Flux tubes at finite temperature

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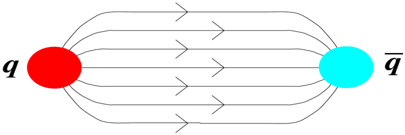
based on arXiv:1511.01783

Outline

The color confinement problem



$q\bar{q}$ pair at distance R in the QCD vacuum

Deconfined phase

$$E_0(R) \xrightarrow{R\to\infty} 2m$$

Confined phase

$$E(R) \longrightarrow \sigma R, \quad \sqrt{\sigma} = 420 \text{ MeV}$$

At the scale of color confinement non perturbative methods are needed

Dual superconductivity

Dual superconductor picture of confinement in QCD by Mandelstam and 't Hooft.
[G. 't Hooft, in High Energy Physics, EPS International Conference, (1975)]
[S. Mandelstam, Phys. Rep. 23, (1976)]

QCD vacuum as a dual superconductor

- Color confinement due to the dual Meissner effect by condensation of chromomagnetic monopoles
- Chromoelectric field connecting a $q\bar{q}$ static pair squeezed inside a tube structure: Abrikosov vortex

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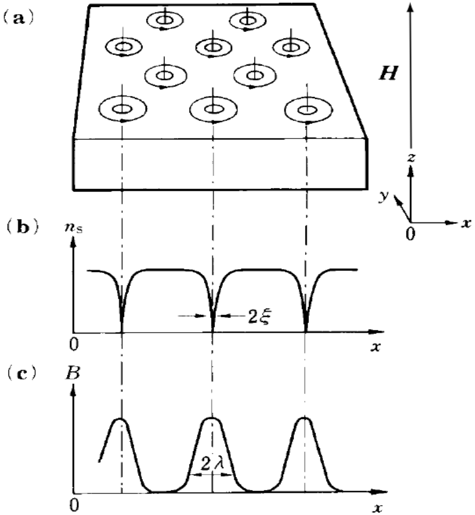
Relevance of nonperturbative study of chromoelectric flux tubes at $T \neq 0$ to clarify the formation of $c\bar{c}$ and $b\bar{b}$ bound states in heavy ion collisions.

Coherence length & London penetration depth



[A. C. Rose-Innes and E. H. Rhoderick, Introduction to Superconductivity (Pergamon Press, Second edition, 1978)]

- λ London penetration depth: characteristic length of the exponential decrease of $\vec{B}$ in a superconductor
- ξ Coherence length: length scale on which the density of Cooper pairs can change appreciably

Fitting functions for $E_l(x_t)$

Here enters the dual superconductor model

- Superconductivity: magnetic field as function of the distance from a vortex line in the mixed state
- Fit functions by dual analogy, from either London or Ginzburg-Landau theory

1. Vortex as a line singularity

$$E_l(x_t) = \frac{\phi}{2\pi}\mu^2 K_0(\mu x_t), \qquad x_t > 0, \qquad \lambda \gg \xi \;\leftrightarrow\; \kappa \gg 1$$

[P. Cea and L. Cosmai, Phys.Rev. D52 (1995)]

2. Cylindrical vortex

$$E_l(x_t) = \frac{\phi}{2\pi}\frac{1}{\lambda\xi_v}\frac{K_0(R/\lambda)}{K_1(\xi_v/\lambda)} ,$$

[J. R. Clem, J. Low Temp. Phys. 18, 427 (1975)]

[P. Cea, L. Cosmai, and A. Papa, Phys. Rev. D 86, (2012)]

Fitting function in our work

$$E_l(x_t) = \frac{\phi}{2\pi}\frac{\mu^2}{\alpha}\frac{K_0[(\mu^2 x_t^2 + \alpha^2)^{1/2}]}{K_1[\alpha]} \qquad x_t \geq 0,$$

$$R = \sqrt{x_t^2 + \xi_v^2}, \qquad \mu = \frac{1}{\lambda}, \qquad \frac{1}{\alpha} = \frac{\lambda}{\xi_v}, \qquad \kappa = \frac{\lambda}{\xi} = \frac{\sqrt{2}}{\alpha}\left[1 - K_0^2(\alpha)/K_1^2(\alpha)\right]^{1/2}.$$

1. ϕ external flux
2. $\mu = 1/\lambda$ London penetration depth inverse
3. $1/\alpha = \lambda/\xi_v$ with ξ_v variational core-radius parameter
4. $\kappa = \lambda/\xi$ Ginzburg-Landau parameter

Outline

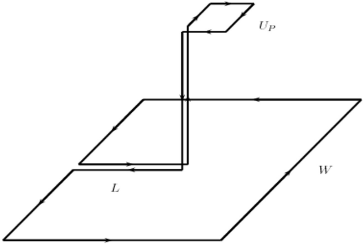
Numerical background

- Theory: $SU(3)$ pure gauge

$$S = \beta \sum_{x,\mu>\nu} [1 - \frac{1}{3} \mathrm{Re}\mathrm{Tr}\, U_{\mu\nu}(x)]$$

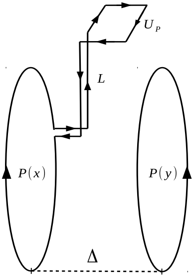
- Degrees of freedom: gauge links $U_\mu(x) \in SU(3)$ in gauge configurations
  - Building blocks for lattice observables
  - Updating: ensemble averages as (simulation-)time averages
    - Cabibbo-Marinari algorithm combined with overrelaxation
  - Smoothing: improve signal-to-noise ratio
    - Single HYP smearing step on links in temporal direction
    - Many APE smearing steps on links in spatial directions
- Code: suited application within MILC collaboration's public lattice gauge theory code (`http://physics.utah.edu/~detar/milc.html`)

Connected correlators



$$\rho_W^{\text{conn}} = \frac{\langle \text{tr}\left(WLU_PL^\dagger\right)\rangle}{\langle \text{tr}(W)\rangle} - \frac{1}{3}\frac{\langle \text{tr}(U_P)\text{tr}(W)\rangle}{\langle \text{tr}(W)\rangle}$$

- W Wilson loop
- L Schwinger line
- U_p Plaquette



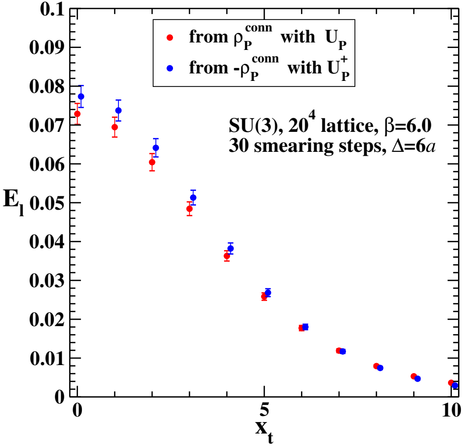
$$\rho_P^{\text{conn}} = \frac{\langle \text{tr}\left(P(x)LU_PL^\dagger\right)\text{tr}P(y)\rangle}{\langle \text{tr}(P(x))\,\text{tr}(P(y))\rangle} - \frac{1}{3}\frac{\langle \text{tr}(P(x))\,\text{tr}(P(y))\,\text{tr}(U_P)\rangle}{\langle \text{tr}(P(x))\,\text{tr}(P(y))\rangle}$$

- $P(x)$, $P(y)$ Polyakov lines at a distance Δ
- L Schwinger line
- U_p Plaquette

[A. Di Giacomo, M. Maggiore, S. Olejnik, Nucl.Phys. B347 (1990); P. Cea, L. Cosmai, Phys.Rev. D52 (1995)]

$\rho_P^{\rm conn}$ MEASURING FIELDS: NUMERICAL EVIDENCE

$\rho_P^{\rm conn}$ changes sign under the transformation $U_P \to U_P^\dagger$



E_l vs x_t, for $\Delta = 6a$, at $\beta = 6.0$ and after 30 smearing steps

$$\rho_P^{\rm conn} \xrightarrow{a\to 0} a^2 g \left[\left\langle n^a F^a_{\mu\nu} \right\rangle_{q\bar{q}} \right],$$

$$F^a_{\mu\nu}(x)\, n^a = \sqrt{\frac{\beta}{6}}\, \rho_P^{conn}(x)\,.$$

Our investigation in few steps

For different $N_s \times N_t$ and β

- Smearing over thermalized configuration
- Measurement of $E_l(x_t)$ through $\rho^{\mathrm{conn}}_{P,W}$
- Check continuum scaling
- Fit of $E_l(x_t)$ to extract ϕ, μ, λ/ξ_v, κ
- Check optimal smearing
- Temperature dependence of λ, ξ and κ

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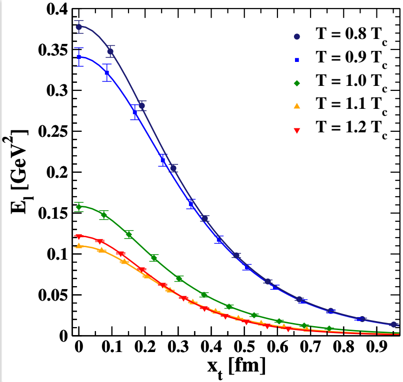

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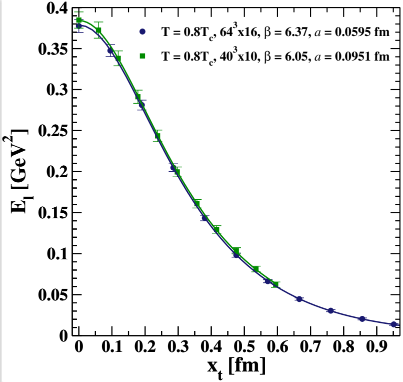

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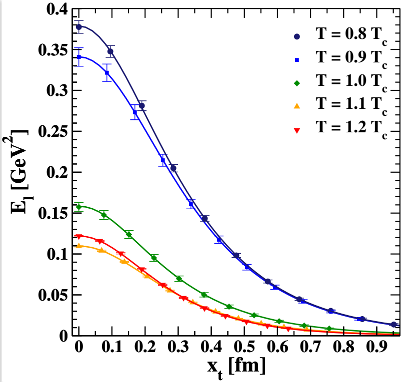

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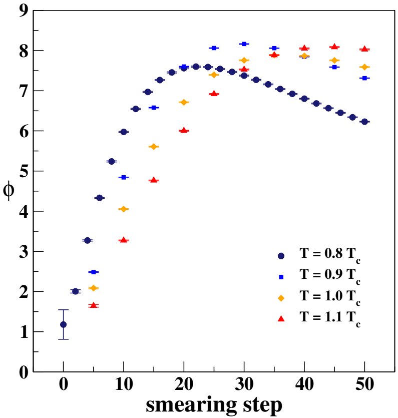

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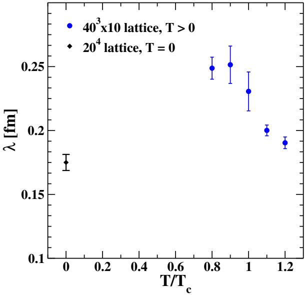
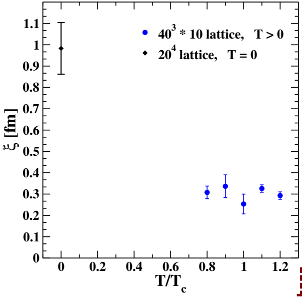
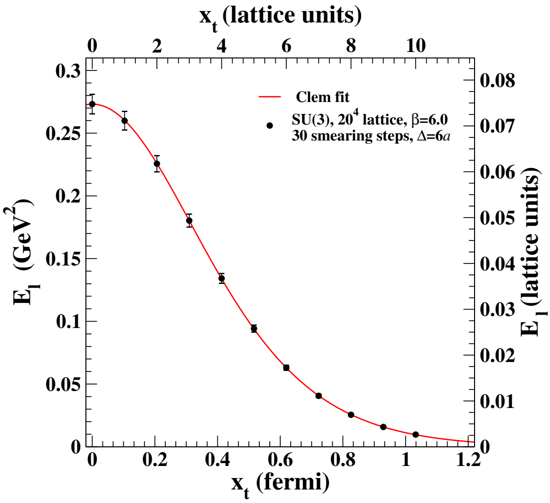




Summary of our previous results



- Connected correlator with Polyakov loops and smearing:
  - $\lambda = 1/\mu = 0.1750(63)$ fm
  - $\xi = 0.983(121)$ fm
  - $\kappa = 0.178(21)$
- λ, ξ, κ in agreement with $T = 0$ studies using Wilson-plaquette connected correlator and cooling
- SU(3) vacuum as a type-I dual superconductor

[P. Cea, L. Cosmai, F. C. and A. Papa, Phys. Rev. D 89, (2014)]

Outline

Polyakov connected correlator

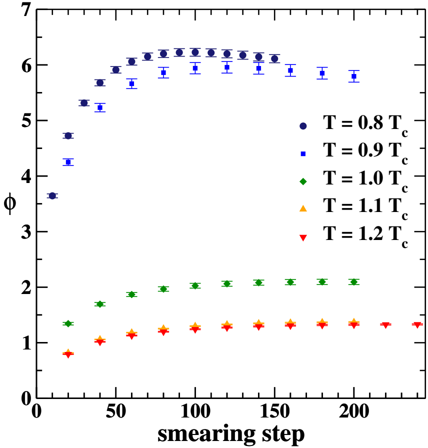
Motivations

- Naive extension of the $T = 0$ study.
- Check of dual superconductor scenario by measuring changes in parameters of flux tubes across the deconfining transition.

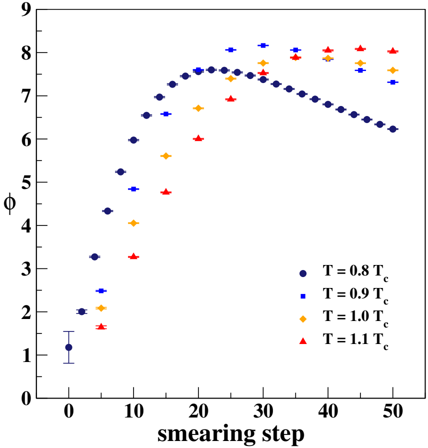
Lattice and correlator features

- Polyakov connected correlator with loops at physical distance $\Delta = 0.76$ fm
- $N_t = 8, 10, 12$ with aspect ratio fixed to 4
- $\beta = 6.050, 6.370$ corresponding to $0.8\,T_c \leq T \leq 1.2\,T_c$

Optimal smearing step: ϕ vs smearing step



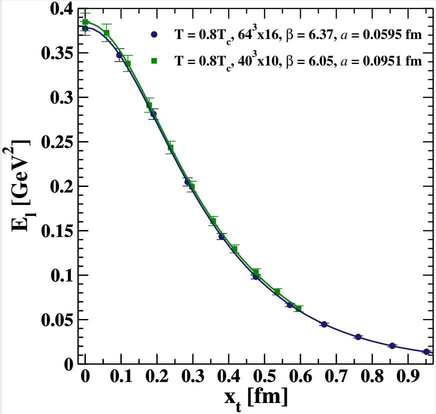
Polyakov connected correlator
$40^3 \times 10$ lattice



Wilson connected correlator
$40^3 \times 10$ lattice.

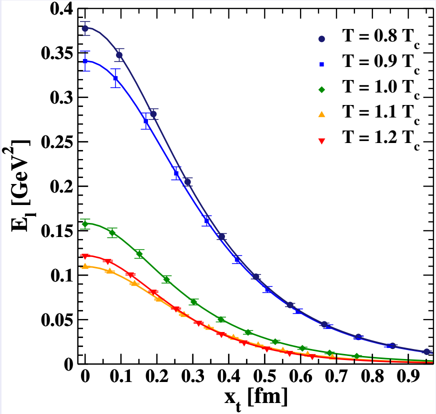
Field in physical units

Continuum scaling



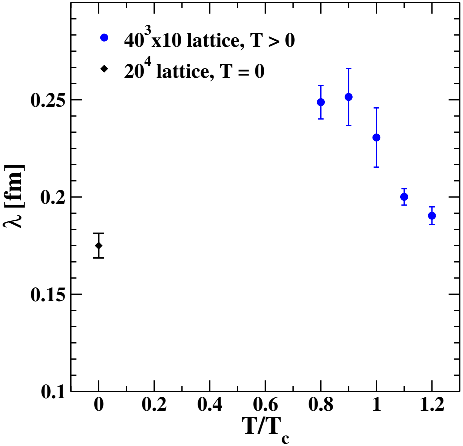
E_l *vs* x_t at $\beta = 6.050(6.370)$, $\Delta \sim 0.76$ fm, after 100(180) smearing steps.

Temperature effect

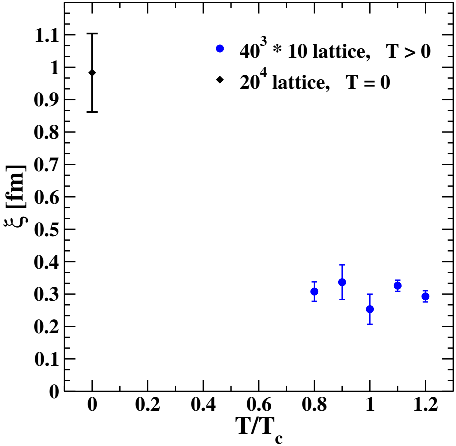


E_l *vs* x_t on $40^3 \times 10$ lattices, and various gauge couplings in the scaling region.

Plateau values vs T/T_c at fixed β



Plateau values for λ vs T/T_c. $T = 0$ value was $\lambda_{T=0} = 0.1750(63)$ fm



Plateau values for ξ vs T/T_c. $T = 0$ value was $\xi_{T=0} = 0.983(121)$ fm

Results & perspective

- Expectations from ordinary superconductivity:
  - T-dependent λ and ξ
$$\lambda, \xi \propto \frac{1}{\sqrt{T_c - T}}$$
  - T-independent $\kappa = {}^{\lambda(T)}/_{\xi(T)}$
- Results:
  - Φ decreases with T
  - λ (ξ) bigger (much smaller) than its $T = 0$ value, but:
    - ⋆ Weak T-dependence for $T \lesssim T_c$
    - ⋆ No signal of divergence for $T \to T_c$: "evaporation" rather than fattening
  - SU(3) vacuum is a type-II dual superconductor (disagreement with $T = 0$ results)
- Outlook:
  - Introduction of dynamical quarks d.o.f.

Outline

Wilson connected correlator: chromomagnetic sector

Motivations

- nonperturbative structure of QCD at high-T due to correlation functions for the spatial components of gauge fields (spatial Wilson loops)
- quantitative description of (reconstructed) spatial string tension

Lattice and correlator features

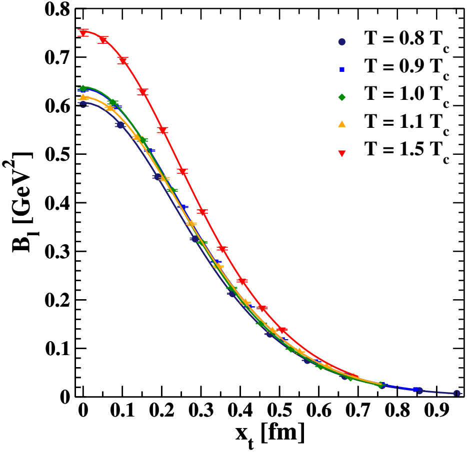
- Wilson connected correlator in the spatial sublattice (loop size $\Delta = 0.76$ fm)
- $N_t = 10$ with aspect ratio fixed to 4

Consistency check: measured spatial string tension $\sqrt{\sigma_s}$

[F. Karsch, E. Laermann, M. Lutgemeier, Phys. Lett. B346 (1995)]

- $N_s = 32$, $N_t = 2, 4$ and $\beta = 6.0$
- Spatial Wilson loops of sizes up to 12×12

Field in physical units: temperature effect



B_l *vs* x_t on $40^3 \times 10$ lattices, and various gauge couplings in the scaling region.

- $B_l(x_t)$ well-fitted by Clem function
- chromomagnetic flux-tube beyond deconfinement transition
- $\sqrt{\sigma_s}$, measured within the same code, in agreement with literature predicting area law for Wilson loops also for $T \geq T_c$

Outline

Results & perspective

- Open questions:
  - The thickness of flux tubes does not diverge towards T_c, while our fit results are reliable at all T
    - ★ "Evaporation" rather than fattening: no counterpart in ordinary SC
  - Clem function well fits measurements of fields in the chromomagnetic sector
- Outlook:
  - Introduction of dynamical quarks d.o.f. with masses at (almost) the physical point: new MILC application written for this purpose.