Time-energy correlations as a hallmark of different branching processes

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Computational approaches in Quantum Field Theory, Statistical Mechanics and Complex Systems

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SOLAR FLARES

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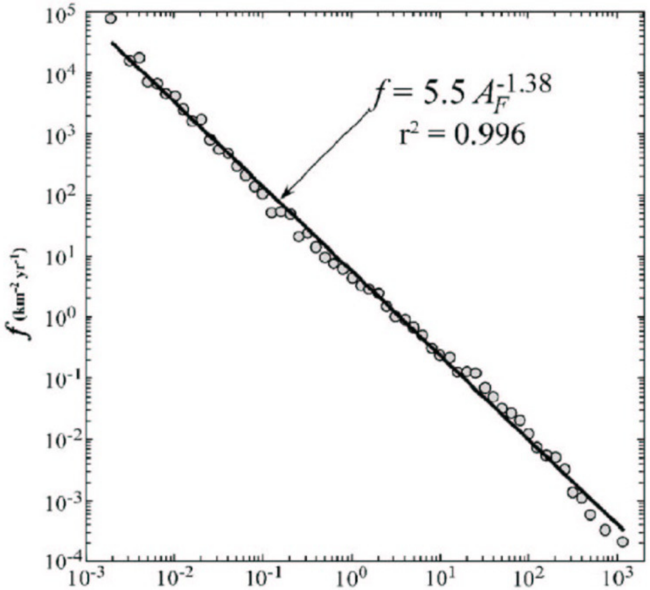
Power laws in natural hazards

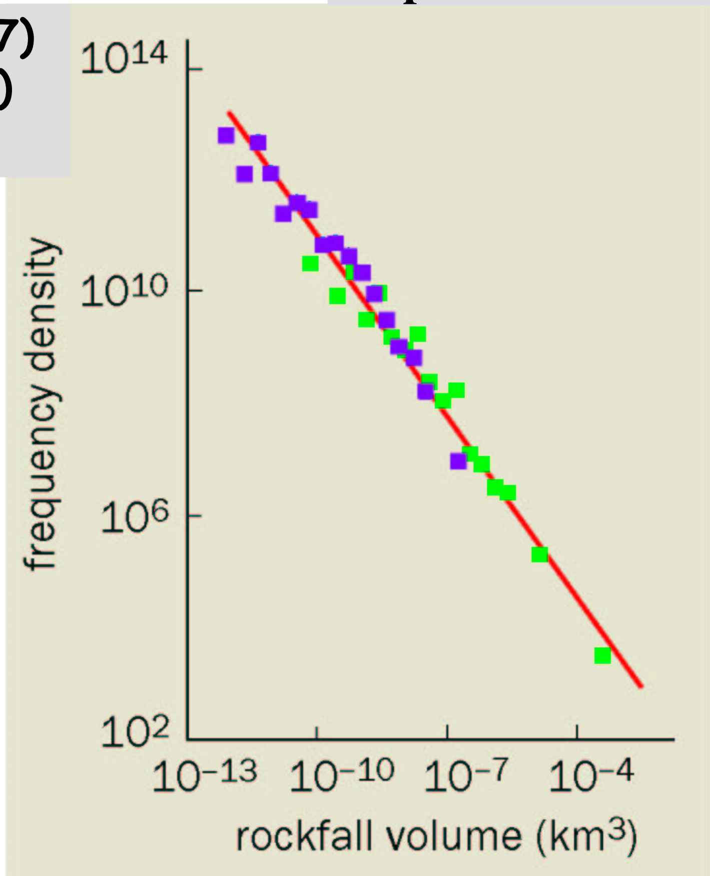


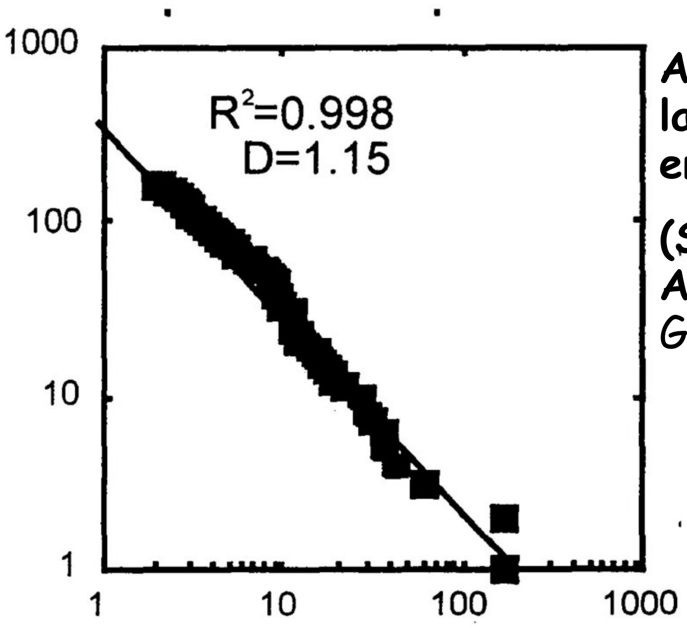
Forest fires in Ontario (Canada) 1976-1996

Turcotte & Malamud 2004

Rockfall in Umbria (1997) & Yosemite (1980-2002)
Malamud 2004

Exponent 1.1

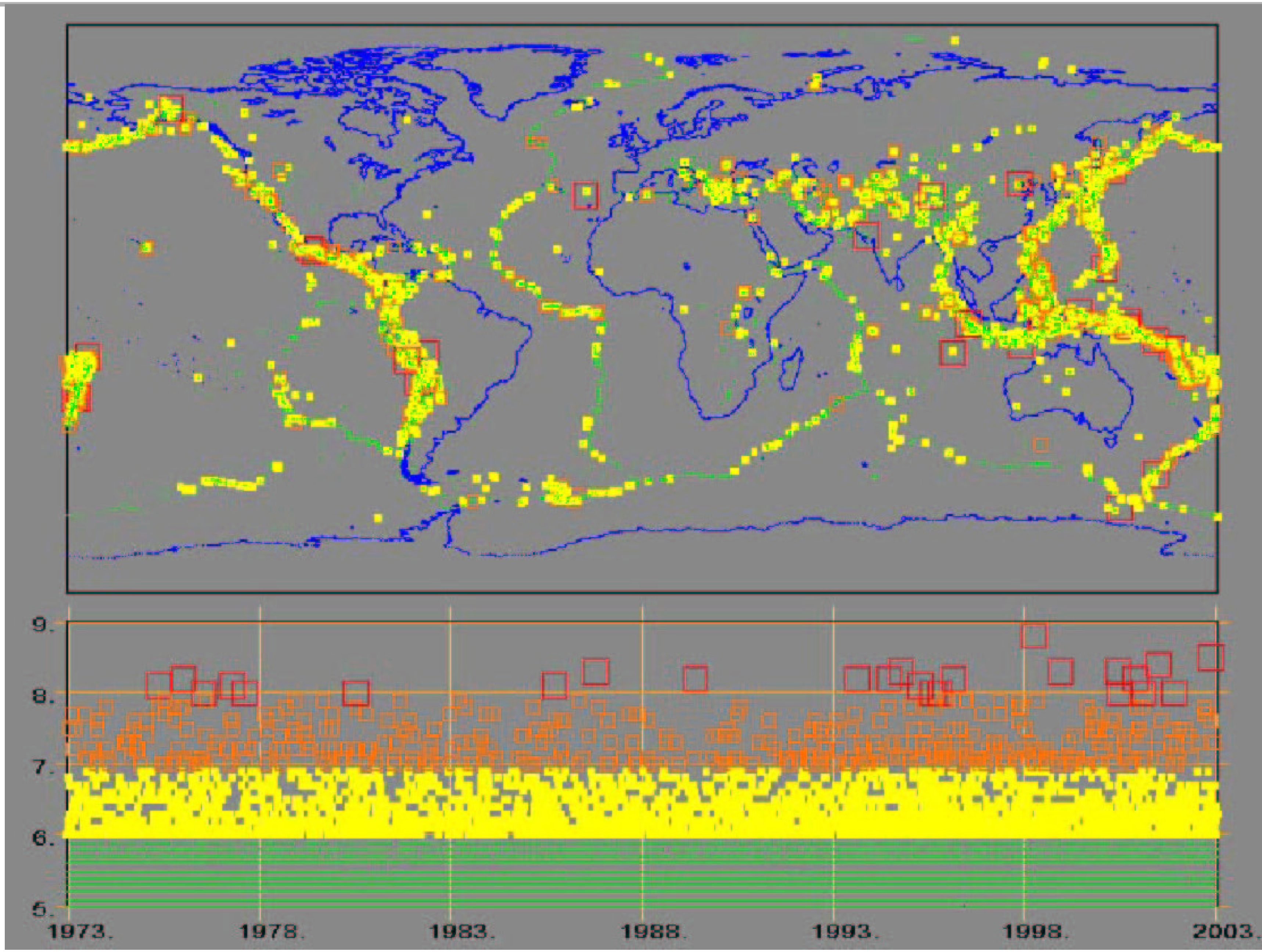




Areas covered by lava in volcanic eruptions

(Springerville, Arizona) Lahaie & Grasso 1998

Earthquakes in the world during 1973-2003



How big can an earthquake be?

Gutenberg-Richter Law (1954)

$$P(>M) \sim 10^{-b\,M} \quad (b\sim 1)$$

Seismic moment $M_0 = \mu A \Delta u$

$M = (2/3)\log(M_0)-6$
(Kanamori,Anderson 1975)

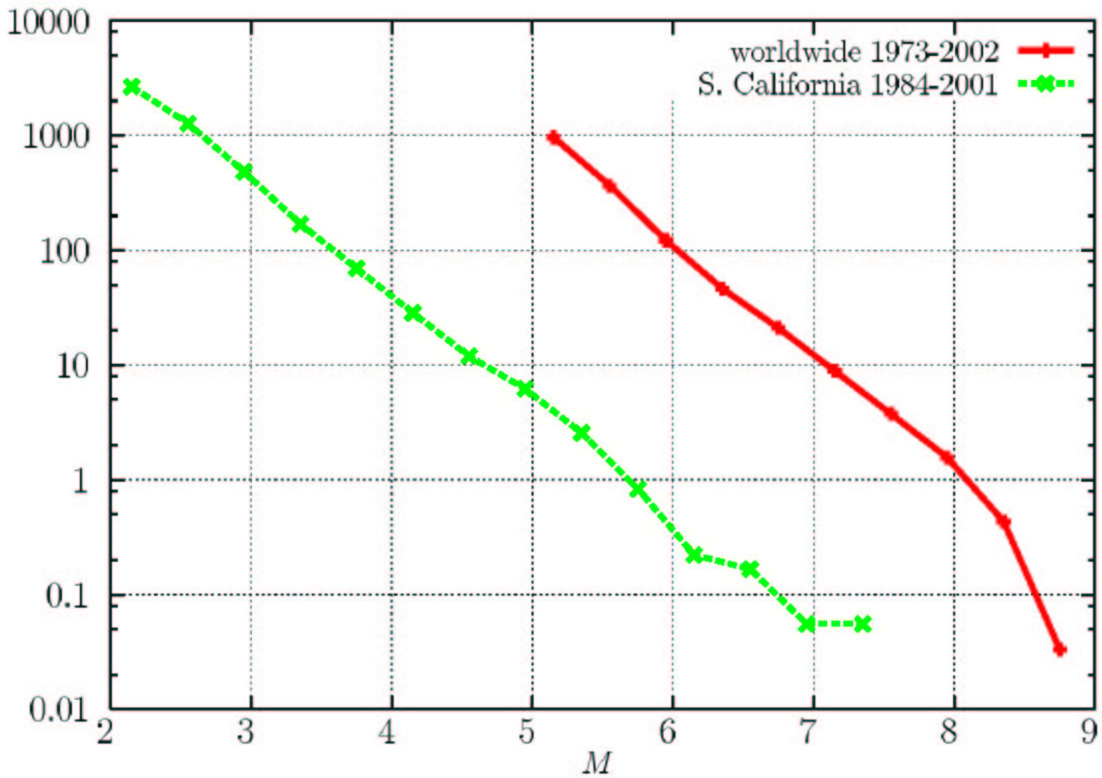
$$P(>M_0) \sim M_0^{-\alpha}$$

Energy

$M = (2/3)\log(E)+cost$

$$P(>E) \sim E^{-\alpha}$$

Universality of $\alpha \sim 0.7$



Temporal correlations: Sequences of aftershocks

Omori law (JCSIUT,1894)

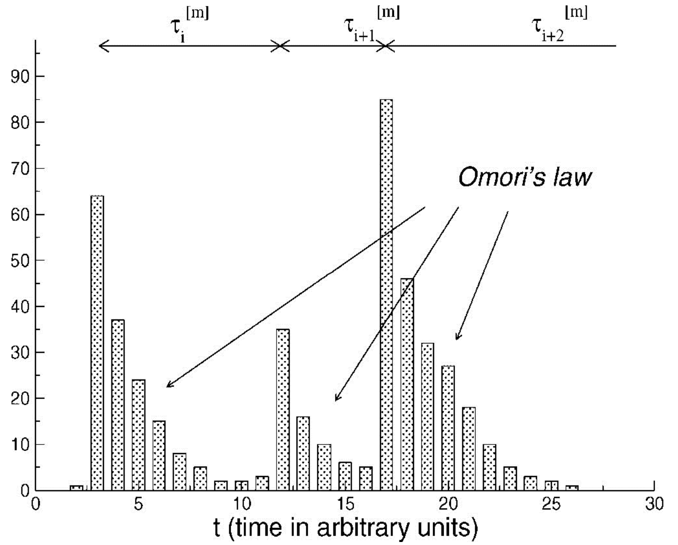
At time t after a main shock at t=0

$$\boxed{n_{AS}(t) \sim (c+t)^{-p}} \qquad p \sim 1$$

c depends on M main shock

and M lower cutoff

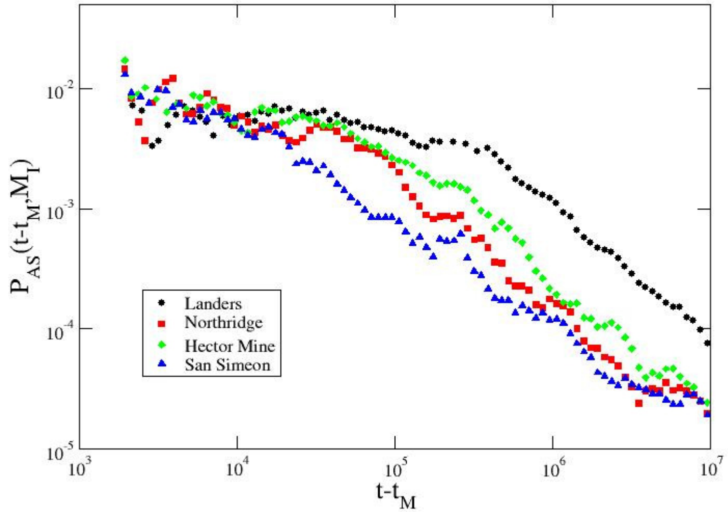
(Kagan 2004, Shcherbakov et al 2004, Lise et al 2004)



Productivity law

$$\boxed{N_{AS}(M) \sim 10^{\alpha M}} \qquad \alpha \sim b$$

(Helmstetter 2003, Felzer et al 2004,
Helmstetter et al 2005, 2006)

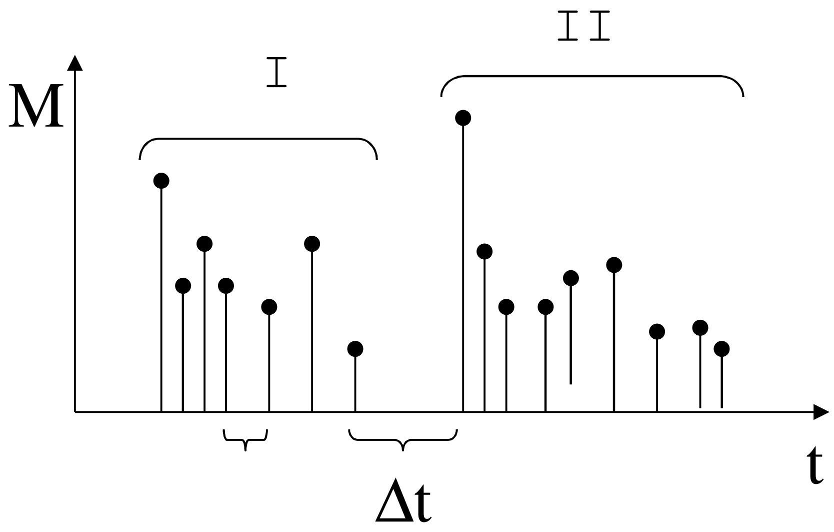


Intertime distribution

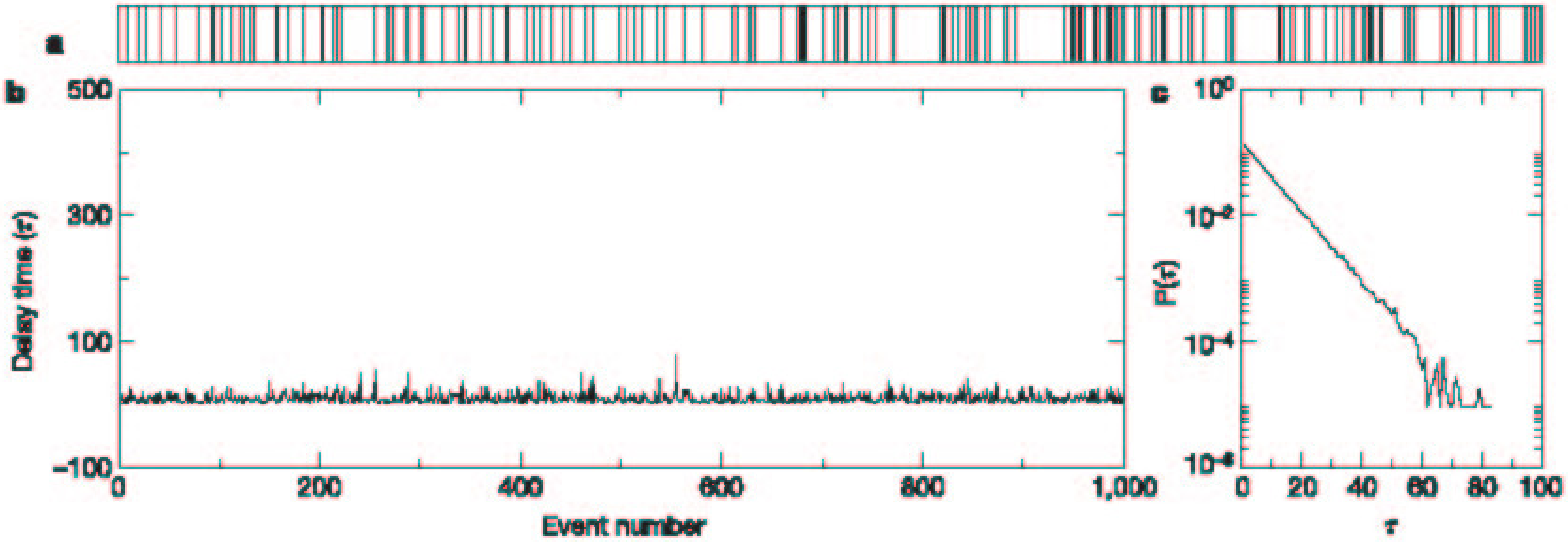
Probability distribution of intertimes

$$\Delta t$$

between consecutive events



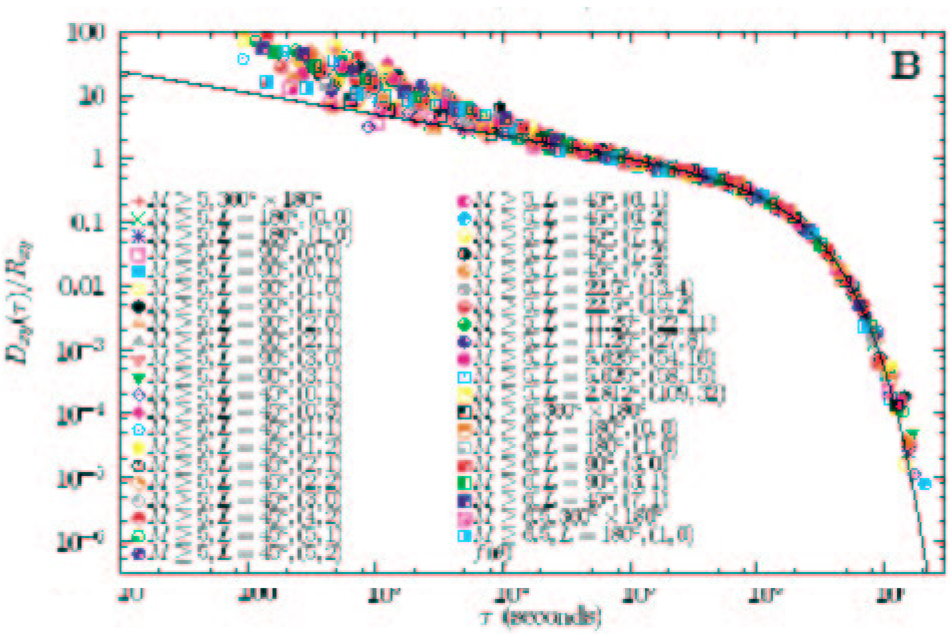
- $P(\Delta t)$ is an exponential for a Poisson process



- It exhibits a more complex structure as temporal correlations are present in the process

➡ Corral (PRL, 2004) rescaling Δt by the average rate in the area obtained a universal scaling law for the probability density

$$D(\Delta t, M_c) = R(M_c) f(R(M_c)\Delta t)$$

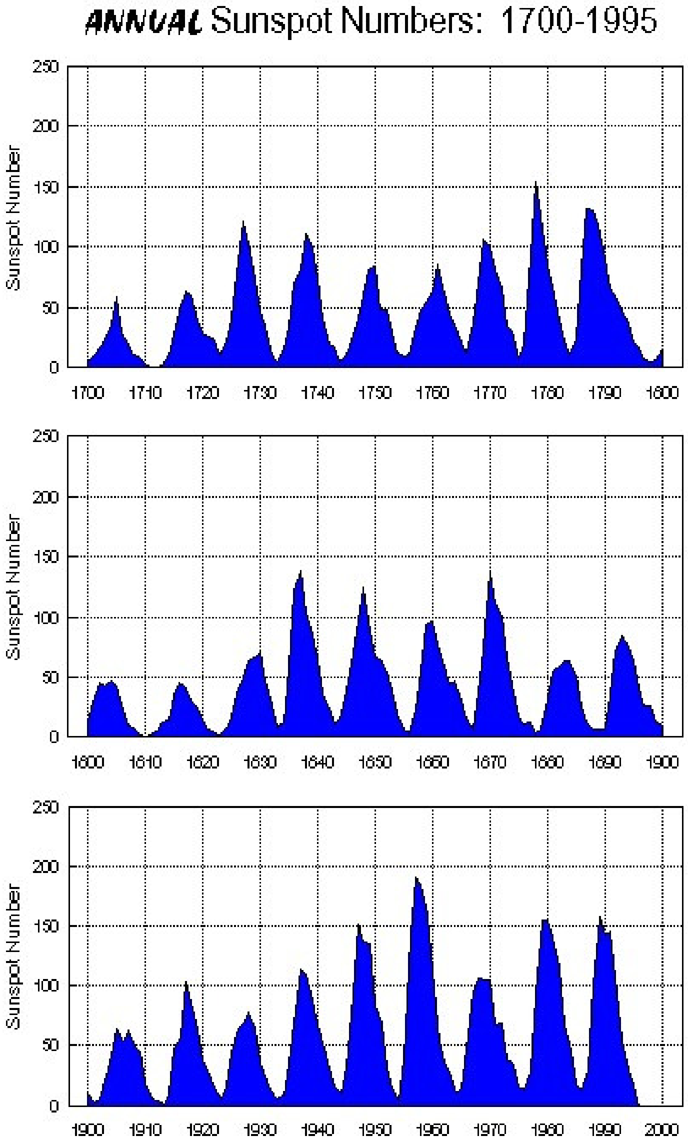


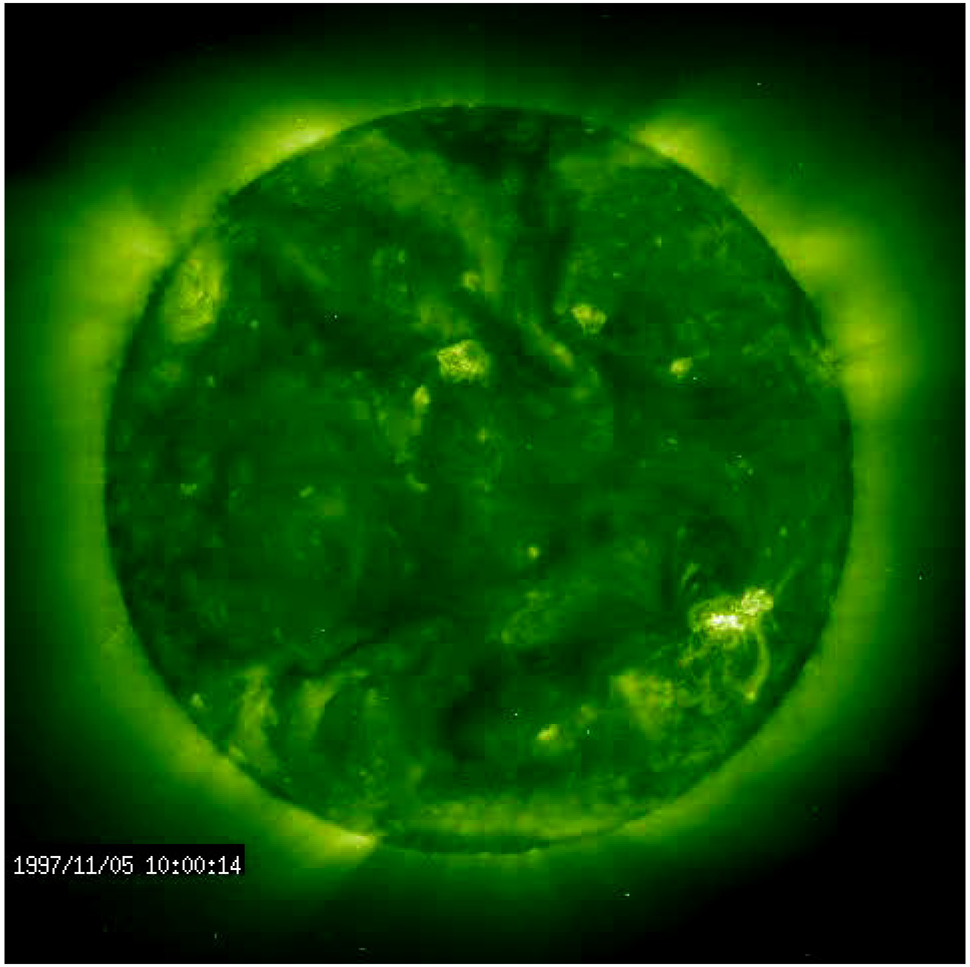
holds also for Japan, Spain, New Zeland…
scaling function not universal
(different areas are characterized by different rates)

Solar flares

Sudden rearrangement of stressed magnetic field lines gives rise to energetic bursts from solar corona.

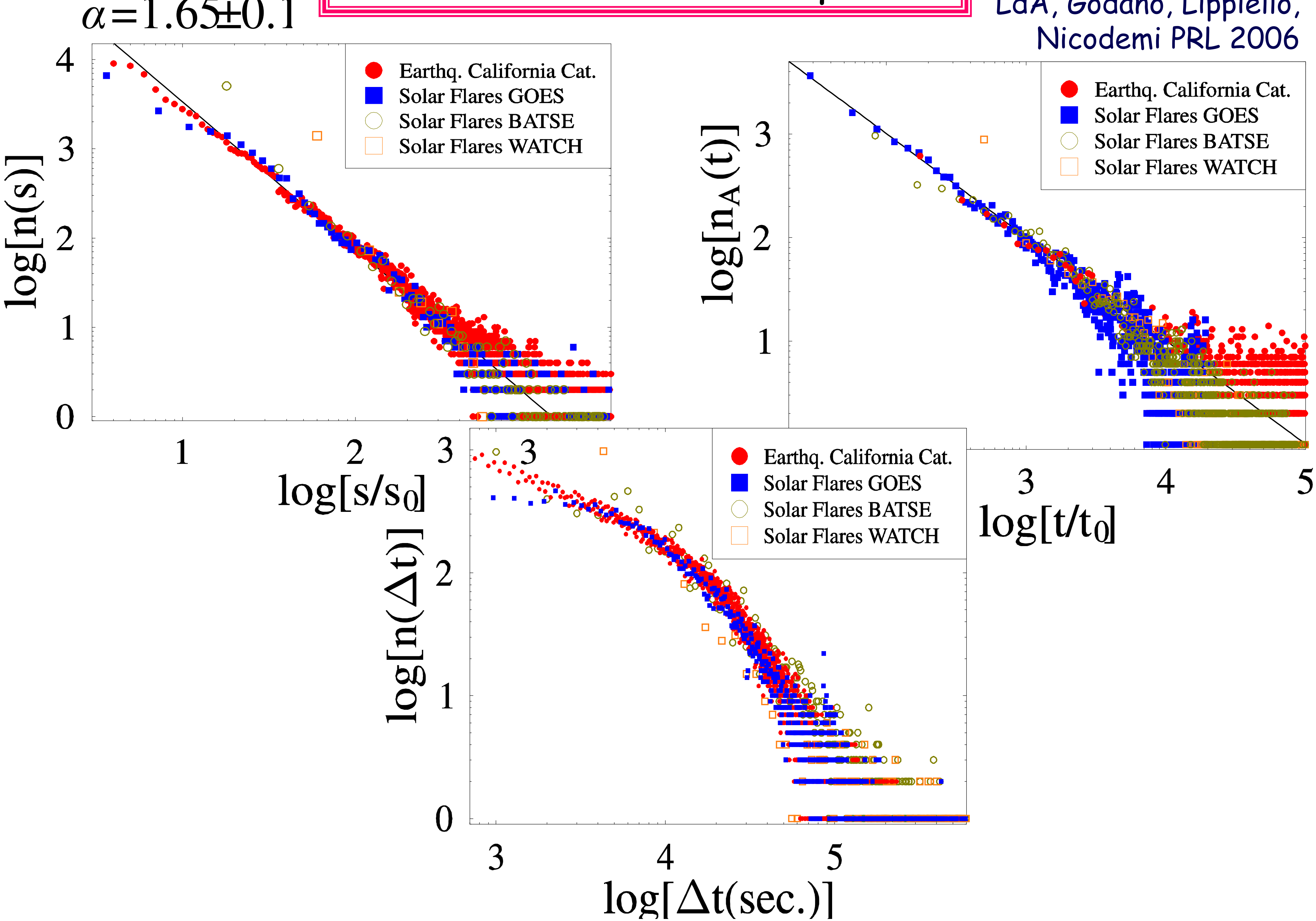
These phenomena take place in active regions identified by sunspots, dark-looking due to the effects of intense magnetic field.





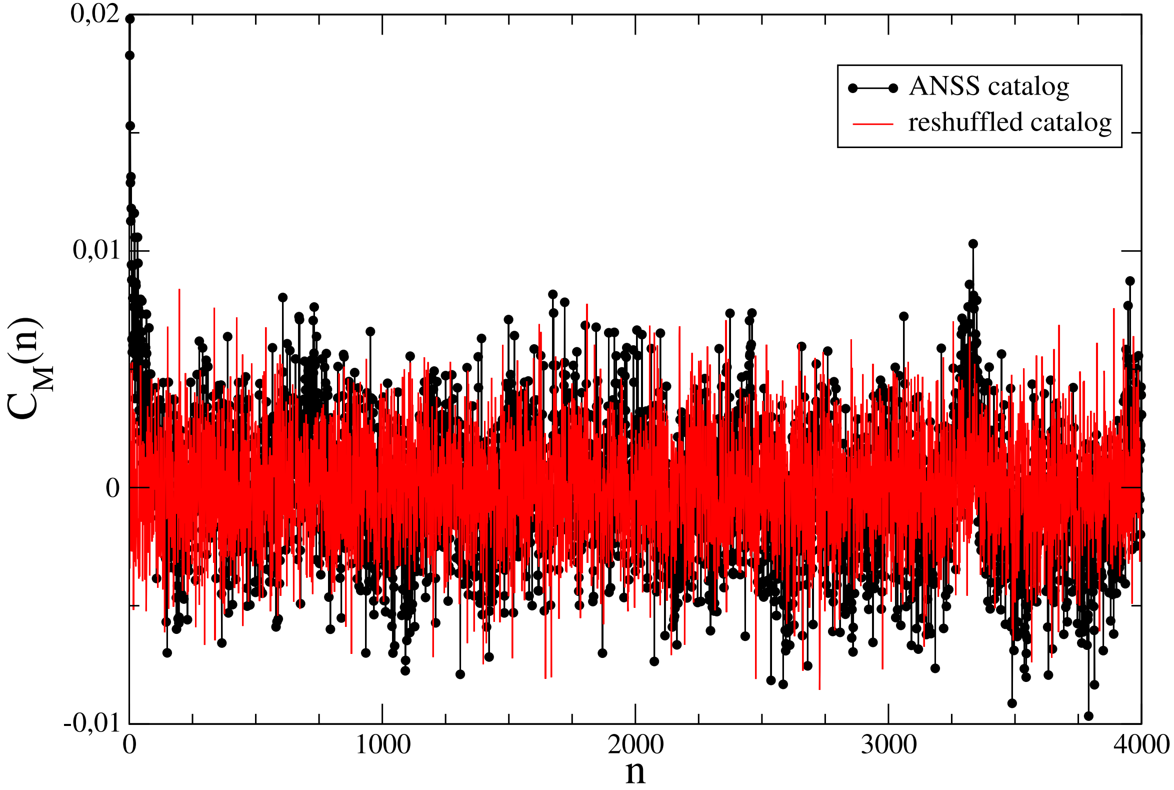
Solar flares and Earthquakes

LdA, Godano, Lippiello, Nicodemi PRL 2006



Is the occurrence of two phenomena as different as earthquakes and solar flares driven by the same physical mechanism?

Can correlations discriminate?



Time-energy correlations

Lippiello, LdA, Godano, PRL 2008

We define for any couple of successive events of the catalog

The time distance $\boxed{\Delta t_i = t_{i+1} - t_i}$ and the magnitude difference

$\boxed{\Delta m_i = m_{i+1} - m_i}$ ....and $\Delta m_i^* = m_{i+1} - m_{i^*}$

for a catalog where we reshuffle the previous magnitude $i^* \neq i$

We evaluate the conditional probability

$$P(\Delta m_i < m_0 \mid \Delta t_i < t_0) = \frac{N(m_0, t_0)}{N(t_0)}$$

couples of subsequent events with both $\Delta m_i < m_0, \Delta t_i < t_0$

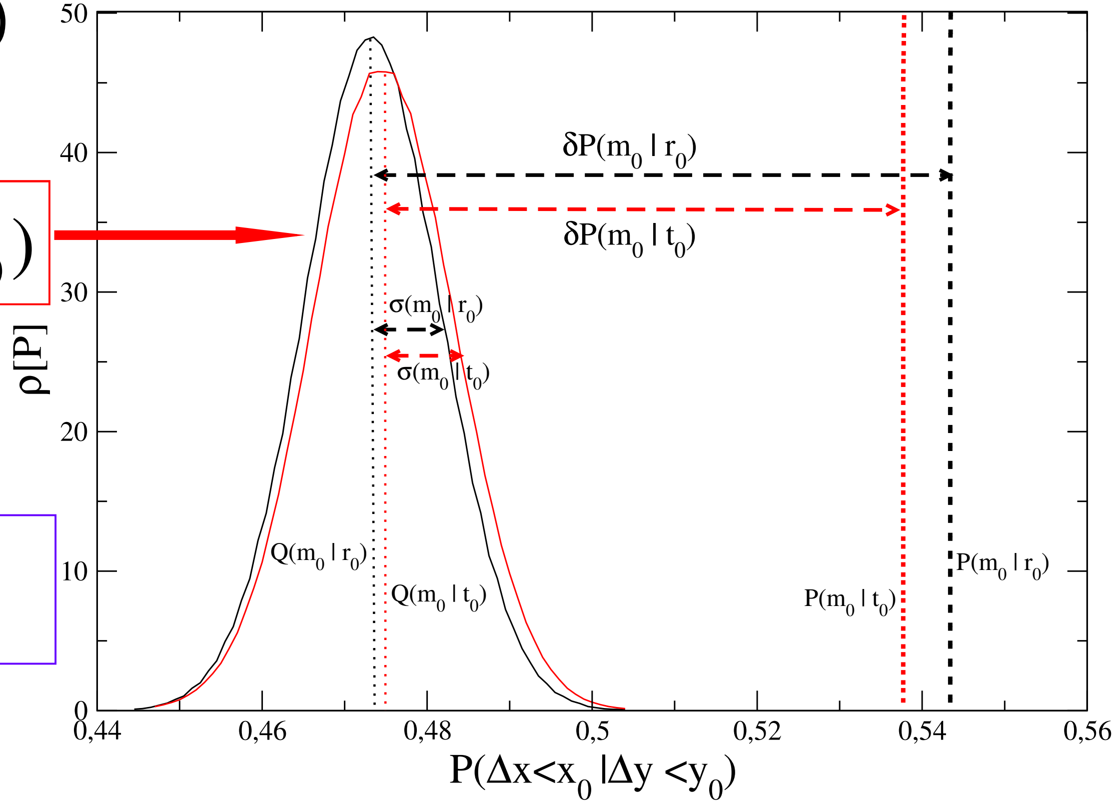
couples of subsequent events with $\Delta t_i < t_0$

➡ We calculate the conditional probabilities in the California catalog

$$P(\Delta m_i < m_0 \mid \Delta t_i < t_0)$$

and for 10^4 realizations of the reshuffled catalog (Gaussian distributed)

$$P(\Delta m_i^* < m_0 \mid \Delta t_i < t_0)$$



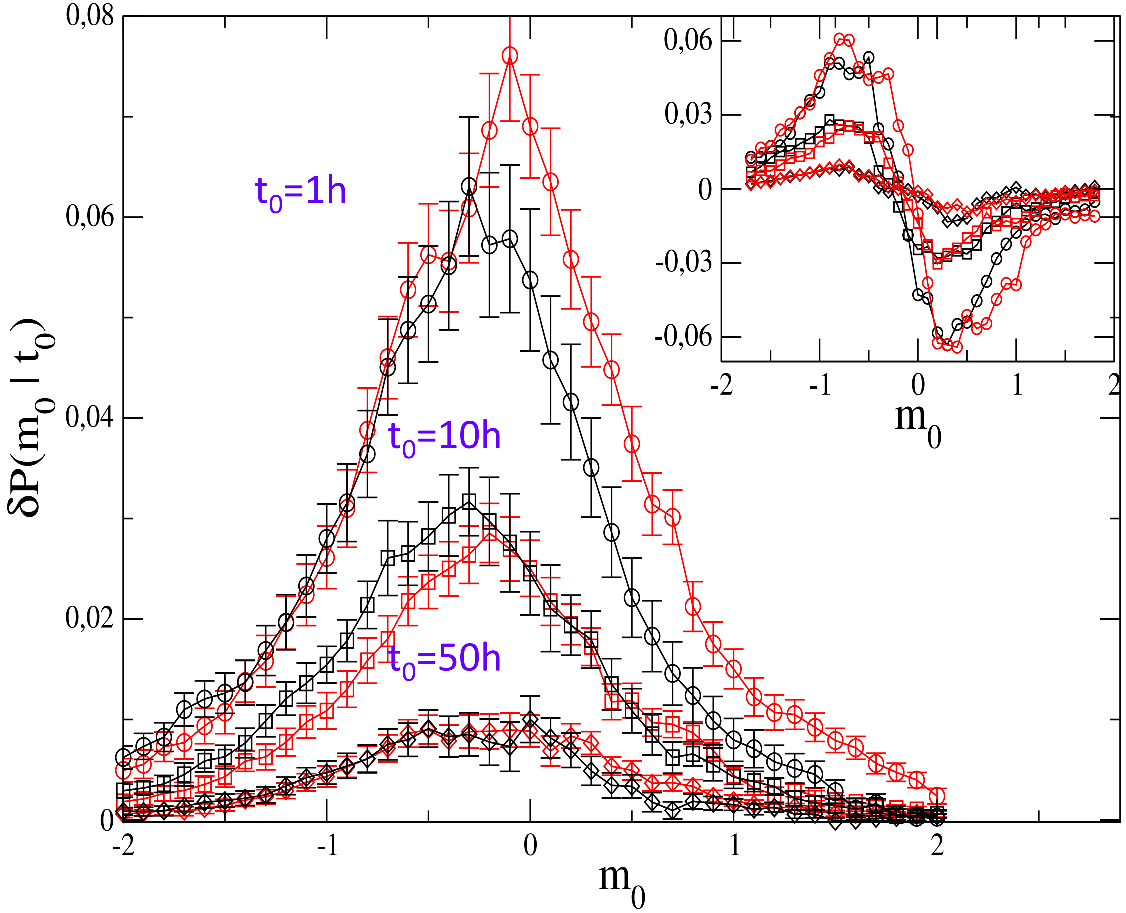
t_0=1h

m_0=0

IF $\delta P(m_0 \mid t_0) = P(\Delta m_i < m_0 \mid \Delta t_i < t_0) - Q(m_0 \mid t_0) > \sigma(m_0 \mid t_0)$

⟶ Evidence for time-energy correlations

Earthquakes in California



$$\frac{d\delta P(m_0 \mid t_0)}{dm_0}$$ probability difference

For $m_0 < 0$ the probability is larger in the real than in the reshuffled catalog, where magnitudes are uncorrelated

Maximum for

m_0 in [-1,-0.5]

Experimental data

Numerical data

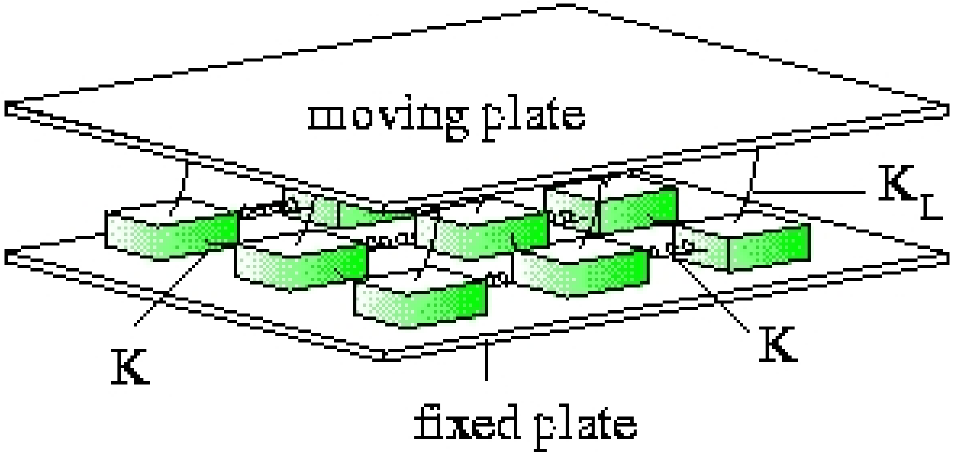
The next earthquake tends to have magnitude

close but smaller than the previous one

In good agreement with models implementing avalanching:

Olami-Feder-Christiensen model

Non conservative spring-block model, where $\alpha<0.25$ is the degree of dissipation



Successive instabilities generated by the stress redistribution

Spatio-temporal organization of foreshocks and aftershocks

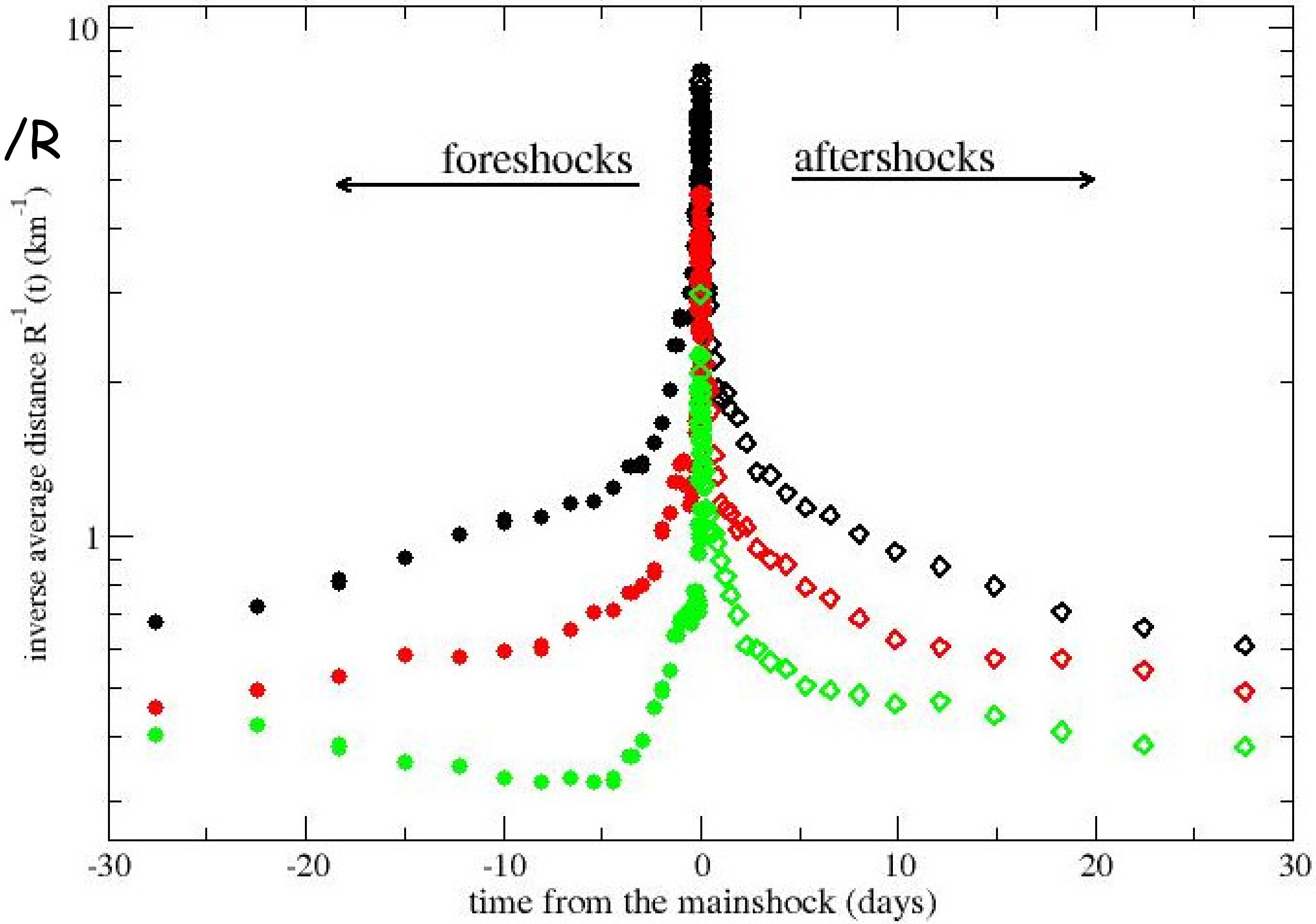
Lippiello et al, Nat Sci Rep 2012

Earthquakes tend to concentrate towards the future mainshock (foreshocks)
Then they spread out after mainshock occurrence (aftershocks)

R=Average distance from the mainshock 1/R

main magnitude
2
3
4

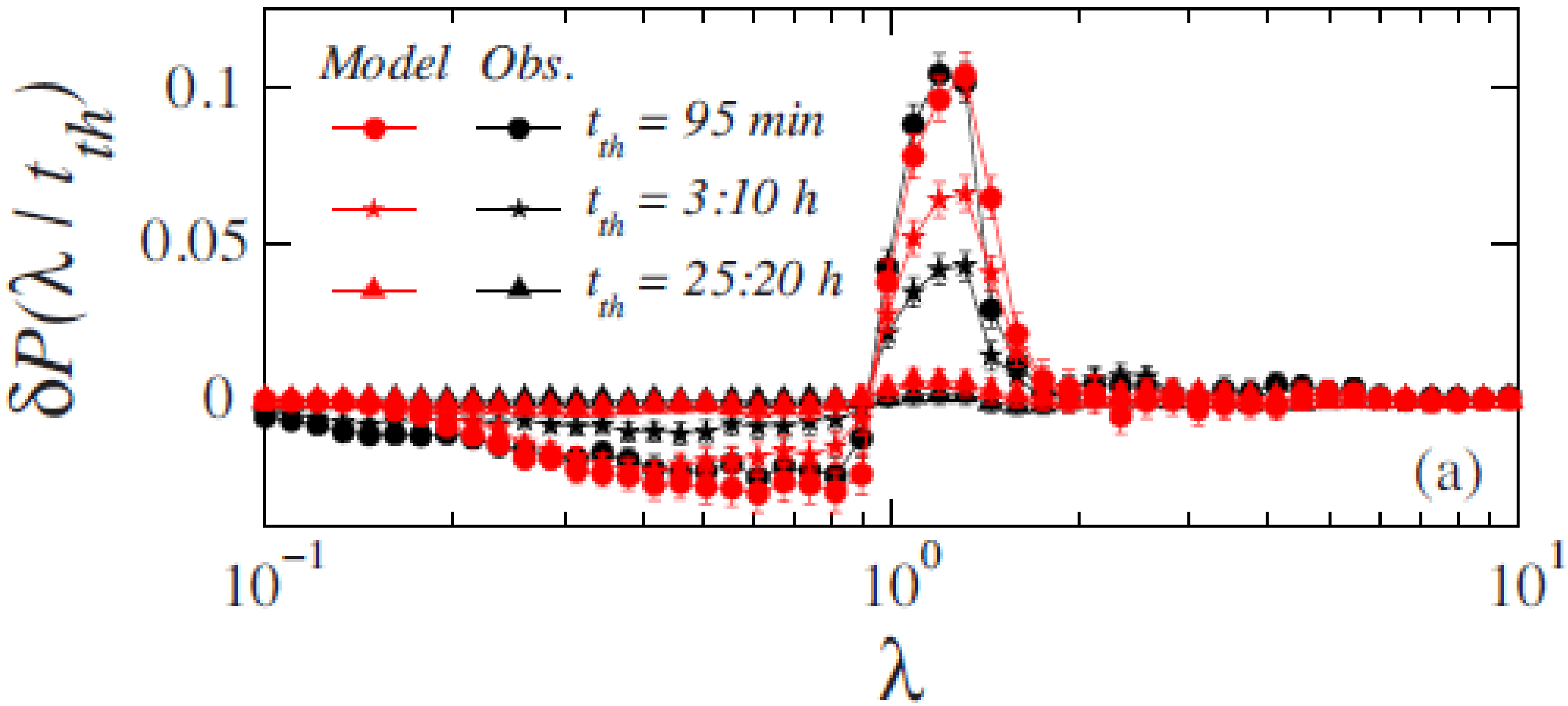
Southern California catalog



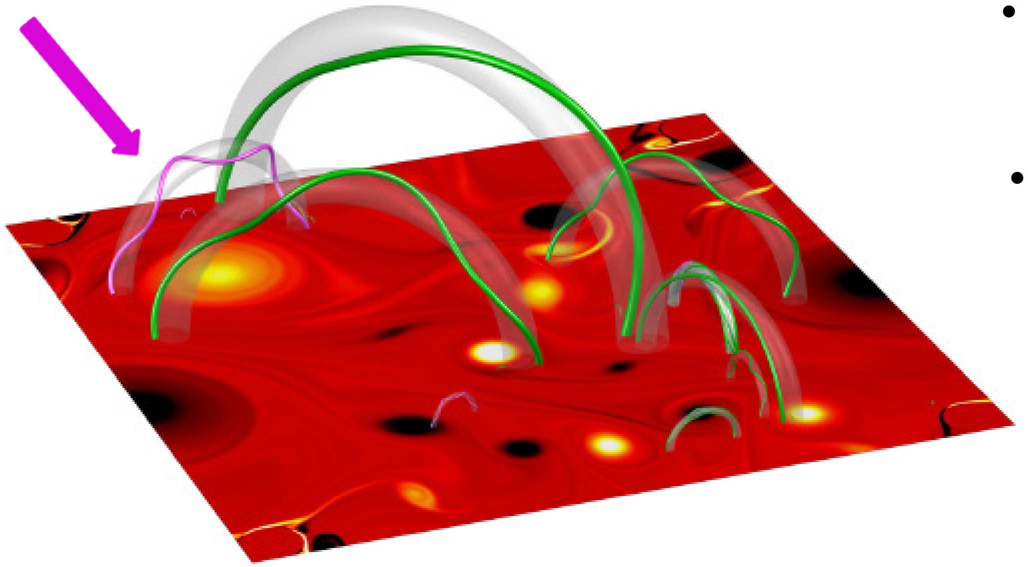
Correlations in solar flare occurrence

$$\delta P(\lambda | t_{th}) = P(E_{i+1}/E_i > \lambda | \Delta t_i < t_{th}) - Q(\lambda | t_{th}) > \sigma(\lambda | t_{th})$$

In consecutive flares (occurring within 3 hours)
the energy of the second flare
is close but larger than the energy of the previous are



Understanding flare triggering

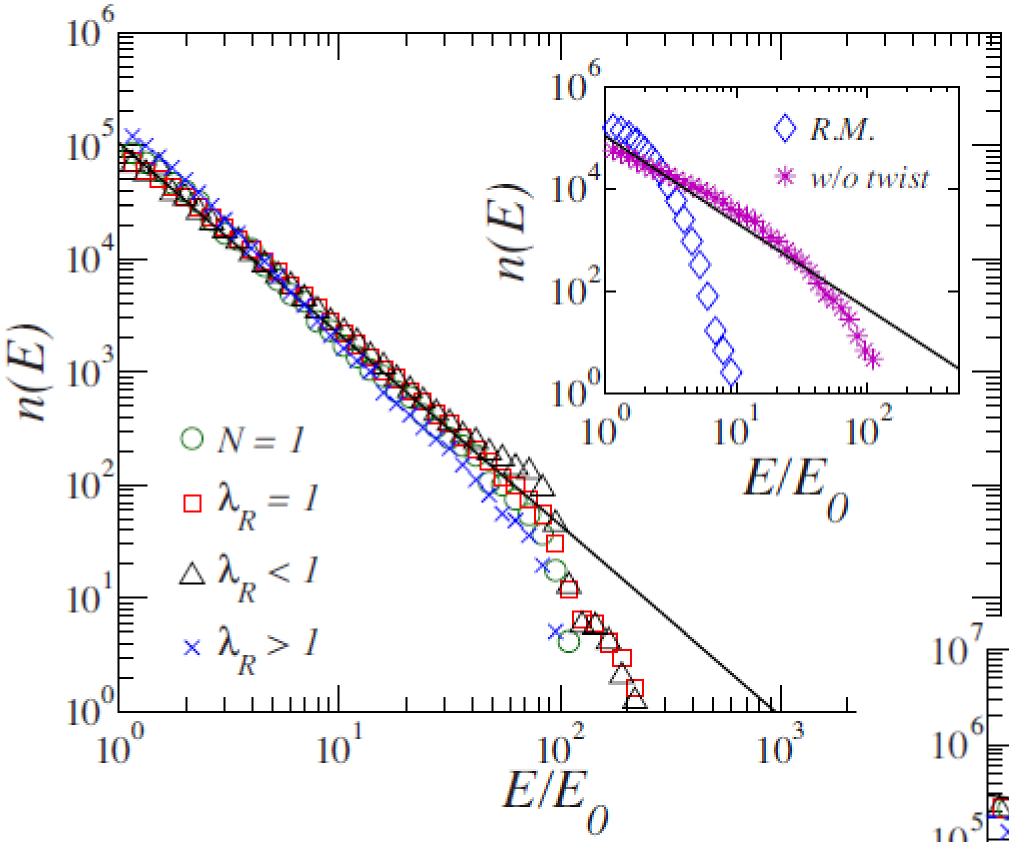


- A flare is due to the reconnection of magnetic flux tubes
  Hughes et al PRL2003
- Footprints of magnetic flux tubes are anchored in the photosphere i.e. plasma in turbulent flow
- Magnetic flux tubes follow the local velocity field and are twisted by the vorticity
- A flare is released as soon as a tube reaches a critical twist (scale free energy distribution)

⇒ Tube-tube interactions:
Reconnection heats up the surrounded plasma increasing the local coronal pressure and the critical twist of the surrounding magnetic flux tubes

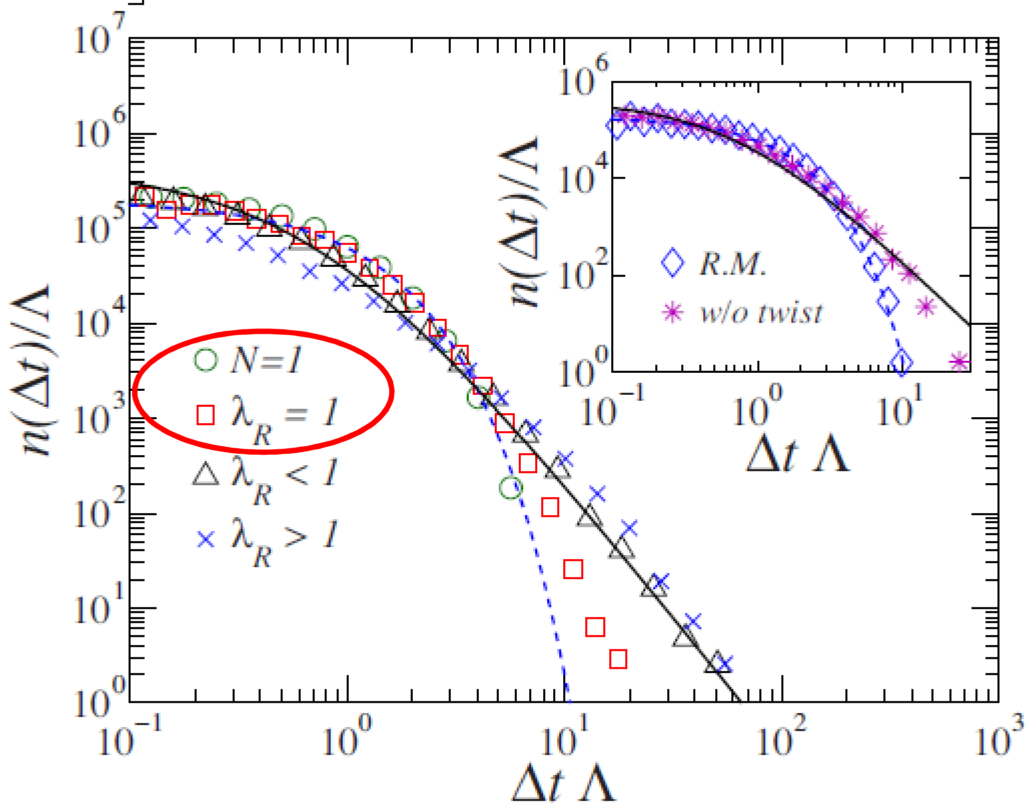
⇒ Rather than avalanching, this leads to delay in the reconnection process
Following flare larger than the previous one!

Mendoza, Kaydul, LdA, Andrade, Herrmann, Nat. Comm. 2014



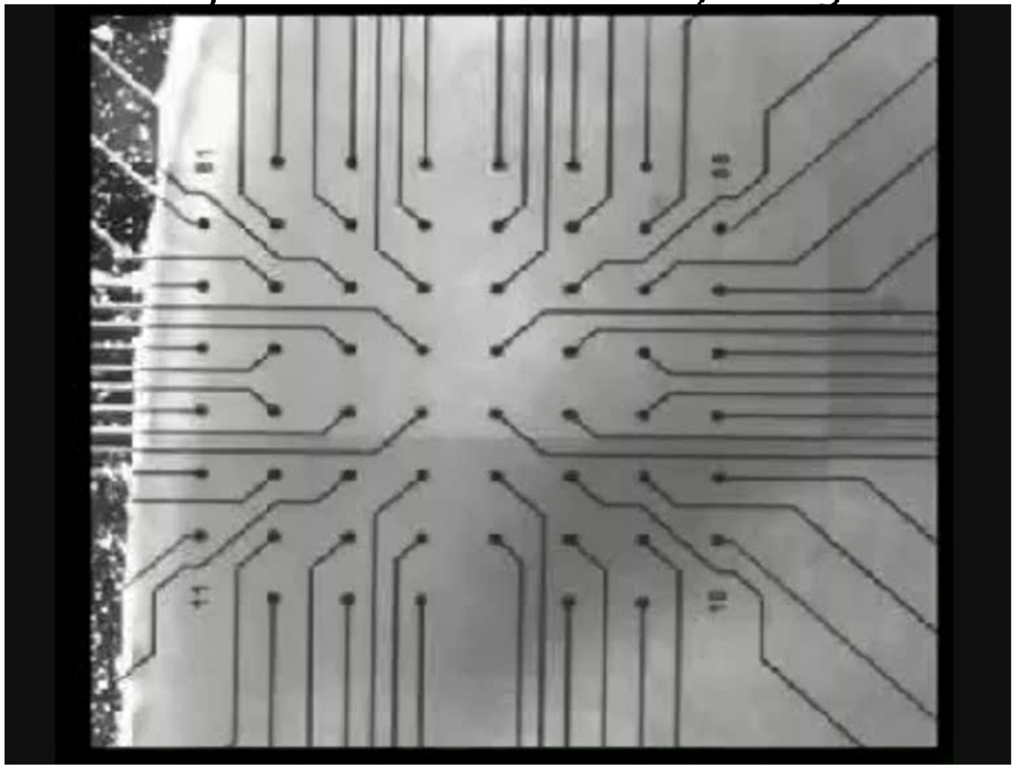
For the size distribution
Fluid vorticity

For the intertime distribution
Tube interactions



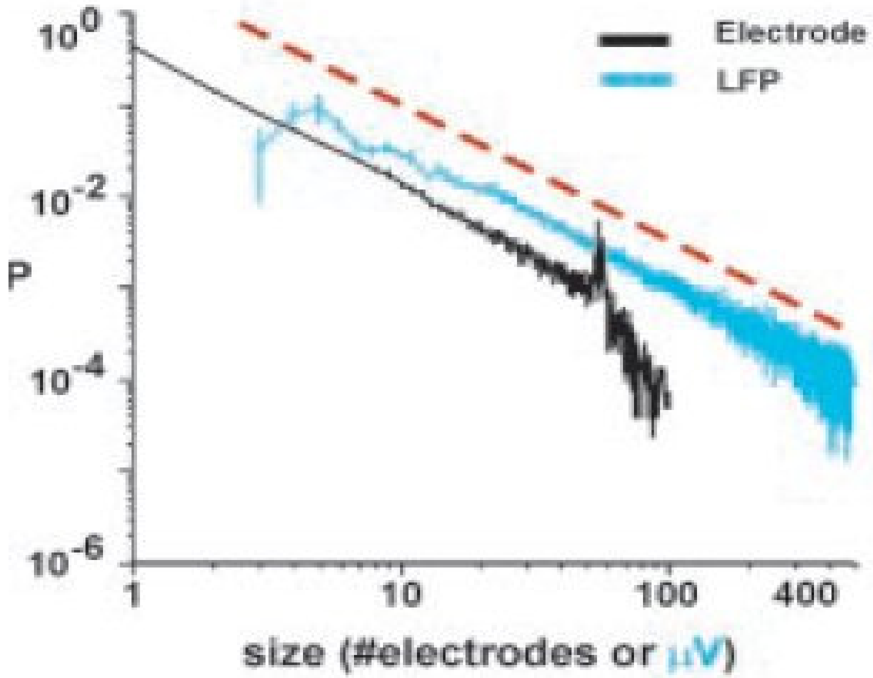
Neuronal avalanches

Beggs & Plenz (J. Neuroscience 2003, 2004) have measured spontaneous local field potentials continuously using a 60 channel multielectrode array in mature organotypic cultures of rat cortex *in vitro* and *in vivo* (rat & monkey) (PNAS 2008, 2009)



- dissociated neurons (V. Pasquale et al, Neurosci. 2008; A. Mazzoni et al PLoS ONE 2007)

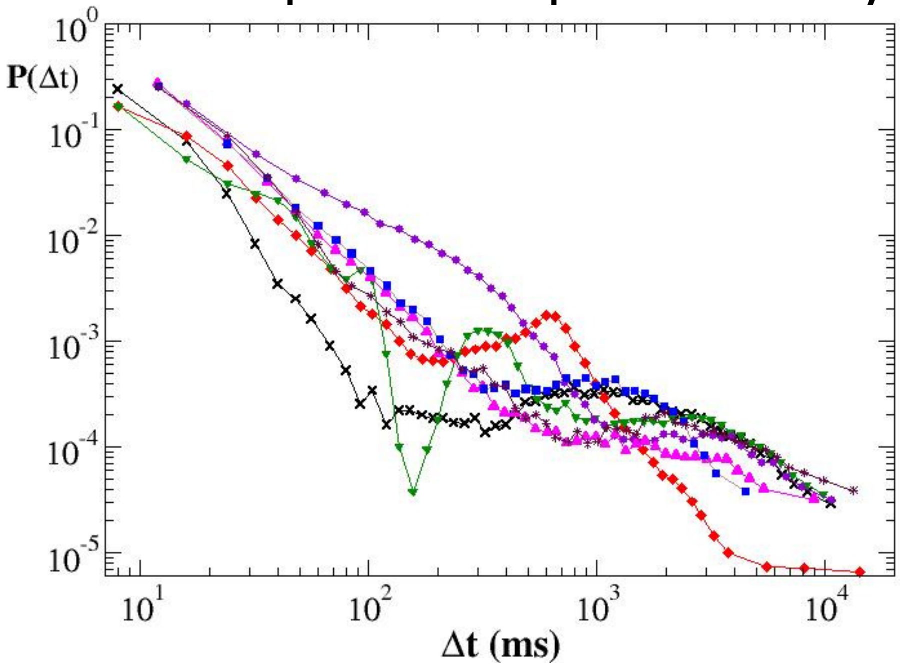
- Avalanche size distribution is a power law with an exponent close to -3/2
- Avalanche duration distribution is a power law with an exponent close to -2.0

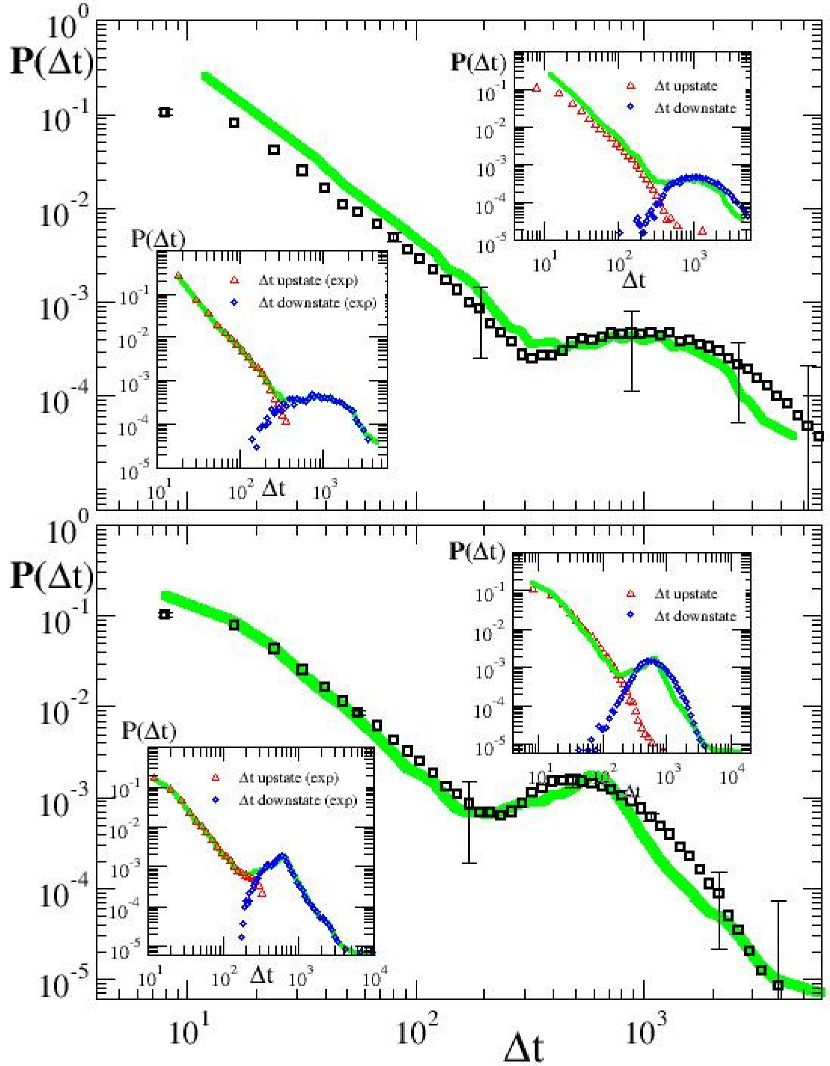


→ Critical state optimizes information transmission

Avalanche inter-time distribution

Experiments performed by D. Plenz (Lombardi et al PRL 2012)





Spontaneous neuronal activity can exhibit slow oscillations between bursty periods, up-states, and quiet periods, down-states

Small correlated avalanches,
neurons depolarized after firing

Disfacilitation period after large avalanche
Neurons hyperpolarized after firing

Implementation of up and down states

- Down-state $\longrightarrow$ After an avalanche with $\boxed{s \geq s_{\min}}$

all neurons active in the last avalanche become hyperpolarized depending on their own activity

$$\boxed{v_i = v_i - h\delta v_i}$$

h > 0 is a hyper-polarization constant

short term memory at neuron level

System is stimulated by a small constant random drive

- Up-state $\longrightarrow$ After an avalanche with $\boxed{s < s_{\min}}$

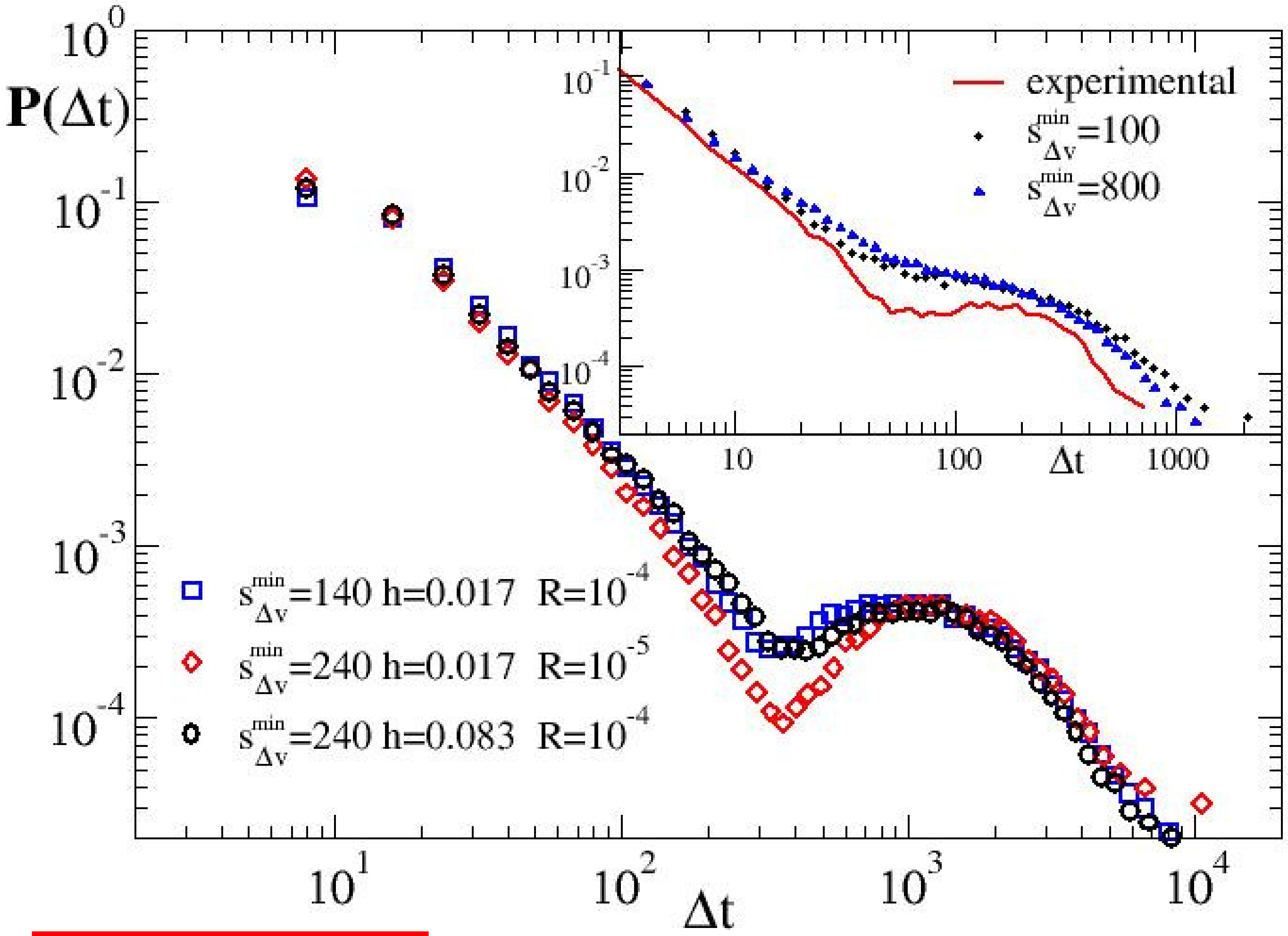
all neurons active in the last avalanche become depolarized depending on the last avalanche size

$$\boxed{v_i = v_{\max}(1 - s/s_{\min})}$$

the smaller the last avalanche
the closer the potential to the firing threshold

Memory at the network level

System is stimulated by a random drive
(network effect which sustains the up-state) $\boxed{\in]0, s_{\min}/s[}$



simple dual drive does not work!

$$\boxed{R = h/s_{\min}}$$

espressing the balance between excitation and inhibition is the unique parameter controlling the distribution

→ Homeostatic regulatory mechanism

Correlations in avalanche activity

Lombardi et al 2015

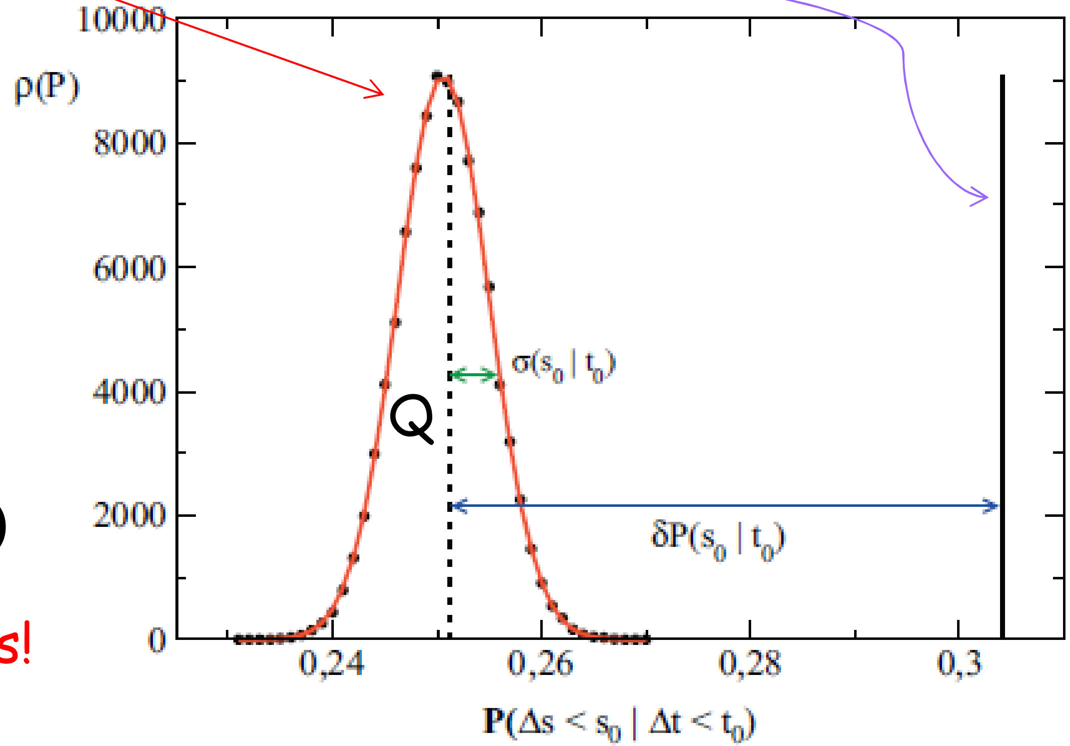
We evaluate the conditional probability $P(s < s_0 \mid \Delta t < t_0)$
with $\Delta t = t' - t$
and s either the preceding or the following avalanche

Lippiello et al PRL 2007

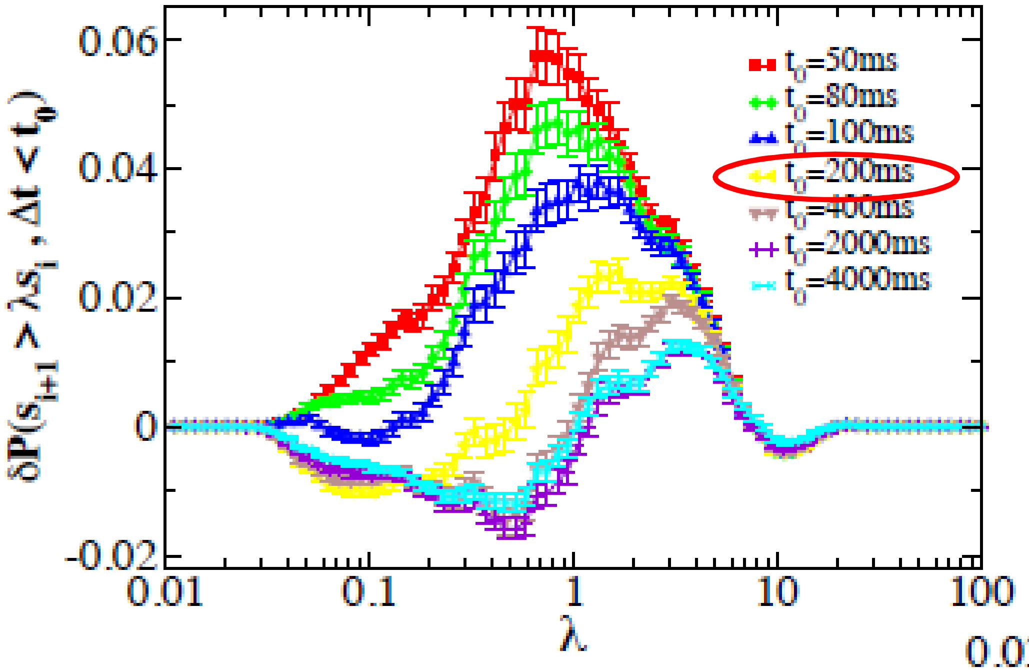
Both in the real and in a reshuffled catalog
where s are uncorrelated

We monitor the conditional probability difference

$$\delta P(s_0 \mid t_0) =$$
$$P(s < s_0 \mid \Delta t_i < t_0) - Q(s_0 \mid t_0)$$
$$> \sigma(m_0 \mid t_0)$$
→ correlations!



Correlations between successive avalanche sizes

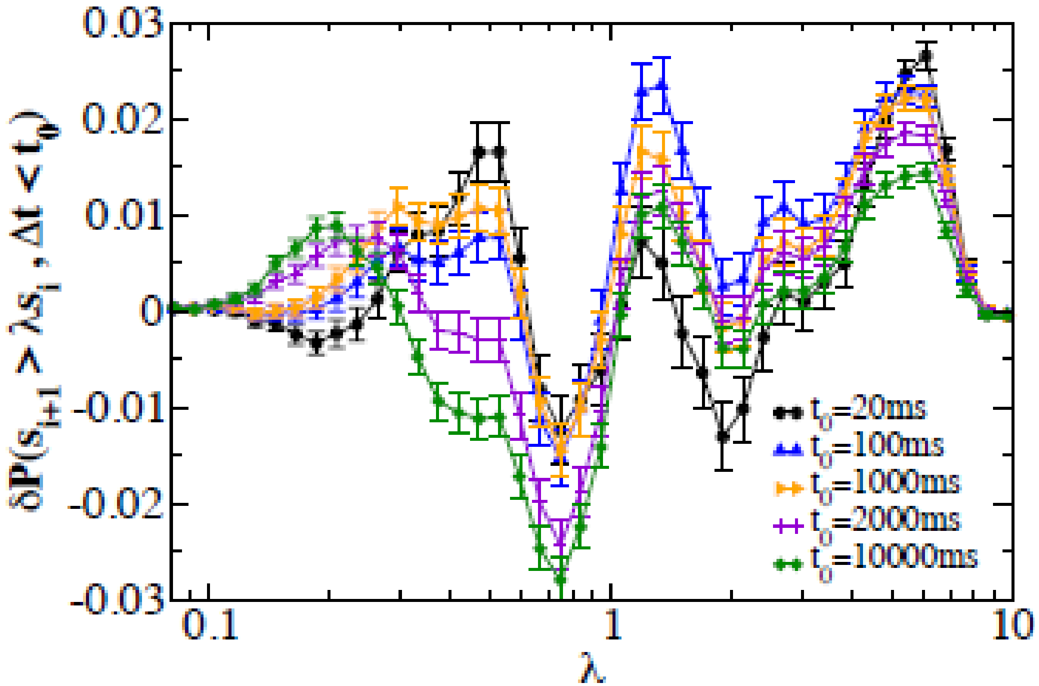


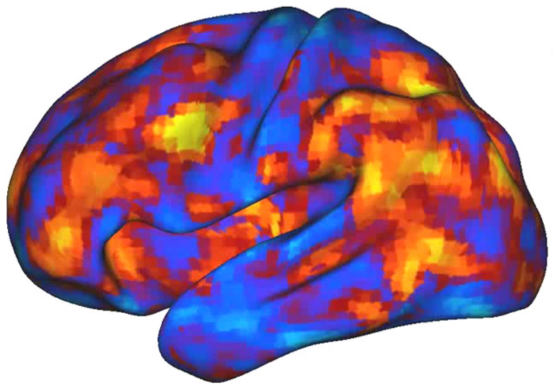
Normal conditions

- For close-in time avalanches the second one tends to be smaller
- For longer separation the second one tends to be larger

Dis-inhibited cultures (PTX)

- Behaviour strongly sample dependent
- Multiple peaks even in a single sample
- Unbalanced behaviour





Correlations in brain signals

Lombardi, Chialvo, Herrmann, LdA CSF 2013

In fMRI data from 7 healthy humans we analyse extreme activity ($B>B_c$)

$$s_i(t) = B(\vec{r}_i, t+\delta t) - B(\vec{r}_i, t)$$ activity variation at each voxel *i*

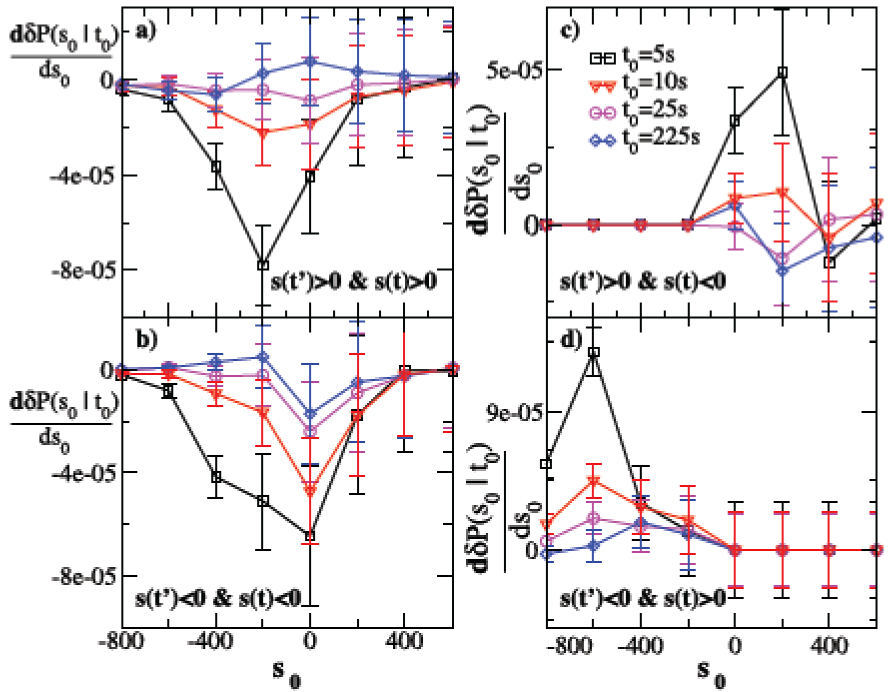
$P(\Delta s < s_0 \mid \Delta t < t_0)$ different than zero ⟶ Consecutive variations with opposite sign are correlated

The derivative $\dfrac{d\delta P(s_0 \mid t_0)}{ds_0}$ represents the probability difference to observe $\Delta s = s_0$ with $\Delta t < t_0$

⬇

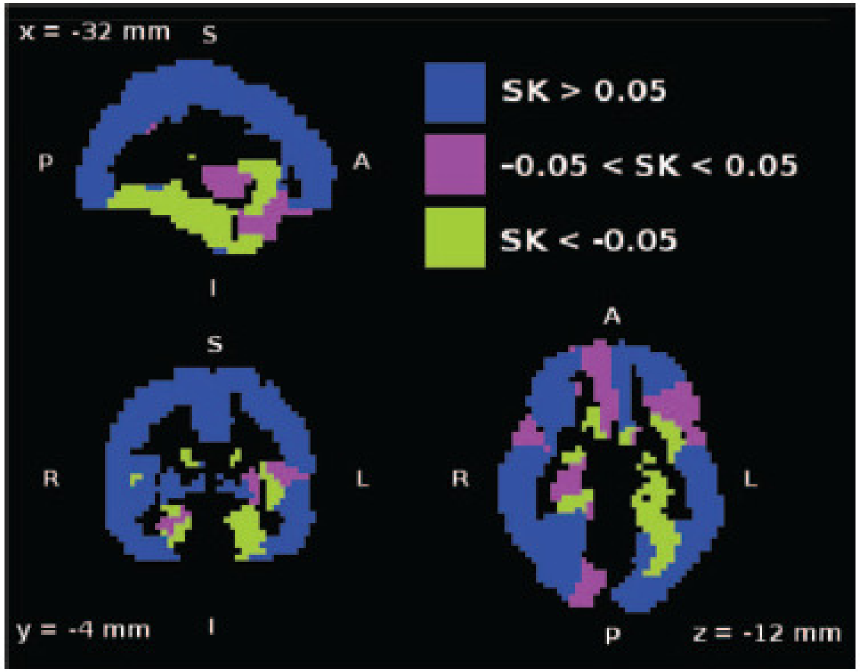
Brain tends to realize activity balance

depressions are compensated by successive enhancements and vice versa



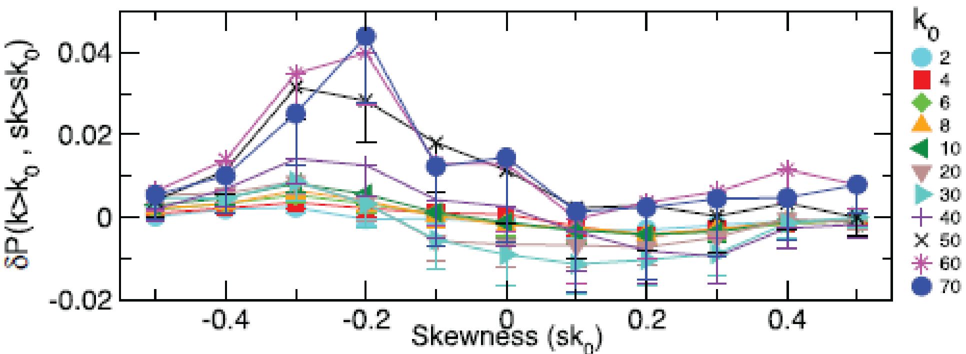
Correlations between structural connectivity and activity

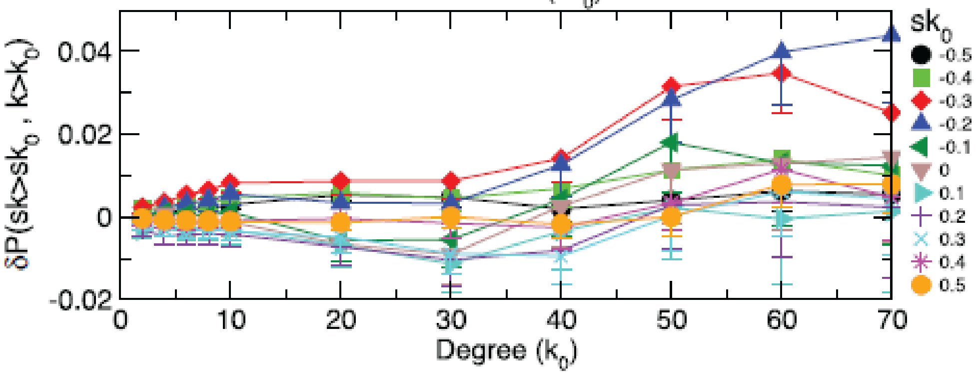
Amor et al, EPL 2015



Activity is measured in terms of Sk skewness of the BOLD signal

Structural connectivity k is the number of fibers connecting 2 ROIs in DTI data





$$\delta P(k > k_0, Sk > Sk_0)$$

Indicates that largest correlations between activity and structure is observed in subcortical areas (large negative skewness)

CONCLUSIONS

- Power law behaviour for event size distribution in many natural

  phenomena $\longrightarrow$ critical activity

- Complex temporal correlations
- Scaling properties of energy/time distributions unable to discriminate among different phenomena
- Conditional probability analysis $\longrightarrow$
- Aftershock triggering controlled by stress diffusion
- Solar flare occurrence driven by kink instability due to turbulent flow
- Balance between excitation and inhibition controls temporal organization in brain activity