

# Classical gluon production amplitude in heavy ion collisions

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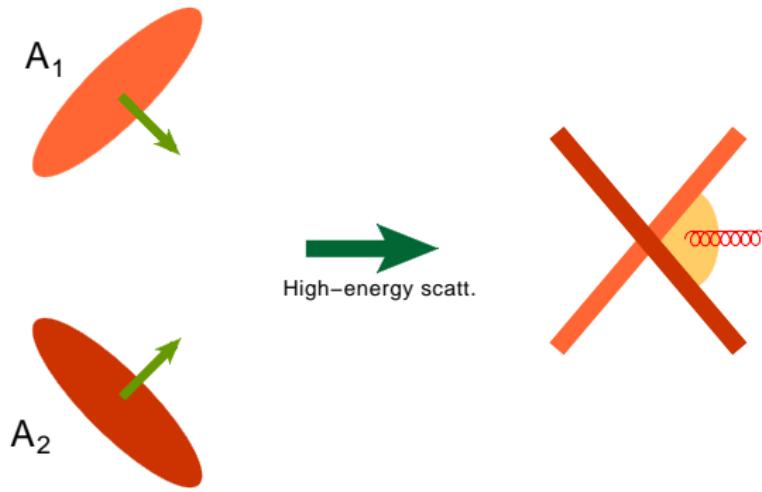
- Motivations for single-gluon cross-section in A-A collisions.
- Simpler case: p-A collisions.
- Simplified problem for A-A collisions:  $1 \ll A_1 \ll A_2$
- Result for the  $g^3$  amplitude.
- Sub-gauge conditions for light-cone propagator.

Result based on

JHEP 1503 (2015) 015    G.A.C., Y. Kovchegov, D. Weretepny

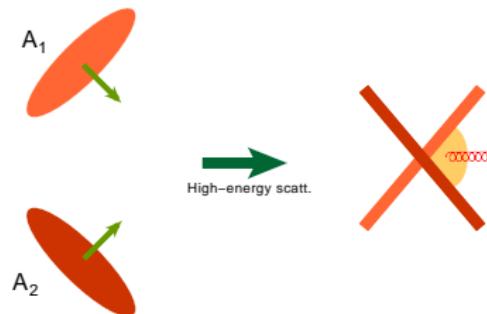
# Goal: Single-gluon cross-section in A-A collisions

$A_1$  and  $A_2$  are the number of nucleons in the two nuclei



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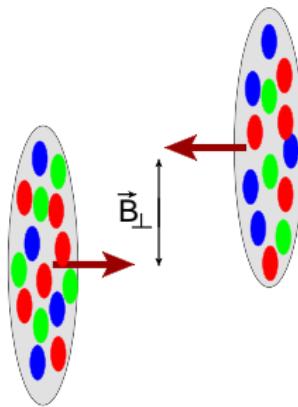


## Motivations

- One would like to obtain the classical gluon produced in heavy-ion collisions: initial condition for Quark-Gluon-Plasma.
- Check validity of  $k_T$ -factorization formula with unintegrated gluon distributions employed in phenomenological applications.
  - Numerical simulations appear to rule out the  $k_T$ -factorization ansatz.

## Set up of the calculation

- Resummation parameters:  $\alpha_s^2 A_1^{1/3}$  and  $\alpha_s^2 A_2^{1/3}$



- Resummation parameters are proportional to the saturation scale squared of each nucleus:  $Q_{s1}^2 \sim \alpha_s^2 A_1^{1/3}$  and  $Q_{s2}^2 \sim \alpha_s^2 A_2^{1/3}$

## Set up of the calculation

Write quasi-classical single-gluon production cross section as

$$\frac{d\sigma}{d^2k d^2B d^2b} = \frac{1}{\alpha_s} f \left( \frac{Q_{s1}^2(\vec{B}_\perp - \vec{b}_\perp)}{k_T^2}, \frac{Q_{s2}^2(\vec{b}_\perp)}{k_T^2} \right)$$

- $\vec{B}_\perp$ : impact parameter between the two nuclei;
- $\vec{b}_\perp$ : transverse position of the produced gluon with respect to the center of the target nucleus;
- $\vec{k}_\perp$  is the transverse momentum of the produced gluon with  $k_T = |\vec{k}_\perp|$ .

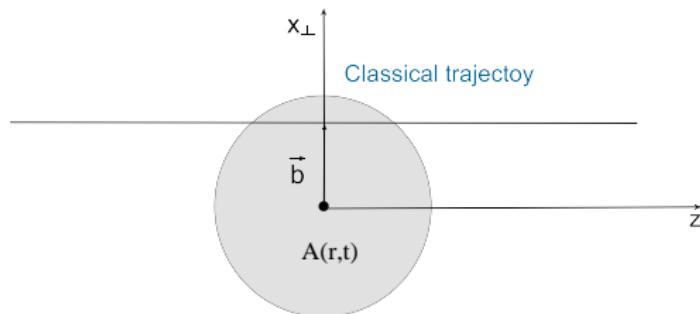
## Set up of the calculation

Expansion of  $f$  in powers of  $\alpha_s^2 A_1^{1/3}$  and  $\alpha_s^2 A_2^{1/3} \Leftrightarrow Q_{s1}^2/k_T^2$  and  $Q_{s2}^2/k_T^2$

$$f\left(\frac{Q_{s1}^2}{k_T^2}, \frac{Q_{s2}^2}{k_T^2}\right) = \sum_{n,m=1}^{\infty} c_{n,m} \left(\frac{Q_{s1}^2}{k_T^2}\right)^n \left(\frac{Q_{s2}^2}{k_T^2}\right)^m$$

- Analytic expression of function  $f(Q_{s1}^2/k_T^2, Q_{s2}^2/k_T^2)$  is not known.
- Knowing analytic expression of function  $f(Q_{s1}^2/k_T^2, Q_{s2}^2/k_T^2)$  would facilitate the inclusion of low- $x$  evolution corrections.
- Coefficient  $c_{1,n}$  is known:  $pA$  collisions.
- Our goal is  $c_{2,n}$ : corresponds to LO contribution for case  $1 \ll A_1 \ll A_2$ .
- Idea: find a pattern to resum class of diagrams to get  $c_{n,m}$ .

# High-energy scattering in QCD

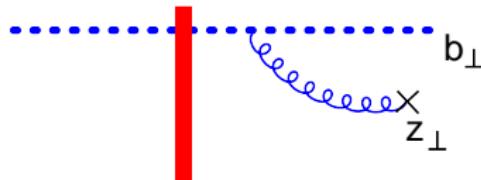
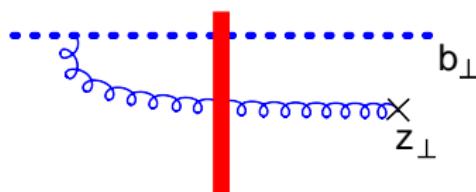


phase factor for the high-energy scattering: Wilson-line operator

$$U(x_{\perp}, v) = \text{Pe}^{\frac{-ig}{c\hbar} \int_{-\infty}^{+\infty} dt \dot{x}_{\mu} A^{\mu}(x(t))}$$

$$\text{Pe}^{\int_{-\infty}^{+\infty} dt A(t)} = 1 + \int_{-\infty}^{+\infty} dt A(t) + \int_{-\infty}^{+\infty} dt A(t) \int_{-\infty}^t dt' A(t')$$

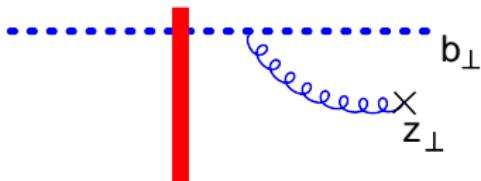
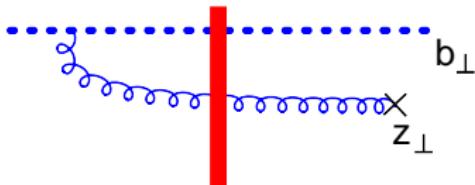
## Simpler case: pA collision



- Power counting

- Projectile: single nucleon  $\Rightarrow \alpha_s^2 A_P^{1/3} \lesssim 1$
- Target:  $\Rightarrow (\alpha_s^2 A_T^{1/3})^N \sim 1$
- $\Rightarrow$  the target reduces to a shock wave (red in the diagram).

## Simpler case: pA collision



$$A(\vec{z}_{\perp}, \vec{b}_{\perp}) = \frac{i g}{\pi} \frac{\vec{\epsilon}_{\perp}^{\lambda*} \cdot (\vec{z}_{\perp} - \vec{b}_{\perp})}{|\vec{z}_{\perp} - \vec{b}_{\perp}|^2} \left[ U_{\vec{z}_{\perp}}^{ab} - U_{\vec{b}_{\perp}}^{ab} \right] \left( V_{\vec{b}_{\perp}} t^b \right)$$

The gluon production cross section is given by

$$\frac{d\sigma}{d^2 k_T dy} = \frac{1}{2(2\pi)^3} \int d^2 z d^2 z' d^2 b e^{-i\vec{k}_{\perp} \cdot (\vec{z}_{\perp} - \vec{z}'_{\perp})} \left\langle A(\vec{z}_{\perp}, \vec{b}_{\perp}) A^*(\vec{z}'_{\perp}, \vec{b}_{\perp}) \right\rangle$$

## Simpler case: pA collision

$$\frac{d\sigma}{d^2k_T dy} = \frac{\alpha_s C_F}{4\pi^4} \int d^2z d^2z' d^2b e^{-ik_\perp \cdot (\vec{z}_\perp - \vec{z}'_\perp)} \frac{\vec{z}_\perp - \vec{b}_\perp}{|\vec{z}_\perp - \vec{b}_\perp|^2} \cdot \frac{\vec{z}'_\perp - \vec{b}_\perp}{|\vec{z}'_\perp - \vec{b}_\perp|^2} \\ \times \left[ S_G(\vec{z}_\perp, \vec{z}'_\perp) - S_G(\vec{b}_\perp, \vec{z}'_\perp) - S_G(\vec{z}_\perp, \vec{b}_\perp) + 1 \right]$$

Kovchegov, Mueller (1998)

$$S_G(\vec{x}_\perp, \vec{y}_\perp) = \frac{1}{N_c^2 - 1} \left\langle U_{\vec{x}_\perp}^{ab} U_{\vec{y}_\perp}^{\dagger ba} \right\rangle$$

In the quasi-classical MV/Glauber–Mueller approximation

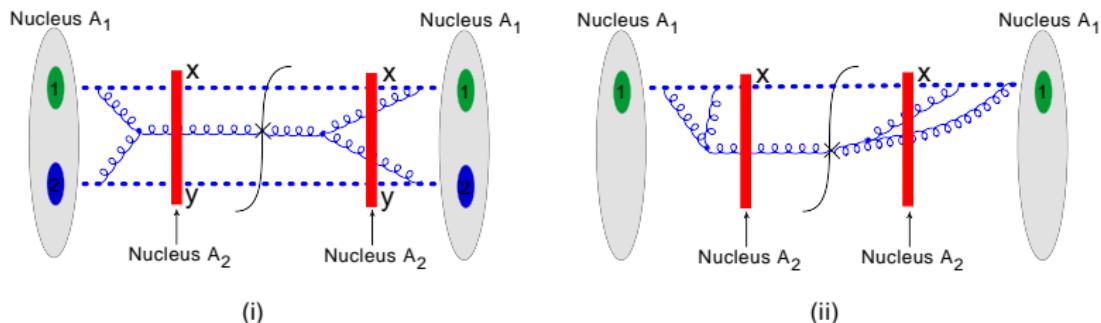
$$S_G(\vec{x}_\perp, \vec{y}_\perp) = \exp \left[ -\frac{1}{4} (\vec{x}_\perp - \vec{y}_\perp)^2 Q_{sG}^2 \left( \frac{\vec{x}_\perp + \vec{y}_\perp}{2} \right) \ln \frac{1}{|\vec{x}_\perp - \vec{y}_\perp| \Lambda} \right]$$

- $Q_{sG}^2 = 4\pi\alpha_s^2 T(\vec{b}_\perp)$  is the square of the gluon saturation scale.
- $T(\vec{b}_\perp)$  is the nuclear profile function.

# Simplified problem for AA collision: $1 \ll A_1 \ll A_2 \Rightarrow Q_{s1} \ll Q_{s2}$

- Nucleus  $A_1$  is considered as a dilute system.
- Only one quark from each nucleon of Nucleus  $A_1$ .
- Nucleus  $A_2$  is densely packed  $\Rightarrow$  shock wave.

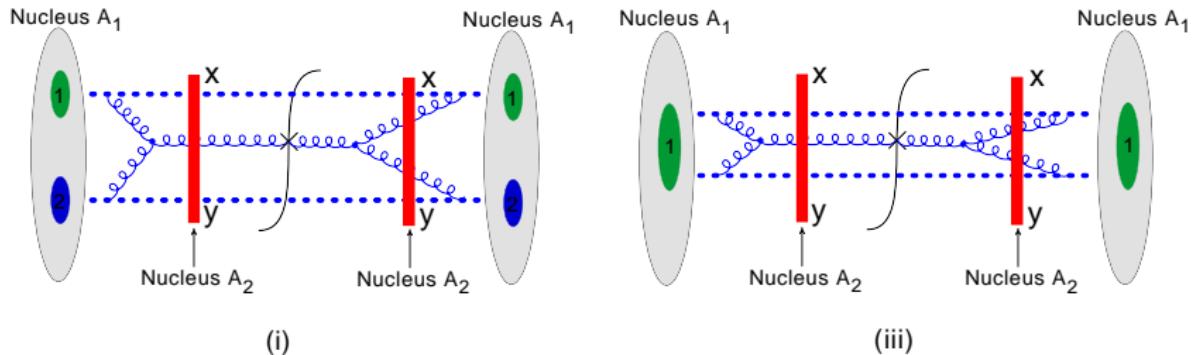
$$\alpha_s^2 A_2^{1/3} \sim 1 \quad \alpha_s^2 A_1^{1/3} \lesssim 1$$



- Contribution from classical field:  $A^\mu \sim \frac{1}{g}$   $\Rightarrow \langle A_\mu A^\mu \rangle \sim \frac{1}{\alpha_s}$
- Power counting of diagram (i):  $\frac{1}{\alpha_s} (\alpha_s^2 A_1^{1/3})^2$  Leading contribution.
- Power counting of diagram (ii):  $\frac{1}{\alpha_s} \alpha_s^4 A_1^{1/3}$  Sub-leading contribution.

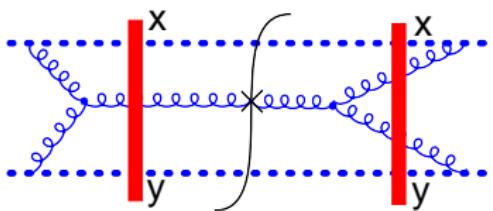
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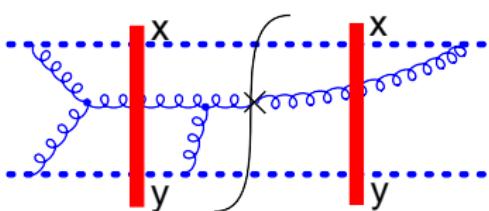


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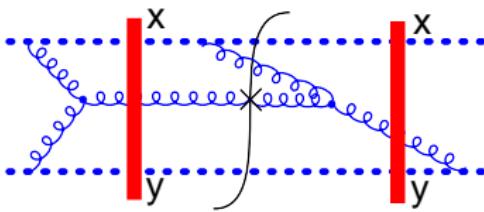
## Sample of diagrams



(a)

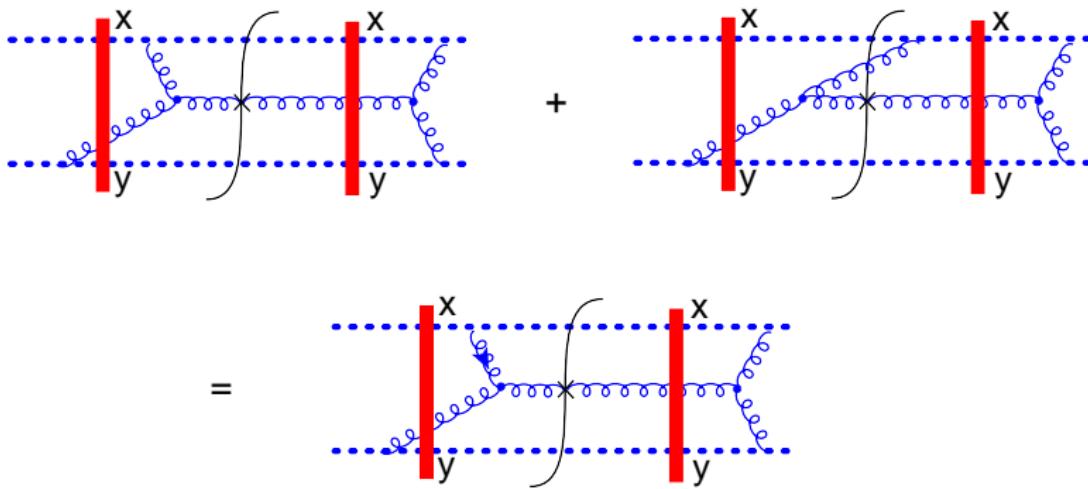


(b)



(c)

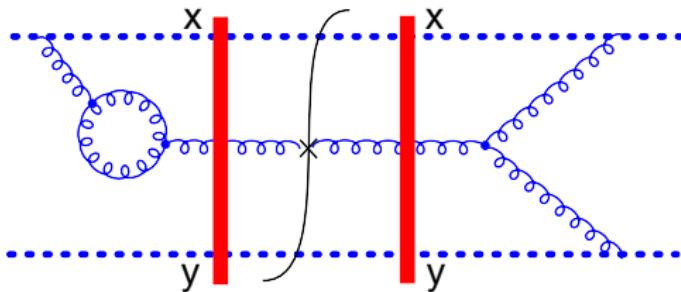
# Retarded Propagator



$$D^{\mu\nu}(k) = g^{\mu\nu} - \frac{k^\mu \eta^\nu + k^\nu \eta^\mu}{k^+} \quad k \cdot \eta = k^+$$

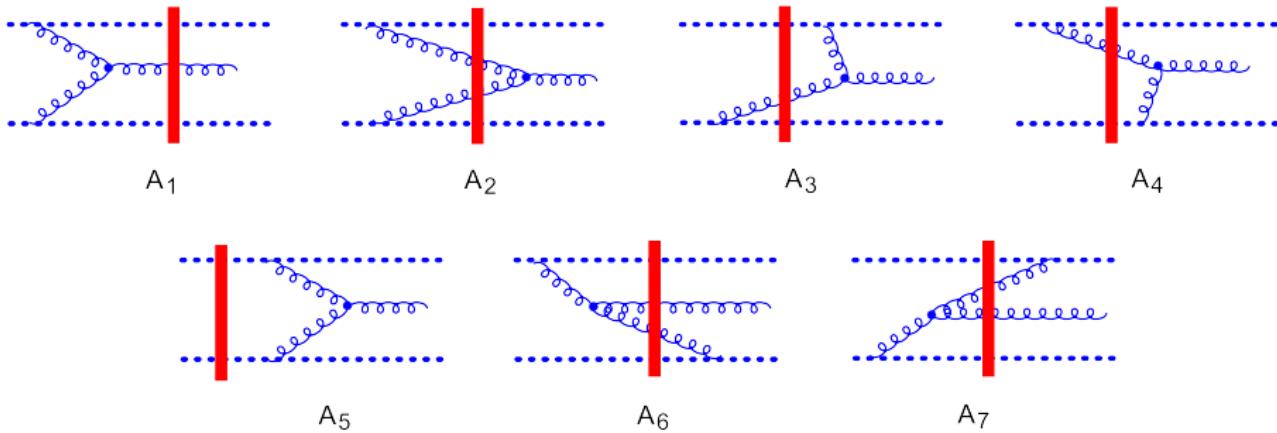
$$\frac{-iD^{\mu\nu}(k)}{k^2 + i\epsilon} + 2\pi\theta(-k^+)\delta(k^2)D^{\mu\nu}(k) = \frac{-iD^{\mu\nu}(k)}{k^2 + i\epsilon k^+}$$

## No quantum corrections

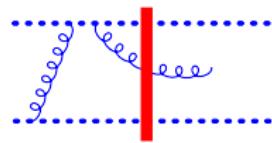


- The diagram is proportional to  $\text{tr}\{U_y U_y^\dagger t^a\} = \text{tr}\{t^a\} = 0$

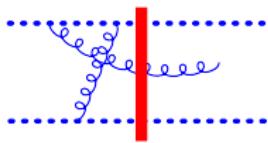
# 3-gluon vertex diagrams



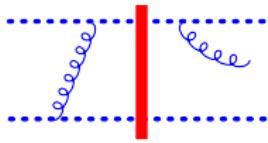
## Box-type diagrams



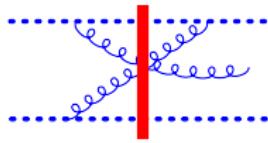
B<sub>1</sub>



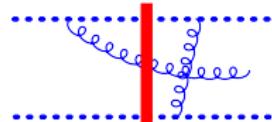
B<sub>2</sub>



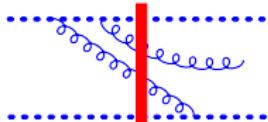
B<sub>3</sub>



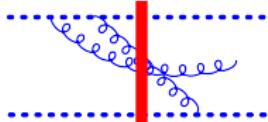
B<sub>4</sub>



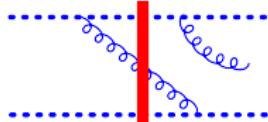
B<sub>5</sub>



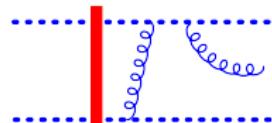
B<sub>6</sub>



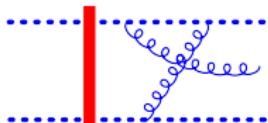
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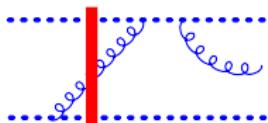
B<sub>8</sub>



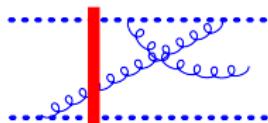
B<sub>9</sub>



B<sub>10</sub>



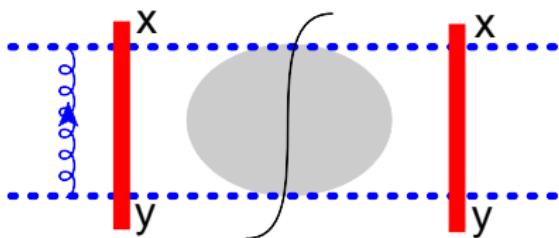
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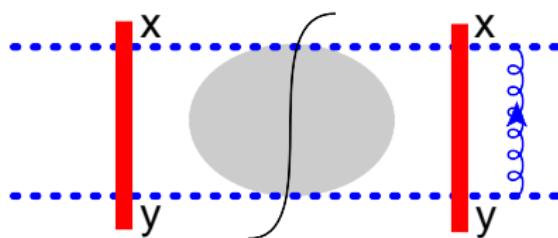
B<sub>12</sub>

# Cancellation of diagrams

shaded area represents any possible interaction.



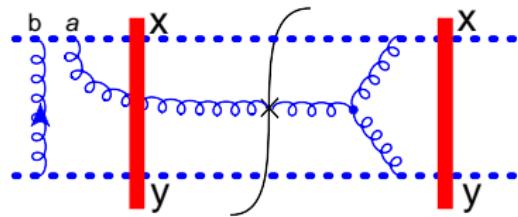
(a)



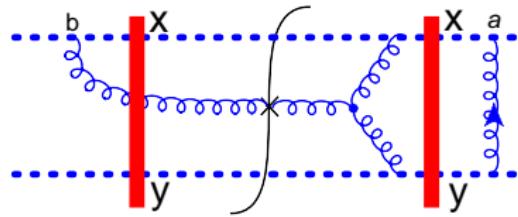
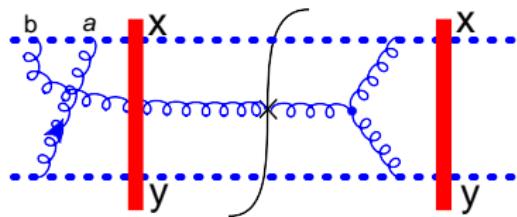
(b)

- Sum of diagram (a) and (b) is zero.

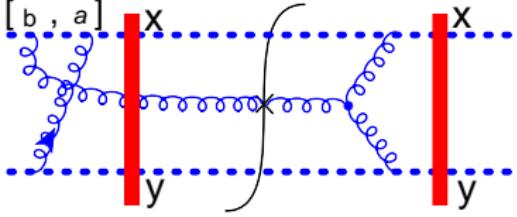
# Commutator: three-gluon vertex



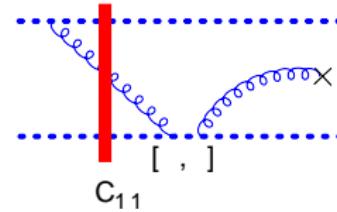
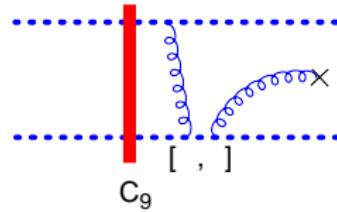
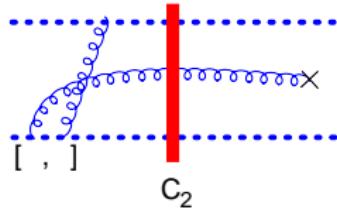
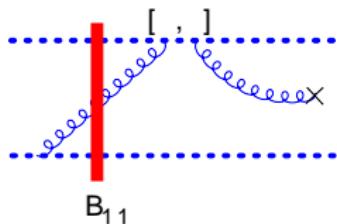
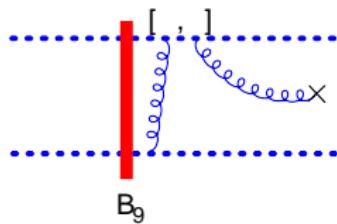
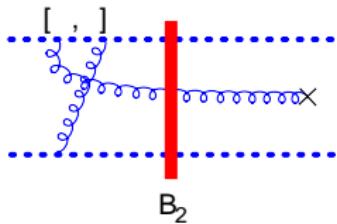
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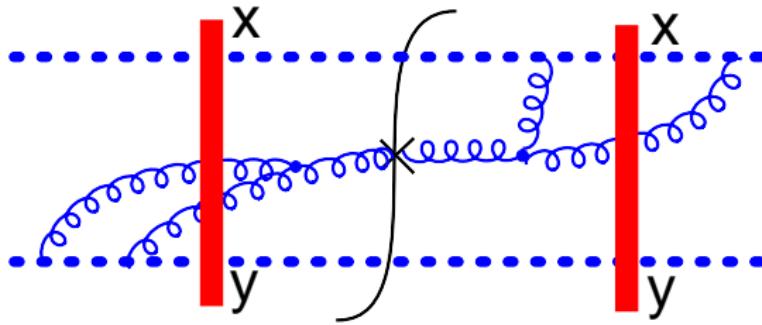
## 3-gluon vertex like diagrams



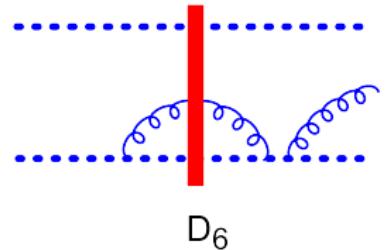
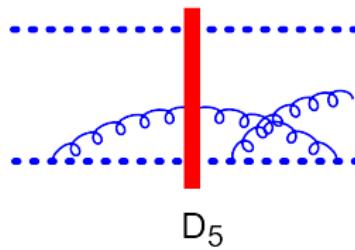
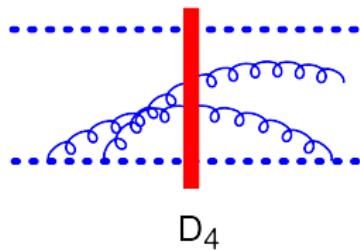
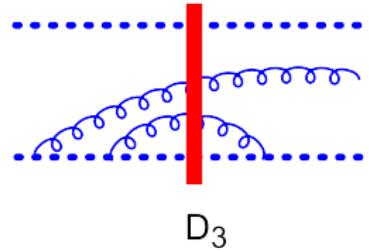
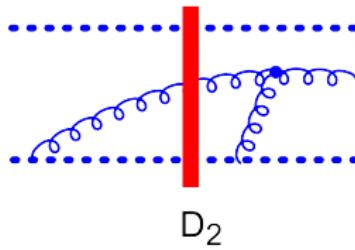
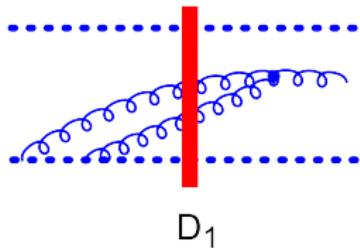
$$\begin{aligned}
& \sum_{i=1}^7 A_i + \sum_{i=1}^{12}' B_i + \sum_{i=1}^{12}' C_i \\
& = -\frac{g^3}{4\pi^4} \int d^2x_1 d^2x_2 \delta[(\vec{z}_\perp - \vec{x}_{1\perp}) \times (\vec{z}_\perp - \vec{x}_{2\perp})] \left[ \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{x}_{2\perp} - \vec{x}_{1\perp})}{|\vec{x}_{2\perp} - \vec{x}_{1\perp}|^2} \frac{\vec{x}_{1\perp} - \vec{b}_{1\perp}}{|\vec{x}_{1\perp} - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_{2\perp} - \vec{b}_{2\perp}}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} \right. \\
& \quad - \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{x}_{1\perp} - \vec{b}_{1\perp})}{|\vec{x}_{1\perp} - \vec{b}_{1\perp}|^2} \frac{\vec{z}_\perp - \vec{x}_{1\perp}}{|\vec{z}_\perp - \vec{x}_{1\perp}|^2} \cdot \frac{\vec{x}_{2\perp} - \vec{b}_{2\perp}}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} + \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{x}_{2\perp} - \vec{b}_{2\perp})}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} \frac{\vec{x}_{1\perp} - \vec{b}_{1\perp}}{|\vec{x}_{1\perp} - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{z}_\perp - \vec{x}_{2\perp}}{|\vec{z}_\perp - \vec{x}_{2\perp}|^2} \Big] \\
& \quad \times f^{abc} \left[ U_{\vec{x}_{1\perp}}^{bd} - U_{\vec{b}_{1\perp}}^{bd} \right] \left[ U_{\vec{x}_{2\perp}}^{ce} - U_{\vec{b}_{2\perp}}^{ce} \right] \left( V_{\vec{b}_{1\perp}} t^d \right)_1 \left( V_{\vec{b}_{2\perp}} t^e \right)_2 \\
& \quad + \frac{i g^3}{4\pi^3} f^{abc} \left( V_{\vec{b}_{1\perp}} t^d \right)_1 \left( V_{\vec{b}_{2\perp}} t^e \right)_2 \int d^2x \left[ U_{\vec{b}_{1\perp}}^{bd} \left( U_{\vec{x}_{1\perp}}^{ce} - U_{\vec{b}_{2\perp}}^{ce} \right) \left( \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{z}_\perp - \vec{x}_\perp)}{|\vec{z}_\perp - \vec{x}_\perp|^2} \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \right. \right. \\
& \quad - \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{z}_\perp - \vec{b}_{1\perp})}{|\vec{z}_\perp - \vec{b}_{1\perp}|^2} \frac{\vec{z}_\perp - \vec{x}_\perp}{|\vec{z}_\perp - \vec{x}_\perp|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} - \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{z}_\perp - \vec{b}_{1\perp})}{|\vec{z}_\perp - \vec{b}_{1\perp}|^2} \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \Big) \\
& \quad - \left( U_{\vec{x}_{1\perp}}^{bd} - U_{\vec{b}_{1\perp}}^{bd} \right) U_{\vec{b}_{2\perp}}^{ce} \left( \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{z}_\perp - \vec{x}_\perp)}{|\vec{z}_\perp - \vec{x}_\perp|^2} \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} - \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \frac{\vec{z}_\perp - \vec{x}_\perp}{|\vec{z}_\perp - \vec{x}_\perp|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \right. \\
& \quad - \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \Big) - \frac{i g^3}{4\pi^2} f^{abc} \left( V_{\vec{b}_{1\perp}} t^d \right)_1 \left( V_{\vec{b}_{2\perp}} t^e \right)_2 \\
& \quad \times \left[ \left( U_{\vec{z}_{\perp}}^{bd} - U_{\vec{b}_{1\perp}}^{bd} \right) U_{\vec{b}_{2\perp}}^{ce} \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{z}_\perp - \vec{b}_{1\perp})}{|\vec{z}_\perp - \vec{b}_{1\perp}|^2} \ln \frac{1}{|\vec{z}_\perp - \vec{b}_{2\perp}| \Lambda} - U_{\vec{b}_{1\perp}}^{bd} \left( U_{\vec{z}_{\perp}}^{ce} - U_{\vec{b}_{2\perp}}^{ce} \right) \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \ln \frac{1}{|\vec{z}_\perp - \vec{b}_{1\perp}| \Lambda} \right] \\
& \quad - \frac{i g^3}{4\pi^3} \int d^2x \left[ U_{x_\perp}^{ab} - U_{z_\perp}^{ab} \right] f^{bde} \left( V_{\vec{b}_{1\perp}} t^d \right)_1 \left( V_{\vec{b}_{2\perp}} t^e \right)_2 \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{z}_\perp - \vec{x}_\perp)}{|\vec{z}_\perp - \vec{x}_\perp|^2} \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \text{Sign}(b_2^- - b_1^-)
\end{aligned}$$

## 3-gluon vertex diagrams with one nucleon

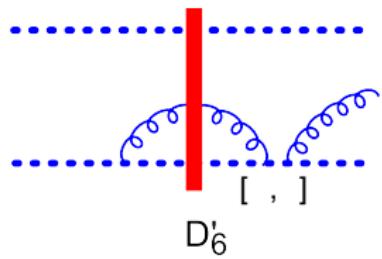
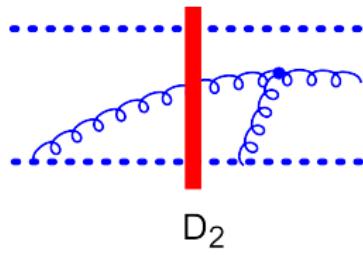
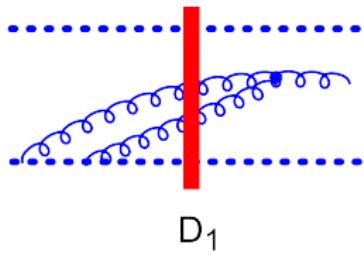
- Power counting:  $\frac{1}{\alpha_s} \left( \alpha_s^2 A_1^{1/3} \right)^2$



# 3-gluon vertex diagrams with one nucleon



## 3-gluon vertex diagrams with one nucleon



$$\sum_{i=1}^6 D_i$$

$$\begin{aligned}
 & = -\frac{g^3}{8\pi^4} \int d^2x_1 d^2x_2 \delta[(\vec{z}_\perp - \vec{x}_{1\perp}) \times (\vec{z}_\perp - \vec{x}_{2\perp})] \left[ \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{x}_{2\perp} - \vec{x}_{1\perp})}{|\vec{x}_{2\perp} - \vec{x}_{1\perp}|^2} \frac{\vec{x}_{1\perp} - \vec{b}_{2\perp}}{|\vec{x}_{1\perp} - \vec{b}_{2\perp}|^2} \cdot \frac{\vec{x}_{2\perp} - \vec{b}_{2\perp}}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} \right. \\
 & \quad \left. - \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{x}_{1\perp} - \vec{b}_{2\perp})}{|\vec{x}_{1\perp} - \vec{b}_{2\perp}|^2} \frac{\vec{z}_\perp - \vec{x}_{1\perp}}{|\vec{z}_\perp - \vec{x}_{1\perp}|^2} \cdot \frac{\vec{x}_{2\perp} - \vec{b}_{2\perp}}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} + \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{x}_{2\perp} - \vec{b}_{2\perp})}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} \frac{\vec{x}_{1\perp} - \vec{b}_{2\perp}}{|\vec{x}_{1\perp} - \vec{b}_{2\perp}|^2} \cdot \frac{\vec{z}_\perp - \vec{x}_{2\perp}}{|\vec{z}_\perp - \vec{x}_{2\perp}|^2} \right] \\
 & \quad \times f^{abc} \left[ U_{\vec{x}_{1\perp}}^{bd} - U_{\vec{b}_{2\perp}}^{bd} \right] \left[ U_{\vec{x}_{2\perp}}^{ce} - U_{\vec{b}_{2\perp}}^{ce} \right] \left( V_{\vec{b}_{1\perp}} \right)_1 \left( V_{\vec{b}_{2\perp}} t^e t^d \right)_2 \\
 & \quad + \frac{i g^3}{4\pi^3} \int d^2x f^{abc} U_{\vec{b}_{2\perp}}^{bd} \left[ U_{\vec{x}_\perp}^{ce} - U_{\vec{b}_{2\perp}}^{ce} \right] \left( V_{\vec{b}_{1\perp}} \right)_1 \left( V_{\vec{b}_{2\perp}} t^e t^d \right)_2 \left( \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{z}_\perp - \vec{x}_\perp)}{|\vec{z}_\perp - \vec{x}_\perp|^2} \frac{1}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \right. \\
 & \quad \left. - \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \frac{\vec{z}_\perp - \vec{x}_\perp}{|\vec{z}_\perp - \vec{x}_\perp|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} - \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \frac{1}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \right) \\
 & \quad + \frac{i g^3}{4\pi^2} f^{abc} U_{\vec{b}_{2\perp}}^{bd} \left[ U_{\vec{z}_\perp}^{ce} - U_{\vec{b}_{2\perp}}^{ce} \right] \left( V_{\vec{b}_{1\perp}} \right)_1 \left( V_{\vec{b}_{2\perp}} t^e t^d \right)_2 \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \ln \frac{1}{|\vec{z}_\perp - \vec{b}_{2\perp}| \Lambda}
 \end{aligned}$$

Setting all  $U = 1$  and all  $V = 1$  we have

$$\sum_{i=1}^7 A_i = 0, \quad \sum_{i=1}^{12}' B_i = 0, \quad \sum_{i=1}^{12}' C_i = 0, \quad \sum_{i=1}^6 D_i = 0, \quad \sum_{i=1}^6 E_i = 0$$

as expected.

## Gauge invariance

Light-cone coordinates:  $x^\pm = \frac{x^0 \pm x^3}{\sqrt{2}}$

Propagator in light-cone gauge  $A^+ = 0$ :

$$\langle A^\mu(x) A^\nu(y) \rangle = \int \frac{d^4 k}{(2\pi)^4} \frac{d^{\mu\nu}(k)}{k^2 + i\epsilon} e^{-ik \cdot (x-y)}$$

Light-cone propagator singularity:

$$d^{\mu\nu}(k) = g^{\mu\nu} - \frac{\eta^\mu k^\nu + \eta^\nu p^\mu}{k^+}$$

Sub-gauge condition will set the prescription for the  $\frac{1}{k^+}$  singularity.

## Which prescription should be used?

- There are several possible choices to regulate light-cone (or temporal) gauge. Why don't we just use one of them?
- If we choose one of the available prescriptions for light-cone pole, how do we know whether quantization of our theory really allows that particular prescription we chose? We might get wrong result for the calculation at hand.
- In calculation at high-energy, PV prescription simplify the calculation reducing a lot the number of diagrams to be calculated.
- Moreover, In processes like two very energetic quarks off a large nucleus, quark lines reduce to Wilson line only if one uses PV prescription for light-cone poles.

## Which prescription should be used?

- Suppose we choose PV prescription for light-cone gauge propagator poles, how do we actually use it when we have multiple light-cone poles? Do we use the same  $\epsilon$ 's or they have to be different?
- Ambiguous case where the use of PV is not clear

$$\int \frac{d^2 k_\perp dk^+}{(2\pi)^3} \frac{d^2 l_\perp dl^+}{(2\pi)^3} e^{-ik^+(x^- - b_2^-) - il^+(b_2^- - b_1^-) + i\vec{k}_\perp \cdot (\vec{x}_\perp - \vec{b}_{2\perp}) + i\vec{l}_\perp \cdot (\vec{b}_{2\perp} - \vec{b}_{1\perp})}$$
$$\times \frac{ig^3 f^{abc} t^a(t^b)_2(t^c)_1}{k_\perp^2 l_\perp^2 (\vec{k}_\perp - \vec{l}_\perp)^2} \left[ \frac{\vec{l}_\perp \cdot (\vec{k}_\perp - \vec{l}_\perp) k_\perp^\mu (k^+ - 2l^+)}{k^+ l^+ (k^+ - l^+)} \right]$$

- If we choose PV prescription as an *ad hoc choice*, then for each pole we have to use different  $i\epsilon$ 's and the result will depend on the order we send the  $i\epsilon$ 's to zero.

# Sub-gauge conditions for light-cone propagator

G.A.C., Y. Kovchegov, D. Wertepny (2015) arXiv:1508.07962

- PV-sub-gauge:  $\partial_{\perp} \cdot A_{\perp}(x^- = +\infty) + \partial_{\perp} \cdot A_{\perp}(x^- = -\infty) = 0$

$$D_{PV}^{\mu\nu}(x, y) \equiv \int \frac{d^4 k}{(2\pi)^4} e^{-ik \cdot (x-y)} \frac{-i}{k^2 + i\epsilon} \left[ g^{\mu\nu} - (k^\mu \eta^\nu + k^\nu \eta^\mu) \text{PV} \left\{ \frac{1}{k^+} \right\} \right]$$

$$\text{PV} \left\{ \frac{1}{k^+} \right\} \equiv \frac{1}{2} \left( \frac{1}{k^+ + i\epsilon} + \frac{1}{k^+ - i\epsilon} \right)$$

- sub-gauge:  $\vec{\partial}_{\perp} \cdot \vec{A}_{\perp}(x^- = +\infty) = 0$

$$D_1^{\mu\nu}(x, y) \equiv \int \frac{d^4 k}{(2\pi)^4} e^{-ik \cdot (x-y)} \frac{-i}{k^2 + i\epsilon} \left[ g^{\mu\nu} - \frac{k^\mu \eta^\nu}{k^+ - i\epsilon} - \frac{k^\nu \eta^\mu}{k^+ + i\epsilon} \right]$$

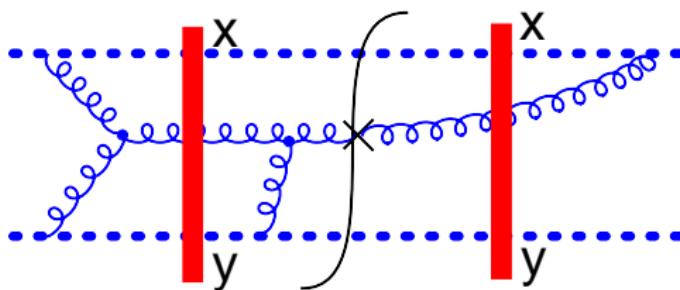
- sub-gauge:  $\vec{\partial}_{\perp} \cdot \vec{A}_{\perp}(x^- = -\infty) = 0$

$$D_2^{\mu\nu}(x, y) \equiv \int \frac{d^4 k}{(2\pi)^4} e^{-ik \cdot (x-y)} \frac{-i}{k^2 + i\epsilon} \left[ g^{\mu\nu} - \frac{k^\mu \eta^\nu}{k^+ + i\epsilon} - \frac{k^\nu \eta^\mu}{k^+ - i\epsilon} \right]$$

## Conclusions

- Result in transverse coordinate space for the  $g^3$  amplitude have been presented.
- The result have been obtained using two different sub-gauge conditions which fix the prescription of the  $k^+$  singularity in the light-cone propagator.
- This result is part of the analytic calculation of the single inclusive gluon production cross-section for Heavy-Light Ion collisions at the classical level.
- Similar calculation have been performed by Balitsky (2004).
- Check conformal invariance in transverse coordinate space of the final result.

- Sample of diagrams:  $g^5$  amplitude



- Final goal:

- Cross-section for gluon production in Nucleus-Nucleus collision.
- Initial condition of Quark Gluon Plasma.