$X(3872)$ And Its Charmonium Content in Heavy Quark Limit[1]

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Motivation and Puzzles

- $X(3872)$ was first observed by the Belle collaboration in the $B^{\pm} \to J/\psi\pi^{+}\pi^{-}K^{\pm}$ in 2003 and later confirmed by CDF, D0, BaBar and LHCb
- Its quantum numbers are determined by LHCb to be $J^{PC} = 1^{++}$
- Some of its properties are hard to reconcile with a single description of $X(3872)$

Motivation and Puzzles

- In the quark models, the mass of the state with $J^{PC} = 1^{++}$ are estimated as 3947 MeV(Ortega'10, Segovia'13), 3906 MeV (Ebert'11), 3925 MeV (Barnes'05)
- The predicted masses are $30 - 80$ MeV above the mass of observed $X(3872)$
- The mass of $X(3872)$ is only 0.16 MeV below the $D^0 \bar{D}^{*0}$ threshold, motivating a description as a $D^0 \bar{D}^{*0}$ molecule
- The mass of the $D^0 \bar{D}^{*0}$ molecule is $m_X = m_{D^0} + m_{D^{*0}} - B$, where B is the binding energy
- In the molecular picture, $X(3872)$ is a loosely bound state
- Another popular description is as a tetraquark formed by a diquark and an anti-diaquark (see e.g. Maiani'05)

Isospin Violating Decays

- $X(3872)$ decays into a final state with $I = 0$ and $I = 1$ with almost equal branching ratio:

$$\frac{B(X(3872) \to J/\psi\rho)}{B(X(3872) \to J/\psi\omega)} \simeq 1 \Rightarrow \frac{A(X(3872) \to J/\psi\rho)}{A(X(3872) \to J/\psi\omega)} \simeq 0.26 \pm 0.07$$

- Isospin symmetry is greatly respected in strong decays, hence hard to reconcile with a conventional charmonium picture
- Can easily be accommodated in the molecular picture where the isospin violation arises due to the mass difference between the neutral $D^0 \bar{D}^{*0}$ and charged $D^+ \bar{D}^{*-}$.
- Mixing between the $I = 0$ and $I = 1$ components in the molecular picture is estimated to be close to maximum: $\theta \simeq 39^\circ$ (Gamermann'10)

Radiative Decays

- $X(3872)$ is observed to decay onto $J/\psi\gamma$ and $\psi(2S)\gamma$ with the ratio:

$$R_{\psi\gamma} = \frac{\text{Br}(X \to \psi(2S)\gamma)}{\text{Br}(X \to J/\psi\gamma)} = 2.46 \pm 0.64 \pm 0.29$$

- $X \to \psi(2S)\gamma$ has a much smaller phase space available.
- This ratio can naturally be accommodated in the conventional picture, since the decay into $\psi(2S)$ is just the $\Delta L = 1$ transition.
- In the molecular picture, to accommodate this measurement, the ratio of the $DD^*\psi'$ coupling to DD^*J/ψ coupling should be around 2. (Guo'15)

Production Rate

- In *pp* collisions, $X(3872)$ has a production rate that is about $1/20$ of the production rate of $\psi(2S)$. (Takizawa'13)
- Hard to explain in the pure molecular picture

Charmonium-Molecule Hibrid

- The “good” aspects of both worlds can be combined assuming that $X(3872)$ is mainly a molecular state with about $5-10\%$ charmonium admixture (Takizawa’13, Dong’11)

EFT To Describe Molecular $X(3872)$

- Molecular states can be described within an effective field theory framework.
- Too many parameters for too few experimental observations
- Solution: (approximate) symmetries to relate properties of different particles
- c quark is a heavy quark $\rightarrow$ heavy quark spin symmetry (HQSS)

Heavy Quark Effective Theory

- In the heavy quark limit, $m_Q \to \infty$, the spin of the heavy quark decouples from the dynamics
- Particles that differ by the spin of the heavy quark form degenerate multiplets, e.g. D and D^*
- These multiplets can be described by the field:

$$H_a^{(Q)} = \frac{1+ \not{v}}{2} \left(P_{a\mu}^{*(Q)} \gamma^\mu - P_a^{(Q)} \gamma_5 \right), \qquad v \cdot P_a^{*(Q)} = 0,$$

$$H^{(\bar{Q})a} = \left(P_\mu^{*(\bar{Q})a} \gamma^\mu - P^{(\bar{Q})a} \gamma_5 \right) \frac{1- \not{v}}{2}, \qquad v \cdot P^{*(\bar{Q})a} = 0.$$

Heavy Quark Effective Theory

- The LO Lagrangian respecting HQSS symmetry is (AlFiky'06):

$$
\begin{aligned}
\mathcal{L}_{4H} = & \; C_A \, \mathrm{Tr}\left[\bar{H}^{(Q)a} H_a^{(Q)} \gamma_\mu\right] \mathrm{Tr}\left[H^{(\bar{Q})a} \bar{H}_a^{(\bar{Q})} \gamma^\mu\right] \\
& + C_A^\tau \, \mathrm{Tr}\left[\bar{H}^{(Q)a} \vec{\tau}^{\,b}_{\;.\,a} H_b^{(Q)} \gamma_\mu\right] \mathrm{Tr}\left[H^{(\bar{Q})c} \vec{\tau}^{\,d}_{\;.\,c} \bar{H}_d^{(\bar{Q})} \gamma^\mu\right] \\
& + C_B \, \mathrm{Tr}\left[\bar{H}^{(Q)a} H_a^{(Q)} \gamma_\mu \gamma_5\right] \mathrm{Tr}\left[H^{(\bar{Q})a} \bar{H}_a^{(\bar{Q})} \gamma^\mu \gamma_5\right] \\
& + C_B^\tau \, \mathrm{Tr}\left[\bar{H}^{(Q)a} \vec{\tau}^{\,b}_{\;.\,a} H_b^{(Q)} \gamma_\mu \gamma_5\right] \mathrm{Tr}\left[H^{(\bar{Q})c} \vec{\tau}^{\,d}_{\;.\,c} \bar{H}_d^{(\bar{Q})} \gamma^\mu \gamma_5\right]
\end{aligned}
$$

- This model predicts 0^{++}, 1^{++}, 2^{++} and 1^{+-} states that have similar binding energies (Nieves'12)

Adding the charmonium

- $L = 1$ charmonia form quadruplets that can be represented by the field:

$$J^{\mu} = \frac{1+ \not{v}}{2} \left(\chi_2^{\mu\alpha}\gamma_\alpha + \frac{i}{\sqrt{2}}\epsilon^{\mu\alpha\beta\gamma}\chi_{1\gamma} v_\alpha \gamma_\beta + \frac{1}{\sqrt{3}}\chi_0(\gamma^\mu - v^\mu) + h^\mu \gamma_5 \right) \frac{1- \not{v}}{2}$$

Adding the charmonium

- At LO, the HQSS respecting interaction vertex between J^μ and $H^{(Q)}$ is unique:

$$\mathcal{L}_{HHQ\bar{Q}} = d\,\mathrm{Tr}[H^{a(\bar{Q})}\bar{J}_\mu H_a^{(Q)}\gamma^\mu] + d\mathrm{Tr}[\bar{H}^{a(Q)}J_\mu H_a^{(\bar{Q})}\gamma^\mu]$$

- The interaction introduces a single additional parameter
- In terms of explicit fields:

$$\begin{aligned}\mathcal{L}_{c\bar{c}} = -2\sqrt{2}d \Big[& -\sqrt{2}\chi_1^{\dagger\eta}\left(P\bar{P}^*_\eta - P^*_\eta\bar{P}\right) \\ & - \sqrt{3}\chi_0^\dagger\left(P\bar{P} + \frac{1}{3}P^*_\eta\bar{P}^{*\eta}\right) + h^{\dagger\eta}\left(P\bar{P}^*_\eta + P^*_\eta\bar{P}\right) \\ & + i\epsilon_{\alpha\mu\rho\eta}v^\alpha h^{\dagger\mu}P^{*\rho}\bar{P}^{*\eta} + 2\chi_2^{\dagger\rho\eta}P^*_\rho\bar{P}^*_\eta\Big] + h.c.\end{aligned}$$

Adding the charmonium

- In the potential quark model (1985), there are infinitely many P-wave charmonia
- Each radial excitation of the charmonia form a quadruplet they can couple to the meson multiplets by a Lagrangian of the same form.
- The charmonia that will mix the most with the molecule is the one that will have its mass closest to the $D\bar{D}^*$ mass (for the $X(3872)$)
- For the case of $X(3872)$, the charmonia that will mix the most with the molecular states are the $2P$ states, i.e. the first radial excitation.

Adding the charmonium

- The unknown parameters:
  - The contact interaction parameters $C_A^{(\tau)}$ and $C_B^{(\tau)}$
  - The coupling parameter d
  - The masses of the charmonium states: 1 parameter in every J^{PC} sector.
- To reduce the parameters:
  - Assume isospin symmetry
  - Concentrate only in the 1^{++} $(D\bar{D}^*)$ and 2^{++} $(D^*\bar{D}^*)$ sectors: depends only on $C_{0A} + C_{0B}$

Forming the Lippmann-Schwinger Equation

- Isoscalar $\left[D^{(*)}\bar{D}^{(*)} \to D^{(*)}\bar{D}^{(*)}\right]$ potential is unique:

$$V(1^{++}) = V(2^{++}) = C_{0A} + C_{0B} \equiv C_0$$

Forming the Lippmann-Schwinger Equation

- $\psi_{c\bar{c}}(2P) \to D^{(*)}\bar{D}^{(*)}$ amplitudes

$$V_{c\bar{c}}(1^{++}) = d \qquad \chi_{c1}(2P) \to [D\bar{D}^*]_+$$

$$V_{c\bar{c}}(2^{++}) = d \qquad \chi_{c2}(2P) \to D^*\bar{D}^*$$

Forming the Lippmann-Schwinger Equation

The T matrix can be constructed as

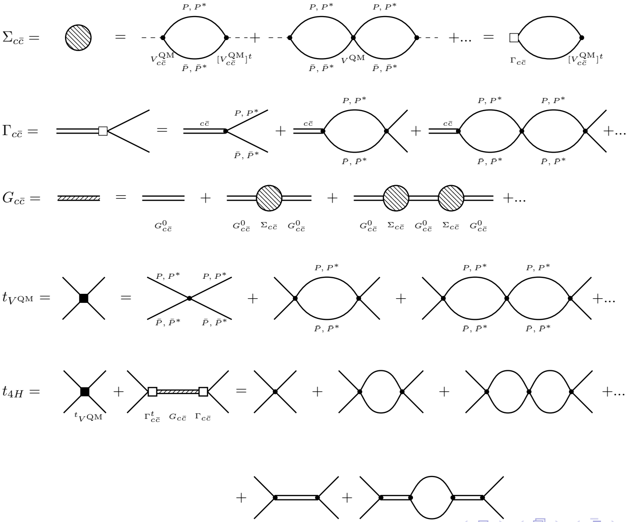
$$\langle \vec{p}\,' | T(E) | \vec{p}\,\rangle = F_\Lambda(\vec{p}\,') \begin{pmatrix} \left[t_V + \Gamma_{c\bar{c}} G_{c\bar{c}} \Gamma^t_{c\bar{c}}\right]_{n\times n} & \left[\dfrac{\Gamma_{c\bar{c}}}{1 - G^0_{c\bar{c}}\Sigma_{c\bar{c}}}\right]_{n\times 1} \\ \left[\dfrac{\Gamma^t_{c\bar{c}}}{1 - G^0_{c\bar{c}}\Sigma_{c\bar{c}}}\right]_{1\times n} & \left[\dfrac{\Sigma_{c\bar{c}}}{1 - G^0_{c\bar{c}}\Sigma_{c\bar{c}}}\right]_{1\times 1} \end{pmatrix} F_\Lambda(\vec{p}\,)$$

where F_Λ, $\Gamma_{c\bar{c}}$ and $\Sigma_{c\bar{c}}$ are the regularisation functions, dressed $c\bar{c} \to D\bar{D}$ vertex, and $c\bar{c}$ self energy

$$t_V = (1 - VG(E))^{-1}\, V$$

is the T matrix in the absence of the charmonium contribution

Forming the Lippmann-Schwinger Equation



Forming the Lippmann-Schwinger Equation

- The resonances are found at the poles of the T matrix
- The couplings of the resonance to various states can be calculated

$$t_{ij}(E \sim E_R) = \frac{g_i g_j}{E - E_R} + \text{ finite terms}$$

where $E_R = m_R - i\frac{\Gamma}{2}$ and g_i is the coupling of the resonance to the i^{th} channel

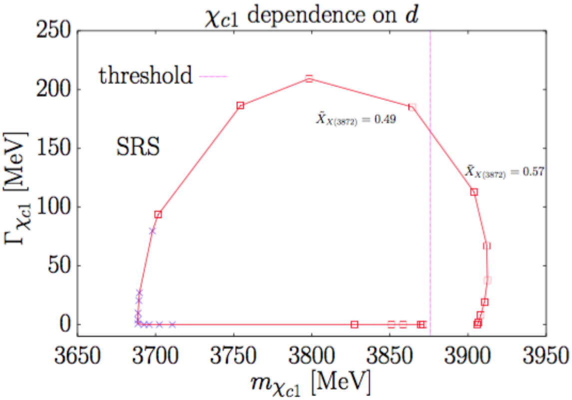
Results for 1^{++} sector

- The bare mass of the 1^{++} is taken to be $m = 3906$ MeV (Ebert'11)
- Mass of $X(3872)$ is fixed at $m_X = 3871.69$ MeV, fixing the value of C_0 for a given value of d
- The only unknown is d.

Results for 1^{++} sector

d [$\mathrm{fm}^{1/2}$]	C_0 [fm^2]	$g_{D\bar{D}^*}^{X(3872)}$ [$\mathrm{GeV}^{-1/2}$]	$\tilde{X}_{X(3872)}$	$(m_{\chi_{c1}}, \Gamma_{\chi_{c1}})$ [MeV]	$g_{D\bar{D}^*}^{\chi_{c1}}$ [$\mathrm{GeV}^{-1/2}$]	$\lvert\tilde{X}_{\chi_{c1}}\rvert$	$\tilde{Z}_{\chi_{c1}}$
0.	-0.789	0.90	1	(3906.0,0)	0.	0.	1.
0.05	-0.774	0.89	0.98	(3906.6, 1.9)	$0.01+0.16\,i$	0.02	$0.99+0.01\,i$
0.1	-0.731	0.87	0.92	(3908.2, 7.9)	$0.03+0.31\,i$	0.06	$0.96+0.05\,i$
0.15	-0.659	0.83	0.84	(3910.5, 19.2)	$0.07+0.44\,i$	0.14	$0.92+0.11\,i$
0.20	-0.559	0.78	0.75	(3912.4, 37.8)	$0.14+0.56\,i$	0.23	$0.87+0.19\,i$
0.25	-0.429	0.73	0.66	(3912.0, 67.0)	$0.24+0.65\,i$	0.36	$0.82+0.31\,i$
0.30	-0.271	0.68	0.57	(3903.9, 112.8)	$0.38+0.73\,i$	0.55	$0.77+0.50\,i$
0.35	-0.084	0.63	0.49	(3864.5, 185.2)	$0.63+0.85\,i$	>1	$0.70+1.01\,i$
$d^{\,\mathrm{crit}}$	0.000	0.61	0.46	(3798.3, 209.4)	$0.93+1.09\,i$	>1	$0.53+2.12\,i$
0.375	0.020	0.61	0.46	(3754.4, 186.4)	$1.21+1.37\,i$	>1	$0.29+3.66\,i$
0.3775	0.031	0.61	0.46	(3701.6, 93.5)	$2.19+2.39\,i$	>1	$-0.44+12.27\,i$
0.40	0.132	0.59	0.43	(3827.1, 0) at SRS	0.96	$\tilde{X}_{\chi_{c1}}<0$	2.07
0.45	0.376	0.55	0.37	(3850.9,0) at SRS	0.63	$\tilde{X}_{\chi_{c1}}<0$	1.52
0.5	0.649	0.51	0.32	(3858.4,0) at SRS	0.51	$\tilde{X}_{\chi_{c1}}<0$	1.36
1.0	4.963	0.29	0.11	(3869.7, 0) at SRS	0.21	$\tilde{X}_{\chi_{c1}}<0$	1.08
2.0	22.217	0.15	0.03	(3871.3, 0) at SRS	0.10	$\tilde{X}_{\chi_{c1}}<0$	1.02
$d \gg d^{\,\mathrm{crit}}$	$\sim \frac{d^2}{\mathring{m}_{\chi_{c1}}-M_X}$	$\mathcal{O}(\frac{1}{d^2})$	$\mathcal{O}(\frac{1}{d^2})$	$(M_X-\mathcal{O}(\frac{1}{d^2}), 0)$ at SRS	$\mathcal{O}(1/d^2)$	$\tilde{X}_{\chi_{c1}}=-\mathcal{O}(\frac{1}{d^2})$	$1+\mathcal{O}(1/d^2)$

Results for 1^{++} sector



Results for the 2^{++} sector

- 2^{++} charmonium is observed experimentally to have a mass $m_{\chi_{c2}} = 3927.2$ MeV
- Use as an input to fix the bare mass of the 2^{++} charmonium
- The predicted masses from various quark models are range around 3970 ± 10 MeV
- A $D^* \bar{D}^*$ molecular state is not observed

Results for the 2^{++} sector

d [fm$^{1/2}$]	$\tilde{X}_{X(3872)}$	$g^{\chi_{c2}}_{D^*\bar{D}^*}$ [GeV$^{-1/2}$]	$\tilde{X}_{\chi_{c2}}$	$\mathring{m}_{\chi_{c2}}$ [MeV]	$M_{X_2} - 2M_{D^*} - i\frac{\Gamma_{X_2}}{2}$ [MeV]	$g^{X_2}_{D^*\bar{D}^*}$ [GeV$^{-1/2}$]	$\tilde{X}_{X_2}$
0.	1	0.0	0.0	3927.2	-5.6	0.97	1.
0.05	0.98	0.27	0.01	3927.8	-4.5	0.90	0.996
0.10	0.92	0.51	0.02	3929.6	-1.8	0.67	0.991
0.15	0.84	0.69	0.04	3932.2	-0.0 at SRS	$-0.12\,i$	> 1
0.20	0.75	0.82	0.05	3935.2	-6.4 at SRS	$-0.76\,i$	> 1
0.22	0.71	0.86	0.06	3936.4	-21.2 at SRS	$-1.24\,i$	> 1
0.25	0.66	0.90	0.06	3938.3	$-28.3 - \frac{72.9}{2}\,i$	$0.23 - 0.65\,i$	$0.47 + 0.32\,i$
0.30	0.57	0.95	0.07	3941.2	$-31.2 - \frac{162.8}{2}\,i$	$0.03 + 0.67\,i$	$0.48 - 0.04\,i$
0.35	0.49	0.96	0.07	3943.8	$-59.5 - \frac{312.6}{2}\,i$	$0.30 + 0.71\,i$	$0.52 - 0.39\,i$

Conclusion and Outlook

- A framework based on HQSS has been constructed which takes into account the charmonium content of $D^{(*)}\bar{D}^{(*)}$ molecules
- $X(3872)$ can easily acquire a sizable charmonium content
- χ_{c1} should have a sizable molecular content
- The charmonium can destabilize the 2^{++} molecule, explaining its non-observation
- Coupling between the charmonium and the molecular components reduces the discrepancies between the potential quark model calculations and (non)observations

Conclusion and Outlook

Outlook:

- Effects of isospin breaking
- Calculate d
- $1/m_Q$ corrections

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