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The pseudocritical line in QCD at nonvanishing chemical potential

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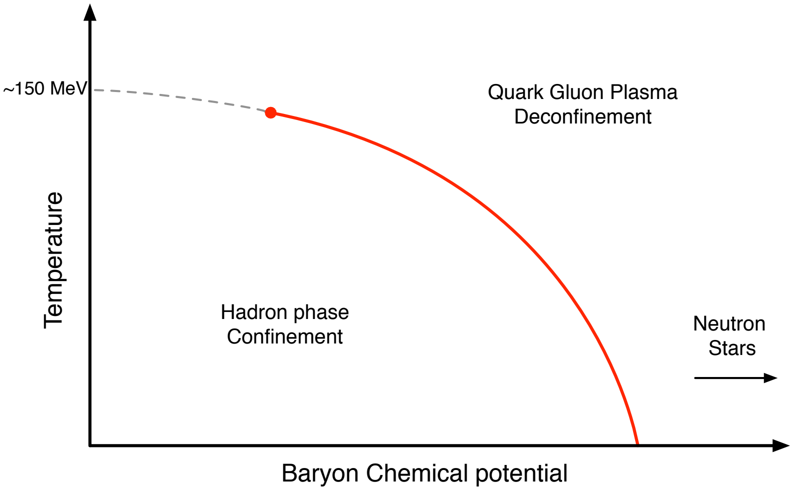
Outline

- The critical line of QCD
- The method of analytic continuation
- Renormalized observables

  Chiral condensate [renormalization (I) and (II)], Chiral susceptibility
- The role of the strange quark chemical potential μ_s
- Numerical setup
- Numerical results
  - Finite size effects on $n_t = 6$ lattices.
  - Determination of $T_c(\mu)$, taking into account systematic uncertainties
  - Effects of $\mu_s \neq 0$
  - Continuum limit extrapolations.
- Conclusions

QCD at nonzero μ_B



The pesudocritical line and analytic continuation

At lowest order in μ, the pseudocritical line can be parametrized as:

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa \left(\frac{\mu_B}{T_c(\mu_B)} \right)^2 + O(\mu^4)$$

The sign problem and analytic continuation

For purely imaginary μ, the fermion determinant is real positive, and the sign problem is non existent.

With the transformation $\mu \to i\mu$, the pseudocritical line parametrization is modified as:

$$\frac{T_c(\mu_B)}{T_c} = 1 + \kappa \left(\frac{\mu_B}{T_c(\mu_B)} \right)^2 + O(\mu^4)$$

Renormalization of the chiral condensate

$$\langle\bar{\psi}\psi\rangle_{ud} = \frac{T}{V}\frac{\partial \log Z}{\partial m_{ud}} = 2\frac{T}{V}\langle \mathrm{Tr} M_l^{-1}\rangle = \langle\bar{u}u\rangle + \langle\bar{d}d\rangle$$

Renormalization (I) [Cheng *et al.*, 08]:

$$\langle\bar{\psi}\psi\rangle^r_{(1)} \equiv \frac{\langle\bar{\psi}\psi\rangle_{ud}(T) - \frac{2m_{ud}}{m_s}\langle\bar{s}s\rangle(T)}{\langle\bar{\psi}\psi\rangle_{ud}(0) - \frac{2m_{ud}}{m_s}\langle\bar{s}s\rangle(0)}$$

Renormalization (II) [Endrodi *et al.*, 11 , Bellwied *et al.*, 15]:

$$\langle\bar{\psi}\psi\rangle^r_{(2)} \equiv \frac{m_{ud}}{m_\pi^4}\left(\langle\bar{\psi}\psi\rangle_{ud} - \langle\bar{\psi}\psi\rangle_{ud}(T=0)\right)$$

Renormalized chiral susceptibility

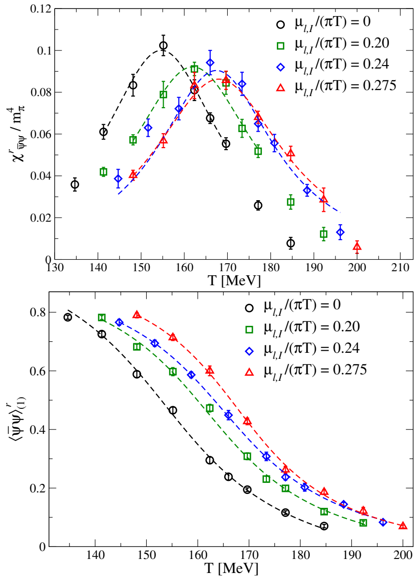
$$\chi_{\bar{\psi}\psi} \equiv \frac{\partial\langle\bar{\psi}\psi\rangle_{ud}}{\partial m_l}$$

Renormalized as [Y.Aoki *et al.*, 06]:

$$\chi^r_{\bar{\psi}\psi}(T) \equiv m_{ud}^2\left[\chi_{\bar{\psi}\psi}(T) - \chi_{\bar{\psi}\psi}(0)\right]$$

We use the dimensionless quantity $\chi^r_{\bar{\psi}\psi}(T)/m_\pi^4$.

Defining T_c and systematic errors



Fit at the peak for the **renormalized chiral susceptibility**:

$$\chi^r_{\bar\psi\psi}(T) = \frac{A_2}{(T - T_c)^2 + B_2^2}$$

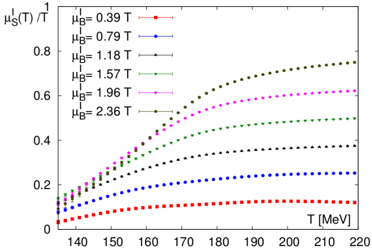
Fit for the **chiral condensates (I) and (II)**:

$$\langle\bar\psi\psi\rangle^r(T) = A_1 + B_1 \arctan\left[C_1\,(T - T_c)\right]$$

Estimates of the systematic errors are obtained by varying the fit range.

The role of the strange chemical potential μ_s

Our aim is to study strongly interacting matter in conditions as similar as possible to the ones found in heavy ion collision experiments, where typically $Q \sim 0.4B$ and $S = 0$ (strangeness neutrality).



Value of the strange quark chemical potential needed to achieve strangeness neutrality. From [Bellwied *et al.*,15]

Due to interactions, $\mu_s = 0 \not\Rightarrow S = 0$ [Bazavov *et al.*,14, Borsanyi *et al.*,13 , Bellwied *et al.*,15].

Physical conditions in heavy ion collision experiments could be matched approximately with $\mu_l \equiv \mu_u = \mu_d$ and $\mu_s \simeq \mu_l/4$ (around the critical temperature).

Our study has been performed mainly in the $\mu_s = 0$ setup, **but we also checked the $\mu_s = \mu_l$ case to obtain an estimate of the effect of a nonzero μ_s.**

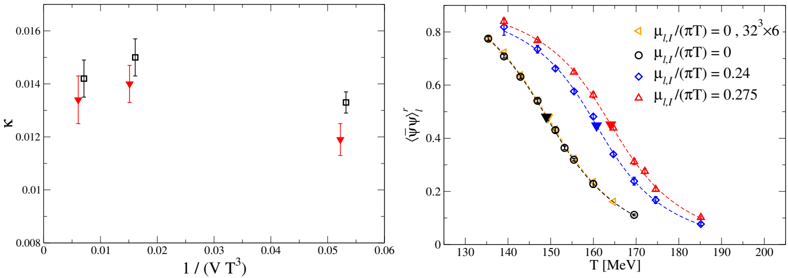
Numerical setup

- Study of the $\mu_s = \mu_l \neq 0$ (32^3x8 only) and $\mu_s = 0$ cases.
- Tree level Symanzik improved gauge action with $N_f = 2 + 1$ flavours of 2-stouted staggered fermions.
- At the physical point (line of constant physics, parameters taken from [Aoki *et al.*, 09]) $N_t = 6, 8, 10, 12$ lattices.
- Also performed simulations at zero temperature for renormalizations (32^4,48^3x96).
- Observables evaluated with noisy estimators, with 8 random vectors per quark.

Simulations run on IBM BG-Q at CINECA (Bologna, Italy) and on the Zefiro cluster in Pisa.

Lattice	$16^3 \times 6$	$24^3 \times 6$	$32^3 \times 6$
$i\mu/(\pi T)$	0.00 0.20 0.24 0.275	0.00 0.24 0.275	0.00 0.24 0.275
Lattice	$32^3 \times 8$	$40^3 \times 10$	$48^3 \times 12$
$i\mu/(\pi T)$	0.00 0.10 0.15 0.20 0.24 0.275 0.30	0.00 0.20 0.24 0.275	0.00 0.20 0.24 0.275

From [Bonati *et al.*, 14] (16^3x6, 24^3x6 and 32^3x6 lattices)

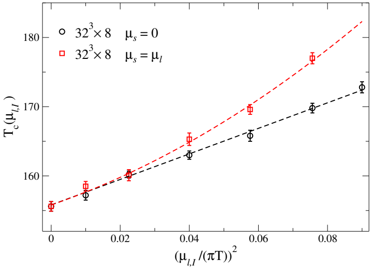


Left: Estimates of κ. **Black**: Renormalized Chiral Condensate (1), **Red**: Renormalized Chiral Susceptibility.

Right: The chiral condensate on the $24^3 \times 6$ lattice, with the data for $\mu_I = 0$ on the $32^3 \times 6$ lattice

$\Rightarrow$ Aspect ratio 4 is enough.

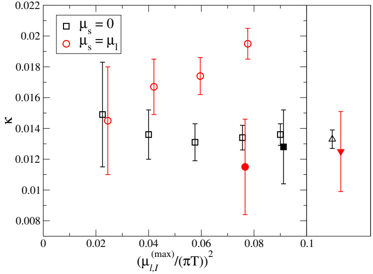
Effects of μ_s
$32^3 \times 8$ Lattice



From Renormalized Chiral Susceptibility, [Bonati *et al.*, 14, Bonati *et al.*, 15]

If we include a quartic term in the $\mu_s = \mu_l$ case, the curvature κ is the same as in the $\mu_s = 0$ case.

Results for κ varying the μ fit range:

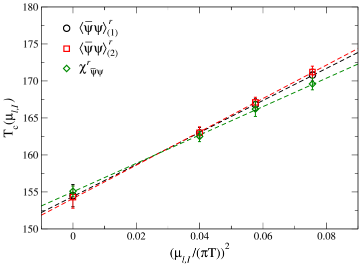


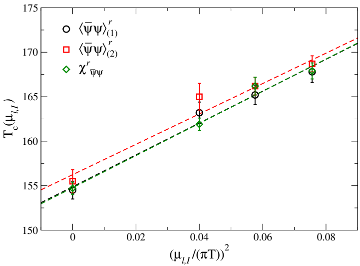
Empty/Full : linear / linear+quadratic

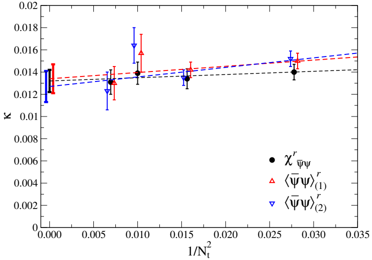
Black/Red: $\mu_s = 0$ / $\mu_s = \mu_l$

Right: κ from combined lin + (lin+quad) fit (keeping $T_C(0)$ equal)

Critical line and Continuum limit I (κ)







$40^3 \times 10$ [up left]

$$\begin{cases} \kappa_{\bar{\psi}\psi,1} = 0.0157(17) \\ \kappa_{\bar{\psi}\psi,2} = 0.0164(16) \\ \kappa_{\chi} = 0.0139(10) \end{cases}$$

$48^3 \times 12$ [up right]

$$\begin{cases} \kappa_{\bar{\psi}\psi,1} = 0.0130(15) \\ \kappa_{\bar{\psi}\psi,2} = 0.0123(17) \\ \kappa_{\chi} = 0.0131(11) \end{cases}$$

We evaluated the curvature κ for each lattice spacing and then performed the continuum limit extrapolation on κ itself.

Continuum limit

$$\begin{cases} \kappa_{\bar{\psi}\psi,1} = 0.0134(13) \\ \kappa_{\bar{\psi}\psi,2} = 0.0127(14) \\ \kappa_{\chi} = 0.0132(10) \end{cases}$$

For the **renormalized chiral condensate**, we used the formula

$$\langle\bar{\psi}\psi\rangle^r(T) = A_1 + B_1 \arctan\left[C_1\left(T - T_c\right)\right]$$

to fit the data from all values of N_t simultaneously. We added a N_t dependency to T_c ($T_c(N_t) = T_c(N_t = \infty) + const./N_t^2$) and a similar one to C_1.

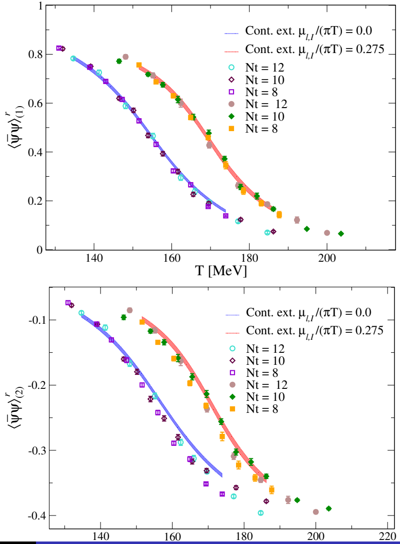
$$\langle\bar{\psi}\psi\rangle^r_{(1)} \equiv \frac{\langle\bar{\psi}\psi\rangle_{ud}(T) - \frac{2m_{ud}}{m_s}\langle\bar{s}s\rangle(T)}{\langle\bar{\psi}\psi\rangle_{ud}(0) - \frac{2m_{ud}}{m_s}\langle\bar{s}s\rangle(0)}$$

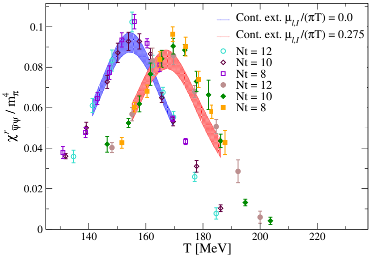
[Cheng *et al.*, 08]

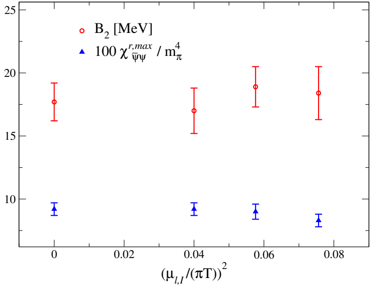
$$\langle\bar{\psi}\psi\rangle^r_{(2)} \equiv \frac{m_{ud}}{m_\pi^4}\left(\langle\bar{\psi}\psi\rangle_{ud} - \langle\bar{\psi}\psi\rangle_{ud}(T=0)\right)$$

[Endrodi *et al.*, 11 , Bellwied *et al.*, 15]

Renormalized chiral condensate (I) and (I)







Continuum limit of the Renormalized Chiral Susceptibility

We used the formula

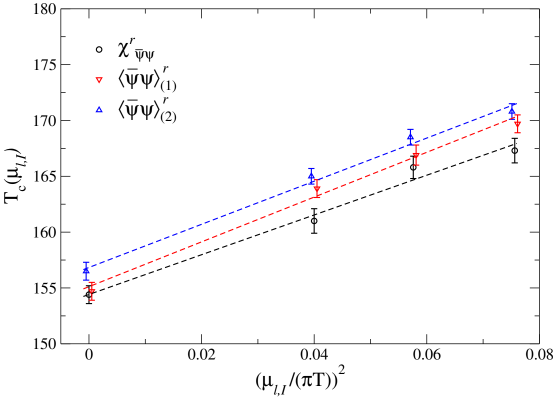
$$\chi^r_{\bar\psi\psi}(T) = \frac{A_2(N_t)}{(T - T_c(N_t))^2 + B_2(N_t)^2}$$

with a dependency on N_t like

$$p(N_t) = p(N_t = \infty) + const./N_t^2$$

for all parameters.

The *maximum value* $\chi^{r,max}_{\bar\psi\psi}$ and the *width of the peak* B_2 of the renormalized Chiral Susceptibility extrapolated to the continuum limit



2nd method:

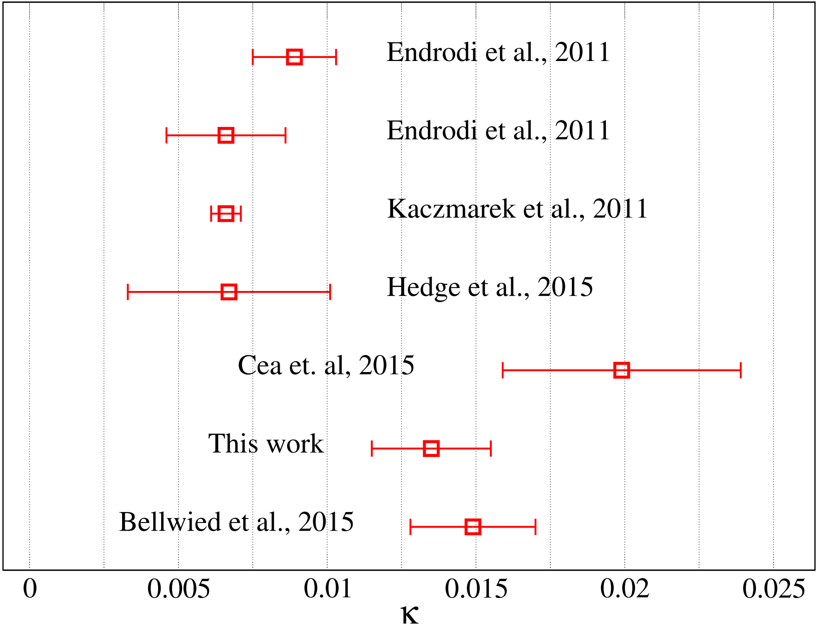
$$\begin{cases} \kappa_{\bar\psi\psi,1} &= 0.0145(11) \\ \kappa_{\bar\psi\psi,2} &= 0.0138(10) \\ \kappa_{\chi} &= 0.0131(12) \end{cases}$$

1st method (see slide 11):

$$\begin{cases} \kappa_{\bar\psi\psi,1} &= 0.0134(13) \\ \kappa_{\bar\psi\psi,2} &= 0.0127(14) \\ \kappa_{\chi} &= 0.0132(10) \end{cases}$$

The difference between the two methods is an estimate of the systematic error associated with the continuum limit.

Comparison with other lattice determinations



Conclusions

- The finite size effects have been studied, and we deemed **aspect ratio** 4 **sufficient** for the present level of accuracy.
- We performed checks to compare our determinations with the ones of other groups [Bonati *et al.*,14].
- We studied **the effects of the strange quark chemical potential** (in the setup $\mu_s = \mu_l = \mu$). In this case, we have observed a contribution of order μ_B^4 to the pseudocritical temperature. Considering such contribution, the curvatures of the critical line (the coefficient of the quadratic term) for $\mu_s = \mu_l$ and $\mu_s = 0$ **are compatible within errors**.
- We performed a continuum scaling analysis in two ways, directly on κ and on the observables. The resulting estimates of κ are in agreement. **Our prudential estimate is** $\kappa = 0.0135(20)$**.**