Real-Time Simulation of Large Open Quantum Spin Systems Driven by Dissipation

Uwe-Jens Wiese

Albert Einstein Center for Fundamental Physics
Institute for Theoretical Physics, Bern University

UNIVERSITÄT BERN
AEC ALBERT EINSTEIN CENTER FOR FUNDAMENTAL PHYSICS

SM&FT 2015
Bari, Italy, December 11

erc European Research Council

Collaboration:
Debasish Banerjee (DESY Zeuthen),
Fu-Jiun Jiang (NTNU, Taipei), Mark Kon (Boston University),
Stephan Caspar, Florian Hebenstreit, David Mesterhazy (Bern)

Members of the Collaboration

Debasish Banerjee

Fu-Jiun Jiang

Mark Kon

Stephan Caspar

Florian Hebenstreit

David Mesterhazy

Outline

Outline

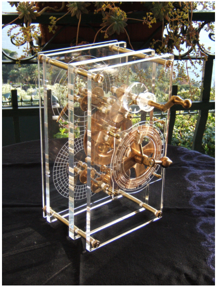
The first "digital computer" in Babylonia about 2400 b.c.

The first “digital computer” in Babylonia about 2400 b.c.

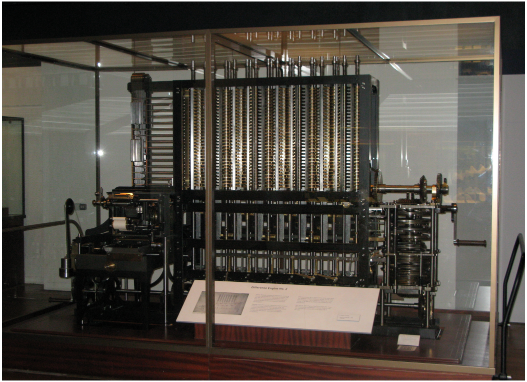
The first “analog computer”: Antikythera for determining the position of celestial bodies, Crete, about 100 b.c.

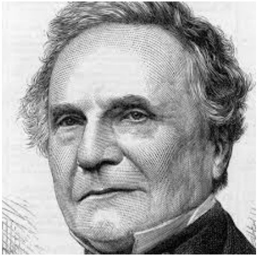
The first “digital computer” in Babylonia about 2400 b.c.

The first “analog computer”: Antikythera for determining the position of celestial bodies, Crete, about 100 b.c.



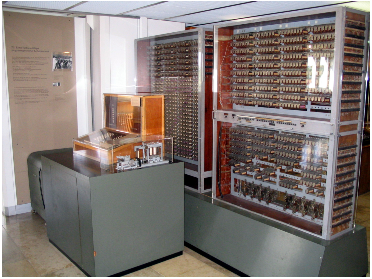
The first programmable computer:
mechanical "difference engine"
Charles Babbage (1791-1871)



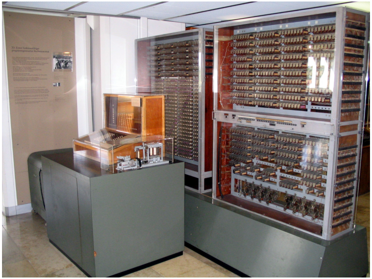


was realized by his son after Babagge's death.

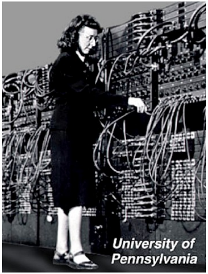
Konrad Zuse's (1910-1992) relay-driven computer Z3



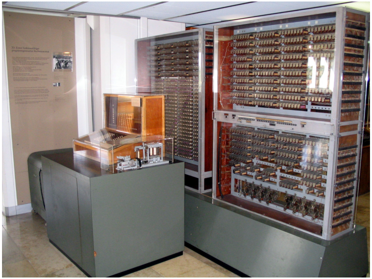
Konrad Zuse's (1910-1992) relay-driven computer Z3



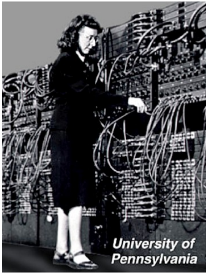
From the vacuum-tube ENIAC to the IBM Blue Gene



Konrad Zuse's (1910-1992) relay-driven computer Z3



From the vacuum-tube ENIAC to the IBM Blue Gene

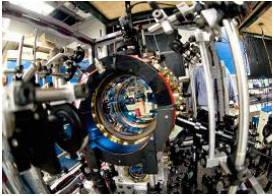


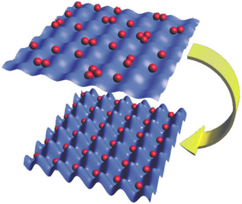
Outline

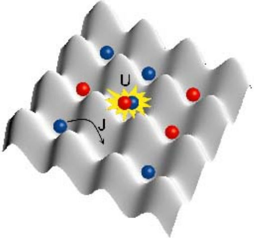
Richard Feynman's vision of 1982

"I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy."

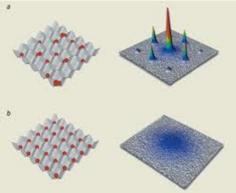
Ultra-cold atoms in optical lattices as analog quantum simulators

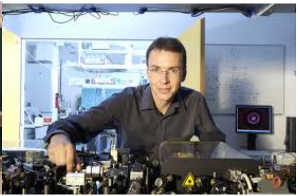






Transition from a superfluid to a Mott insulator





M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, I. Bloch, Nature 415 (2002) 39.

Outline

Real-time path integral describing a quench from H_0 to H

$$
\begin{aligned}
p_{\rho_0}[m(t)] &= \frac{1}{Z_0} Tr[\exp(-\beta H_0)\exp(iHt)|m(t)\rangle\langle m(t)|\exp(-iHt)] \\
&= \frac{1}{Z_0}\sum_{[n_0,n]} \exp(-S_{0E}[n_0])\exp(i(S_R[n]+iS_I[n]))\delta_{n(t),m(t)} \\
&= \frac{Z}{Z_0}\langle\cos(S_R)\delta\rangle, \qquad Z_0 = \sum_{[n_0]}\exp(-S_E[n_0])
\end{aligned}
$$

Path integral for a corresponding Euclidean ensemble

$$
Z = \sum_{[n_0,n]} \exp(-S_E[n_0] - S_I[n])
$$

Large error to signal ratio:

$$
\Delta p_{\rho_0}[m(t)] = \frac{Z}{Z_0}\sqrt{\langle\cos^2(S_R)\delta^2\rangle - \langle\cos(S_R)\delta\rangle^2} \approx \frac{Z}{Z_0}\sqrt{\langle\delta\rangle/2}
$$

$$
\frac{\Delta p_{\rho_0}[m(t)]}{p_{\rho_0}[m(t)]} = \frac{\sqrt{\langle\delta\rangle/2}}{\langle\cos(S_R)\delta\rangle} = \frac{Z\sqrt{\langle\delta\rangle/2}}{Z_0 p_{\rho_0}[m(t)]} \sim \frac{\exp(\Delta f V t)\sqrt{\langle\delta\rangle/2}}{p_{\rho_0}[m(t)]}
$$

Real-time evolution of the density matrix of an isolated quantum system

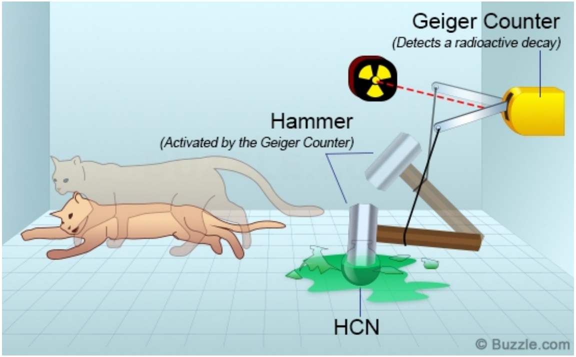
$$\partial_t \rho(t) = i[\rho(t), H(t)], \quad \rho(t) = U(t, t_0)\rho(t_0)U(t_0, t),$$
$$U(t_0, t) = \mathcal{T} \exp\left(-i \int_{t_0}^{t} dt' \; H(t')\right)$$

Why is this so difficult to compute?

Real-time evolution of the density matrix of an isolated quantum system

$$\partial_t \rho(t) = i[\rho(t), H(t)], \quad \rho(t) = U(t, t_0)\rho(t_0)U(t_0, t),$$
$$U(t_0, t) = \mathcal{T} \exp\left(-i \int_{t_0}^{t} dt' \; H(t')\right)$$

Why is this so difficult to compute?



It should be easier to compute the real-time evolution when the system is under observation.

Outline

Measurement process of an observable O_k

$$O_k|\lambda\rangle = o_k|\lambda\rangle, \quad P_{o_k} = \sum_\lambda |\lambda\rangle\langle\lambda|,$$

$$P_{o_k}^2 = P_{o_k}, \quad \mathrm{Tr} P_{o_k} = g_{o_k}, \quad \sum_{o_k} P_{o_k} = \mathbb{1}$$

Evolution of the density matrix after one measurement

$$\rho_{o_1} = P_{o_1}\rho P_{o_1}, \quad \rho' = \sum_{o_1}\rho_{o_1} = \sum_{o_1} P_{o_k}\rho P_{o_1},$$

$$\mathrm{Tr}\rho' = \sum_{o_1}\mathrm{Tr}(P_{o_1}\rho P_{o_1}) = \sum_{o_1}\mathrm{Tr}(\rho P_{o_1}) = 1$$

Evolution of the density matrix after N measurements

$$\begin{aligned}
\rho_{o_1,o_2,\ldots,o_N} &= P_{o_N}U(t_N,t_{N-1})\ldots U(t_3,t_2)P_{o_2}U(t_2,t_1)P_{o_1}\rho \\
&\times P_{o_1}U(t_1,t_2)P_{o_2}U(t_2,t_3)\ldots U(t_{N-1},t_N)P_{o_N}, \\
\rho' &= \sum_{o_1,o_2,\ldots,o_N}\rho_{o_1,o_2,\ldots,o_N}
\end{aligned}$$

Outline

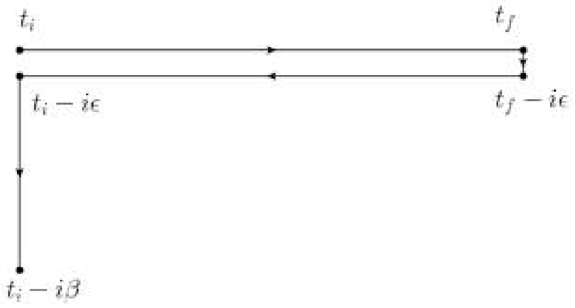
Path integral with a Schwinger-Keldysh contour

$$p_{\rho_0 f}(o_1, o_2, \ldots, o_N) =$$

$$\sum_i p_i \langle ii|(P_{o_1} \otimes P^*_{o_1})(P_{o_2} \otimes P^*_{o_2}) \ldots (P_{o_N} \otimes P^*_{o_N})|ff\rangle =$$

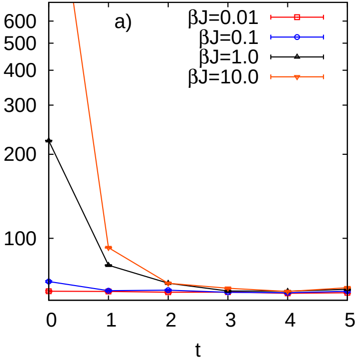
$$\sum_i p_i \sum_{n_1, n'_1} \ldots \sum_{n_{N-1}, n'_{N-1}} \prod_{k=1}^{N} \langle n_{k-1}|P_{o_k}|n_k\rangle \langle n'_{k-1}|P_{o_k}|n'_k\rangle^*,$$

$$\langle n_0 n'_0| = \langle ii|, \quad |n_N n'_N\rangle = |ff\rangle$$



Antiferromagnetic spin $\frac{1}{2}$ quantum Heisenberg model, $H = J \sum_{\langle xy \rangle} \vec{S}_x \cdot \vec{S}_y$, driven by measurements of the total spin $S \in \{0, 1\}$ of adjacent spin pairs $\vec{S} = \vec{S}_x + \vec{S}_y$

$$P_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad P_0 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



Continuous monitoring described by a Lindblad process

$$
\begin{aligned}
\partial_t \rho &= i[\rho, H] + \frac{1}{\varepsilon} \sum_{k,o_k} \left(L_{o_k} \rho L_{o_k}^{\dagger} - \frac{1}{2} L_{o_k}^{\dagger} L_{o_k} \rho - \frac{1}{2} \rho L_{o_k}^{\dagger} L_{o_k} \right) \\
&= \gamma \sum_{k} \Big(\sum_{o_k} P_{o_k} \rho P_{o_k} - \rho \Big)
\end{aligned}
$$

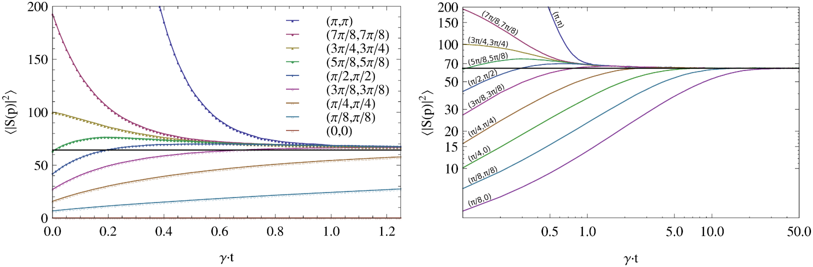
Lindblad or Kraus quantum jump operators

$$
L_{o_k} = \sqrt{\varepsilon \gamma} P_{o_k}, \quad (1 - \varepsilon \gamma N) \mathbb{1} + \sum_{k,o_k} L_{o_k}^{\dagger} L_{o_k} = \mathbb{1}
$$

G. Lindblad, Commun. Math. Phys. 48 (1976) 119.
K. Kraus, States, Effects and Operations,
Fundamental Notions of Quantum Theory, Academic, Berlin (1983).

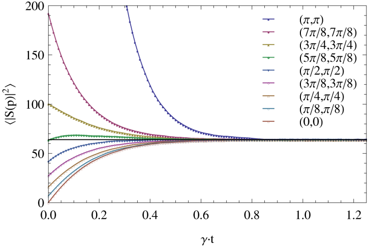
Equilibration of the Fourier modes of the magnetization in a dissipative process that "measures" $\vec{S}_x \cdot \vec{S}_y$

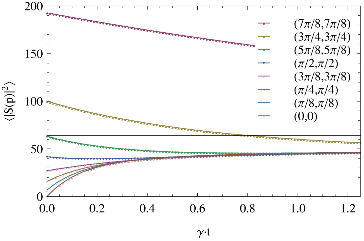
$$\widetilde{S}(p) = \sum_x S_x^3 \exp(ip_1 x_1 + ip_2 x_2)$$

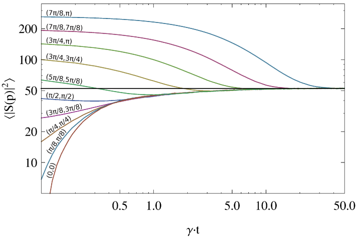


D. Banerjee, F.-J. Jiang, M. Kon, UJW, Phys. Rev. B90 (2014) 241104(R).

Equilibration of the Fourier modes of the magnetization in dissipative processes that “measure” $S_x^1 S_y^1$ or $S_x^+ S_y^+ + S_x^- S_y^-$

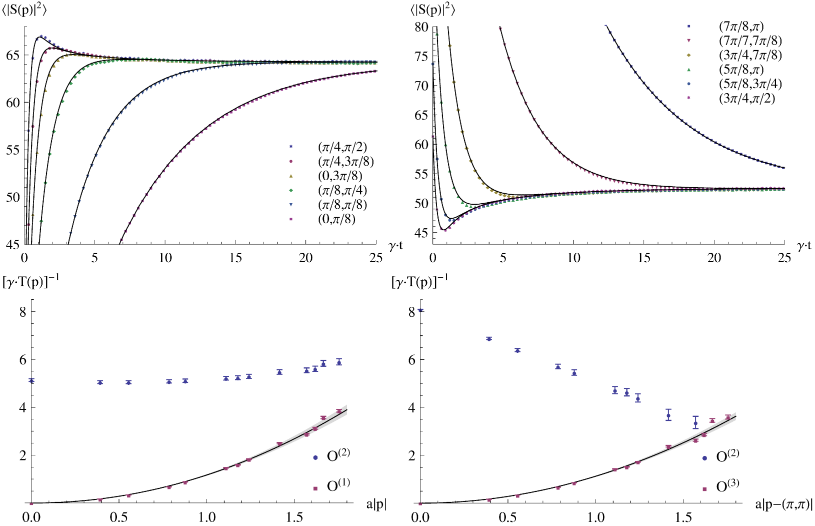




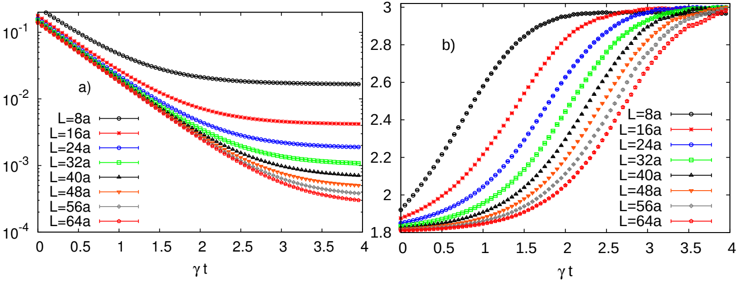


F. Hebenstreit, D. Banerjee, M. Hornung, F.-J. Jiang, F. Schranz, UJW, Phys. Rev. B92 (2015) 3, 035116.

Equilibration times

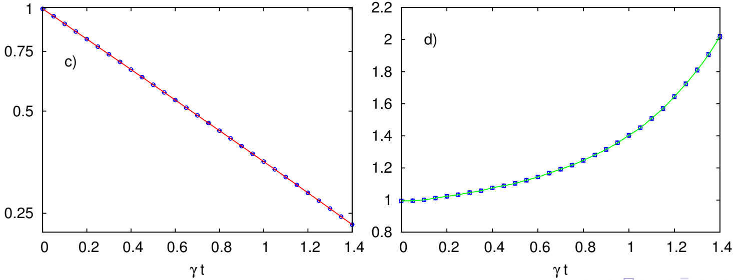


Staggered susceptibility $\langle M_s^2 \rangle / L^4$ and Binder ratio $\langle M_s^4 \rangle / \langle M_s^2 \rangle^2$



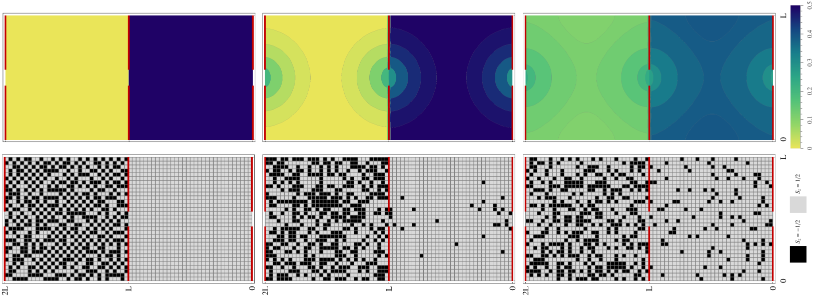
Staggered magnetization $\mathcal{M}_s$ and length scale $\xi = c/(2\pi\rho_s)$,

$$\langle M_s(t)^2 \rangle = \mathcal{M}_s(t)^2 L^4 / 3 \sum_{n=0}^{3} c_n (\xi(t)/L)^n$$

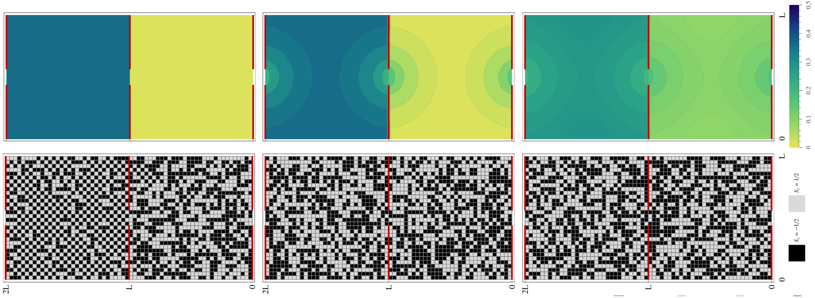


Outline

Diffusion of uniform magnetization through a hole



Diffusion of staggered magnetization through a hole



Outline

Cooling of $(3+1)$-d hard-core bosons

$$H = J \sum_{\langle xy \rangle} (S_x^+ S_y^- + S_x^- S_y^+)$$

driven by non-Hermitean Lindblad operator

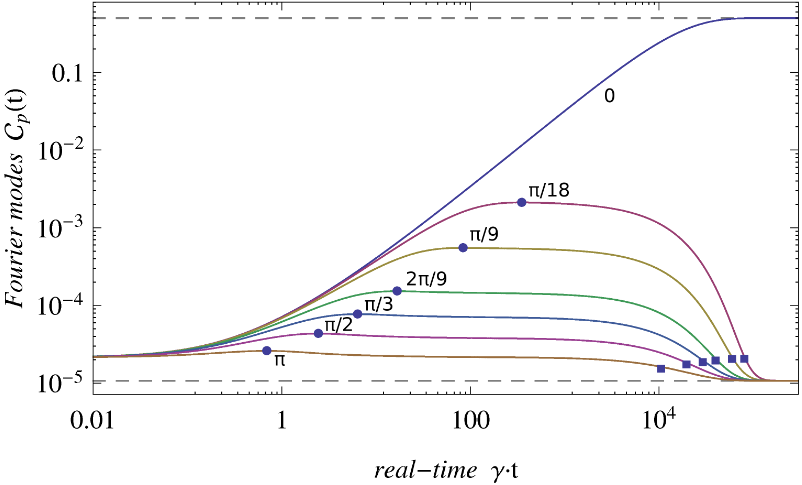
$$L_1 = (S_x^+ + S_y^+)(S_x^- - S_y^-) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad L_1^\dagger \neq L_1,$$

$$L_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad L_2^\dagger = L_2$$

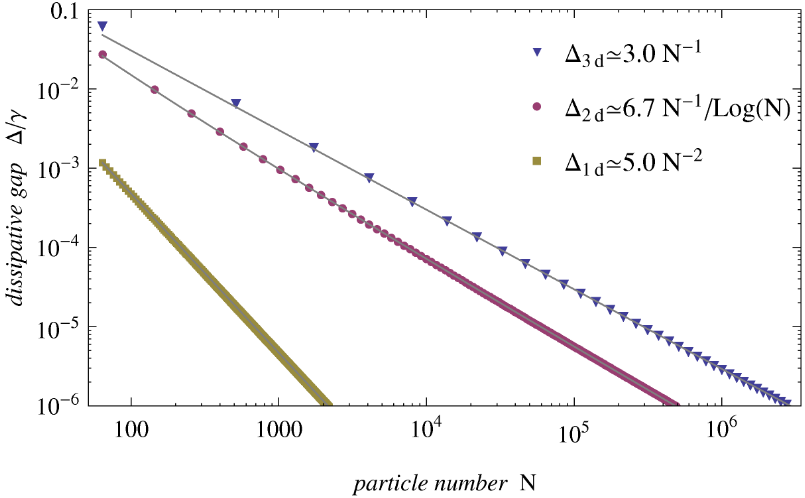
S. Caspar, F. Hebenstreit, D. Mesterhazy, UJW, arXiv1511.08733

Momentum modes of 2-point correlation function

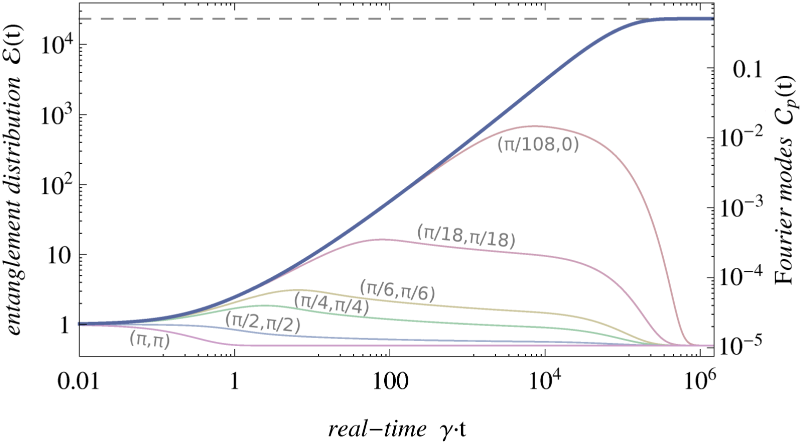
$$C_p = \sum_x (S_0^+ S_x^- + S_0^- S_x^+) \exp(ipx)$$



Dissipative gap as a function of system size



Time-evolution of entanglement



Outline

Conclusions

• Real-time simulations of some large open quantum entirely driven by dissipation or by measurement processes are sign-problem-free and can be performed using importance sampling quantum Monte Carlo.

• Such simulations have allowed us to study the time-dependence of different dissipative processes which is slowed down by conserved quantities.

• Transport processes in dissipation driven strongly correlated large open quantum spin systems lead to diffusion of magnetization or staggered magnetization from one reservoir to another.

• Lindblad processes with non-Hermitean quantum jump operators which describe cooling of bosons into a dark state can also be simulated. Different momentum modes of the Bose-Einstein condensate equilibrate at different time scales.