Heterogenous mean field approach to neural networks dynamics

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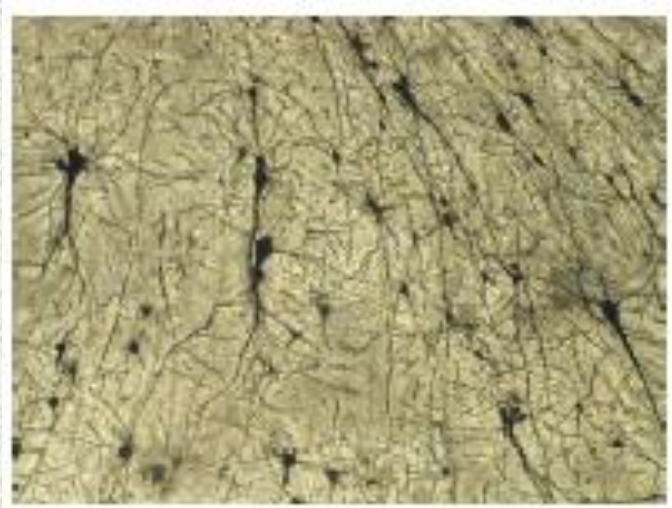
- Neural network model: Leaky integrate and fire with short-term plasticity (TUM)
- Application Heterogeneous mean field (HMF) to TUM model: numerical results and analytic insight
- Synchronous-quasi-periodic regime
- Inverse problem
- Phase diagram
- Bursty regime
- Inhibition

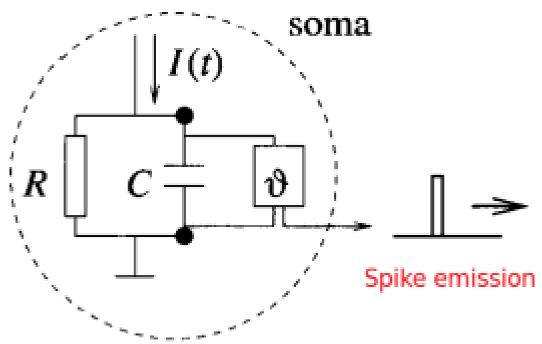
Neural Networks

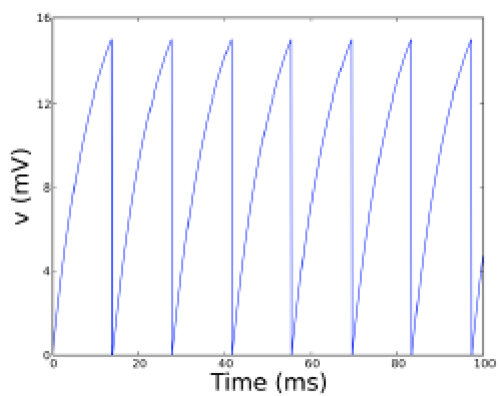


-Sites represent neurons: electrical circuits described by non linear oscillators

-Directed links represents synapses: synaptic dynamics describes the evolution of neurotransmitters

Neural dynamics





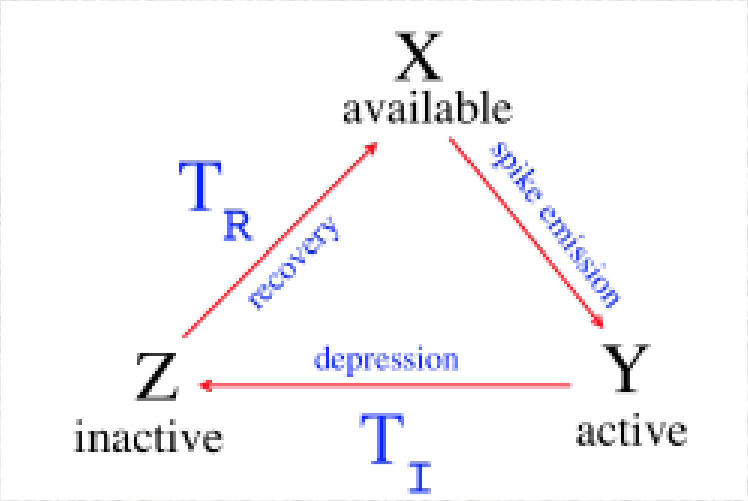
M. Tsodyks et all. Neurosci. **20, 825** (2000)

$$\dot{v}_i = a - v_i + I_i(t)$$

$I_i(t)$ current due to interactions with other neurons
For $I_i(t)=0$ exponential charging with reset at a critical value $v_{cr}=1$

Single neuron dynamics: periodic for $a > v_{cr}$ fixed point $v_i=a$ for $a < v_{cr}$

Synaptic dynamics



$$\dot{y}_i = -\frac{y_i}{\tau_{in}} + u x_i S_i(t)$$

$$\dot{x}_i = \frac{z_i}{\tau_r} - u x_i S_i(t)$$

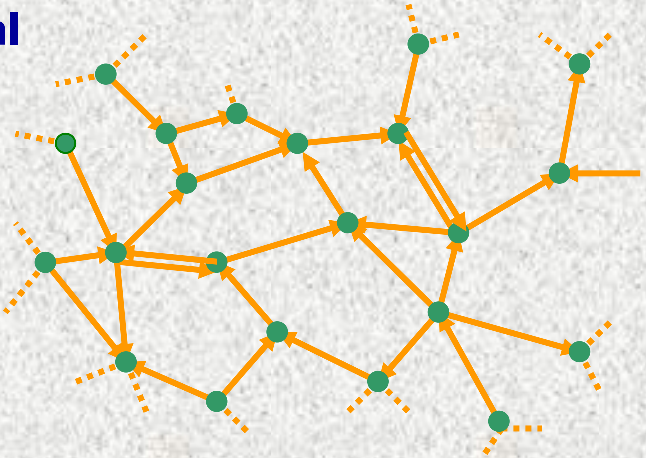
$$\dot{z}_i = \frac{y_i}{\tau_{in}} - \frac{z_i}{\tau_r}$$

$$S_i(t) = \sum_m \delta(t - t_i(m))$$

M. Tsodyks et all., Proc. Natl. Acad. Sci. USA **94,** 719 (1997).

$S_i(t)$ spike train emitted by neuron i
$t_i(m)$ times when the potential $v_i(t)$ is set to 0

Synapses outgoing from the same neuron display the same dynamic TUM Model (Tsodycs, Uziel, Markam) describing chemical synapses
y active resources
x available resources
z inactive resources
$y + x + z = 1$



$A_{i,j}$ adjacency matrix of the directed network

$A_{i,j}$ = 1 only if site j is connected to site i

$k_i = \Sigma_j A_{i,j}$ in degree of site i

$$I_i(t) = g \sum_j A_{ij} y_j(t)$$

The current induced by the networks on neuron i is proportional to the available resources $y_i(t)$ of its neighboring sites

Heterogeneous mean field

Originally introduce for studying epidemic speading on networks

Pastor–Satorras, R. & Vespignani, A. Phys. Rev. Lett. 86, 3200 (2001) ,
Vespignani, A. Nat. Phys. 8, 39 (2012)

$$g\sum_{j}^{N} A_{i,j} y_j = g k_i \frac{\sum_{j=1..k_i}^{j\in I(i)} y_j}{k_i} \approx g k_i \frac{\sum_{j=1}^{N} y_j}{N} = g k_i Y(t)$$

Plug the approximation into the dynamical equation
The dynamics depends on the site i only through the in-degree k_i
Replace the site index with the connectivity index k

$$\dot{y}_k = -\frac{y_k}{\tau_{in}} + u x_k S_k(t)$$

$$\dot{v}_k = a - v_k + kgY(t)$$

$$\dot{x}_k = \frac{z_k}{\tau_r} - u x_k S_k(t)$$

$$Y(t) = \int P(k) y_k(t) dk$$

$$\dot{z}_k = \frac{y_k}{\tau_{in}} - \frac{z_k}{\tau_r}$$

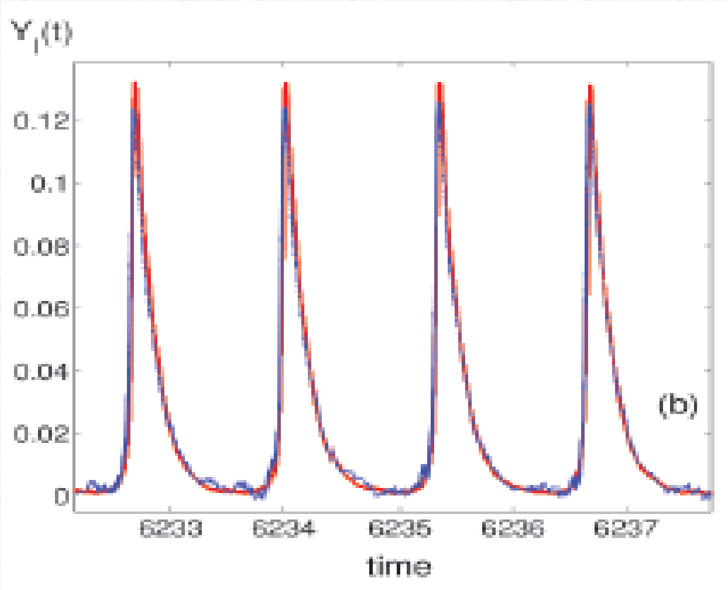
$$S_k(t) = \sum_m \delta(t - t_k(m))$$

An equation for each in degree k

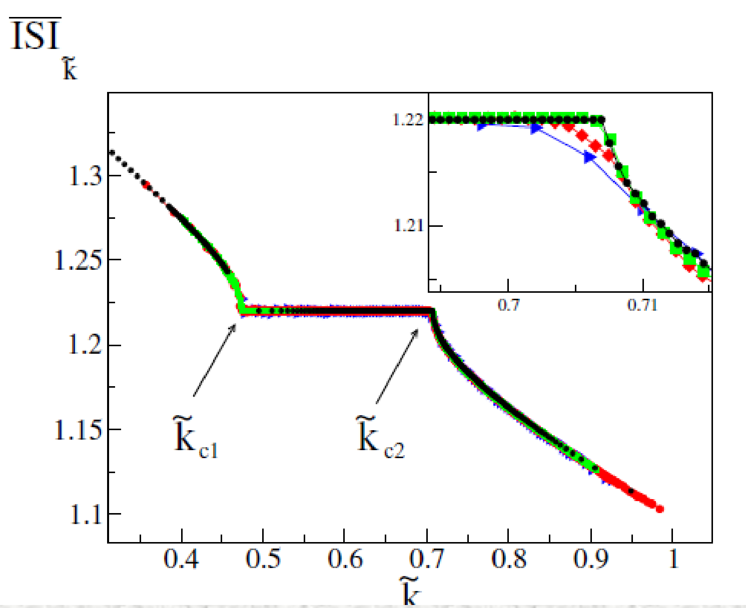
Structure of the graph remains only in the indegree distribution $P(k)$

Numerical simulation of HMF

Very efficient Event driven simulations with important sampling of the indegree distribution $P(k)$



HMF provides a very good description of dynamics on graphs as soon as the connectivity is of order of one hundred

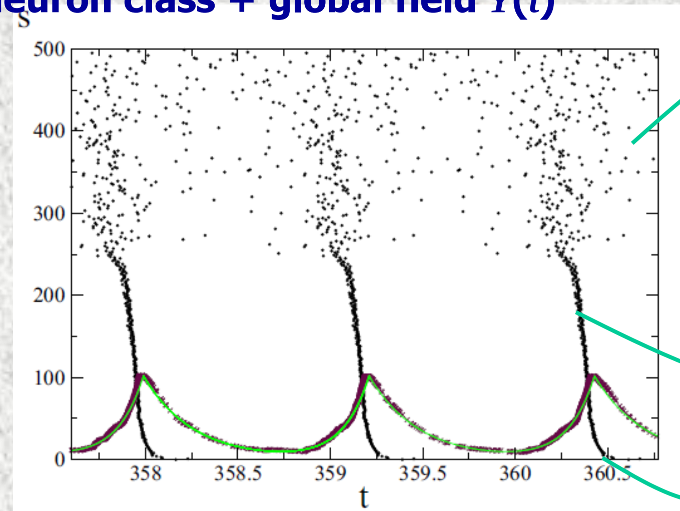


$\overline{ISI}_k$ average time spacing between two firing events of neuron of indegree k

Numerical efficiency:
black line HMF 300 classes
color graphs (500-5000-20000 sites)

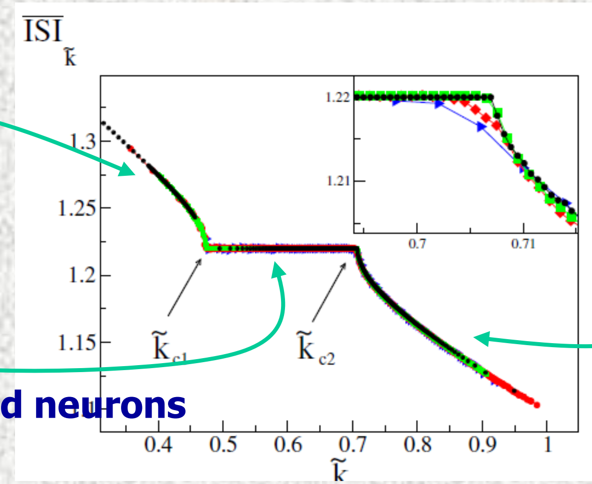
Synchronous, quasi periodic regime

Raster plot: firing event of each neuron class + global field $Y(t)$



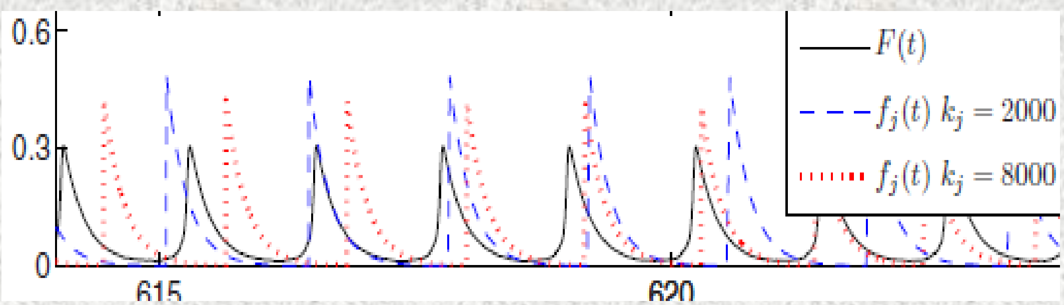
Locked periodic with the same period of the global coupling $Y(t)$

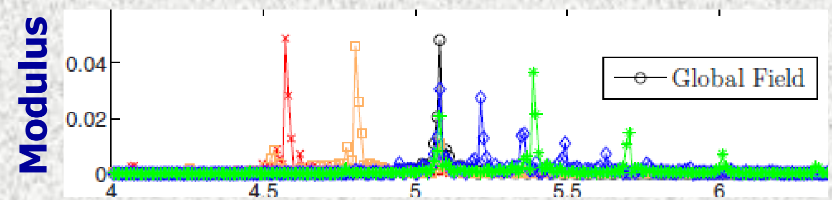
Unlocked neurons



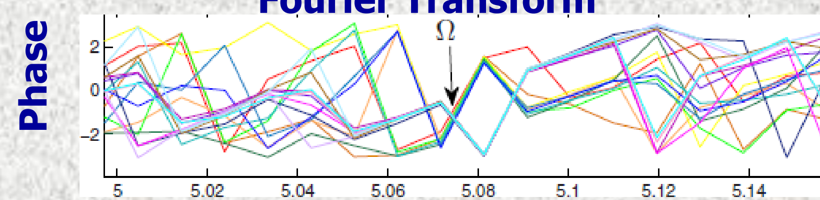
Locked neurons

Global $Y(t)$-local $y_i(t)$ fields





Fourier Transform



Phase locking at the frequency of the global field

Inverse problem

From the knowledge of the global field $Y(t)$ we can reconstruct the in-degree distribution $P(k)$

Solve for each k $\dot{v}_k = a - v_k + gkY(t)$

Obtain the raster plot $S_k(t) = \sum_m \delta(t - t_k(m))$

Solve the equation to find $y_k(t)$

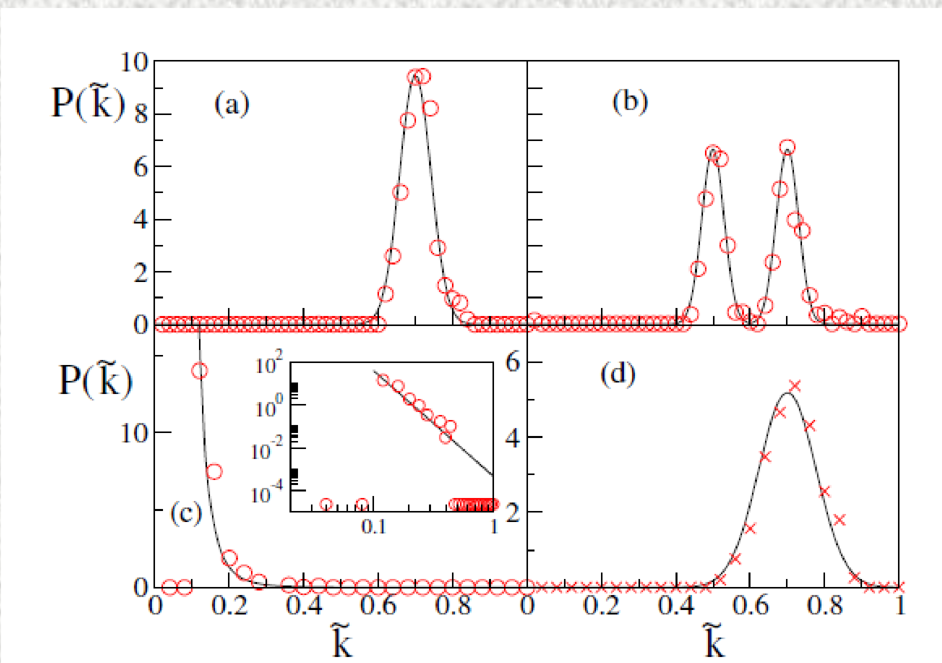
$$\dot{y}_k = -\frac{y_k}{\tau_{in}} + u x_k S_k(t)$$

$$\dot{x}_k = \frac{z_k}{\tau_r} - u x_k S_k(t)$$

$$\dot{z}_k = \frac{y_k}{\tau_{in}} - \frac{z_k}{\tau_r}$$

Invert the Fredholm equation e.g. with a Montecarlo technique

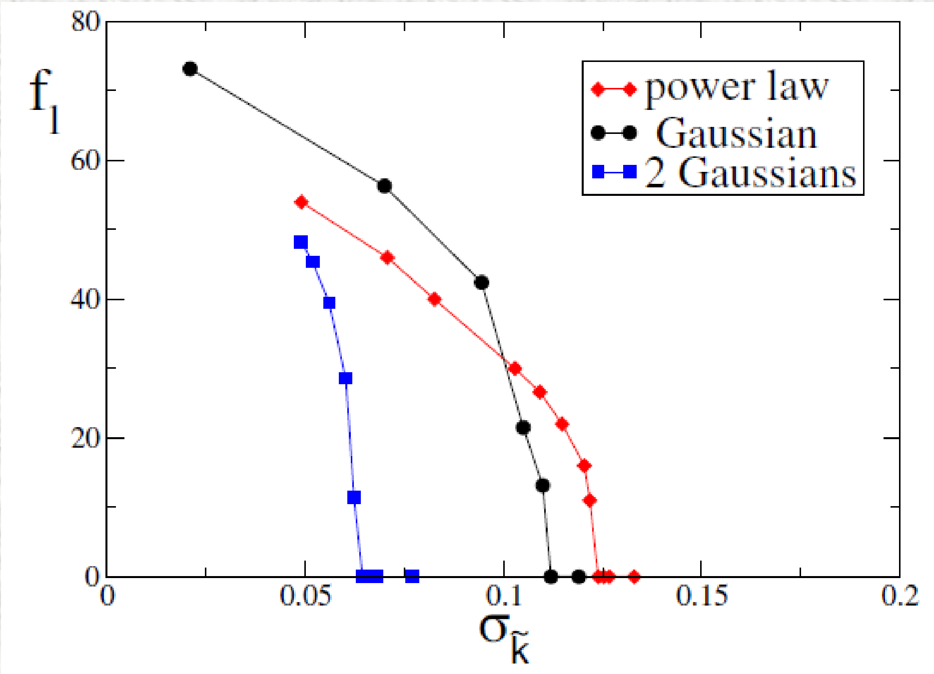
$$Y(t) = \int P(k) y_k(t) dk$$



Reconstruction of $P(k)$ for different type of distribution

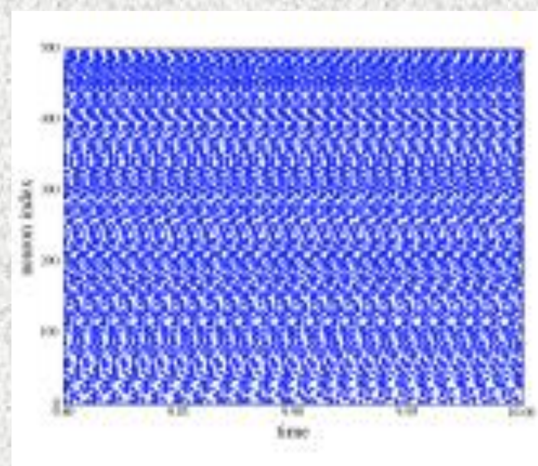
In panel (d) the field $I(t)$ is generated by a the model defined on the graph and not using HMF equation

Phase diagram



The deviation σ of f $P(k)$ for different type of distributions is varied by changing:

-the exponent $P(k) \approx k^{-\alpha}$
-the Gaussian width
- the distance between the Gaussian peaks



Values of the parameter in the simulations

$a = 1.3$

$\tau_{in} = 0.2$

$g = 30$

$\tau_R = 133$

$u = 0.5$

Many parameters: what is the phase diagram of the system?

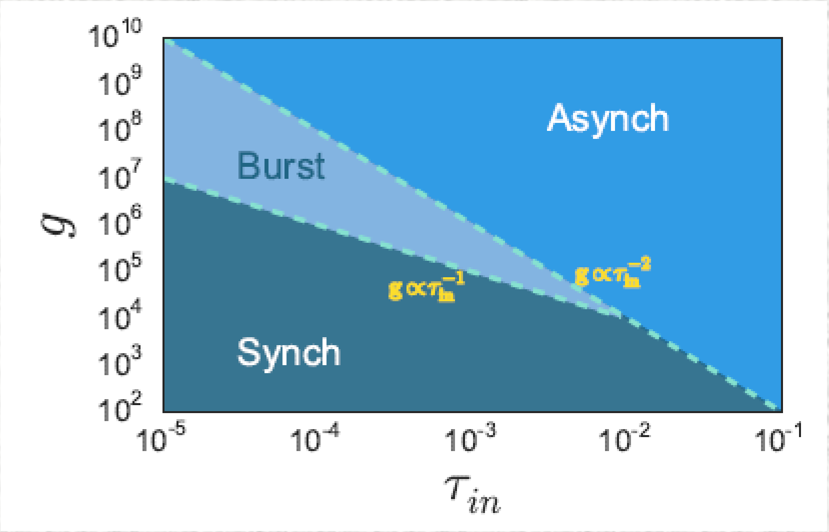
Typical raster plot in the asynchronous regime
Constant field $Y(t)$ independent of t
Periodic neurons with random phases
Inverse problem does not apply

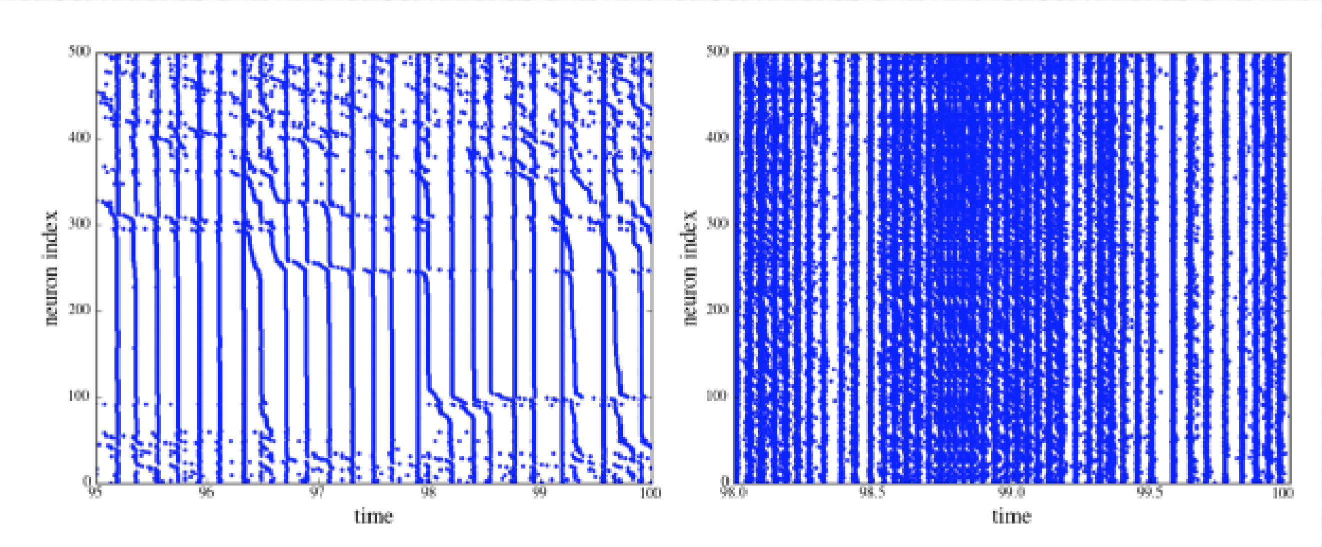
Phase diagram- bursty regime

Many parameters: what is the phase diagram of the system?

Transition from quasi-periodic to asynchronous regimes is typical of system with large τ_{in} i.e. $\tau_{in} \geq 10^{-2}$

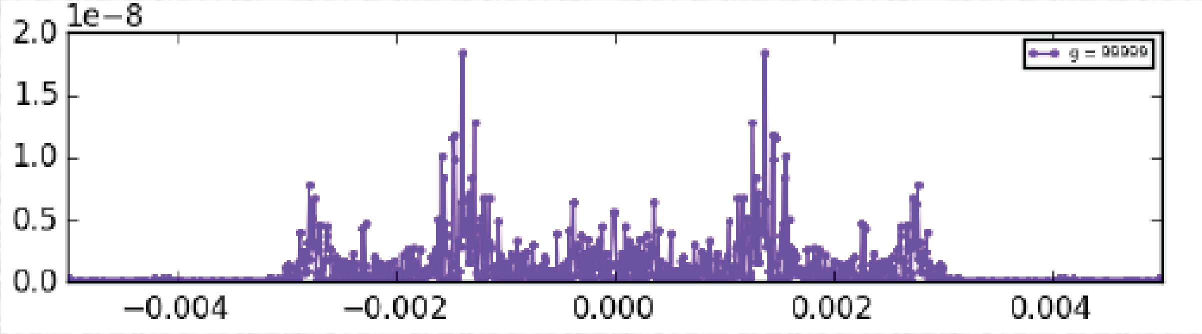
For small $\tau_{in} \leq 10^{-2}$ also a bursty phase is present



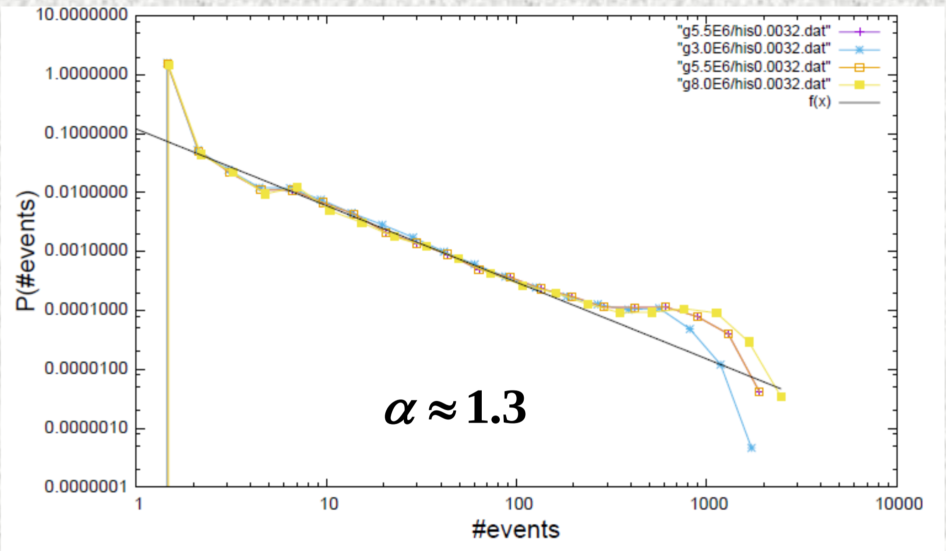


$\tau_{in} = 10^{-3}$, g=5 10^{-5} and g=5 10^{-6}

Characterization of bursty regime



Fourier transform of the global field $Y(t)$



Number of firing in a time interval power-law $\alpha \approx 1.3$, independent of g and of the time interval

Conclusions

HMF for neural network dynamics:

synchronous: explanation of mechanism, effective inverse problem

asynchronous: periodic random phase neurons, no inverse problem

bursty regime: ?????

- Basic mechanism for bursty regime. Noise is not present in the system: disorder, non-linearity, plasticity? – Need of simpler model
- Inverse problem in bursty regimes

Inhibition

Mixing of positive and negative interactions; important point in order to describe real system – published or in publications

- HMF for inhibitory-excitatory networks
- Inverse problem for inhibitory HMF systems
- Synchronizing effect of inhibition

- HMF inverse problem Scirep 4 4336 (2014), PRE 90 022812 (2014)
- quasiperiodic regimes PRE 90 042918 (2014)
- HMF and inverse problem with inhibition arXiv:1507.08183
- Bursty regimes in preparation
- Phase diagram with inhibitio in preparation

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THANKS FOR YOUR ATTENTION