Curvature of the pseudocritical line in (2+1)-flavor QCD

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Based on
P. Cea, L. Cosmai, A.P., arXiv:1508.07599

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Outline

1. Introduction
   - QCD phase diagram
   - QCD with non-zero baryon density and the sign problem
   - The method of analytic continuation
2. Critical line of QCD with $n_f = 2 + 1$
   - Lattice setup and numerical simulations
   - Continuum limit
   - Comparison with other analyses
3. Conclusions

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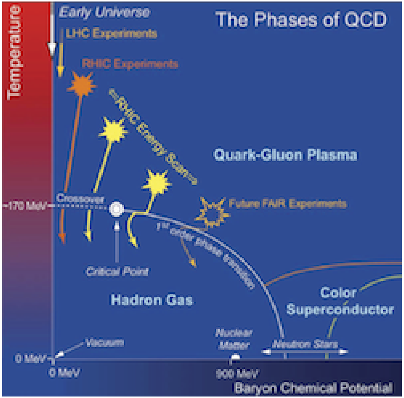
QCD phase diagram



(from bnl.gov)

$$\frac{T(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T(\mu_B)} \right)^2 + \dots$$

Important implications in cosmology, in the physics of compact stars and in relativistic heavy-ion collisions.

Outline

QCD at non-zero temperature and density

- Lattice is the main non-perturbative tool for the investigation of the QCD phase diagram
- Non-zero temperature:  $T = \frac{1}{N_\tau\, a(\beta)}\ , \quad \beta = \frac{2N}{g^2}$
- Non-zero density:  sign problem!

  Importance sampling requires positive weights, but in (*e.g.* Wilson fermions, $n_f = 1$)

$$Z(T,\mu) = \int [dU]\ e^{-S_G[U]}\ \det[M(\mu)]$$

  the fermionic determinant $\det[M(\mu)]$ is complex for $\mu \neq 0$ in SU(3).

  Exceptions:
  - imaginary chemical potential: $\mu = i\mu_I$
  - SU(2) or two-color QCD
  - isospin chemical potential: $\mu_u = -\mu_d$

Ways around I

- Perform simulations at μ=0 and take advantage of physical fluctuations in the thermal ensemble for extracting information at (small) non-zero μ, after suitable **reweighting**

  [I.M. Barbour *et al.*, 1997] [Z. Fodor, S.D. Katz, 2002 $\rightarrow$]

- **Taylor-expand** in μ the v.e.v. of interest and calculate the coefficients of the expansion by numerical simulations at $\mu = 0$

  [S.A. Gottlieb, 1988] [QCD-TARO coll., 2001] [C.R. Allton *et al.*, 2002-2003-2005] [R.V. Gavai, S. Gupta, 2003-2005] [S. Ejiri *et al.*, 2006]

- Build **canonical partition** functions by Fourier transform of the grand canonical function at imaginary chemical potential

  [A. Hasenfratz, D. Toussaint, 1992] [M.G. Alford, A. Kapustin, F. Wilczek, 1999] [P. de Forcrand, S. Kratochvila, 2004-2005-2006] [A. Alexandru *et al.*, 2005]

- Reorder the path integral representation of the partition function, by first calculating expectation values with constrained parameters and then weighting over the **density of states**

  [G. Bhanot *et al.*, 1987] [M. Karliner *et al.*, 1988] [A. Gocksch, 1988] [V. Azcoiti, G. Di Carlo, A.F. Grillo, 1990] [X.-Q. Luo, 2001] [J. Ambjorn *et al.*, 2002] [Z. Fodor, S.D. Katz, C. Schmidt, 2005-2007]

Ways around II

- Allow the field variables to take value in the **complexified configuration space** (complex Langevin dynamics, integration along Lefschetz thimbles)
  [G. Aarts, 2012]
  [G. Aarts, L. Bongiovanni, E. Seiler, D. Sexty, I.-O. Stamatescu, 2013]
  [M. Cristoforetti, F. Di Renzo, A. Mukherjee, L. Scorzato, 2013]

- Use the **strong-coupling expansion** (worldline representation of lattice QCD, heavy dense approximation)
  [P. Rossi, U. Wolff, 1984] [F. Karsch, K. Mutter, 1989]
  [P. de Forcrand, M. Fromm, 2010]
  [P. de Forcrand, J. Langelage, O. Philipsen, W. Unger, 2013]
  [H. Vairinhos, P. de Forcrand, 2014]
  [J. Langelage, M. Neuman, O. Philipsen, 2014]
  [T. Rindlisbacher, P. de Forcrand, 2015]

- Simulate the theory in some **dual representation**
  [Y. Delgado Mercado, C. Gattringer, A. Schmidt, 2013]
  [O. Borisenko's talk]

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The method of analytic continuation

- Perform Monte Carlo numerical simulations at some selected **imaginary** values of the chemical potential, $\mu = i\mu_I$, thus getting data points with their statistical uncertainties
- Interpolate the results obtained by a suitable function of μ_I^2
- Analytically continue to **real** chemical potentials: $\mu_I \to -i\mu$

Some historical remarks:

- Idea of formulating a theory at imaginary chemical potential [M.G. Alford, A. Kapustin, F. Wilczek, 1999]
- test of effectiveness in strong-coupling QCD [M.P. Lombardo, 2000]
- thereafter, a lot of applications to QCD and tests in QCD-like theories and in spin models

- Applications in QCD:

  - $n_f = 2$ staggered [Ph. de Forcrand, O. Philipsen, 2002] [M. D'Elia, F. Sanfilippo, 2009] [P. Cea, L. Cosmai, M. D'Elia, A.P., F. Sanfilippo, 2012]
  - $n_f = 3$ staggered [Ph. de Forcrand, O. Philipsen, 2003]
  - $n_f = 4$ staggered [M. D'Elia, M.P. Lombardo, 2003-2004] [V. Azcoiti *et al.*, 2004-2005] [M. D'Elia, F. Di Renzo, M.P. Lombardo, 2007] [P. Cea, L. Cosmai, M. D'Elia, A.P., 2010]
  - $n_f = 2 + 1$ staggered [Ph. de Forcrand, O. Philipsen, 2007] [P. Cea, L. Cosmai, A.P., 2014-2015] [C. Bonati, M. D'Elia, M. Mariti, M. Mesiti, F. Negro, F. Sanfilippo, 2014-2015] [R. Bellwied, S. Borsanyi, Z. Fodor, J. Günther, S.D. Katz, C. Ratti, K.K. Szabo, 2015]
  - $n_f = 2$ Wilson [L.-K. Wu, X.-Q. Luo, H.-S. Chen, 2007] [A. Nagata, K. Nakamura, 2011]
  - $n_f = 4$ Wilson [H.-S. Chen, X.-Q. Luo, 2005]

- Tests:
  - 3*d* SU(3) + adj. Higgs [A. Hart, M. Laine, O. Philipsen, 2001]
  - SU(2), $n_f = 8$ staggered [P. Giudice, A.P., 2004] [P. Cea, L. Cosmai, M. D'Elia, A.P., 2007-2008] [P. Cea, L. Cosmai, M. D'Elia, C. Manneschi, A.P., 2009]
  - SU(3), $n_f = 8$ staggered [S. Conradi, M. D'Elia, 2007]
  - SU(2) via chiral RMT model [Y. Shinno, H. Yoneyama, 2009]

  - 3*d* 3-state Potts model [S. Kim *et al.*, 2005]
  - 2*d* Gross-Neveu at large *N* [F. Karbstein, M. Thies, 2006]

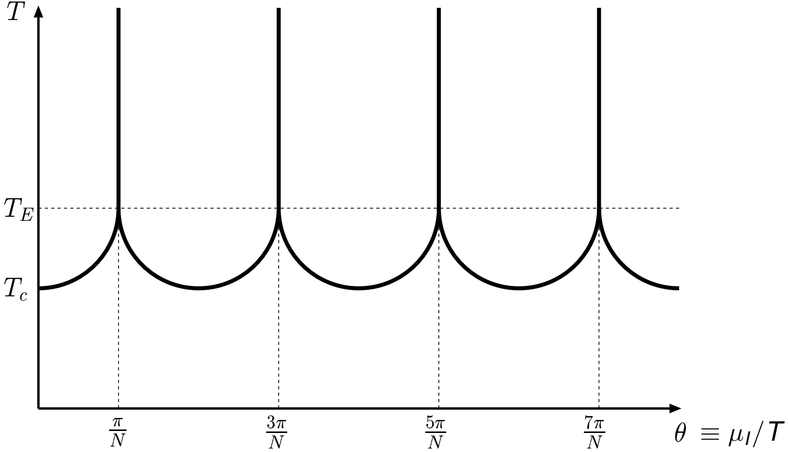
Drawbacks

1. a practical one: Monte Carlo simulations yield data points with statistical uncertainties at fixed values of the imaginary chemical potential; the interpolation of these points is not unambiguous

2. a principle one: the theory at imaginary chemical potential has its own non-analyticities and is periodic in the variable $\theta = \mu_I/T$ (period $2\pi/N$) [A. Roberge, N. Weiss, 1986]

$\Rightarrow$ the region effectively available for Monte Carlo simulations is limited by the condition $\mu_I/T \lesssim 1$

- The combination of these two drawbacks implies that the analytic continuation is expected to work for real chemical potentials satisfying $\mu_R/T \lesssim 1$.


T
T_E
T_c
$\frac{\pi}{N}$
$\frac{3\pi}{N}$
$\frac{5\pi}{N}$
$\frac{7\pi}{N}$
$\theta \equiv \mu_I/T$

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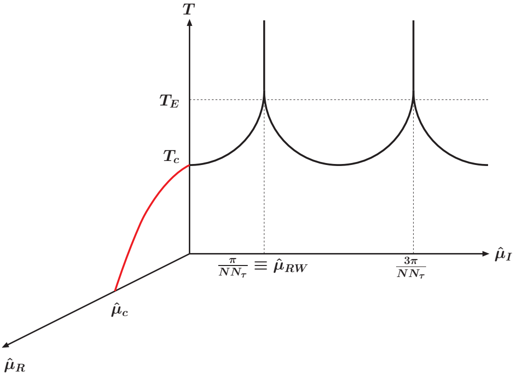
Analytic continuation of the critical line

- Locate $T_c(\mu_I)$ for some values of the imaginary chemical potential μ_I, looking for peaks in the susceptibilities of a given observable

- Interpolate the values of $T_c(\mu_I)$ with an analytic function of μ^2 and extrapolate to real chemical potential



$$\hat{\mu}_I \equiv a\,\mu_I\ , \quad \hat{\mu}_R \equiv a\,\mu_R\ , \qquad T = \frac{1}{N_\tau a}$$

$$\frac{\mu_{RW}}{\pi T} = \frac{1}{3}$$

(valid for $n_F = 1$ QCD or for QCD with same μ for all quarks)

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Lattice setup and numerical simulations

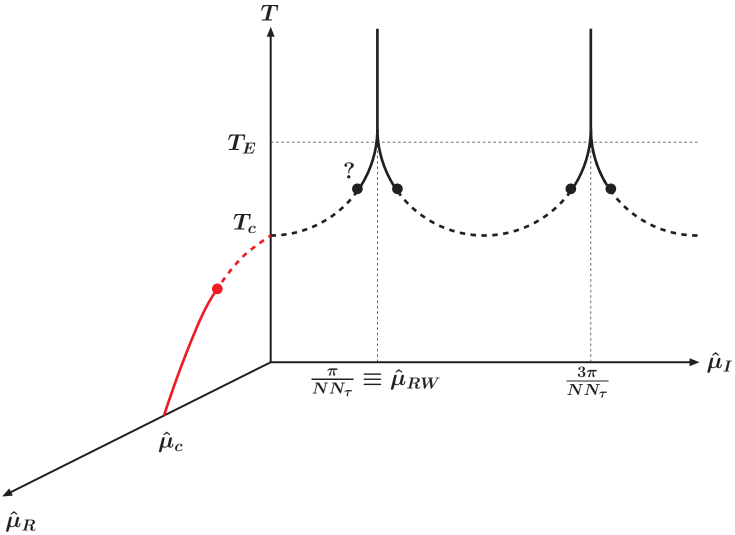
- Highly improved staggered quark action with tree-level improved Symanzik gauge action (**HISQ/tree**) with 2+1 flavors:

$$Z = \int [DU] e^{-S_{\text{gauge}}} \prod_{q=u,d,s} \det(D_q[U, \mu_q])^{1/4}$$

- Same quark chemical potential for the three quark species:

$$\mu_u = \mu_d = \mu_s \equiv \mu = \frac{\mu_B}{3}$$

- **Line of constant physics (LCP)** with physical strange quark mass at each value of the coupling β and light-quark mass fixed at $m_l = m_s/20$ ($M_\pi = 160$ MeV)
[A. Bazavov *et al.* (HotQCD coll.), 2012]


T
T_E
?
T_c
$\hat{\mu}_I$
$\frac{\pi}{NN_\tau} \equiv \hat{\mu}_{RW}$
$\frac{3\pi}{NN_\tau}$
$\hat{\mu}_c$
$\hat{\mu}_R$

- To probe the crossover transition at $\mu^2 < 0$ we adopted the renormalized disconnected susceptibility of the light quark chiral condensate over T^2:

$$\chi_{l,\mathrm{ren}} = \frac{1}{Z_m^2}\,\chi_{l,\mathrm{disc}}\ , \qquad \chi_{l,\mathrm{disc}} = \frac{n_f^2}{16 L_s^3 L_t}\left\{\langle(\mathrm{Tr}D_q^{-1})^2\rangle - \langle \mathrm{Tr}D_q^{-1}\rangle^2\right\}\ ,$$

$$Z_m(\beta) = \frac{m_l(\beta)}{m_l(\beta^*)}\ , \qquad \beta^*\ \text{reference point}$$

[A. Bazavov *et al.* (HotQCD coll.), 2010]

- Modified **MILC** public code (**http://physics.utah.edu/~detar/milc.html**): forward and backward temporal links entering the discretized Dirac operator multiplied by $e^{ia\mu}$ and $e^{-ia\mu}$, respectively.

- Rational hybrid Monte Carlo (**RHMC**) simulation algorithm, with length of each trajectory set to 1.0 in molecular dynamics time units.

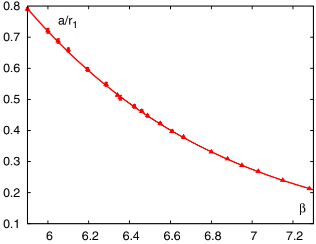
- Typically not less than 1000 trajectories for each run discarded to ensure thermalization and from 4000 to 8000 trajectories collected for measurements.

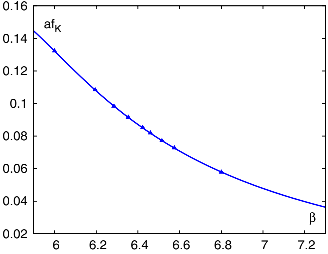
- Two different procedures to set the lattice scale in order to get the physical temperature at a given gauge coupling.

Setting the lattice scale

From (i) slope of the $q\bar{q}$ potential at $T = 0$ and (ii) decay constant f_K

[A. Bazavov *et al.* (HotQCD coll.), 2012]





$$\frac{a}{r_1}(\beta) = \frac{c_0 f(\beta) + c_2(10/\beta)f^3(\beta)}{1 + d_2(10/\beta)f^2(\beta)}$$

$c_0 = 44.06$, $c_2 = 272102$, $d_2 = 4281$,
$r_1 = 0.3106(20)$ fm

$$af_K(\beta) = \frac{c_0^K f(\beta) + c_2^K(10/\beta)f^3(\beta)}{1 + d_2^K(10/\beta)f^2(\beta)}$$

$c_0^K = 7.66$, $c_2^K = 32911$, $d_2^K = 2388$,
$r_1 f_K \simeq 0.1738$.

$f(\beta)$ is the two-loop beta function: $f(\beta) = (b_0(10/\beta))^{-b_1/(2b_0^2)} \exp(-\beta/(20b_0))$

(b_0 and b_1 universal coefficients)

Determination of $T_c(\mu)$

$$\chi_{l,\mathrm{ren}} = \frac{1}{Z_m^2}\chi_{l,\mathrm{disc}}$$

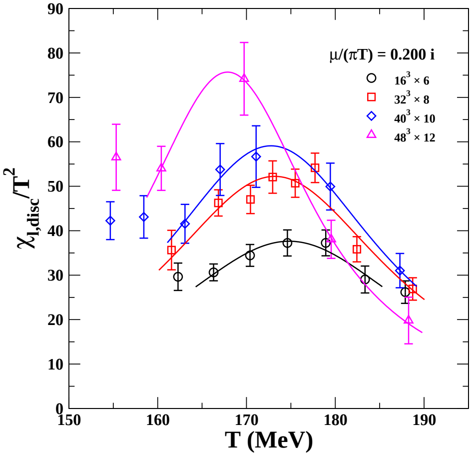
$$Z_m(\beta) = \frac{m_l(\beta)}{m_l(\beta^*)}\,, \qquad \frac{r_1}{\beta^*} = 2.37$$

$\beta^* = 6.54706$ (r_1 scale)

$\beta^* = 6.56778$ (f_K scale)

To localize the peak, a Lorentzian fit has been used:

$$\frac{a_1}{1 + a_2(T - T_c)^2}$$



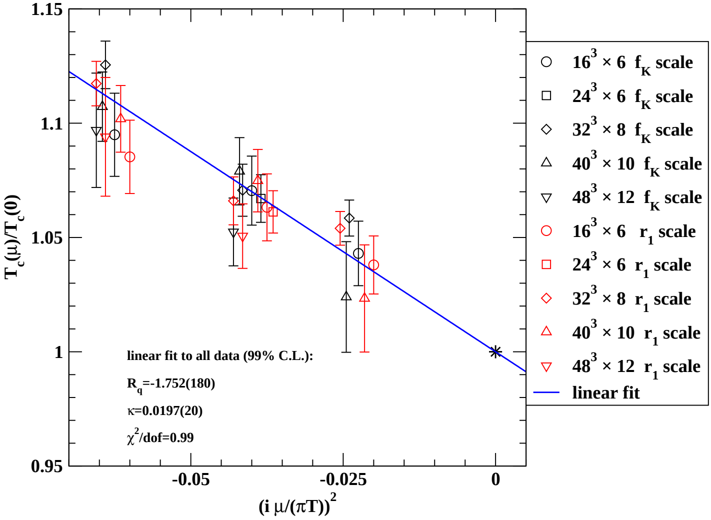
Summary of results for of $T_c(\mu)/T_c(0)$

lattice	$\mu/(\pi T)$	$T_c(\mu)/T_c(0)$ (r_1 scale)	$T_c(\mu)/T_c(0)$ (f_K scale)
$16^3 \times 6$	$0.15i$	1.038(13)	1.043(14)
	$0.2i$	1.063(15)	1.070(15)
	$0.25i$	1.085(16)	1.095(18)
$24^3 \times 6$	$0.2i$	1.061(9)	1.067(10)
$32^3 \times 8$	$0.15i$	1.054(7)	1.059(8)
	$0.2i$	1.066(10)	1.071(11)
	$0.25i$	1.117(10)	1.126(10)
$40^3 \times 10$	$0.15i$	1.023(23)	1.024(24)
	$0.2i$	1.075(14)	1.079(15)
	$0.25i$	1.102(15)	1.107(15)
$48^3 \times 12$	$0.15i$	1.013(31)	1.013(33)
	$0.20i$	1.051(14)	1.052(15)
	$0.25i$	1.094(26)	1.097(25)

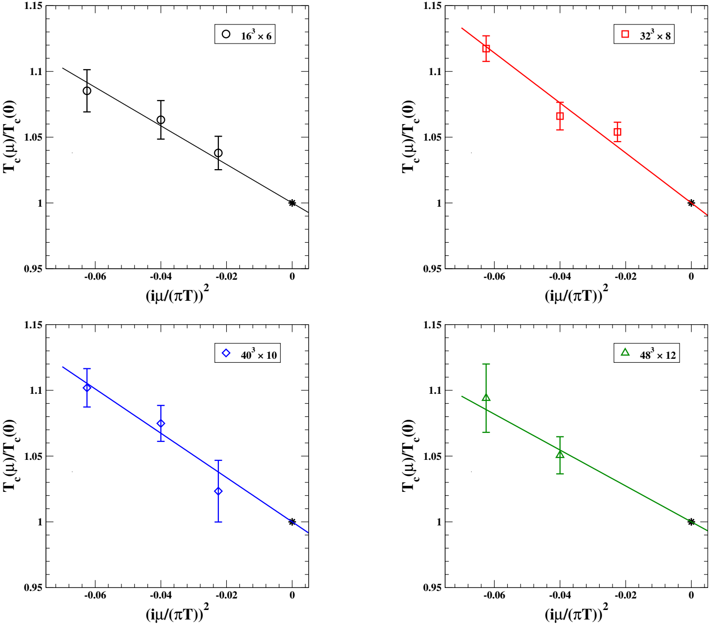
$T_c(0)$ determined using data for disconnected light chiral susceptibility obtained by the HotQCD collaboration [A. Bazavov *et al.* (HotQCD coll.), 2012, 2014]

Fit linear in μ^2 to **all data**: $\frac{T_c(\mu)}{T_c(0)} = 1 + R_q \left(\frac{i\mu}{\pi T_c(\mu)}\right)^2$

$R_q = -1.752(180)\,,\qquad \kappa = -\mathbf{R_q}/(9\pi^2) = \mathbf{0.0197(20)}$

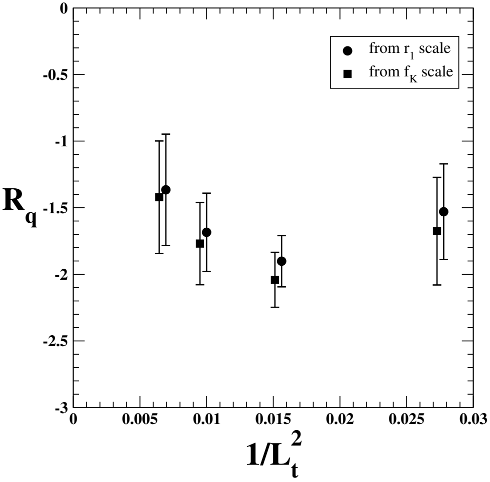


Fit linear in μ^2 to **data at fixed** L_t (r_1 scale)



Outline

Curvature in the continuum limit



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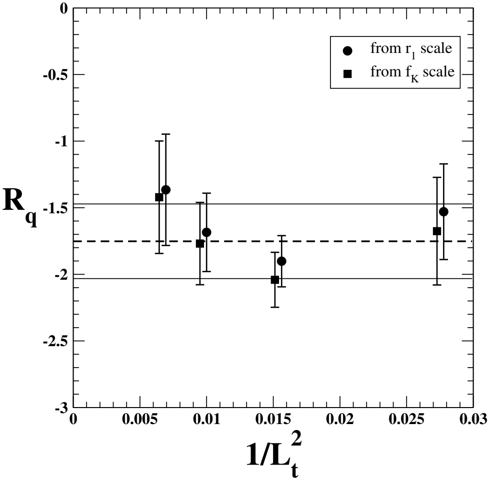


$$R_q = -1.7518(2802)$$

$$\chi_r^2 = 0.99$$

$$\kappa = -\frac{R_q}{9\pi^2} = 0.0197(32)$$

Outline

Comparison with other analyses

- **Caveats** in the comparison with other lattice studies

  • different choices for discretization, lattice size, quark masses, chemical potentials, procedure to circumvent the sign problem, *etc.*, lead to different systematics

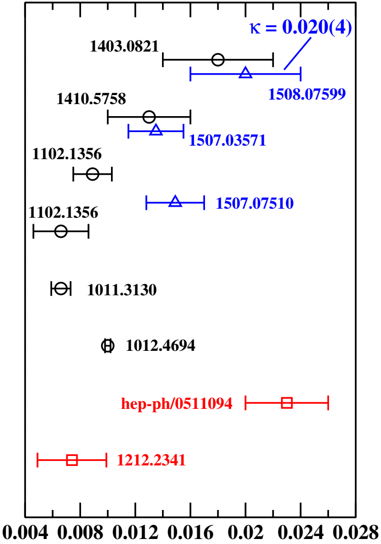
  • since QCD exhibits a smooth crossover rather than a true phase transition, different probe observables lead to different values of $T_c(\mu)$, *even with the same lattice setup*

- **Caveats** in the comparison with the curvature of the freeze-out curve

  • no *a priori* reason for the coincidence of the QCD pseudocritical line with the chemical freeze-out curve: the quark-gluon plasma fireball (if created) *first* rehadronizes, *then* reaches the chemical freeze-out

  • in heavy-ion collisions strangeness neutrality, $\langle n_s \rangle = 0$, is satisfied; this implies that, near $T_c(0)$, we should have $\mu_u \simeq \mu_d$, $\mu_s \simeq \mu_{u,d}/4$

  • the freeze-out curve is determined through thermal-statistical models, subjected to their own systematic effects



`arXiv:1403.0821` - P. Cea, L. Cosmai, A. P.,
analytic continuation, HISQ/tree action, $\mu_l = \mu_s$, disconnected chiral susceptibility

`arXiv:1508.07599` - P. Cea, L. Cosmai, A. P.,
same, with continuum extrapolation

`arXiv:1410.5758` - Pisa group,
analytic continuation, stout action, $\mu_s = 0$, chiral condensate, chiral susceptibility

`arXiv:1507.03571` - Pisa group,
same, with continuum extrapolation

`arXiv:1102.1356` - Budapest-Wuppertal group,
Taylor expansion, stout action, $\mu_s = 0$, (1) chiral condensate, (2) strange quark number susceptibility

`arXiv:1507.07510` - Budapest-Wuppertal group,
analytic continuation, stout action, $\langle n_s \rangle = 0$, chiral condensate, chiral susceptibility and strange susceptibility

`arXiv:1011.3130` - Bielefeld group,
Taylor expansion, p4-action, $\mu_s = 0$, chiral susceptibility

`arXiv:1012.4694` - R. Falcone, E. Laermann, M.P. Lombardo,
analytic continuation, p4-action, $\mu_l = \mu_s$, Polyakov loop

`hep-ph/0511084` - J. Cleymans *et al.*,
freeze-out curvature, from standard statistical hadronization model

`arXiv:1212.2341` - F. Becattini *et al.*,
same + effect of inelastic collisions after freeze-out

Extrapolation of the critical line to real μ_B

Caveats:

- reliable up to $\frac{\mu}{\pi T} \simeq 0.25$, *i.e.* $\mu_B \simeq 0.4$ GeV
- effect of $\mu_s \neq 0$ at the larger μ_B in this range not assessed

$T_c(\mu_B) = a - b\mu_B^2$

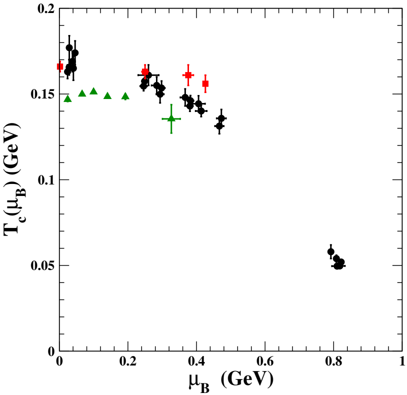
$a = T_c(0)\ , \quad b = \frac{\kappa}{T_c(0)}$

Using our result $\kappa = 0.020(4)$
and $T_c(0) = 154(9)$ MeV
[A. Bazavov *et al.* (HotQCD coll.), 2012]

$\longrightarrow \ b = 0.128(25)\ \text{GeV}^{-1}$

to be compared with

$b = 0.139(16)\ \text{GeV}^{-1}$
[J. Cleymans *et al.*, 2006]



- hep-ph/0511094 - J. Cleymans *et al.*
- arXiv:1212.2341 - F. Becattini *et al.*
- arxiv:1403.4903 - P. Alba *et al.*

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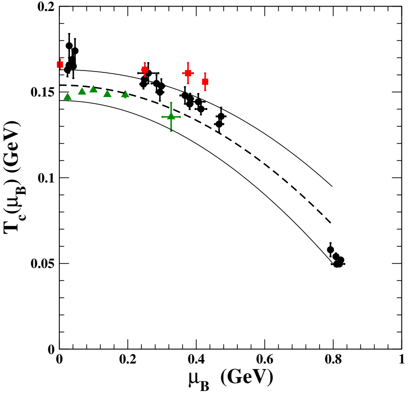

● hep-ph/0511094 - J. Cleymans *et al.*
■ arXiv:1212.2341 - F. Becattini *et al.*
▲ arxiv:1403.4903 - P. Alba *et al.*

Conclusions

- We have simulated on a space-time lattice **QCD with 2+1 flavors** at almost physical masses, in a setup with the same chemical potential for the three quark species
- By analytic continuation, we have estimated the **continuum limit of the curvature** of the QCD pseudocritical line at zero baryon density
- Our result **agrees at 1σ level** with the most recent determinations of the same quantity, with a slightly higher central value
- Within statistical and systematic uncertainties, the extrapolated pseudocritical line extrapolated **nicely compares** with most determinations of the **freeze-out curve** at small μ_B

Acknowledgements

• Work in part based on the MILC collaboration's public lattice gauge theory code (**http://physics.utah.edu/˜detar/milc.html**)

• Work partially supported by the **INFN SUMA project**.

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