cdot \vec{\epsilon}_2}_{\text{Born term}} \\
 &+ \underbrace{8 \pi M_\pi \left\{ \alpha_\pi \omega_1 \omega_2 \vec{\epsilon}_1 \cdot \vec{\epsilon}_2 + \beta_\pi (\vec{\epsilon}_1 \times \vec{q}_1) \cdot (\vec{\epsilon}_2 \times \vec{q}_2) \right\}}_{\text{el-mag polarizabilities}} + \dots
 \end{aligned}$$

- The electric, α_π , and magnetic, β_π , polarizabilities characterize the response of hadrons to their two-photon interactions at low energies
- These quantities are analogous to electromagnetic radii and magnetic moments which characterize the response of hadrons to their single-photon interactions at low energies



- The concept of the polarizability of molecules, atoms and nuclei was applied for the first time to hadrons in
 - A. Klein, Phys. Rev. 99 (1955) 998,
 - A.M. Baldin, Nucl. Phys. 18 (1960) 310,
 - V.A. Petrun'kin, JETP 13 (1961) 804
- Many theoretical papers afterwards
- Only a few experiments

The units of measurement

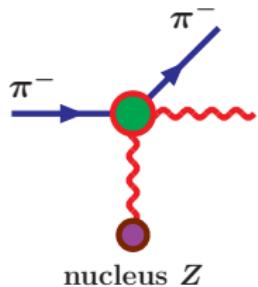
As follows from the definition, the dipole pion polarizabilities are proportional to

$$\alpha_\pi(\beta_\pi) \sim \frac{\alpha}{M_\pi} \frac{1}{\Lambda^2} \approx 4 \times 10^{-4} \text{ fm}^3$$

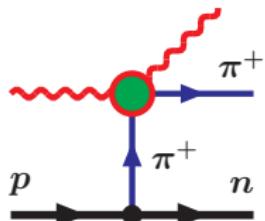
where the hadronic scale $\Lambda \sim 4\pi F_\pi \sim 1 \text{ GeV}$.

Then a natural choice of units for the dipole polarizabilities is 10^{-4} fm^3

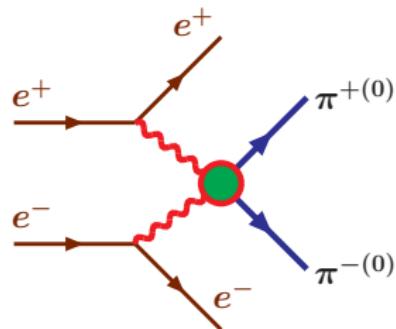
Experiment



(a)



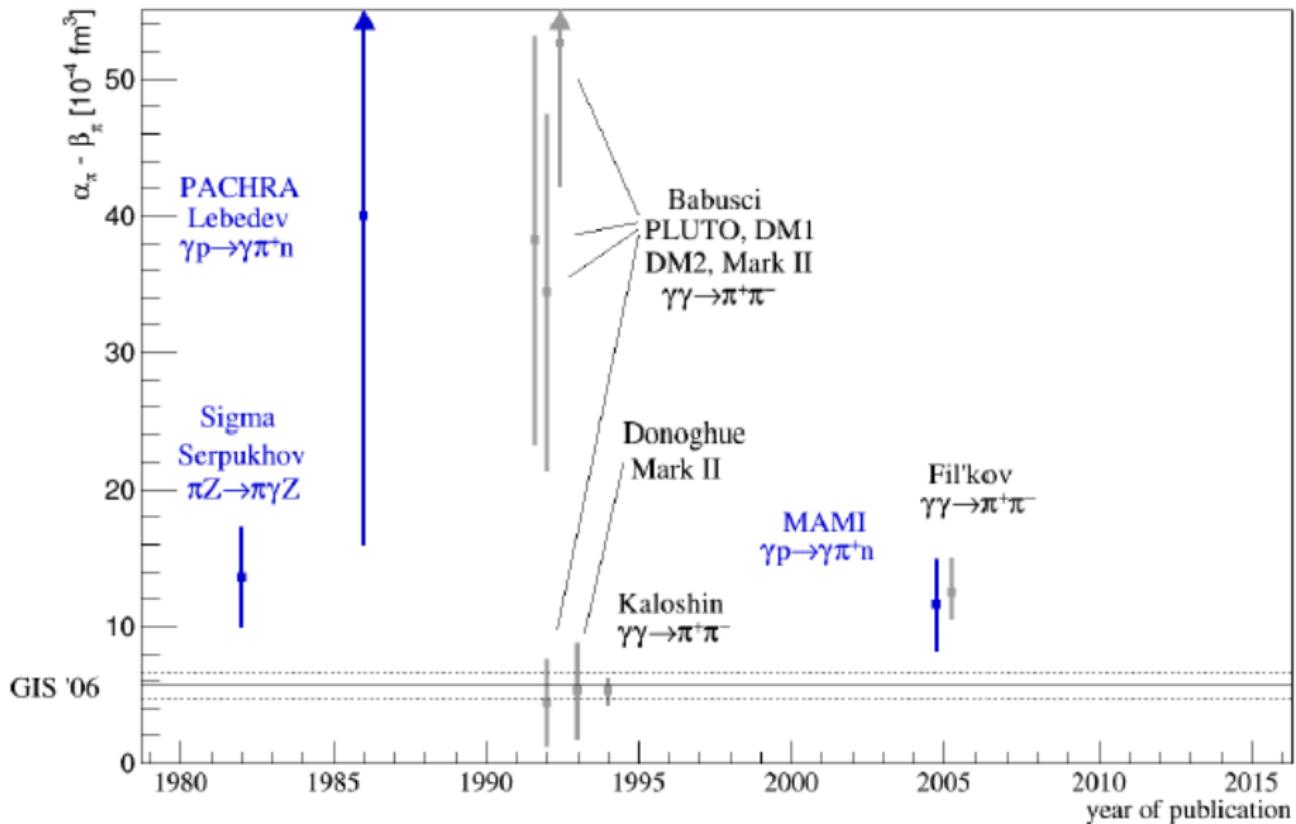
(b)



(c)

- (a) The scattering of high energy pions off the Coulomb field of heavy nucleus.**
- (b) The radiative pion photoproduction from the proton.**
- (c) The pion pair production in photon-photon collisions.**

Plot: T.Nagel, PhD TUM, 2012



GIS'06 = Gasser, Ivanov, Sainio, Nucl. Phys. B745 (2006) 84

General properties of pion polarizabilities

- Classical sum rule (Petrunk'kin'64):

$$\alpha_\pi = \frac{\alpha}{3m} \langle r_\pi^2 \rangle + 2\alpha \sum_{n \neq 0} \frac{|\langle n | \mathcal{D} | 0 \rangle|^2}{E_n - E_0}$$

where \mathcal{D} is the electric dipole operator.

$$\alpha_{\pi^\pm} \mapsto (3.5 - 6.8)$$

- The optical theorem relates the sum of polarizabilities to an unsubtracted forward dispersion relation

$$(\alpha + \beta)_\pi = \frac{M_\pi^2}{\pi^2} \int_{4M_\pi^2}^{\infty} \frac{ds'}{(s' - M_\pi^2)^2} \sigma_{\text{tot}}^{\gamma\pi}(s') > 0$$

Low Energy Theorem

- Using current algebra/PCAC gives the relation of $\alpha_\pi(\beta_\pi)$ with the vector F_V and axial F_A structure constants for radiative pion decays $\pi^- \rightarrow e\nu\gamma$
(Terent'ev'73):

$$\alpha_{\pi^\pm} = -\beta_{\pi^\pm} = \frac{\alpha}{8\pi^2 F_\pi^2 M_\pi} \frac{F_A}{F_V} = 2.7 \pm 0.4$$

Update: PIBETA Coll.: PRL 103, 051802 (2009): = 2.78 (10)

Effective Lagrangians: $\mathcal{L}_{\text{QCD}} \longrightarrow \mathcal{L}_{\text{eff}}$ for $E \ll M_\rho$

Weinberg'1979; Gasser, Leutwyler 1984,1985

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \dots$$

\mathcal{L}_{eff} expressed in observed hadron fields,
has the same symmetry as QCD.

- The leading order in chiral $SU(2)$ (pions and photons only):

$$\mathcal{L}_2 = \frac{F^2}{4} \langle D_\mu U D^\mu U^\dagger + M^2 (U + U^\dagger) \rangle ,$$

$$D_\mu U = \partial_\mu U - i(QU - UQ)A_\mu ,$$

$U \in SU(2)$, contains the pion fields

- $M^2 = (m_u + m_d)B$
- F, B are low-energy constants (LECs) not fixed by chiral symmetry
- \mathcal{L}_2 - nonrenormalizable quantum field theory

- Higher order Lagrangians

$$\mathcal{L}_4 = \sum_{i=1}^{10} \ell_i K_i = \frac{\ell_1}{4} \langle D_\mu U D^\mu U^\dagger \rangle^2 + \dots,$$

$$\mathcal{L}_6 = \sum_{i=1}^{57} c_i P_i, \quad (57 \rightarrow 56) \text{ Haefeli, Ivanov, Schmid, Ecker 2007}$$

- Local monomials K_i, P_i are known

Gasser, Leutwyler 1984,1985; Bijnens, Colangelo,Ecker 1999

- LECs ℓ_i, c_i absorb the divergences at order p^4 and p^6
- Notation later on: $\ell_i, c_i \rightarrow \ell_i^r, c_i^r$ UV finite parts of ℓ_i, c_i ;
 $\ell_i^r, c_i^r \rightarrow \ell_i, \bar{c}_i$ scale independent parts of ℓ_i, c_i .

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- Calculations with \mathcal{L}_{eff} give an expansion in quark masses and external momenta.

Chiral perturbation theory (ChPT)

Gasser, Leutwyler 1984,1985

Pion polarizabilities in ChPT to one-loop

- Chiral expansion to one-loop

Bijnens, Cornet 1988

Donoghue, Holstein 1989

$$\alpha_\pi = -\beta_\pi = \frac{\alpha}{8\pi^2 F_\pi^2 M_\pi} \cdot \frac{1}{6} (\bar{\ell}_6 - \bar{\ell}_5)$$

- The LECs $\bar{\ell}_{5,6}$ also arise in $\pi \rightarrow e\nu\gamma$ -amplitude

- This gives a low energy theorem

Terent'ev 1973

$$\alpha_\pi = -\beta_\pi = \frac{\alpha}{8\pi^2 F_\pi^2 M_\pi} \frac{F_A}{F_V} \left\{ 1 + O(M_\pi^2) \right\}$$

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- Chiral expansion to two-loops

$$\gamma\gamma \rightarrow \pi^0\pi^0$$

$$\gamma\gamma \rightarrow \pi^+\pi^-$$

Recalculation of the amplitudes

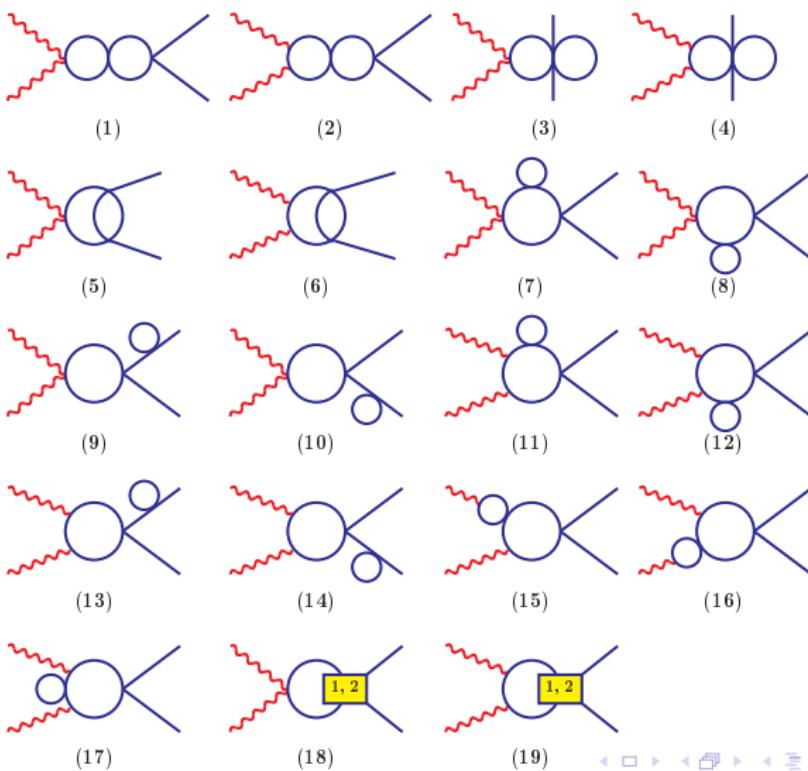
Bellucci,Gasser,Sainio 1994

Burgi 1996

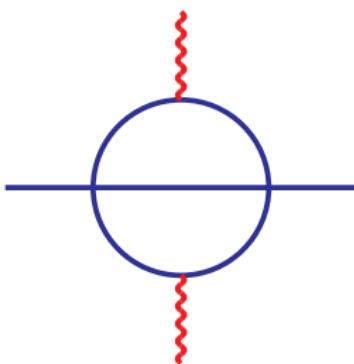
Gasser, Ivanov, Sainio 2005,2006

Pion polarizabilities in ChPT to two-loop

Gasser, Ivanov, Sainio 2005,2006



Acnode diagram

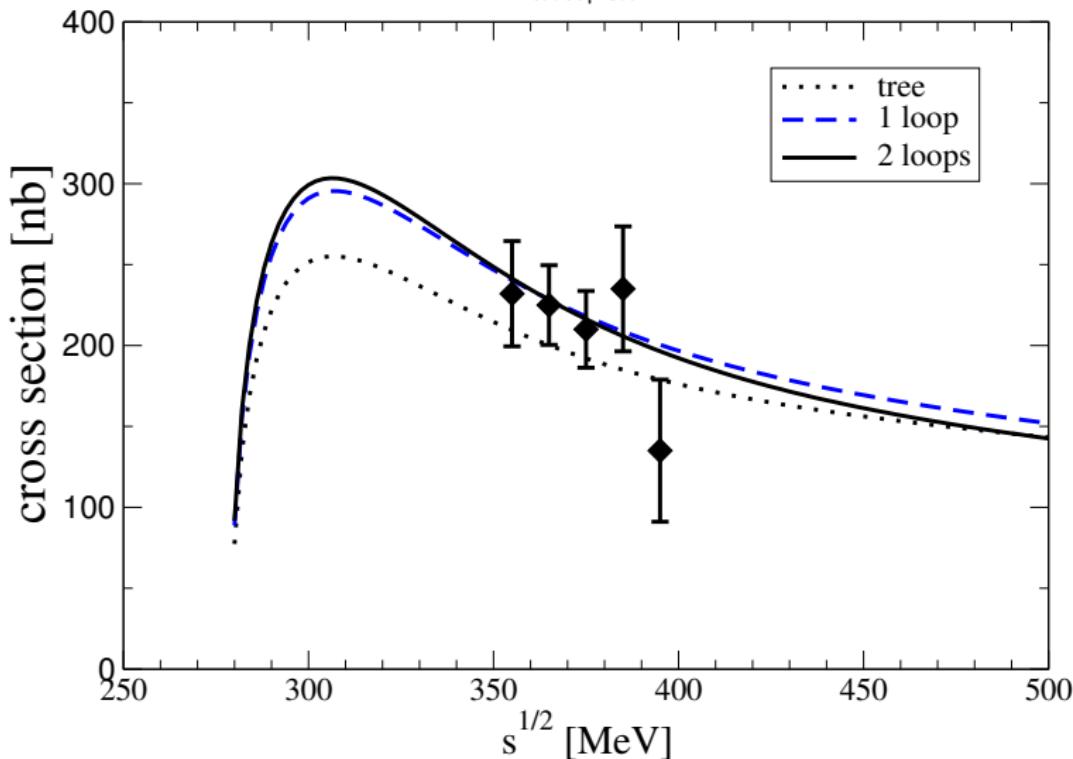


Invoke a dispersion relation for the function

$$\begin{aligned} I(\mu, n; s) &= \int_0^1 dx [x(1-x)]^n [1 - s x(1-x)]^\mu \\ &= \int_4^\infty \frac{d\sigma \rho(\mu, n; \sigma)}{\sigma - s} \quad (-1 < \mu < 0) \end{aligned}$$

Cross section $\gamma\gamma \rightarrow \pi^+ \pi^-$

$|\cos\theta| < 0.6$



Experimental data from MARK II (SLAC) 1990

Charged pion polarizabilities: analytic results

$$(\alpha_1 \pm \beta_1)_{\pi^+} = \frac{\alpha}{16 \pi^2 F_\pi^2 M_\pi} \left\{ c_{1\pm} + \frac{M_\pi^2 d_{1\pm}}{16 \pi^2 F_\pi^2} + O(M_\pi^4) \right\},$$

$$c_{1+} = 0, \quad c_{1-} = \frac{2}{3} \bar{\ell}_\Delta,$$

$$d_{1+} = 8 b^r - \frac{4}{9} \left\{ \ell \left(\ell + \frac{1}{2} \bar{\ell}_1 + \frac{3}{2} \bar{\ell}_2 \right) - \frac{53}{24} \ell + \frac{1}{2} \bar{\ell}_1 + \frac{3}{2} \bar{\ell}_2 + \frac{91}{72} + \Delta_+ \right\}$$

$$\begin{aligned} d_{1-} = & a_1^r + 8 b^r - \frac{4}{3} \left\{ \ell \left(\bar{\ell}_1 - \bar{\ell}_2 + \bar{\ell}_\Delta - \frac{65}{12} \right) - \frac{1}{3} \bar{\ell}_1 - \frac{1}{3} \bar{\ell}_2 + \frac{1}{4} \bar{\ell}_3 - \bar{\ell}_\Delta \bar{\ell}_4 \right. \\ & \left. + \frac{187}{108} + \Delta_- \right\} \end{aligned}$$

Charged pion polarizabilities: analytic results

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$$\Delta_+ = \frac{8105}{576} - \frac{135}{64} \pi^2 = \underbrace{-6.75}_{\text{Burgi: -8.69}}, \quad \Delta_- = \frac{41}{432} - \frac{53}{64} \pi^2 = \underbrace{-8.08}_{\text{Burgi: -8.73}}$$

Numerics

Numerical values of LECs

$$\bar{\ell}_1 = -0.4 \pm 0.6, \quad \bar{\ell}_2 = 4.3 \pm 0.1, \quad \bar{\ell}_3 = 2.9 \pm 2.4, \quad \bar{\ell}_4 = 4.4 \pm 0.2$$

Colangelo, Gasser, Leutwyler 2001

$$\bar{\ell}_{\Delta} \doteq \bar{\ell}_6 - \bar{\ell}_5 = 3.0 \pm 0.3.$$

Bijnens, Talavera 1997

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$$\bar{\ell}_\Delta \doteq \bar{\ell}_6 - \bar{\ell}_5 = 3.0 \pm 0.3.$$

Bijnens, Talavera 1997

$$a_1^r = -4096\pi^4 (6c_6^r + c_{29}^r - c_{30}^r - 3c_{34}^r + c_{35}^r + 2c_{46}^r - 4c_{47}^r + c_{50}^r)$$

$$b^r = -128\pi^4 (c_{31}^r + c_{32}^r - 2c_{33}^r - 4c_{44}^r)$$

Resonance ρ , a_1 , b_1 exchange at $\mu = M_\rho$

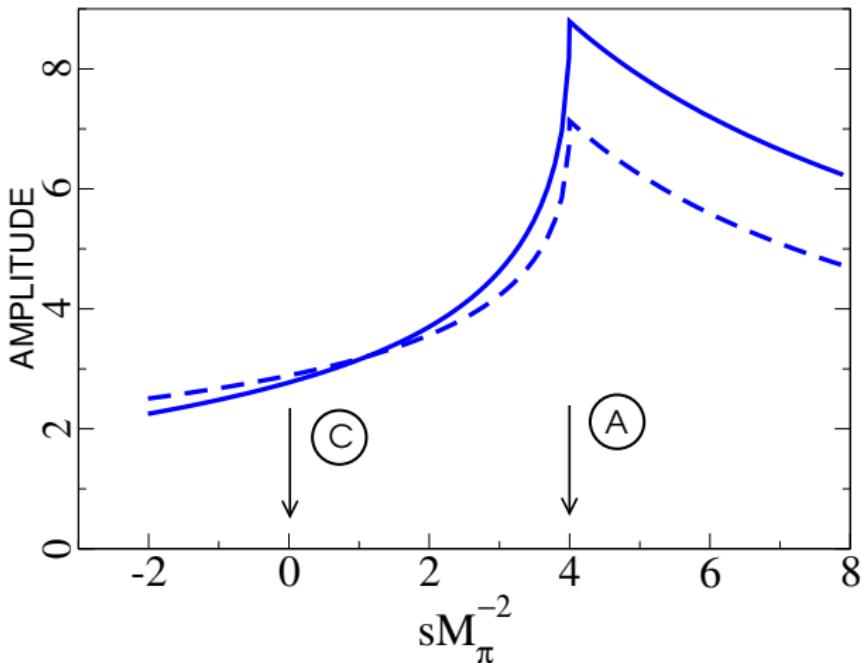
$$(a_1^r, a_2^r, b^r) = (-3.2, 0.7, 0.4)$$

ENJL model with large N_c (Bijnens & Prades)

$$(a_1^r, a_2^r, b^r) = (-8.7, 5.9, 0.38)$$

We use $b^r = 0.4 \pm 0.4$ and vary a_1^r from -10 to 0.

Chiral expansion at the Compton threshold



Example: spin non-flip amplitude. Solid line \rightarrow two-loops, dashed line \rightarrow one-loop

Charged pion polarizabilities

J. Gasser, M. A. Ivanov, M. E. Sainio, Nucl. Phys. B 745 (2006) 84-108

	ChPT to one loop	ChPT to two-loops
$(\alpha - \beta)_{\pi^+}$	6.0 ± 0.6	5.7 ± 1.0
$(\alpha + \beta)_{\pi^+}$	0	0.16 ± 0.14

Pion polarizability via Primakoff reaction

A.G. Galperin, G.V. Mitselmakher, A.G. Olshevski and V.N. Pervushin, Yad. Fiz. 32 (1980) 1053

- The first observation of the Compton scattering off pion at SIGMA spectrometer (Serpukhov)
- The first measurement of pion polarizabilities

Main advantages of COMPASS

- One can use pion and muon beams of the same momentum with the same setup configuration.

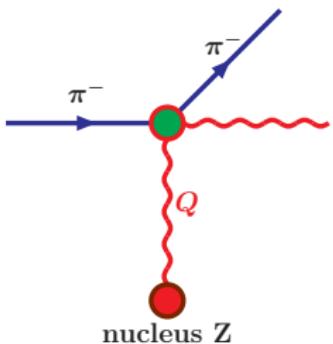
$$\pi^-(A, Z) \rightarrow \pi^-(A, Z)\gamma$$

$$\mu^-(A, Z) \rightarrow \mu^-(A, Z)\gamma$$

- Muon is the point-like particle and corresponding cross section for muon is known with high precision.
- So, muon data can be used as reference to control the systematics.

COMPASS

C. Adolph *et al.* Phys. Rev. Lett. 114 (2015) 062002



Primakoff reaction:



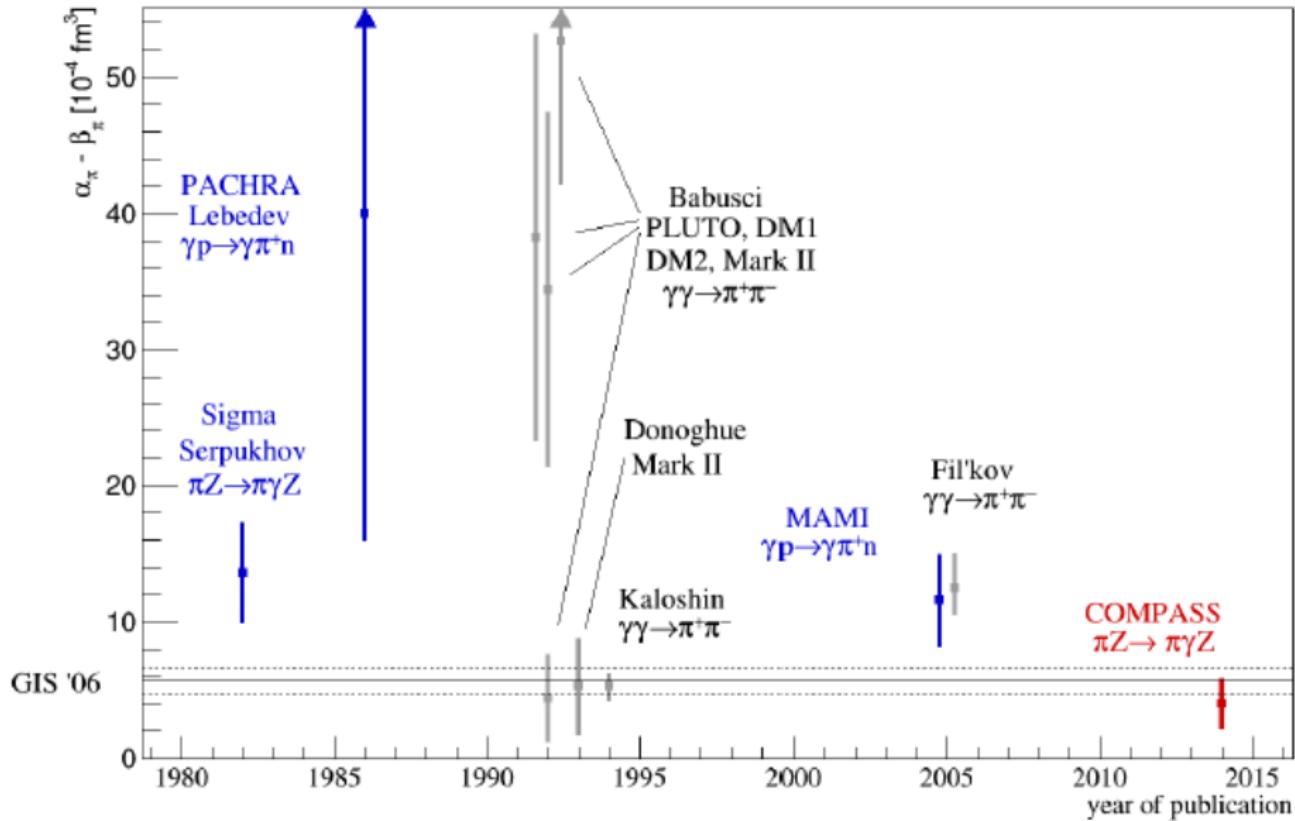
$E_\pi = 190 \text{ GeV}$, $Q^2 < 0.0015 \text{ GeV}^2$

$63 \cdot 10^3$ events

$$\alpha_\pi = 2.0 \pm 0.6 \text{ (stat)} \pm 0.7 \text{ (syst)}$$

assumption: $\alpha_\pi + \beta_\pi = 0$

Plot: B. Badelek (COMPASS) 2015



COMPASS: C. Adolph et al. Phys. Rev. Lett. 114 (2015) 062002

Experimental information

Experiments	$(\alpha - \beta)_{\pi^\pm}$
$\gamma p \rightarrow \gamma \pi^+ n$ Mainz (2005)	$11.6 \pm 1.5_{\text{stat}} \pm 3.0_{\text{syst}} \pm 0.5_{\text{mod}}$
L. Fil'kov, V. Kashevarov (2005)	$13.0^{+2.6}_{-1.9}$
$\gamma\gamma \rightarrow \pi^+\pi^-$ available data	
A. Kaloshin, V. Serebryakov (1994)	5.25 ± 0.95
$\gamma\gamma \rightarrow \pi^+\pi^-$ MARK II	
J.F. Donoghue, B. Holstein (1993)	5.4
$\gamma\gamma \rightarrow \pi^+\pi^-$ MARK II	
D. Babusci et al. (1992)	
$\gamma\gamma \rightarrow \pi^+\pi^-$ PLUTO	$38.2 \pm 9.6 \pm 11.4$
DM 1	34.4 ± 9.2
DM 2	52.6 ± 14.8
MARK II	4.4 ± 3.2
$\gamma p \rightarrow \gamma \pi^+ n$ Lebedev Inst. (1986)	40 ± 24
$\pi^- Z \rightarrow \gamma \pi^- Z$ Serpukhov (1983)	$15.6 \pm 6.4_{\text{stat}} \pm 4.4_{\text{syst}}$
COMPASS (2015)	$4.0 \pm 1.2_{\text{stat}} \pm 1.4_{\text{syst}}$

Summary

- ChPT is successful tool to analyse low-energy physics
- Chiral expansion for the $\gamma\gamma \rightarrow \pi\pi$ amplitude at the Compton threshold converges quite rapidly
- Two-loop result for the charged pion polarizability

$$(\alpha - \beta)_{\pi^+} = 5.7 \pm 1.0$$

is in agreement with very well known low-energy theorem

- However, there was a clash almost a factor of 2 (!) between this result and several experiments
- The last precise measurement of performed by COMPASS

$$(\alpha - \beta)_{\pi^+} = 4.0 \pm 1.8$$

is found in agreement with ChPT.