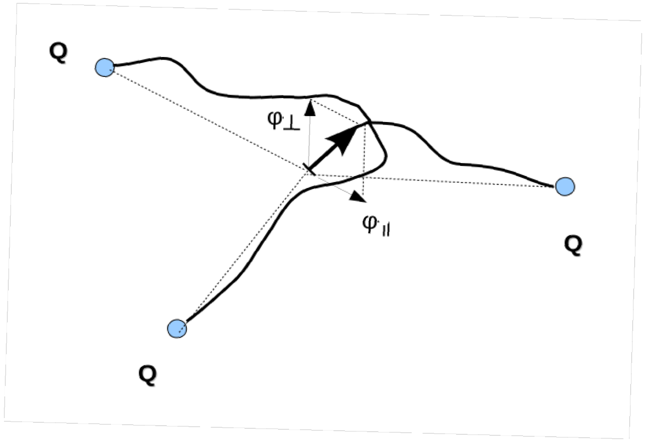
SM&FT 2015, The XVI workshop on Statistical Mechanics and non Perturbative Field Theory, Bari, Italy, December 9-11

Long Range Force in Baryons and QCD strings



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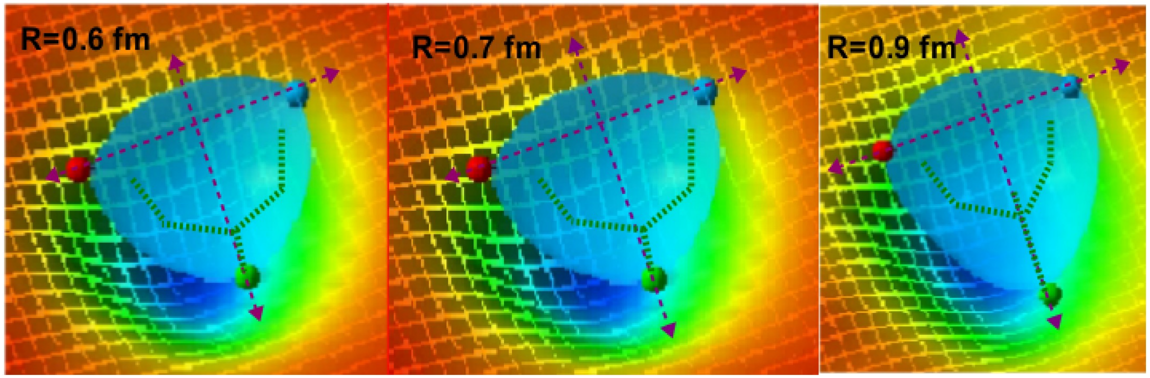


Outline

-- Confining potential of Y-strings.

-- Lattice 3Q potential at finite T.

-- The broadening of the baryonic junction.

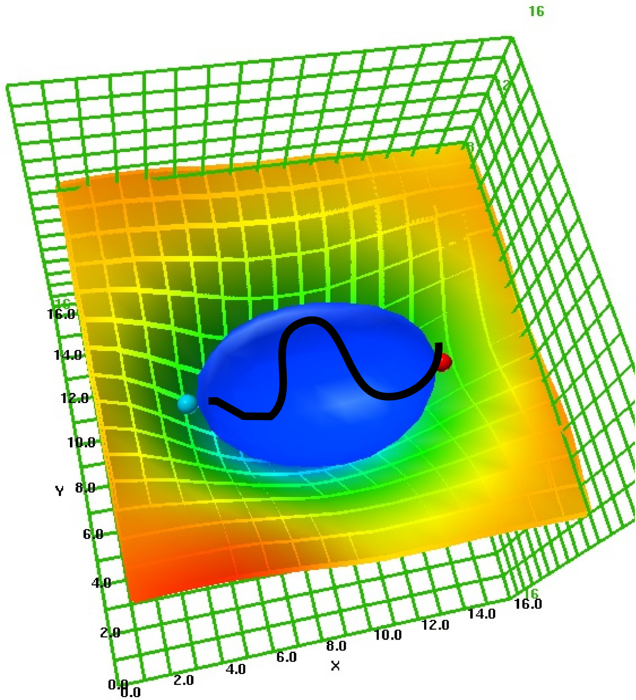
-- Analysis of the energy density.

String picture of confinement

The origin of the linearly rise has been identified to be due to the formation of a thin *string-like* colour-electric flux tube between the quark colour sources.

The quantum fluctuations of the string result in sub-leading correction to the QQ potential known as the Lüscher term.

Simulations of many confining gauge models have tested and verified the existence of this term and the subsequent growth properties of the tube.



M. Luscher and P. Weisz, JHEP0207 049 (2002).

M. Caselle, M. Pepe, and A. Rago, JHEP 10, 5 (2004).

M. Caselle, F. Gliozzi, U. Magnea, and S. Vinti, Nucl.Phys. B460, 397 (1996).

M. Caselle, F. Gliozzi, U. Magnea, S. Vinti, Nucl. Phys. B460, 397 (1996).

F. Gliozzi, M. Pepe, U.J. Wiese, Phys.Rev.Lett. 104, 232001 (2010).

C. Bonati, Phys. Lett. B 703 (2011) 376.

A. Allais and M. Caselle, J. High Energy Phys. 01 (2009)073.

QCD string signatures

A- Zero Temperature T=0

* Luscher- term.

M. Luscher, K. Symanzik, and P. Weisz, Nucl. Phys. B173, 365 (1980).

* Log growth for the flux tube width.

M. Luscher, G. Munster, and P. Weisz, Nucl. Phys. B180,1(1981).

B- Near the Deconfinment point Tc

* Logarithmic term.

P. de Forcrand, G. Schierholz, H. Schneider, and M. Teper, Phys. Lett. 160B, 137 (1985).
M. Gao, Phys. Rev. D 40, 2708 (1989).

* Linear growth at large distances.

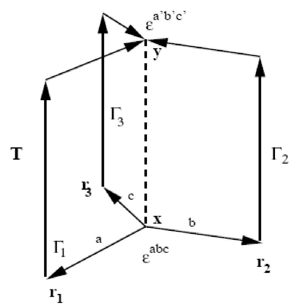
A. Allais and M. Caselle, J. High Energy Phys. 01 (2009)073.

Baryonic Potential and Phenomenology

Perturbative QCD provides good description to the short-distance aspects of the 3Q potential as a two-body (Coulombic) one-gluon-exchange (OGE) interaction potential.

More recently, it has been shown that the breakdown of the two body force in the short-range happens at two loops when the first genuine three-body force appears.

N. Brambilla, J. Ghiglieri, and A. Vairo, Phys. Rev. D81, 054031 (2010), 0911.3541.



Discussions concerning the intermediate and large distance are usually carried out making use of

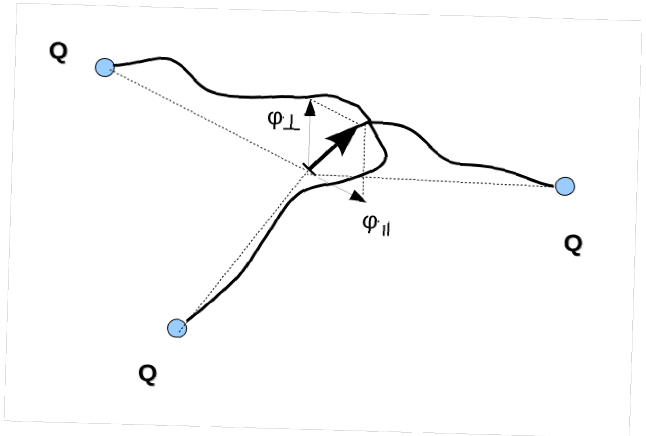
---The strong-coupling expansion arguments.
---Lattice calculations of the three-quark Wilson loop.

C. Alexandrou, P. de Forcrand, and O. Jahn, Nucl. Phys. Proc. Suppl. 119, 667 (2003), hep-lat/0209062.
T. T. Takahashi, H. Suganuma, Y. Nemoto, and H. Matsufuru, Phys. Rev. D 65, 114509 (2002).

on phenomenological basis

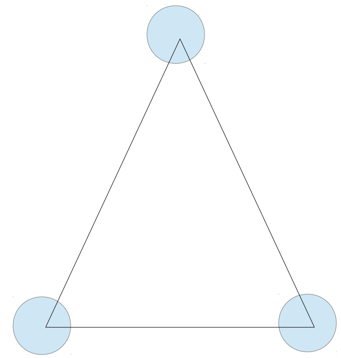
---Baryonic string models.

P. de Forcrand and O. Jahn, Nucl. Phys. A755, 475 (2005).
M. Pfeuffer, G. S. Bali, and M. Panero, Phys. Rev. D 79, 025022 (2009).



Lattice QCD findings regarding the 3-quark potential are settled about a confining potential that admits two possible models depending on the inter-quark separation distances.
The so-called Delta parametrization for small quark

$$V_{qqq}(\vec{r}_1, \vec{r}_2, \vec{r}_3) \approx \frac{1}{2} \sum_{i<j} V_{q\bar{q}}(r_{ij})$$

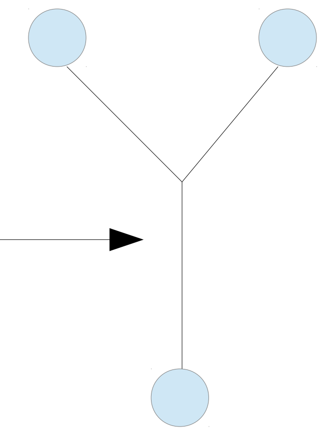


separation distances of **L<0.7 fm.**
and the Y-ansatz for **L>0.7 fm.**

C. Alexandrou, P. de Forcrand, and O. Jahn, Nucl. Phys. Proc. Suppl. 119, 667 (2003), hep-lat/0209062.

T. T. Takahashi, H. Suganuma, Y. Nemoto, and H. Matsufuru, Phys. Rev. D 65, 114509 (2002).

$$V_{3\mathrm{Q}} = -A_{3\mathrm{Q}} \sum_{i<j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} + \sigma_{3\mathrm{Q}} L_{\min} + C_{3\mathrm{Q}},$$



Genuine three-body force

Baryonic String Picture

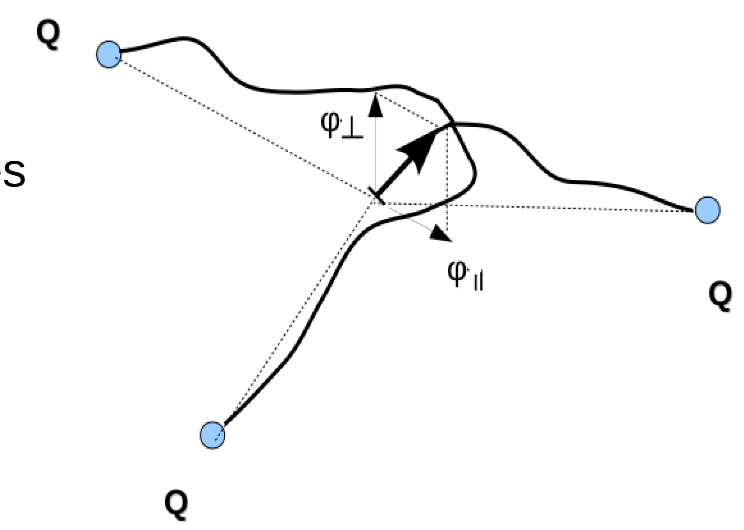
Model assumptions

--The elementary constituents of hadronic matter (quarks) are confined together due to formation of very thin flux tubes.

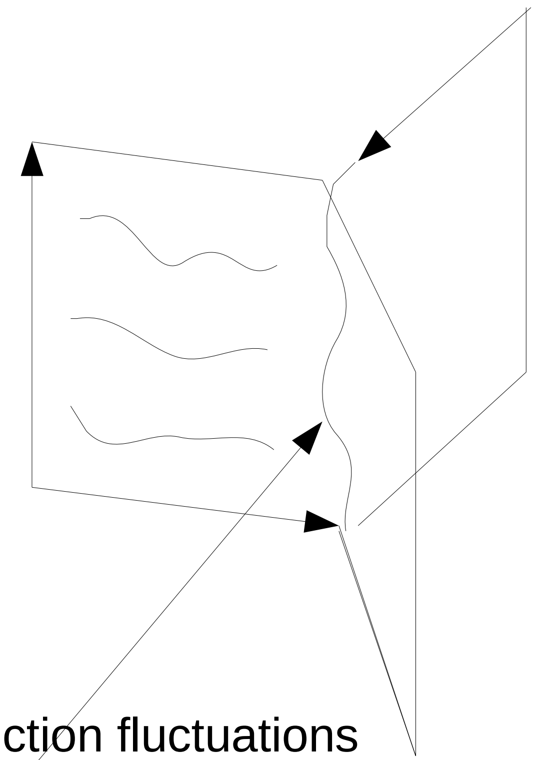
--In the Y-baryonic string model, the quarks are connected by three strings that meet at a junction.

--The classical configuration is the one that minimizes the area of the string world sheets.

--The position of the junction is determined by the requirement of minimal total string length.



- The most natural choice for the string action S is the Nambu–Goto action which is proportional to the surface area of the world sheet.



$$S[X] = \sigma \int d\zeta_1 \int d\zeta_2 \sqrt{g},$$

$g_{\alpha\beta}$ is the two dimensional induced metric on the blade world sheet embedded in the background R4.

$$g_{\alpha\beta} = \frac{\partial X}{\partial \zeta_\alpha} \cdot \frac{\partial X}{\partial \zeta_\beta}, \quad (\alpha, \beta = 1, 2),$$

$$g = \det(g_{\alpha\beta}).$$

The string partition function is a Gaussian integral over the junction fluctuations

$$Z = e^{-(\sigma L_Y + m) L_T} \int D\varphi \; \exp\left(-\frac{m}{2} \int dt \, |\dot{\varphi}|^2\right) \prod_{i=1}^{3} Z_i(\varphi),$$

$$Z_i(\varphi) = \int_\varphi D\xi_i \; \exp\left(-\frac{\sigma}{2} \int |\partial \xi_i|^2\right)$$

The partition function for the fluctuations of a given blade that is bounded by the junction worldline φ(t)

The 3Q potential after the calculations of the Laplacian of the determinant reads

$$\mathrm{V}_{qqq}(L_1, L_2, L_3) = \sigma \sum_i L_i + V_{\|} + V_{\perp} + O(L_i^{-2}) \,,$$

with

$$V_{\|} = -\frac{\pi}{24} \sum_i \frac{1}{L_i} + \int_0^{\infty} \frac{dw}{2\pi} \ln \left[\frac{1}{3} \sum_{i<j} \coth(wL_i) \coth(wL_j) \right] \,,$$

$$V_{\perp} = -\frac{\pi}{24} \sum_i \frac{1}{L_i} + \int_0^{\infty} \frac{dw}{2\pi} \ln \left[\frac{1}{3} \sum_i \coth(wL_i) \right] \,. \qquad ($$

The first test of the baryonic string model predictions with LGT at zero temperature has been reported by Deforcrand and Jahn considering 3 Potts model for a three quark potential.

O Jahn and Ph. Deforcrand, hep-lat/0209062, Ph. De Forcrand and O. Jahn, N phys A 755(2005)

The numerical measurements of the 3-state Potts gauge model are consistent with the predicted Lüscher-like corrections and the formation of a Y-system of three flux tubes.

Now let us consider the finite temperature effects....

The 3Q potential after the calculations of the Laplacian of the determinant reads

$$\mathrm{V}_{qqq}(L_1, L_2, L_3) = \sigma \sum_i L_i + V_{\parallel} + V_{\perp} + O(L_i^{-2}) \, ,$$

with

$$V_{\parallel} = -\frac{\pi}{24} \sum_i \frac{1}{L_i} + \int_0^{\infty} \frac{dw}{2\pi} \ln \left[\frac{1}{3} \sum_{i<j} \coth(wL_i) \coth(wL_j) \right] \, ,$$

$$V_{\perp} = -\frac{\pi}{24} \sum_i \frac{1}{L_i} + \int_0^{\infty} \frac{dw}{2\pi} \ln \left[\frac{1}{3} \sum_i \coth(wL_i) \right] \, . \qquad ($$

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The numerical measurements of the 3-state Potts gauge model are consistent with the predicted Lüscher-like corrections and the formation of a Y-system of three flux tubes.

Jahn and de Forcrand calculated the Casimir energy for the baryonic configuration

$$\begin{aligned} V_{3Q}(L_i) =& \sigma L_Y + V_{\parallel} + 2V_{\perp} + O(L_i^{-2}) \, , \\ V_{\parallel}(L_i) =& -\sum_i \frac{1}{L_T}\eta(iL_T/L_i) + \\ & \sum_{w=0} \frac{1}{L_T} \ln\left[\frac{1}{3}\sum_{i<j} \coth(wL_i)\coth(wL_j)\right] \, , \end{aligned}$$

for the in-plane component and

$$V_{\perp}(L_i) = -\sum_i \frac{1}{L_T}\eta\left(\frac{iL_T}{L_i}\right) + \sum_{w=0} \frac{1}{L_T} \ln\left[\frac{1}{3}\sum_i \coth(wL_i)\right] \, , \tag{1}$$

for the perpendicular fluctuations.

A consistency check in the mesonic limit

$$V_{\perp} = -\frac{1}{L_T}\eta\left(\frac{2iL_T}{L_1}\right) - \frac{1}{L_T}\eta\left(\frac{2iL_T}{L_2}\right) + \sum_{w=0} \frac{1}{L_T} \ln\left[\frac{1}{2}\coth(wL_1) + \coth(wL_2)\right]. \quad (2)$$

The quark anti-quark potential would read

$$V_{Q\bar{Q}} = \sigma(L_1 + L_2) - \frac{1}{L_T} \ln\left[\eta\left(\frac{2iL_T}{L_1 + L_2}\right)\right], \quad (3)$$

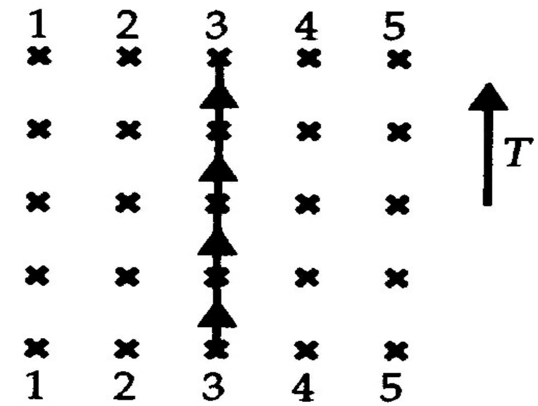
Expressing the sum in equations (1) in terms of Dedekind η functions, the potential in the $3Q$ channel would read

$$V_{3Q} = \sigma L_{\mathrm{Y}} - \frac{\gamma}{L_T} \ln\left[\eta\left(\frac{iL_T}{2L_{\mathrm{Y}}}\right)\right], \quad (4)$$

where γ is a geometrical factor that can be evaluated numerical by solving equations (1).

Baryonic operators

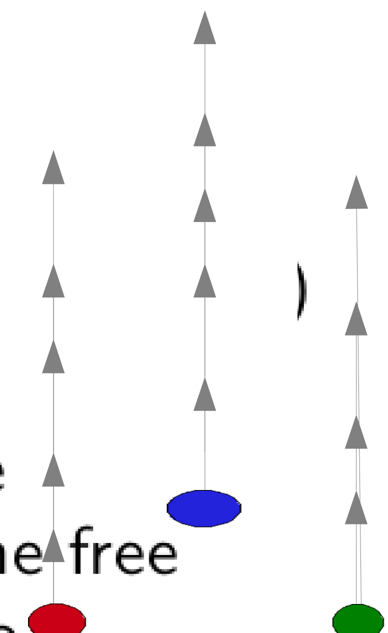
A transfer matrix interpretation to the Polyakov loops correlator allow to obtain the $3Q$ static potential V_{3Q} with the center symmetry preserving operator



$$
\begin{aligned}
\langle \mathcal{P}_{3Q} \rangle &= \langle \mathcal{P}(\vec{r}_1)\mathcal{P}(\vec{r}_2)\mathcal{P}(\vec{r}_3) \rangle, \\
&= \exp(-V_{3Q}(\vec{r}_1, \vec{r}_2, \vec{r}_3; T)),
\end{aligned}
$$

where the Polyakov loop is given by

$$
P(\vec{r}) = \frac{1}{3}\mathrm{Tr}\left[\prod_{n_t=1}^{N_t} U_{\mu=4}(\vec{r}, n_t)\right] .
$$



In the context of the Polyakov loop method we have contributions of temperature-dependent effects to the free energy of a system of three static charges coupled to a heatbath.

Simulation setup

The SU(3) gluonic gauge configurations has been generated employing a pseudo-heatbath algorithm “Fabricius,Kennedy” updating the corresponding three SU(2) subgroup elements.

The gauge configurations were generated using the standard Wilson gauge action.

The two lattices employed in this investigations are being of a typical spatial size of N_s=36,N_t=10 ,N_t=8 --->Temperatures T = 0.8, 0.9 Tc,

Each update step consists of one heat bath and 5 micro-canonical reflections. We choose to perform our analysis with lattices as fine as a = 0.1 fm by adopting a coupling of value Beta 6.00.

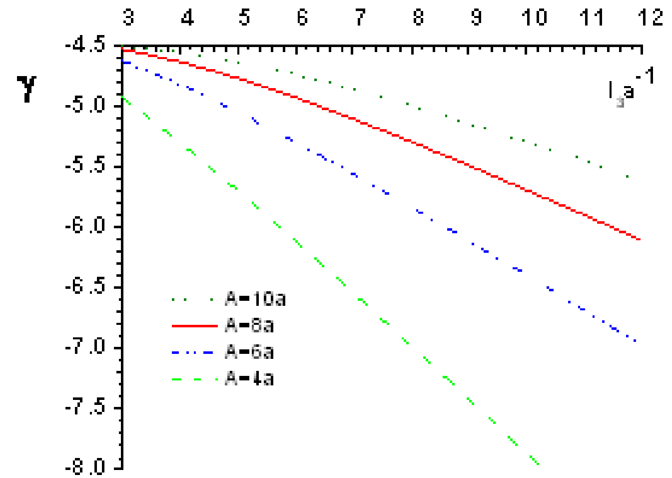
We perform a set of measurements n=20 separated by 70 sweeps of updates. Each set of measurments is taken following a 2000 of updating sweeps. The total measurements taken on 500 bins.

In this investigation, we have taken 10,000 measurement at each temperature.

Potential of a Y-string Model at finite temperature

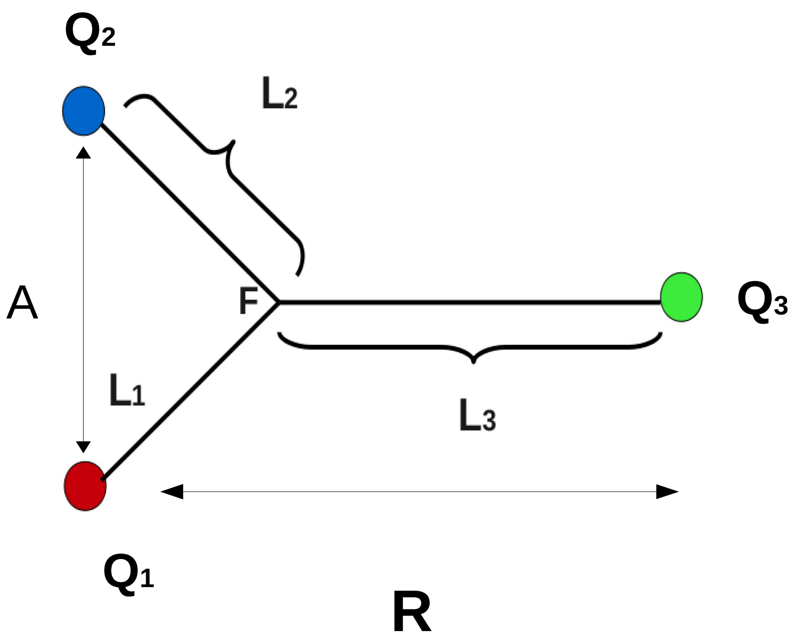
It can be shown that the 3Q potential is given by a sum of term proportional to minimal length of the Y-string in addition to Dedkind eta function accounting for the Gaussian string fluctuations.

$$V(L_Y, T) = \sigma L_Y + \frac{\gamma}{T} Log[\eta(iT/2L_Y)]$$



This formula should be distinguished from the the bare Y-ansatz, which does not include the string corrections.

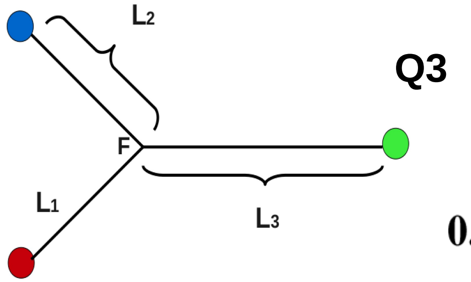
$$V_{3Q} = -A_{3Q} \sum \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} + \sigma_{3Q} L_{\min} + C_{3Q},$$



The string model's geometrical factor calculated for four different isosceles bases versus the third string length.

O. Jahn and P. D. Forcrand, Nucl. Phys. B, Proc. Suppl. 129–130, 700 (2004).

In general, the fits show strong dependency on the fit range with the inclusion of the points at small Q3 sources separations.



Good fits at large distances and same string Tension as the meson.

The points at small isosceles height R = 0.5 fm and R = 0.6 fm deviates from the Y-string curve.

Deviations at relatively small heights

-- self-interactions of the two strings connecting the two quarks at the base.

--The interaction of the node with the third quark Q3 .

That is, higher order effects are expected to be more pronounced at higher temperatures.

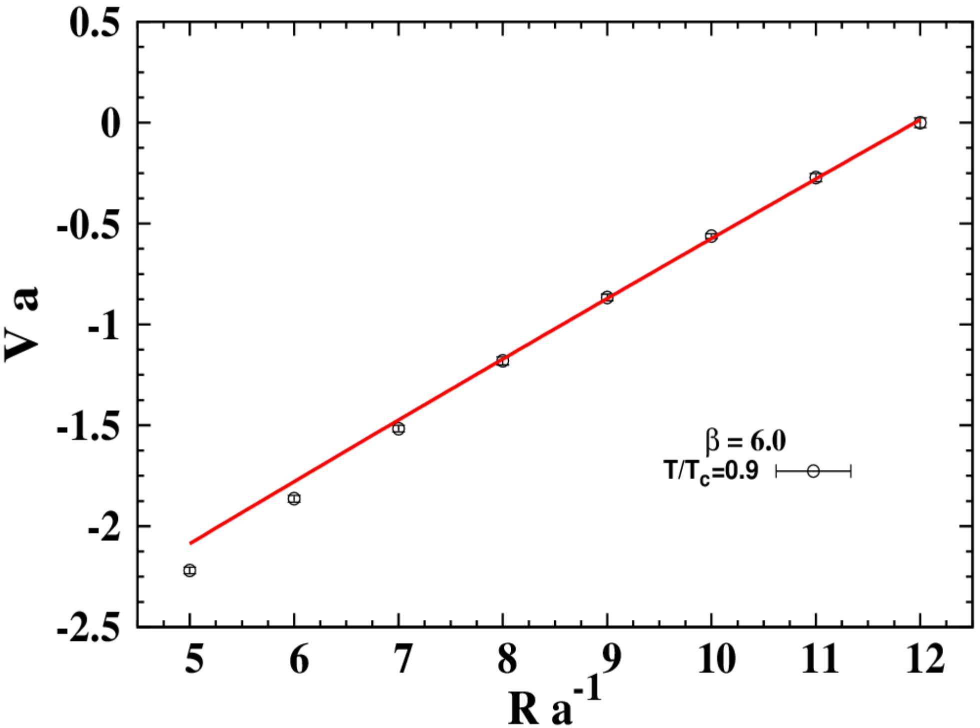
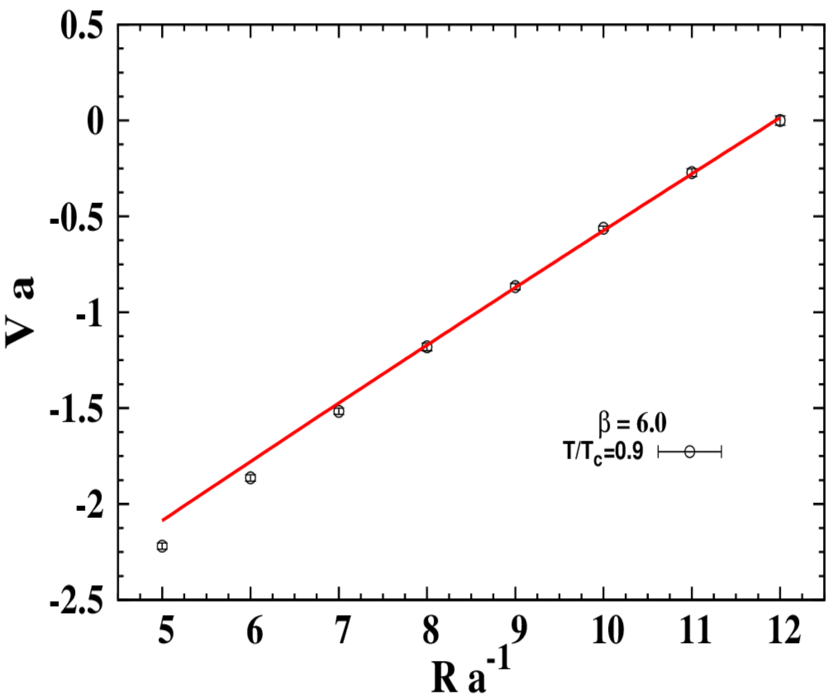


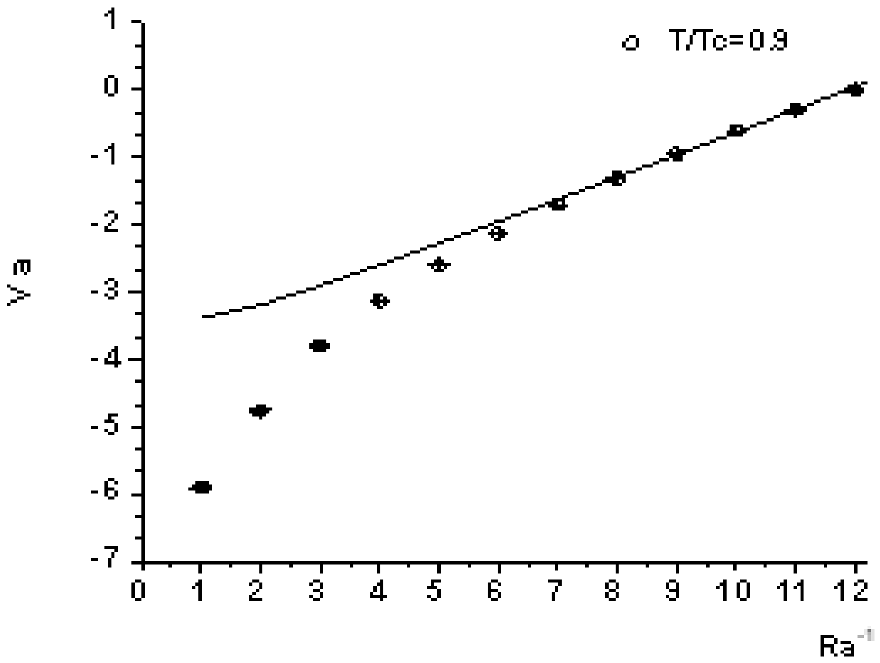
Figure above depicts the potential data corresponding to an isosceles A = 0.6 fm and the string model fit.

Returned fits of the 3Q potential to Y-string Model

Triangle 3Q base length A-0.6 fm



Mesonic string potential



For the meson, the best fits are returned if only the last four points are included, i.e., from quark separation distances R = 0.8 fm to R = 1.2 fm.

The string length in the case of the Y-string linking any two quark sources is greater than the mesonic string of a corresponding QQ pair.

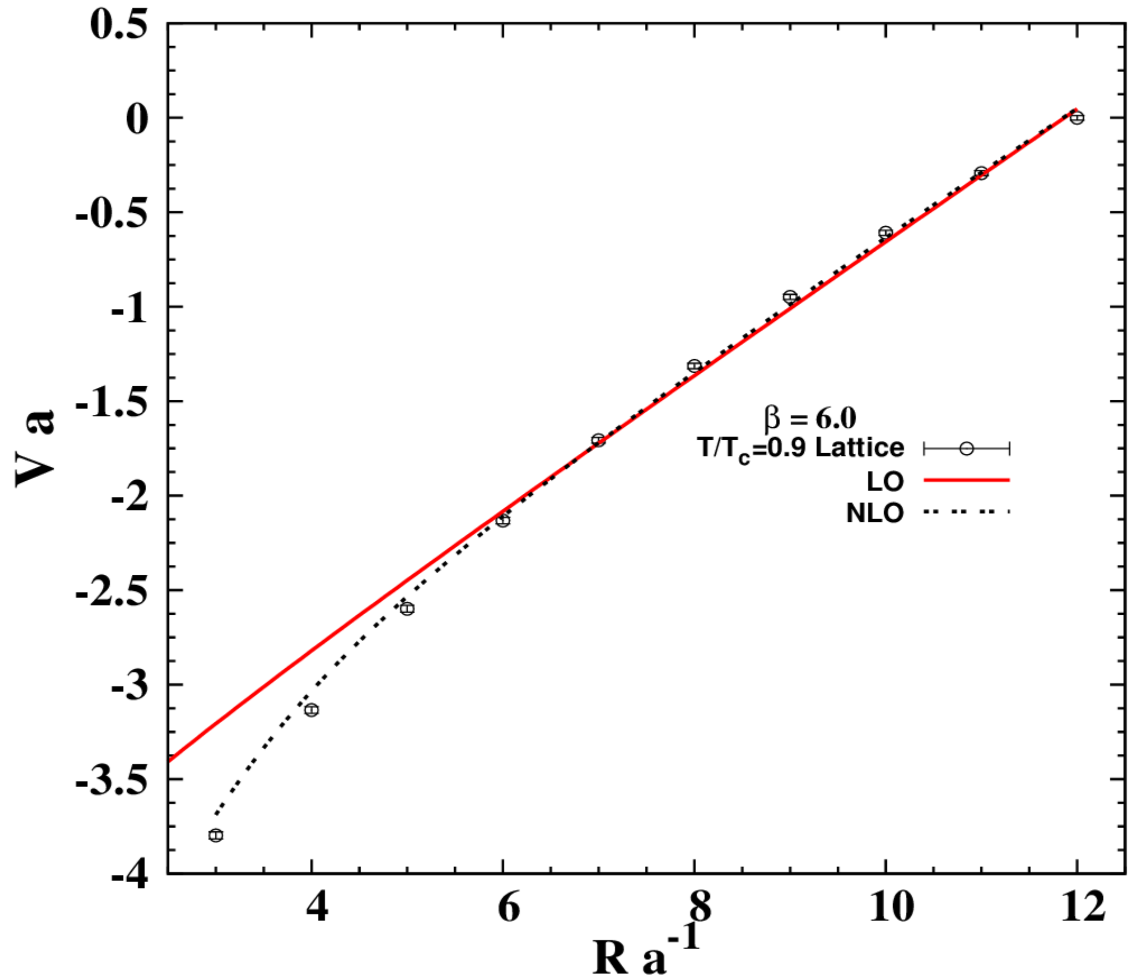
The value returned for the string tension σ = 0.032 is the one used as input for the string tension in formula Eq. 5 for the baryonic Y-string potential.

The inclusion of string-self interactions in the meson improves the match with the lattice data.

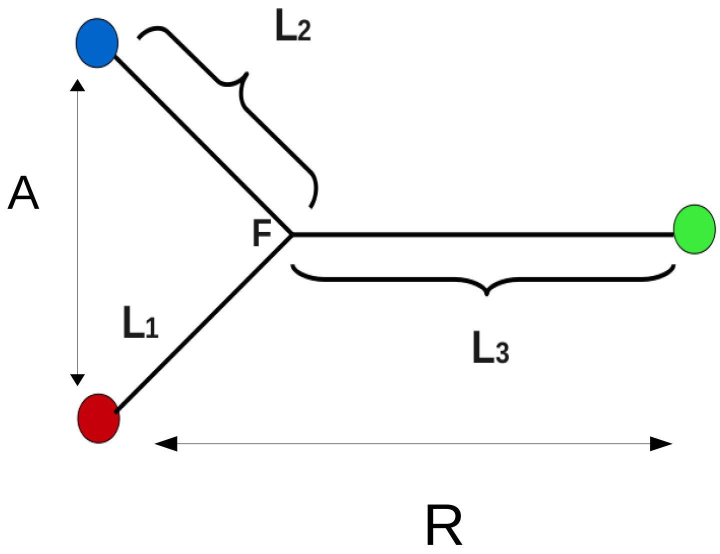
M. Caselle, F. Gliozzi, U. Magnea, S. Vinti, Nucl. Phys. B460, 397 (1996).

And also the string's width profile

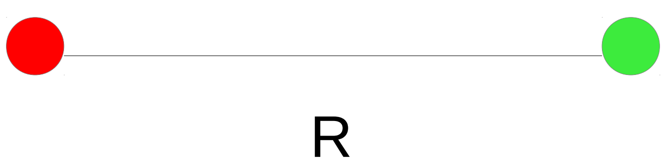
F. Gliozzi, M. Pepe, U.J. Wiese, Phys.Rev.Lett. 104, 232001 (2010)

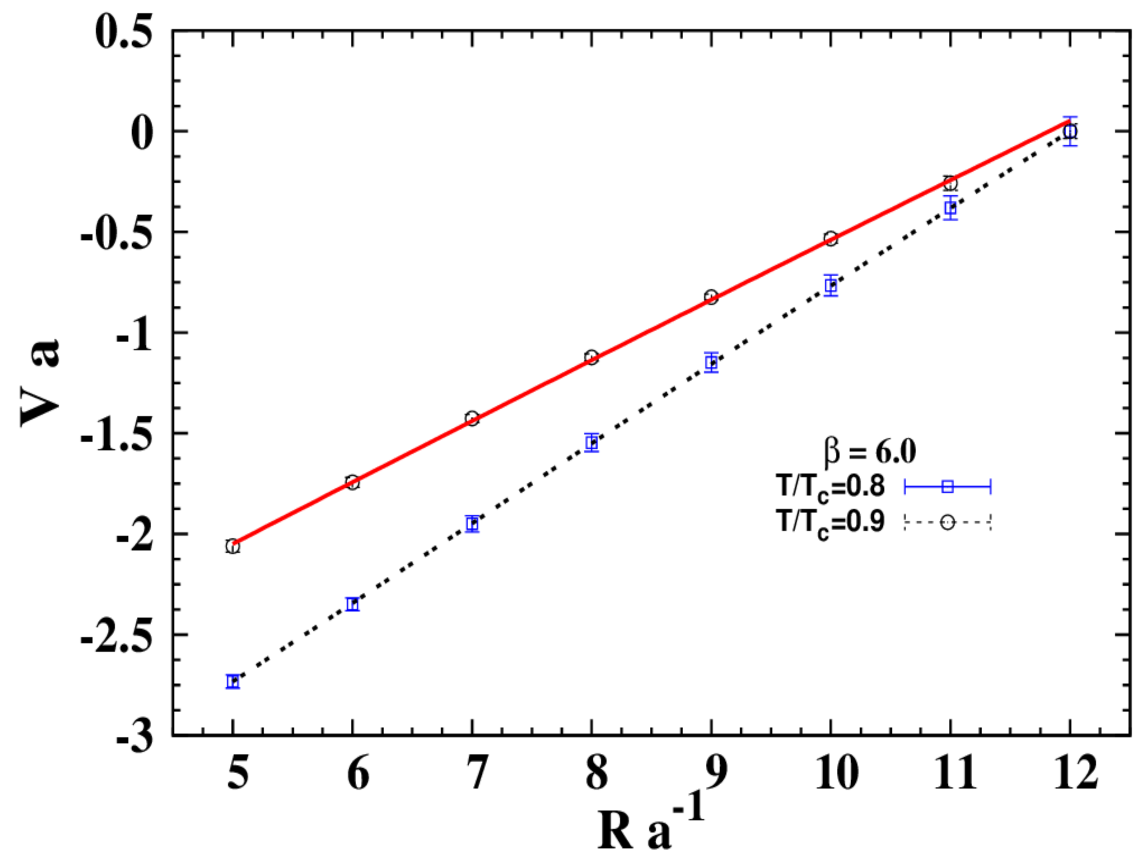


The LO and NLO string potential for mesonic configuration.



Triangle base length A=0.8 fm





Increasing the width of the base to A = 0.8 fm the values of χ2 reduce even for small isosceles heights of R = 0.5 fm and R = 0.6 fm.

A wider base of the isosceles triangle would increase the length of the string linking any two quarks L1 + L3 . This approaches the free mesonic string length which matches the lattice data.

One can also add to this the observation that the self-interactions of the adjacent strings in the Y-string configuration are expected to be negligible at wider base.

To appreciate the effects of the junction fluctuations we draw comparison between
Δ-ansatz given by the form

$$V_{3\mathrm{Q}}(\vec{r}_1, \vec{r}_2, \vec{r}_3) = -\frac{1}{2} A_{Q\bar{Q}} \sum_{i<j} \frac{1}{|\vec{r}_i - \vec{r}_j|} + \frac{1}{2} \sigma_{Q\bar{Q}} \sum_{i<j} |\vec{r}_i - \vec{r}_j|,$$

the constant $A_{Q\bar{Q}}$ determines the strength of the OGE Coulombic term derived from perturbative QCD.
Y-string Model

$$V_{3Q} = \sigma L_{\mathrm{Y}} - \frac{\gamma}{L_T} \ln \left[\eta \left(\frac{i L_T}{2 L_{\mathrm{Y}}} \right) \right].$$

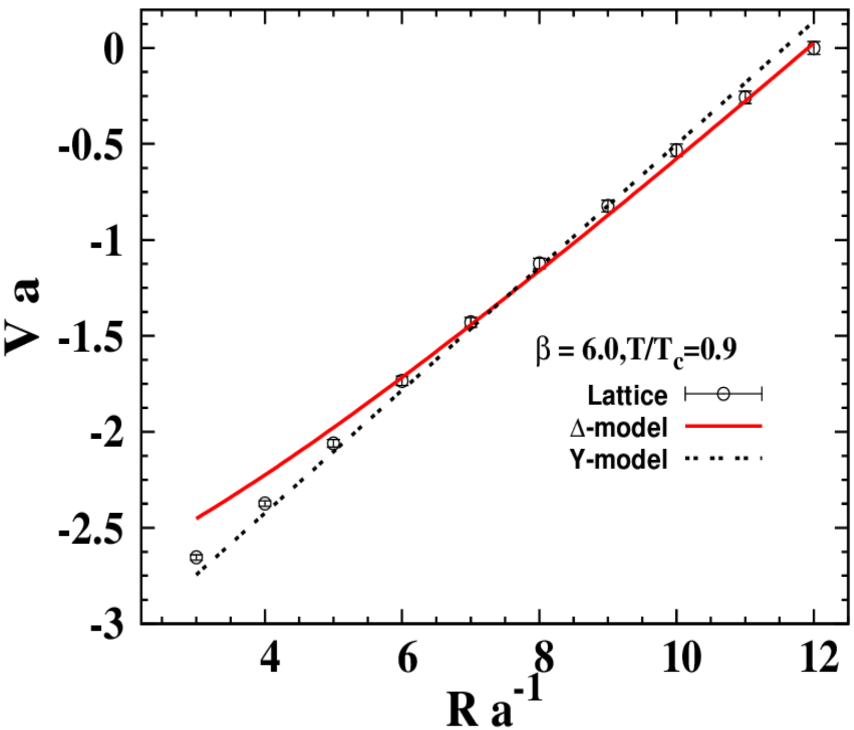
We consider also the fits to a Y-ansatz which includes only the linearly confining term corresponding to the classical configuration of the string $L_{Y_{min}}$

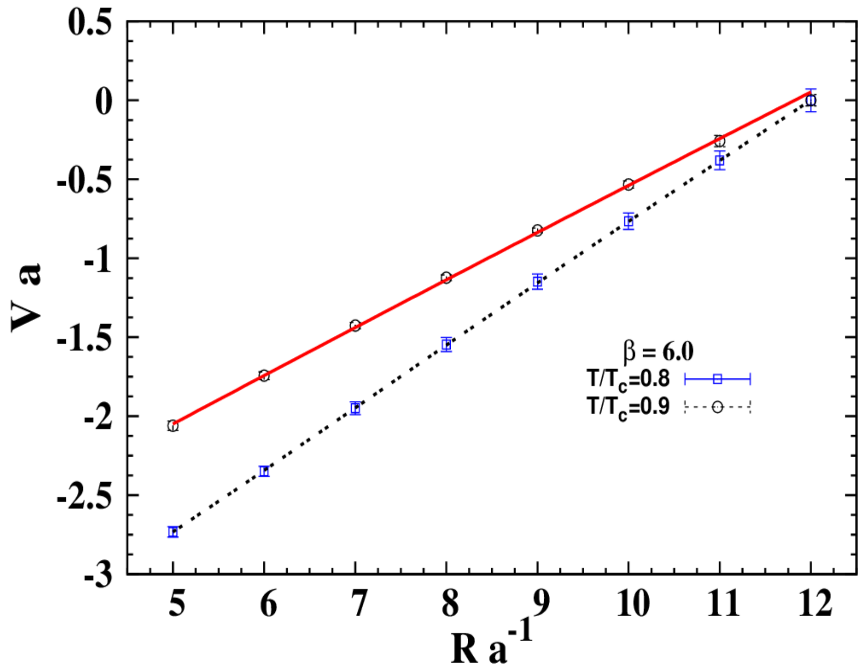
$$V_{3\mathrm{Q}}(\vec{r}_1, \vec{r}_2, \vec{r}_3) = -\frac{1}{2} A_{Q\bar{Q}} \sum_{i<j} \frac{1}{|\vec{r}_i - \vec{r}_j|} + \sigma_{\mathrm{Q}\bar{\mathrm{Q}}} L_{\mathrm{Y}}.$$

Y-string model versus Delta model and bare Y-model

The returned $\chi^2_{\text{dof}}(x)$ for fits of the lattice data to $3Q$ isosceles of width $A = 0.8$ fm at $T/T_c = 0.9$. The fits compare the Δ-ansatz, the Y-string model and a bare Y-ansatz.

Fit Range	R=5-8	R=8-12	R=5-12
Fit Parameters	χ^2	χ^2	χ^2
Δ-ansatz	27.43	9.63	40.5
Y-string model	1.25	6.85	8.09
Y-bare ansatz	2.36	40.84	89.71

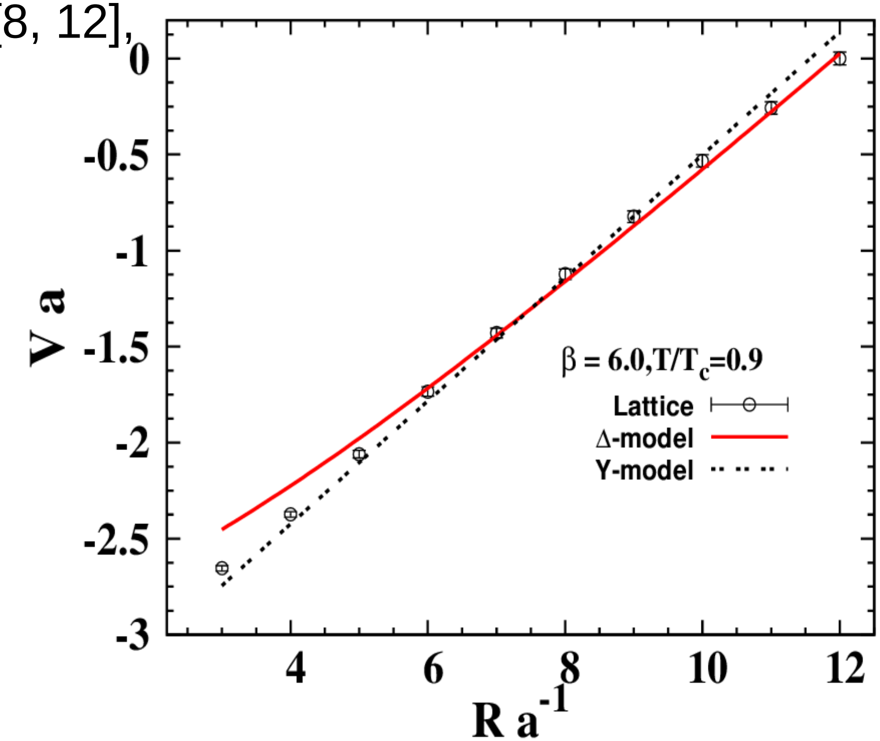




The Y-string model provides the best description of the lattice numerical data as indicated
For all considered fit range.

Fit Range	R=5-8	R=8-12	R=5-12
Fit Parameters	χ^2	χ^2	χ^2
Δ-ansatz	27.43	9.63	40.5
Y-string model	1.25	6.85	8.09
Y-bare ansatz	2.36	40.84	89.71

For points corresponding to larger heights R ∈ [8, 12], we interestingly observe that the bare Y-model, returns high values of χ2.



On the other hand, the χ2 corresponding to the Δ-parametrization which is relatively smaller and closer to that of the Y-string model.

At large distances and temperatures the Y-string assumes a higher energy content and broadening profile .

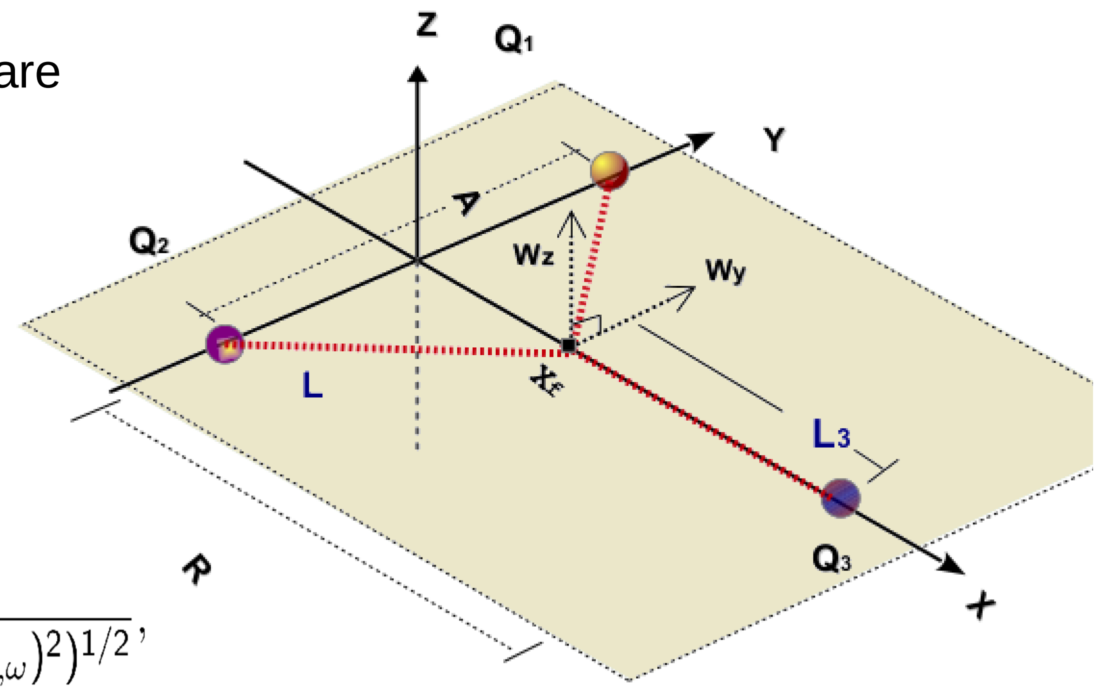
Summary of the results for the 3Q potential

- We discussed the Y-string model at finite temperature.

- We found that the fit to the Y-law reproduces the quark anti-quark string tension provided a Log term (Dedekind η function) accounting for string fluctuation is included in the fit ansatz.

- Comparison with the fit with the Δ-ansatz and even bare Y-ansatz, in some limits, shows that the Y-string model provides the best fits for the confining part of the 3Q potential.

M. Pfeuffer, G. S. Bali, and M. Panero, Phys. Rev. D 79, 025022 (2009).



Extended the calculations to the mean-square width of the Y-junction

For the perpendicular fluctuations

$$\langle \phi_z^2 \rangle = \frac{2}{L_T} \sum_{\omega>0} \frac{1}{k\omega^2 + \sigma\omega \sum_i \coth(\omega L_i)\psi(\omega, L_i)}.$$

and parallel fluctuations

$$\langle \phi_x^2 \rangle = \frac{2}{L_T} \sum_{\omega>0} \frac{1}{Q_{x,\omega} + Q_{y,\omega} - (Q_{xy,\omega}^2 + (Q_{x,\omega} - Q_{y,\omega})^2)^{1/2}},$$

$$\langle \phi_y^2 \rangle = \frac{2}{L_T} \sum_{\omega>0} \frac{1}{Q_{x,\omega} + Q_{y,\omega} + (Q_{xy,\omega}^2 + (Q_{x,\omega} - Q_{y,\omega})^2)^{1/2}}.$$

to the 3 quark plane would read as a sum over Fourier modes as above.

$$Q_x = \left(k\omega^2 + \sigma\omega \sum_i \coth(\omega L_i)\psi(\omega, L_i) \right) +$$

$$\left(\frac{\sigma}{2}\omega + \frac{\omega^3}{12\pi} \right) \left[\sum_i \eta_{i,x}^2 \coth(\omega L_i)\psi(\omega, L_i) \right] ,$$

The string fluctuations are smoothed and decoupled to make it more convenient to compare with lattice data at finite temperature.

Lattice Measurements: Field correlators

The form of the correlation function for the action density,

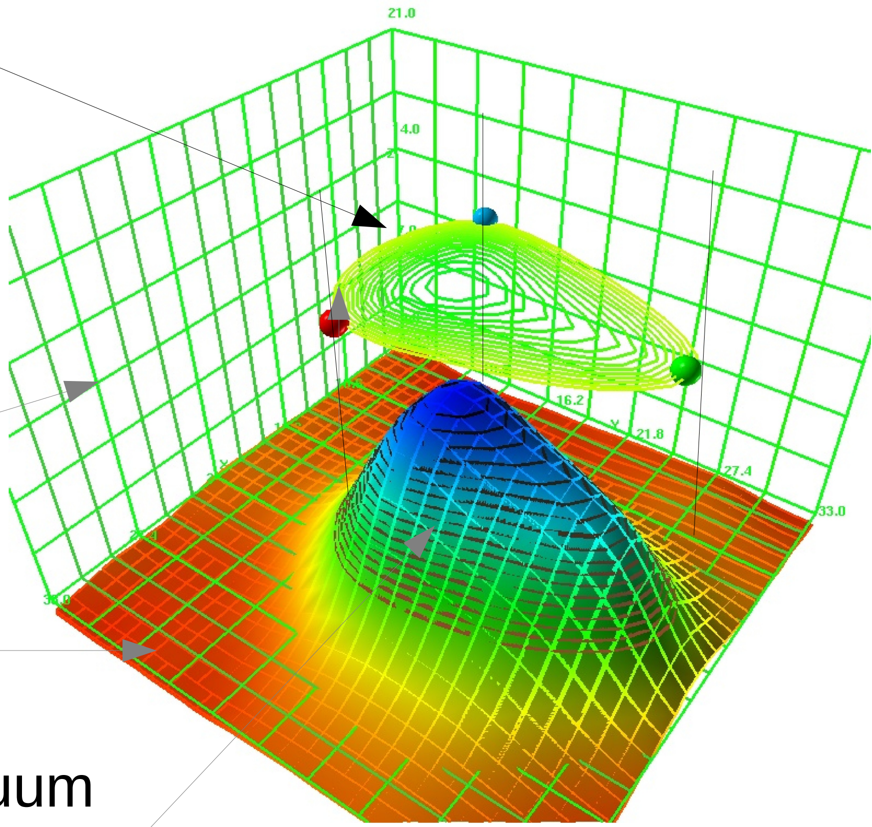
$$C(\vec{y},\vec{r}_1,\vec{r}_2,\vec{r}_3) = \frac{\langle \mathcal{P}_{3Q}(\vec{r}_1,\vec{r}_2,\vec{r}_3)\, S(y,t)\rangle}{\langle \mathcal{P}_{3Q}(\vec{r}_1,\vec{r}_2,\vec{r}_3)\rangle\, \langle S(y,t)\rangle}$$

where $\mathcal{P}_{3Q}(\vec{r}_1,\vec{r}_2,\vec{r}_3) = P(\vec{r}_1)\, P(\vec{r}_2)\, P(\vec{r}_3)$.

C is a scalar field in three dim.

C = 1

C < 1



- C < 1, signaling the expulsion of the vacuum fluctuation from the interior of the Baryonic system.

$\vec{r}_i$

Noise Reduction

A- Link averaging

Link Integration (meson)-or UV filtering (Baryon)

(Appropriate levels of UV filtering --->The physics is intact)

B- Gauge-independent approach.

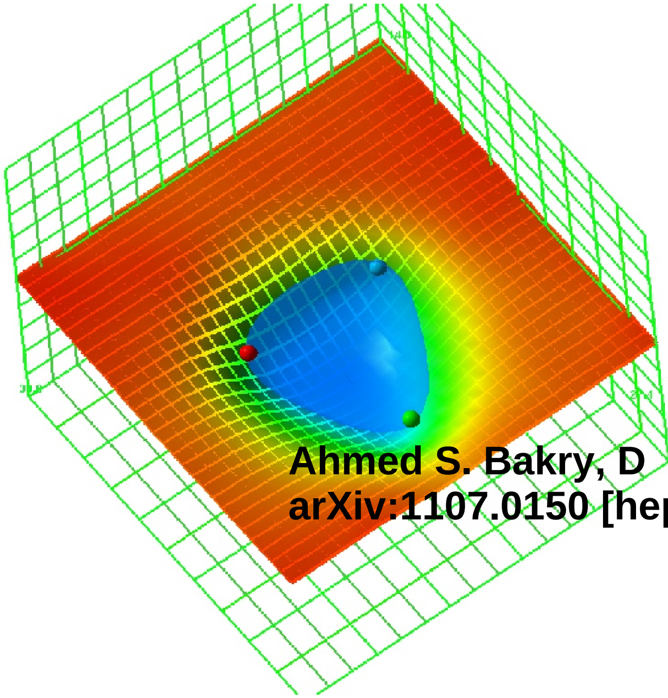
(Lattice symmetries are exploited to reduce the statistical error)

C- Monte-Carlo updates.

(Evaluating the correlation functions on large number of configs with independent bins)

Action density analysis

Unexpected filled Δ-shaped flux arrangement surprisingly persists to large inter-quark separations.



Ahmed S. Bakry, D Leinweber, Tony Williams, Phys Rev D91, 094512(2015)
arXiv:1107.0150 [hep-lat]

- This action density profile is compared to the predictions of the baryonic string model
- Qualitatively

1-Maximum action density is inside triangle.

2- The profile is composed of two-Gaussian

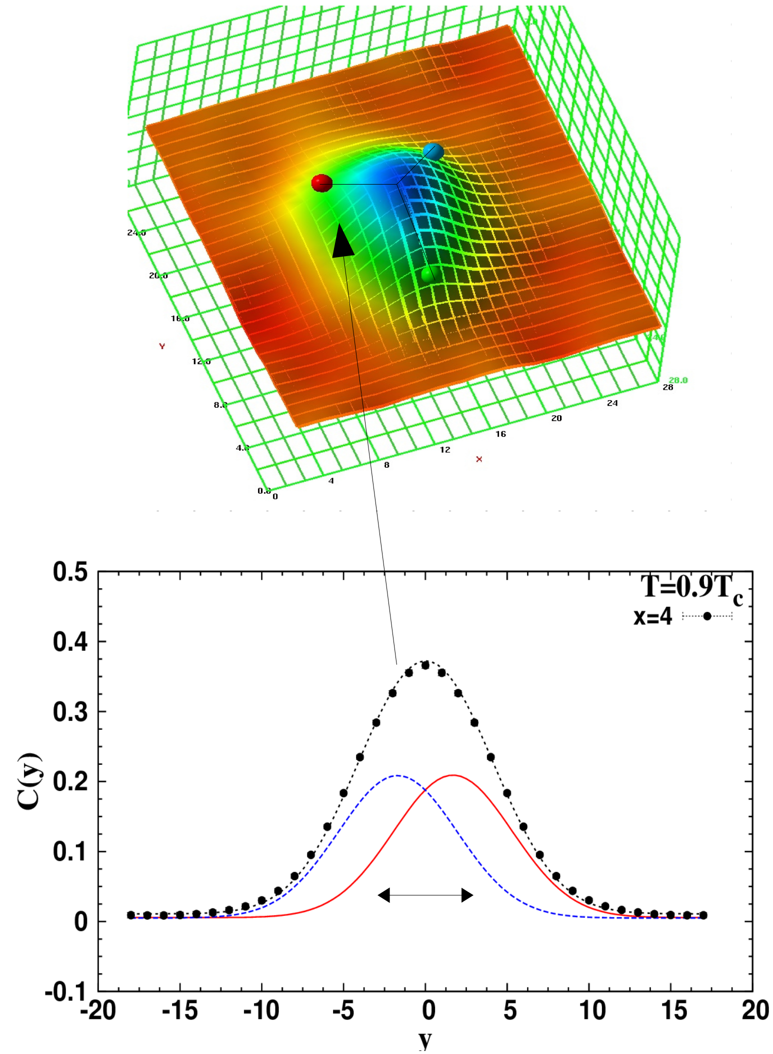
3-Asymmetric Width profile (in-plane and perpendicular directions)

The Delta-action is a sum of three Y “Gaussian-Like flux tubes

To unravel the configuration of the strings in the baryon, we explore the structure of the gluonic distribution with a general ansatz consisting of a two Gaussians

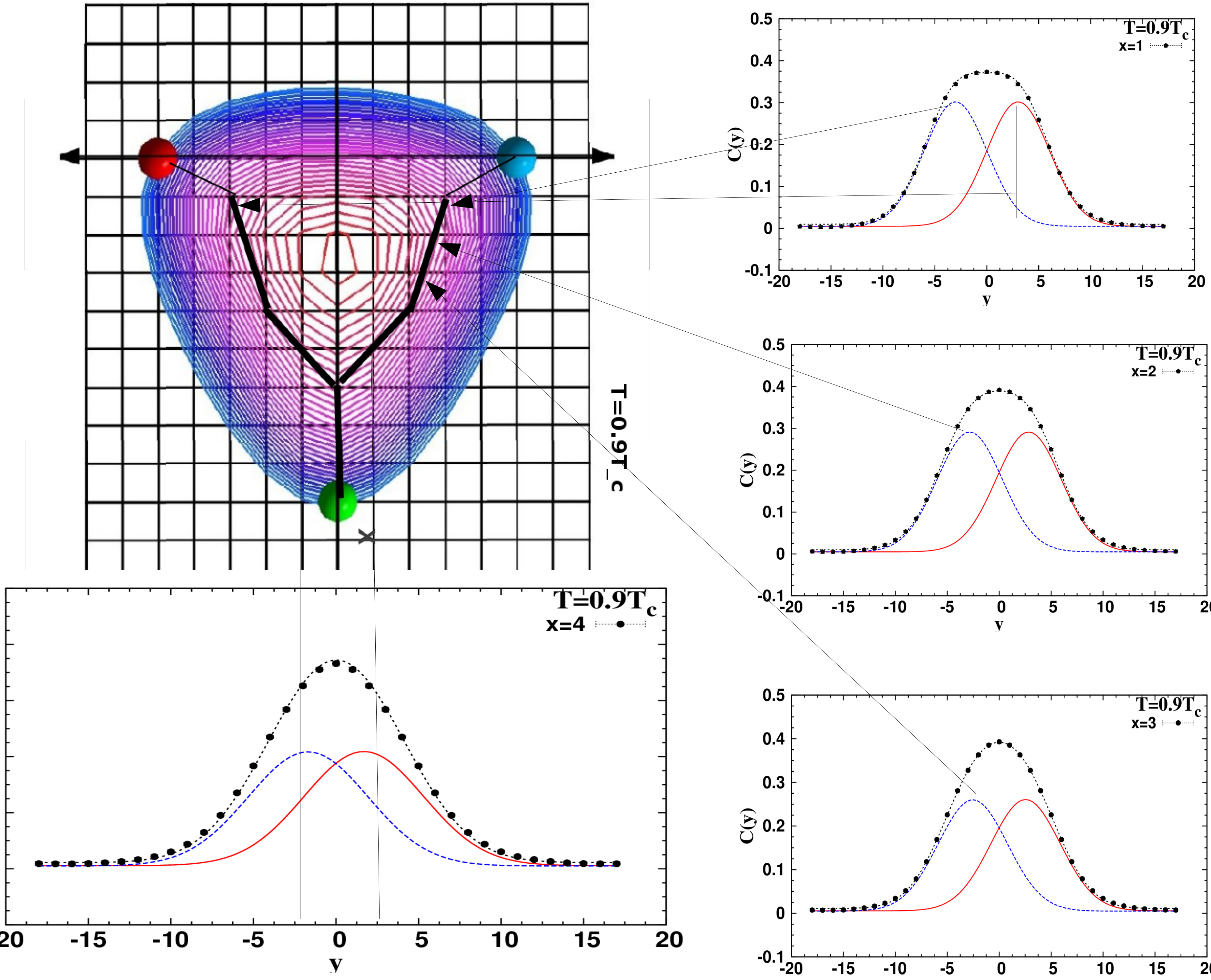
$$G(y; a, w) = A\exp(-(y-u)^2/W^2) + A\exp(-(y+u)^2/W^2).$$

The form assumes a region consisting of a system of two overlapping strings of the same strength A, and mean-square width.



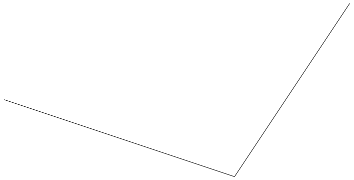
We scan the gluonic domain with the above fit function for all the distances x from the base A connecting the quarks Q1 and Q2 .

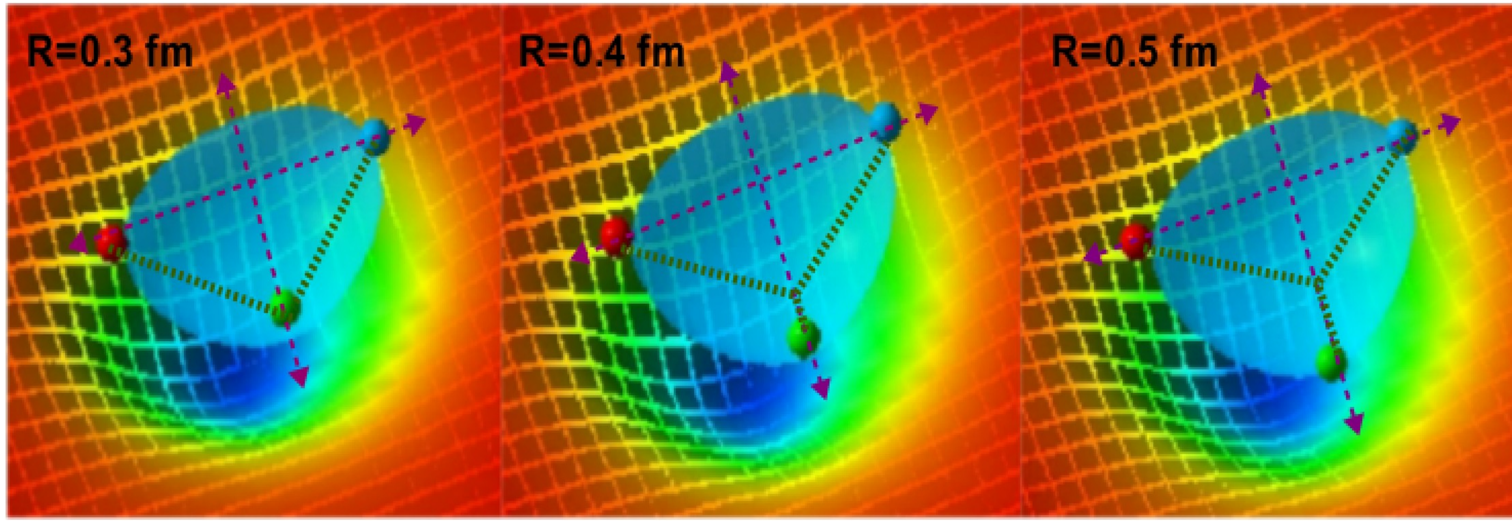
The interesting behavior comes from the returned values of the parameter u.

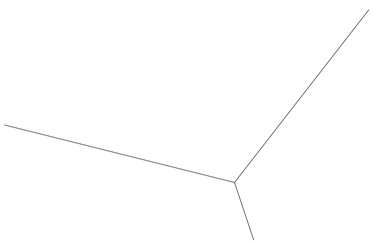


T=0.9T_c
T=0.9T_c
x=1
x=2
x=3
x=4
C(y)
y

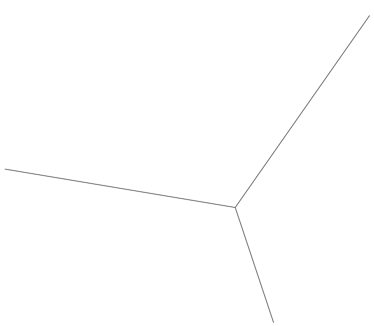
A formation of the Y-shaped confining strings in a static baryon at finite T in the action density o
pure SU(3) YM theory.

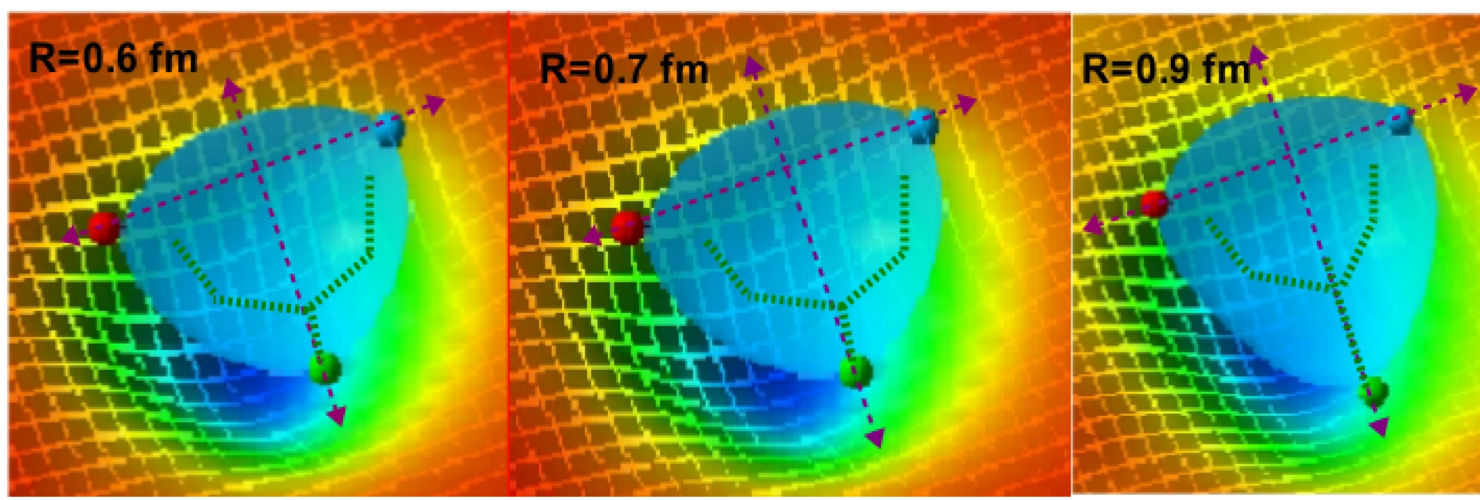






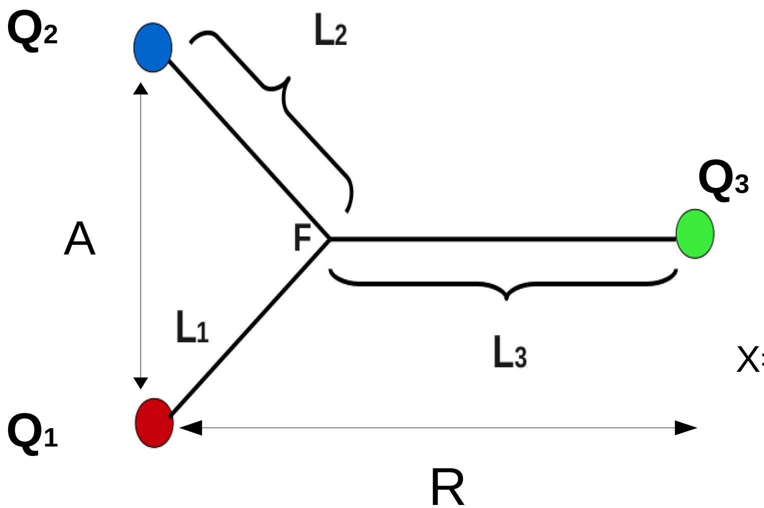
This a schematic plot of the profile of the two Gaussians superimposed over the rendered action density.



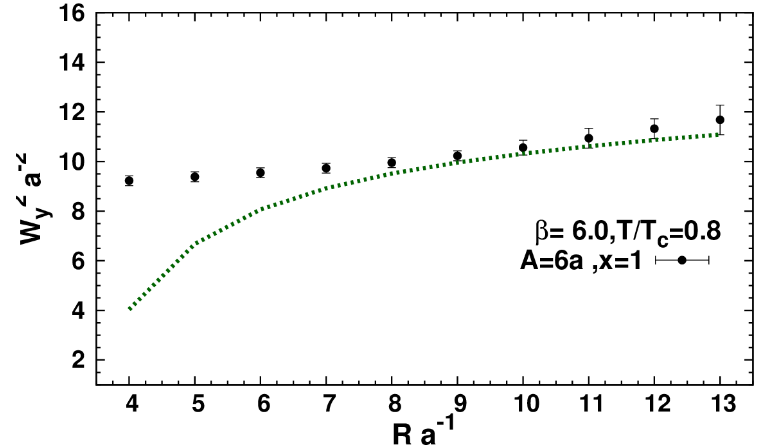


Returned fits of the gluon flux to the string width profile

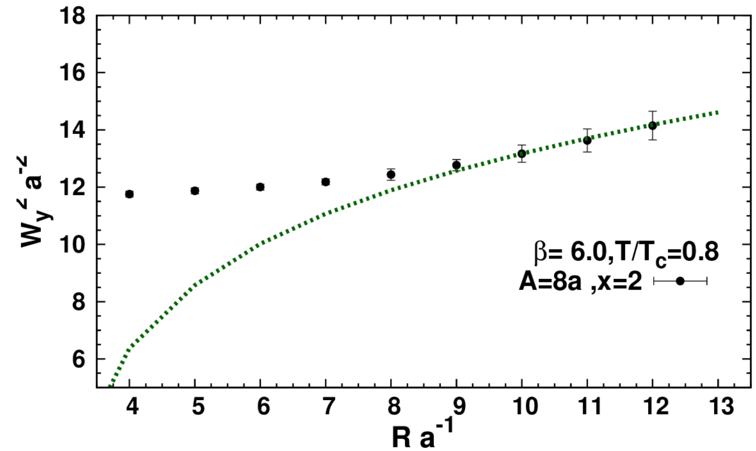
a) In-plane fluctuations



X=1, A=0.6 fm, X_f =1.7



X=2, A=0.8 fm, X_f =2.3

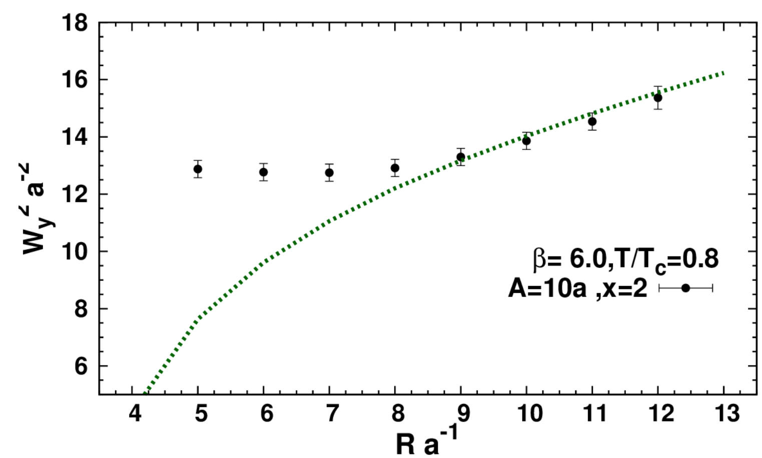


Fits of the junction's profile from string picture to the lattice data.

The broadening of the width versus the third quark's separation R.

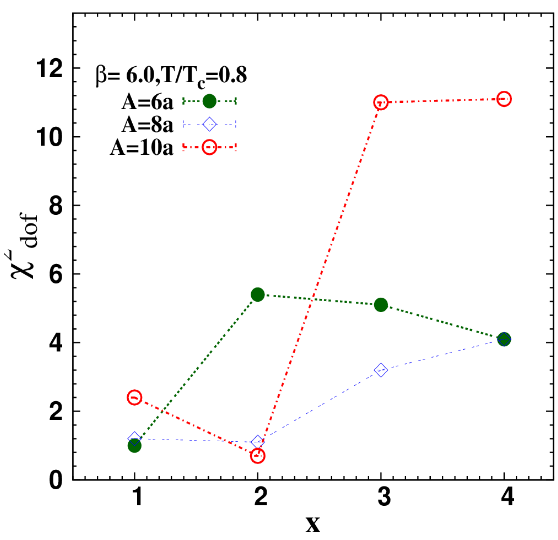
The best fits for the Y-string model for the action density profile are returned for large quark source separation.

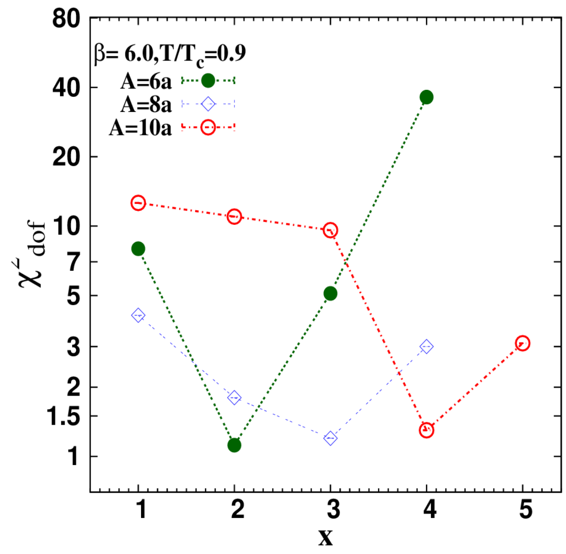
X=2, A=1.0 fm, X_f =2.9

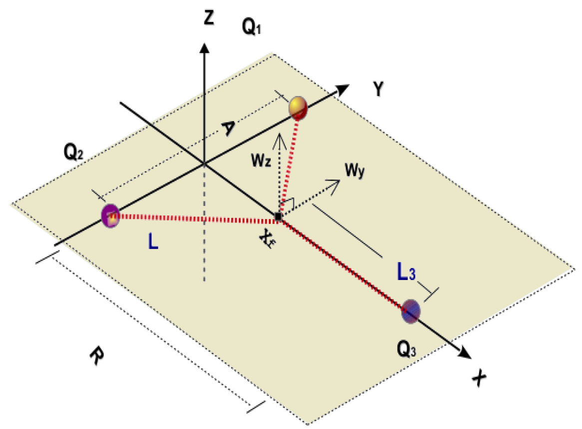


A. S. Bakry, X. Chen, and P.-M. Zhang (2014), Phys. Rev. D 91,114506(2015).

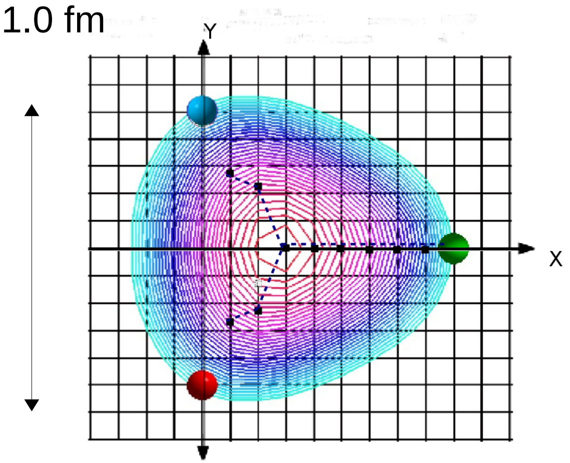
We observe that the planes of best fits manifest in accord with the profile of the two Gaussian shown rather than junction’s classical position at the Fermat point.

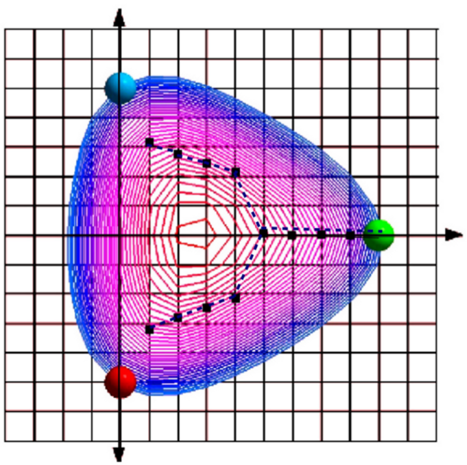






A=1.0 fm

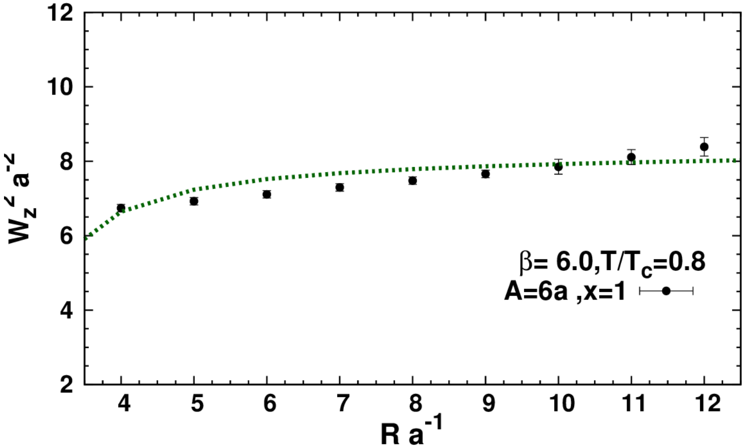


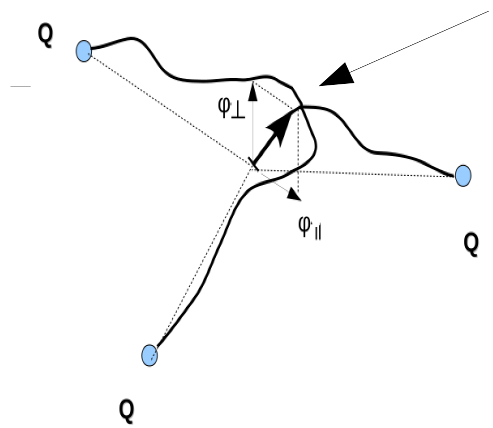


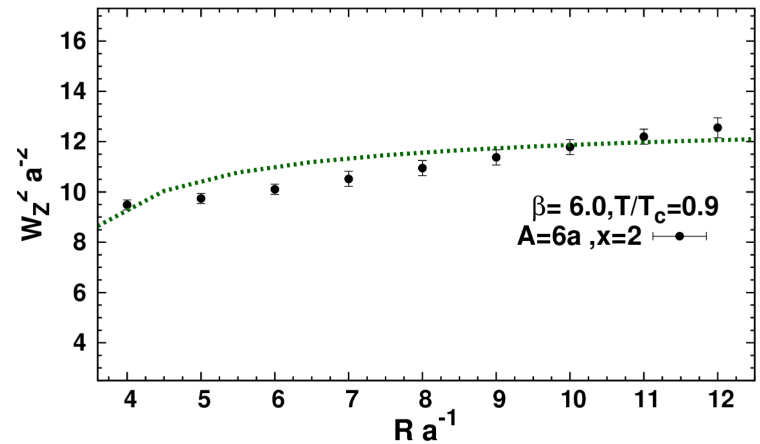
A. S. Bakry, X. Chen, and P.-M. Zhang (2014), Phys. Rev. D 91,114506(2015).

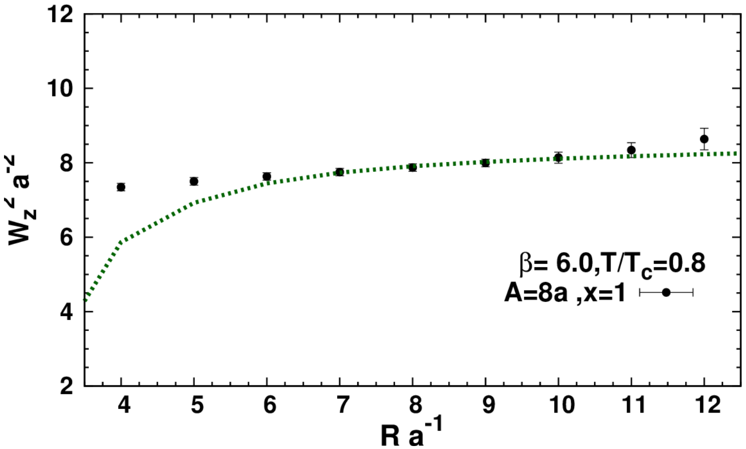
The greatest contribution of the junction appears to be in one lattice spacing immediately before the plane at which $u(x_0) = 0$.

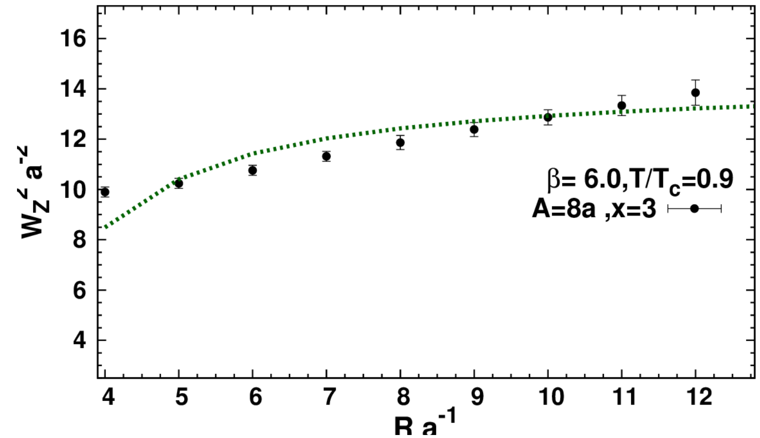
b)Perpendicular Fluctuation

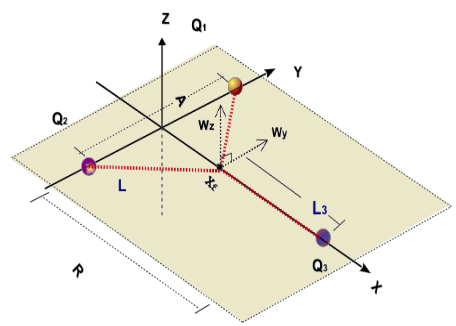


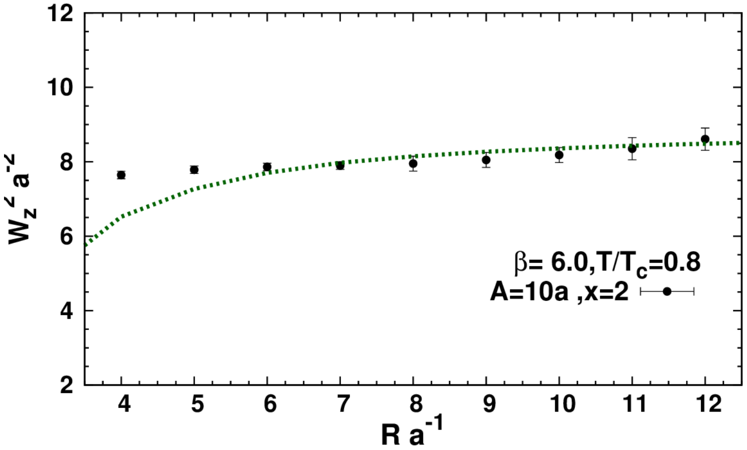


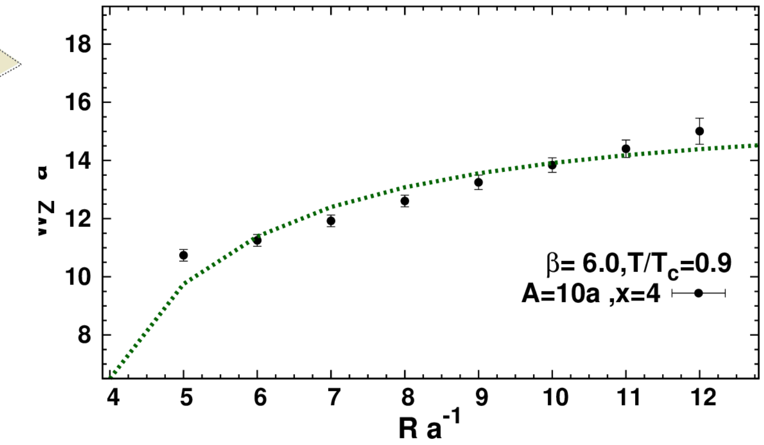












Conclusion

- At finite temperature, we found here that the data corresponding to the 3Q potential are in match with the Y-string model at large quark separation.

- The baryonic gluonic fields are always of a filled Δ-type with characteristics that have been found consistent with the Y-string model as well describing a system of fluctuating strings.

- These two results could be interesting to the study of QCD strings and the gluonic fields in general and may be promoted into a novel picture for the properties of the confining force in the baryon even at low and zero temperatures.