Condensation of Fluctuations

Marco Zannetti

Università di Salerno

SM&FT - Bari 9-11 Dicembre 2015

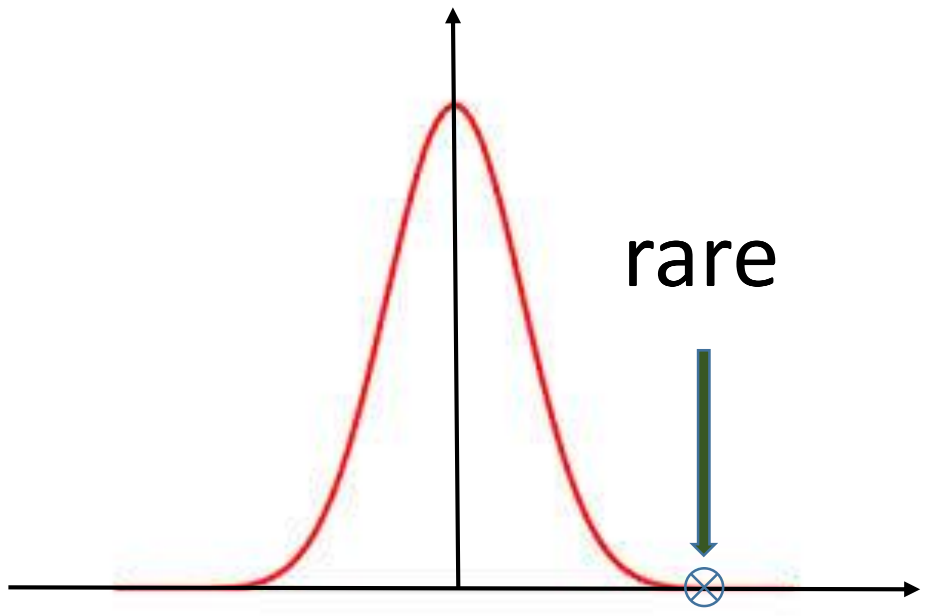
- F. Corberi
- G. Gonnella
- A. Piscitelli

OUTLINE:

- duality of rare and typical events
- IBG in the grand canonical ensemble: condensation of fluctuations as a rare event
- Observability: mean canonical ensemble
- grand canonical catastrophe
- photon condensation

rarity vs typicality

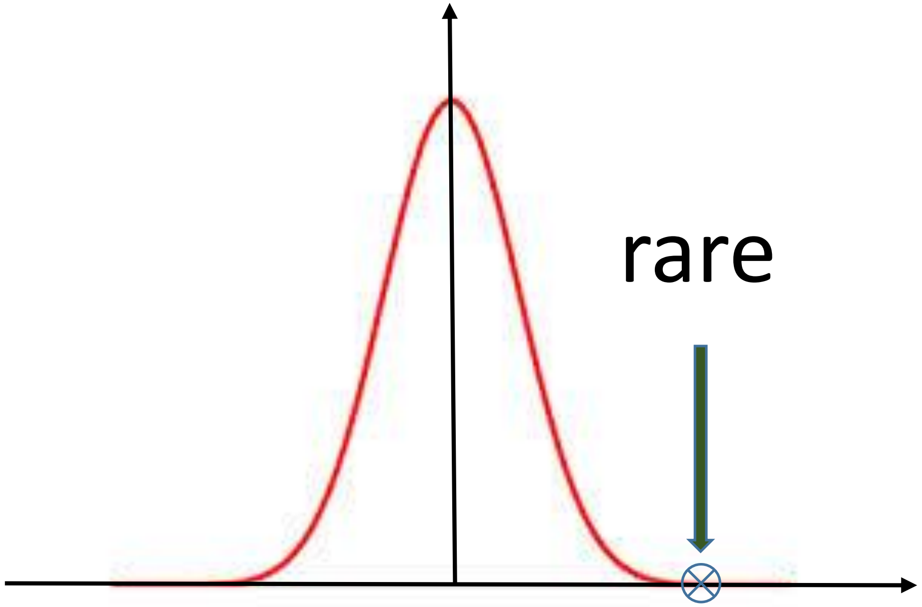
given environment

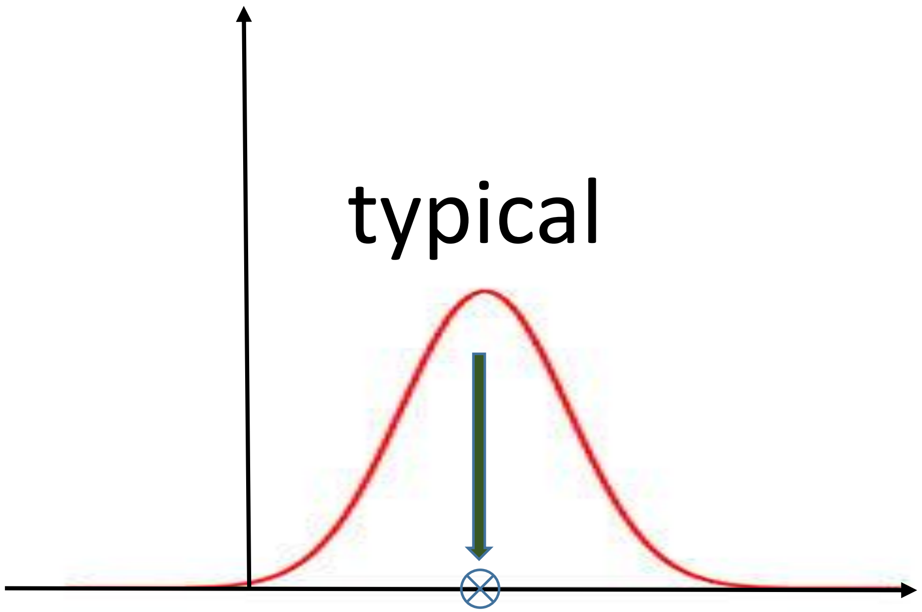


rarity vs typicality

given environment

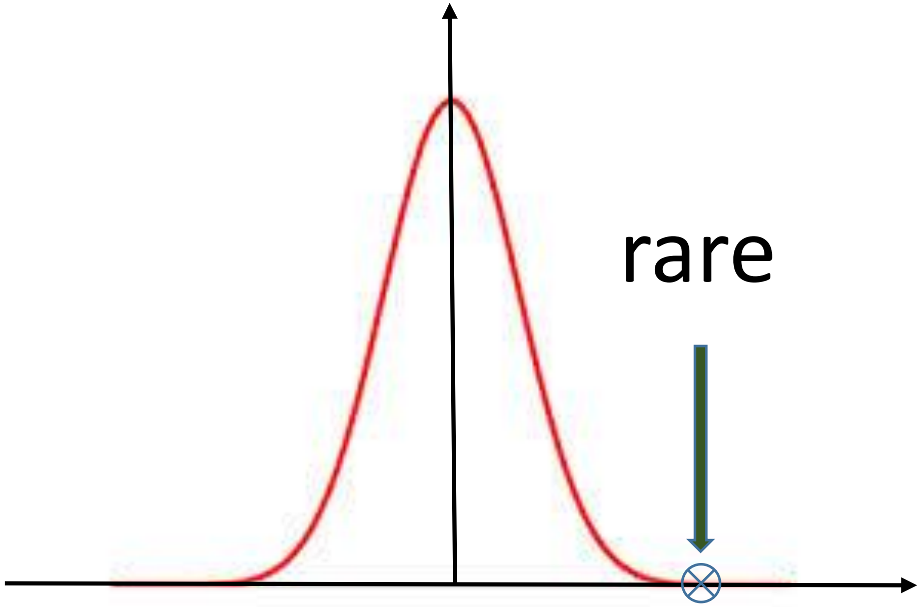
modified environment



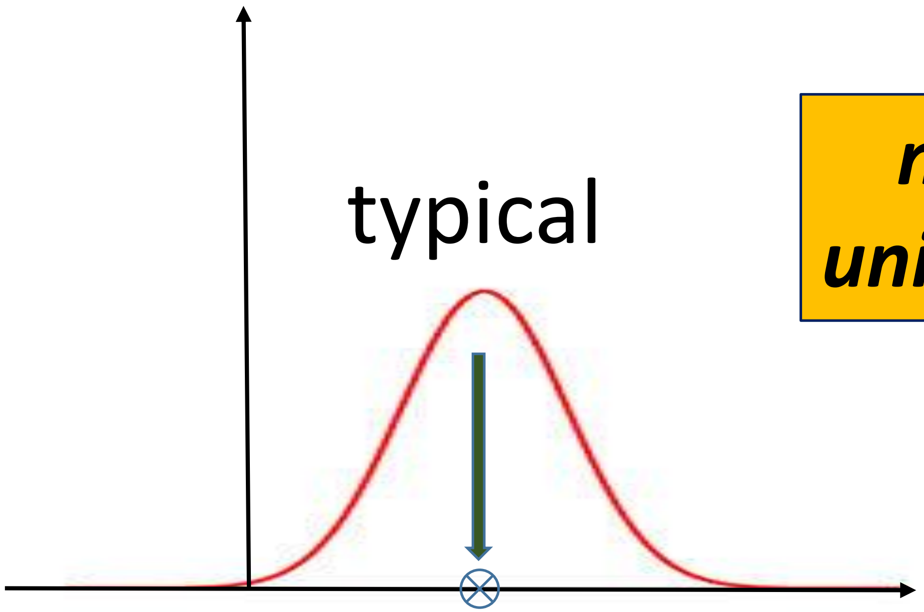


rarity vs typicality

given environment

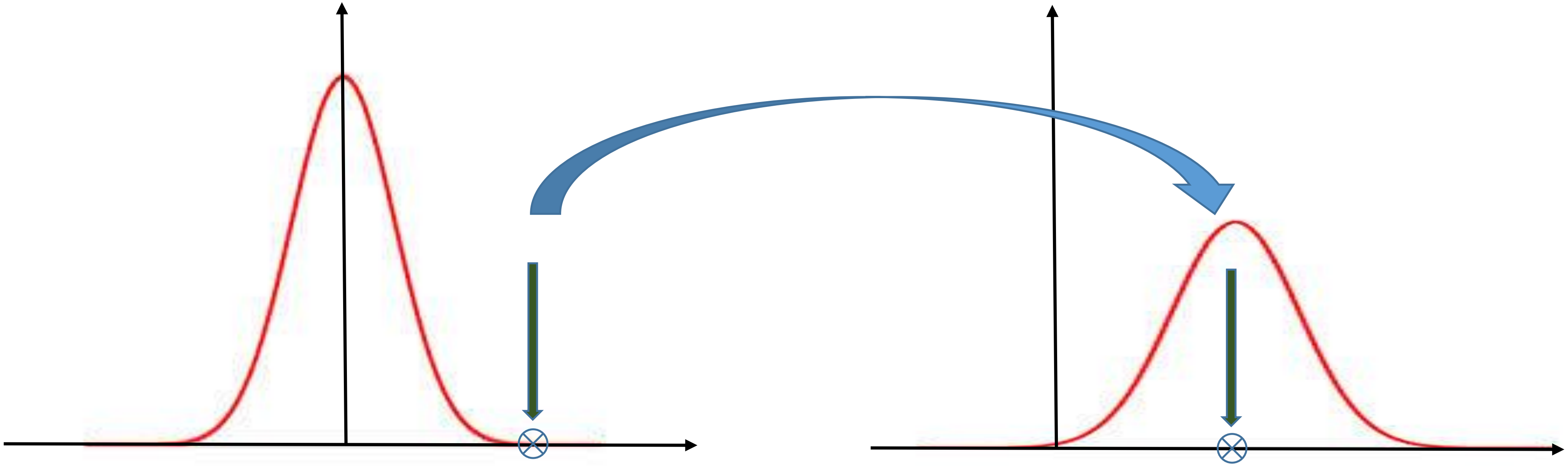


modified environment



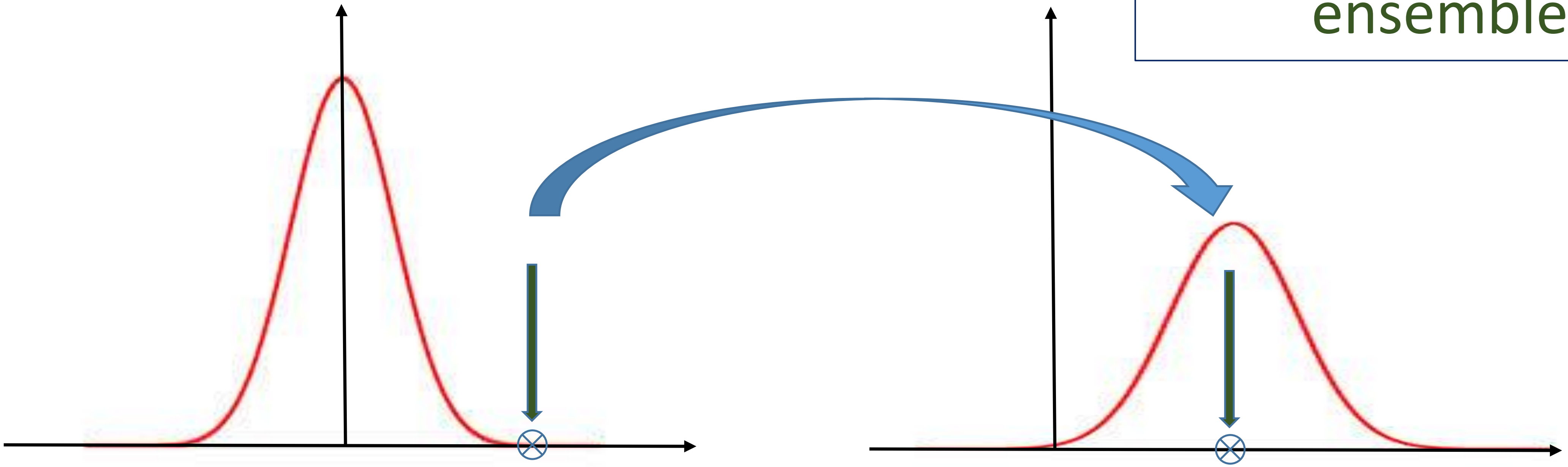
not unique!

statistics of fluctuations select typical environment



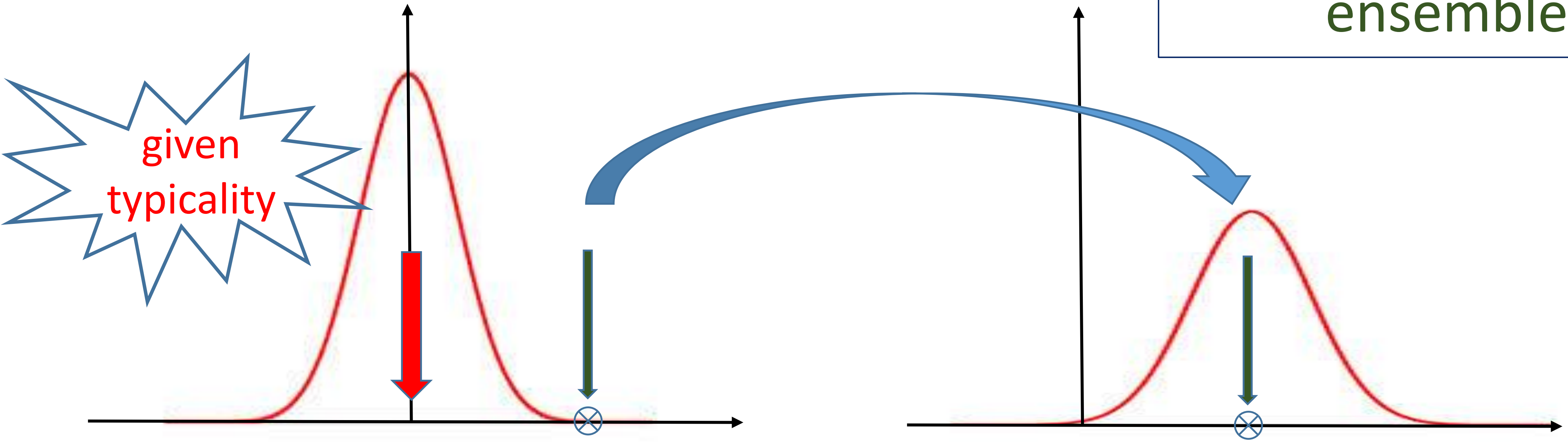
statistics of fluctuations select typical environment

large deviation function ⟷ *duality* ⟷ «free energy» ⬇ ***constrained*** or ***biased*** ensemble



statistics of fluctuations select typical environment

large deviation function ⟷ *duality* ⟷ «free energy» ⬇ ***constrained*** or ***biased*** ensemble



Out of equilibrium

- *Thermodynamics of trajectories*

  L. R. Jack and P. Sollich, Progr. Theor. Phys. Supp. **184**, 304 (2010)
- *Conditioned stochastic process*

  R. Chetrite and H. Touchette, Phys. Rev. Lett. **111**, 120601 (2013)
- *Optimization*

  T. Nemoto and S. Sasa, Phys. Rev. Lett. **112**, 090602 (2014)
- …

non interacting systems

- *constraints induce correlations*
- *fluctuations may display non trivial features even if average behavior is trivial*

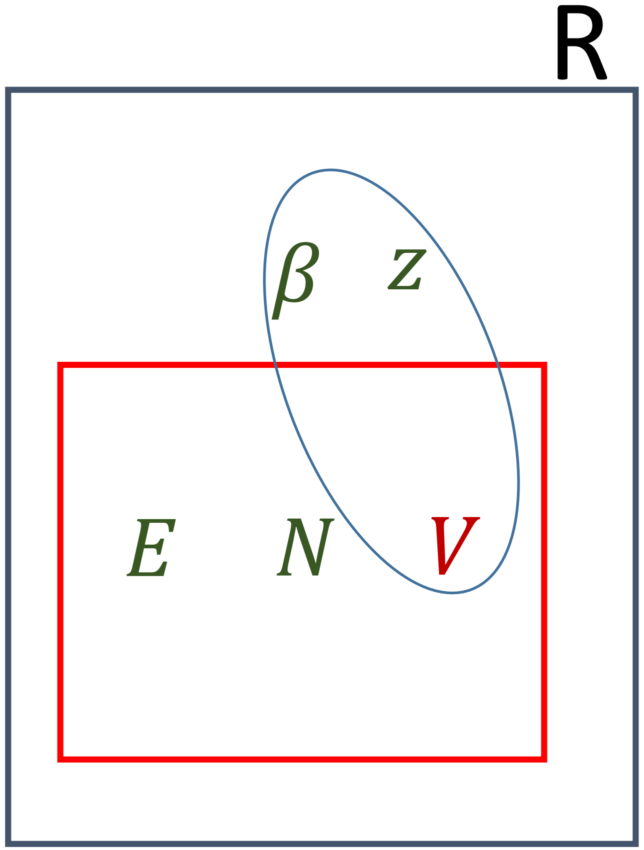
F.Corberi, G.Gonnella, A.Piscitelli and M.Zannetti, *J. Phys. A* **46**, 042001 (2013)
J.Szavits-Nossan, M.R.Evans and S. N. Majumdar, *Phys. Rev. Lett*. **112**, 020602 (2014); *J. Phys. A* **47**, 455004 (2014)
M.Zannetti, F.Corberi and G.Gonnella , *Phys. Rev. E* **90**, 012143 (2014)
M. Zannetti, F.Corberi, G.Gonnella and A.Piscitelli, *Comm. Theor. Phys*. **62**, 555 (2014)
F. Corberi, arXiv:1505.01025

Ideal Bose Gas in the Grand Canonical Ensemble:

MZ EPL **111**, 20004 (2015)

$$\omega = [n_{\vec{p}}] \qquad \boldsymbol{\mathcal{H}}(\omega) = \sum_{\vec{p}} \epsilon_{\vec{p}} n_{\vec{p}} \qquad \boldsymbol{\mathcal{N}}(\omega) = \sum_{\vec{p}} n_{\vec{p}}$$

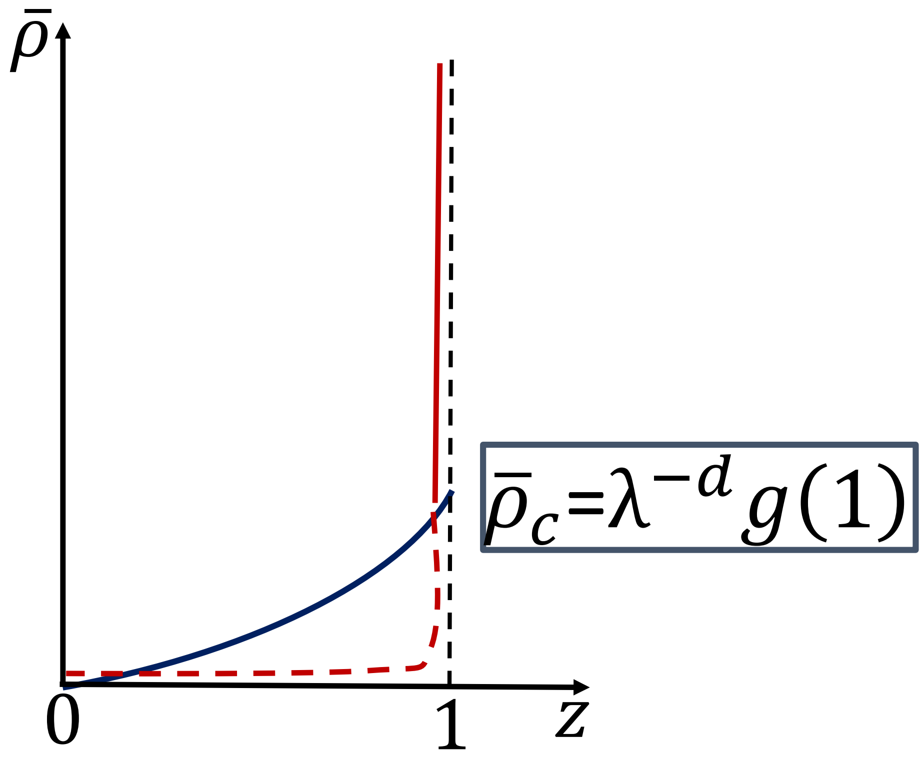
$$J = (\beta, z, V), \quad z = e^{\beta\mu}$$



given typical behavior

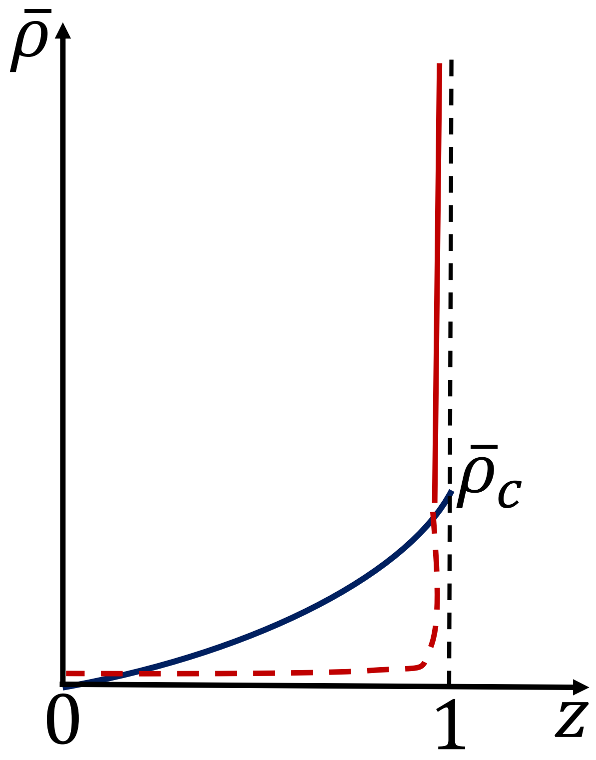
$$\bar{\rho} = \frac{1}{V}\sum_{\vec{p}}\langle n_{\vec{p}}\rangle \cong \frac{1}{V}\left(\frac{z}{1-z}\right) + \lambda^{-d} g(z)$$

thermal wavelength

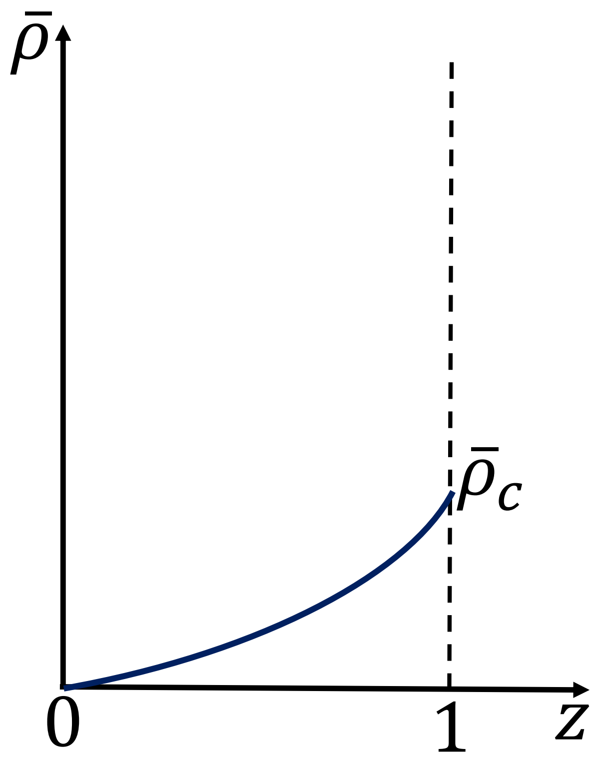


given typical behavior

$$\bar{\rho} = \frac{1}{V}\sum_{\vec{p}} \langle n_{\vec{p}} \rangle \cong \frac{1}{V}\left(\frac{z}{1-z}\right) + \lambda^{-d} g(z)$$

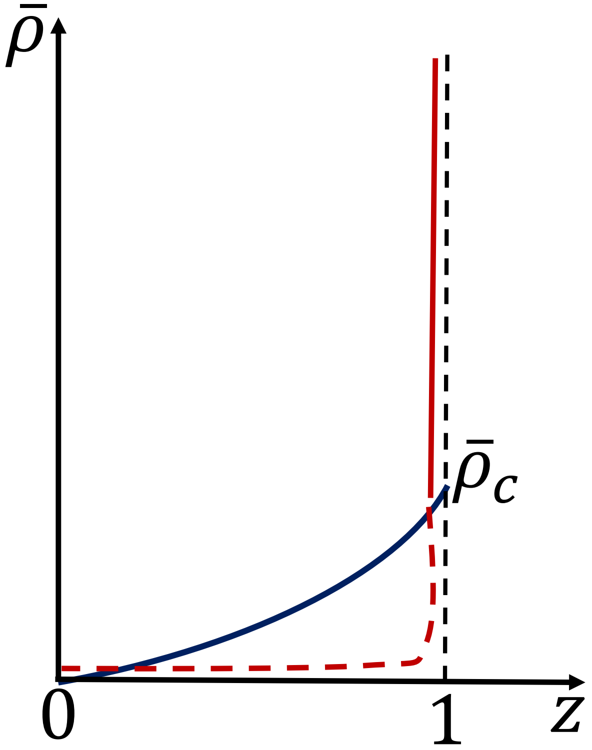


$$V \to \infty$$

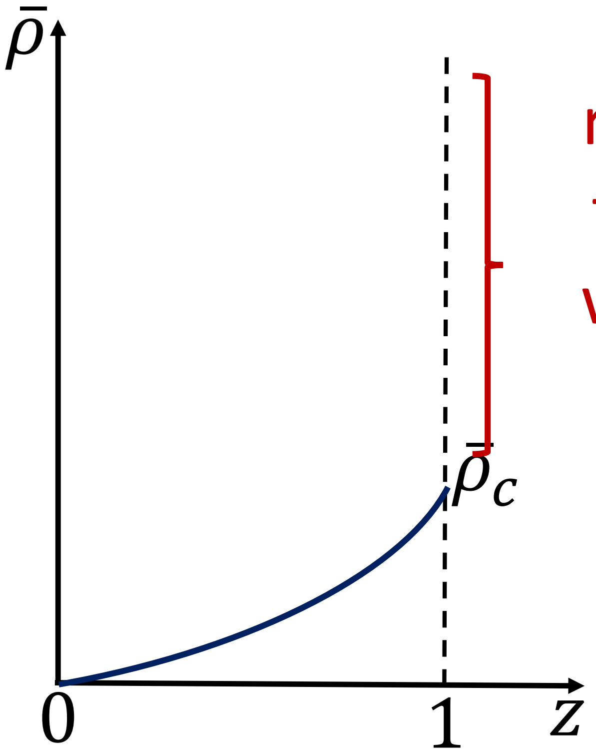


given typical behavior

$$\bar{\rho} = \frac{1}{V}\sum_{\vec{p}}\langle n_{\vec{p}}\rangle \cong \frac{1}{V}\left(\frac{z}{1-z}\right) + \lambda^{-d} g(z)$$



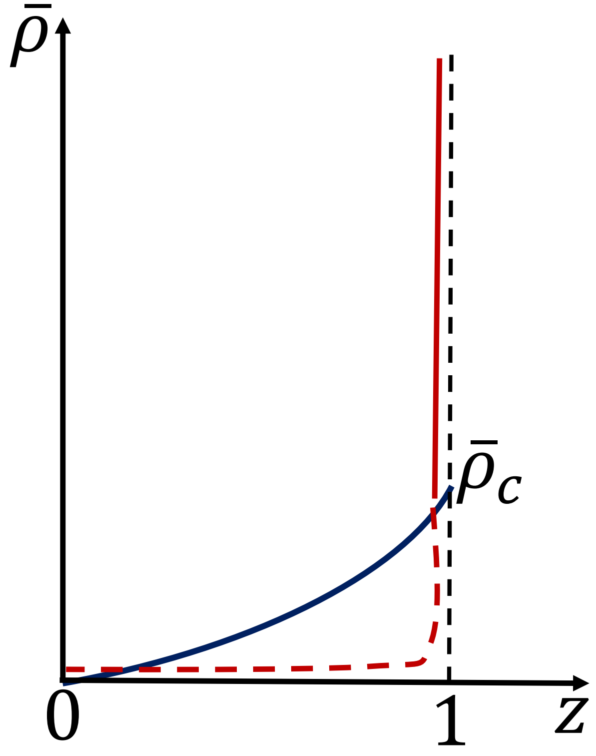
$V \to \infty$



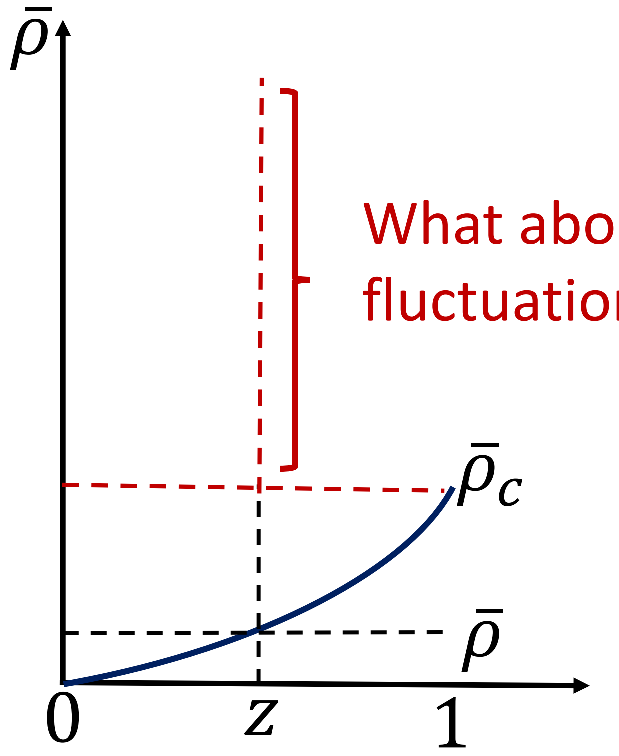
region forbidden
to typical values
when controlling
via J=(β,z,V)

EQUATION OF STATE

$$\bar{\rho} = \frac{1}{V}\sum_{\vec{p}} \langle n_{\vec{p}} \rangle \cong \frac{1}{V}\left(\frac{z}{1-z}\right) + \lambda^{-d} g(z)$$

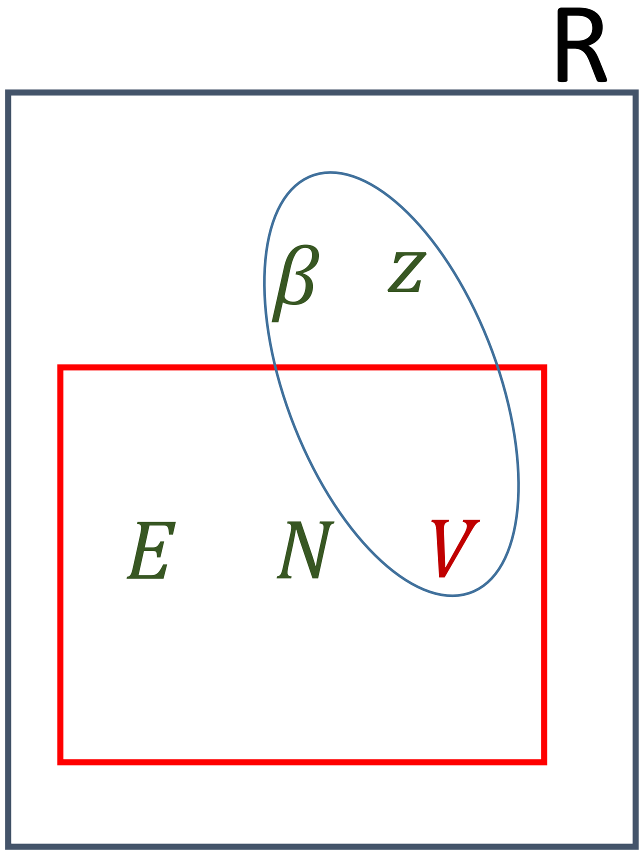


$$V \to \infty$$



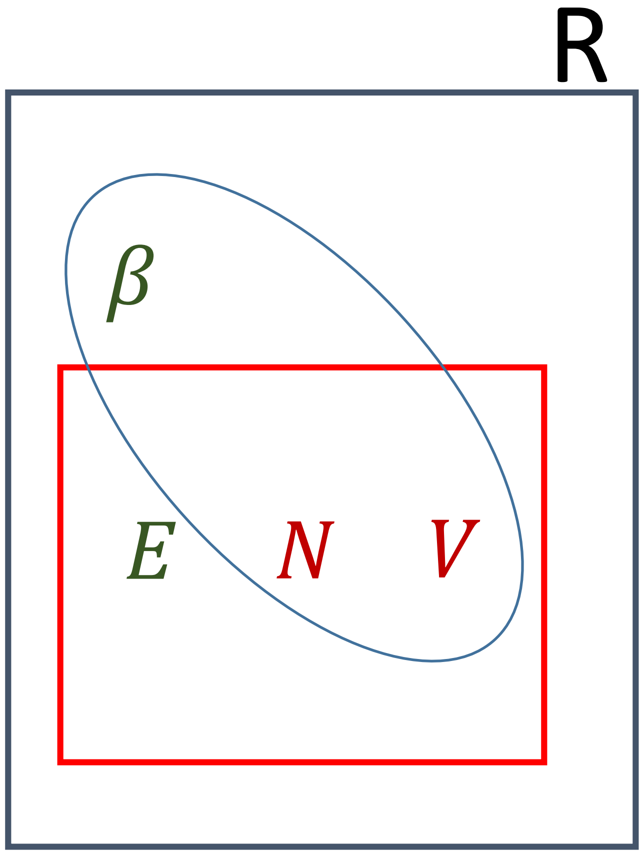
$$P_{gc}(N|\beta, z, V) \sim e^{-V I_{gc}(\rho|\beta,z)}$$

given environment



$$P_{gc}(N|\beta, z, V) \sim e^{-V I_{gc}(\rho|\beta,z)}$$

$$I_{gc}(\rho|\beta, z) = \beta\left[-\mu\rho + f_C\underbrace{(\beta,\rho)}_{\hat{J}} - f_{gc}\underbrace{(\beta, z)}_{J}\right]$$



modified environment

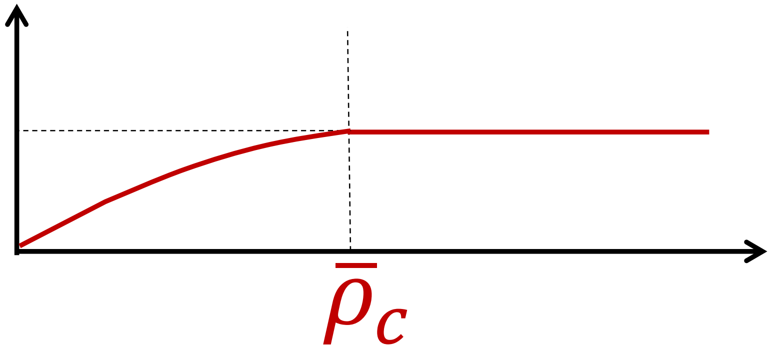
$$P_{gc}(N|\beta,z,V) \sim e^{-V I_{gc}(\rho|\beta,z)}$$

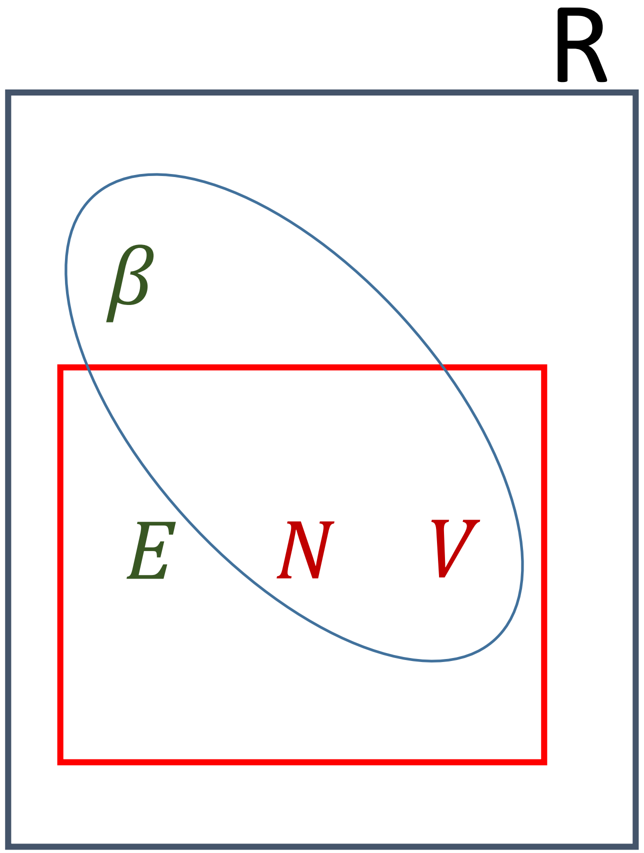
$$\boxed{I_{gc}(\rho|\beta,z) = \beta\left[-\mu\rho + f_C(\beta,\rho) - f_{gc}(\beta,z)\right]}$$

BEC in the canonical ensemble

$$\langle n_0 \rangle = V(\rho - \bar{\rho}_c)$$

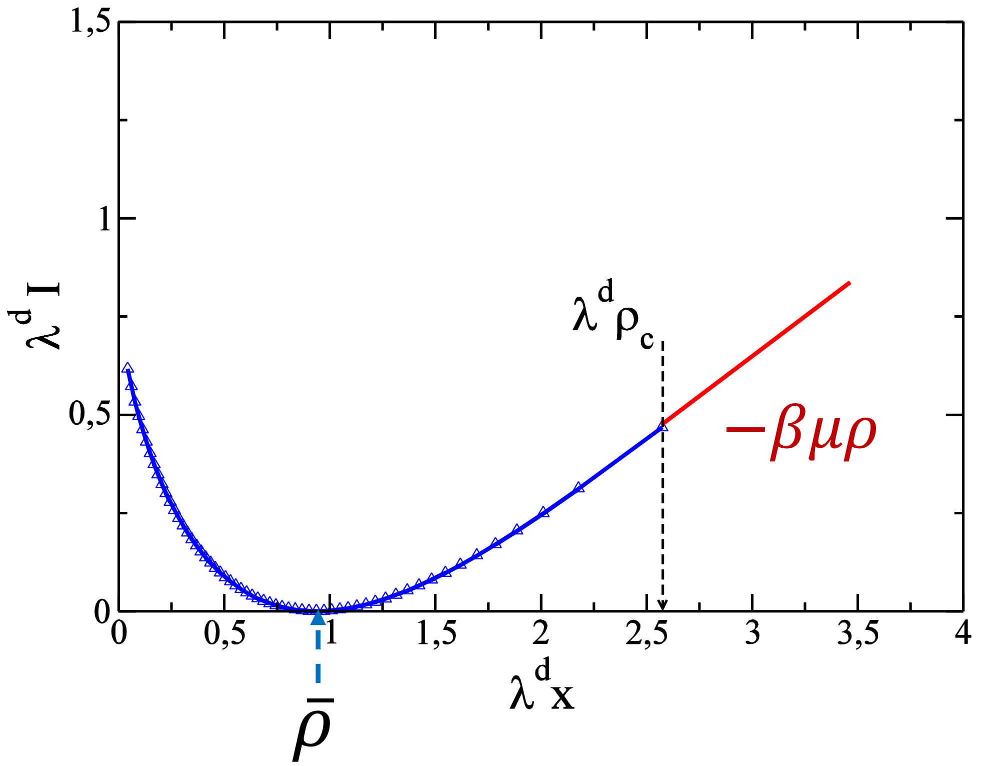
$f_C(\beta,\bar{\rho}_c)$





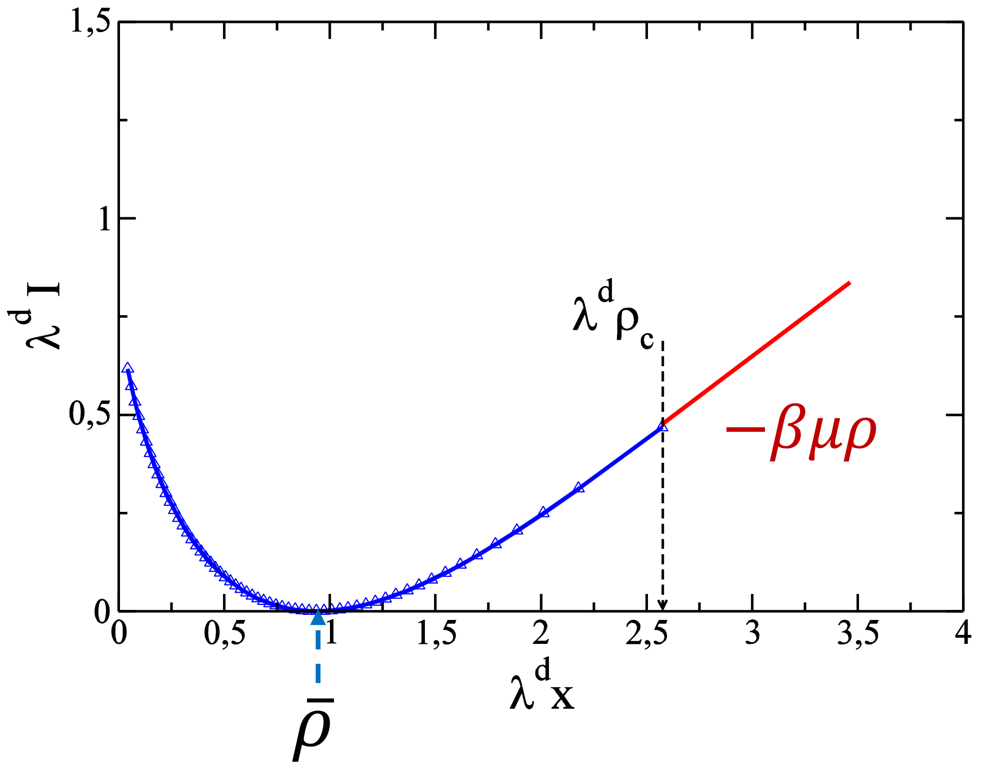
$$P_{gc}(N|\beta,z,V) \sim e^{-V I_{gc}(\rho|\beta,z)}$$

$$I_{gc}(\rho|\beta,z) = \beta\left[-\mu\rho + f_C(\beta,\rho) - f_{gc}(\beta,z)\right]$$



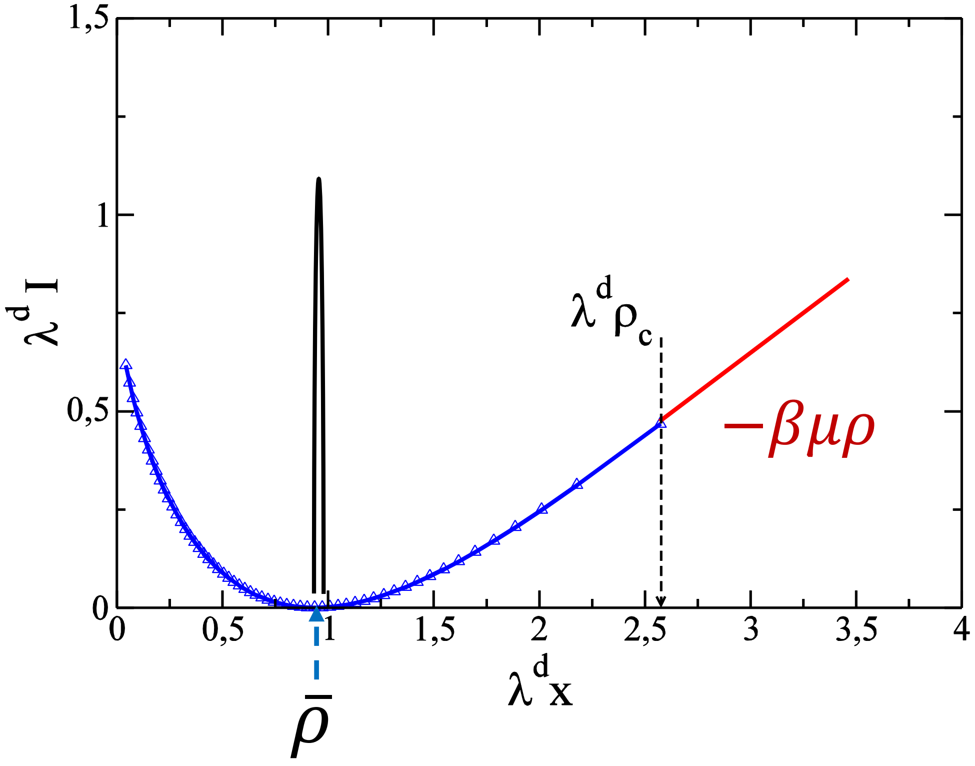
$$P_{gc}(N|\beta,z,V) \sim e^{-V I_{gc}(\rho|\beta,z)}$$

$$I_{gc}(\rho|\beta,z) = \beta\left[-\mu\rho + f_C(\beta,\rho) - f_{gc}(\beta,z)\right]$$



condensation of fluctuations:

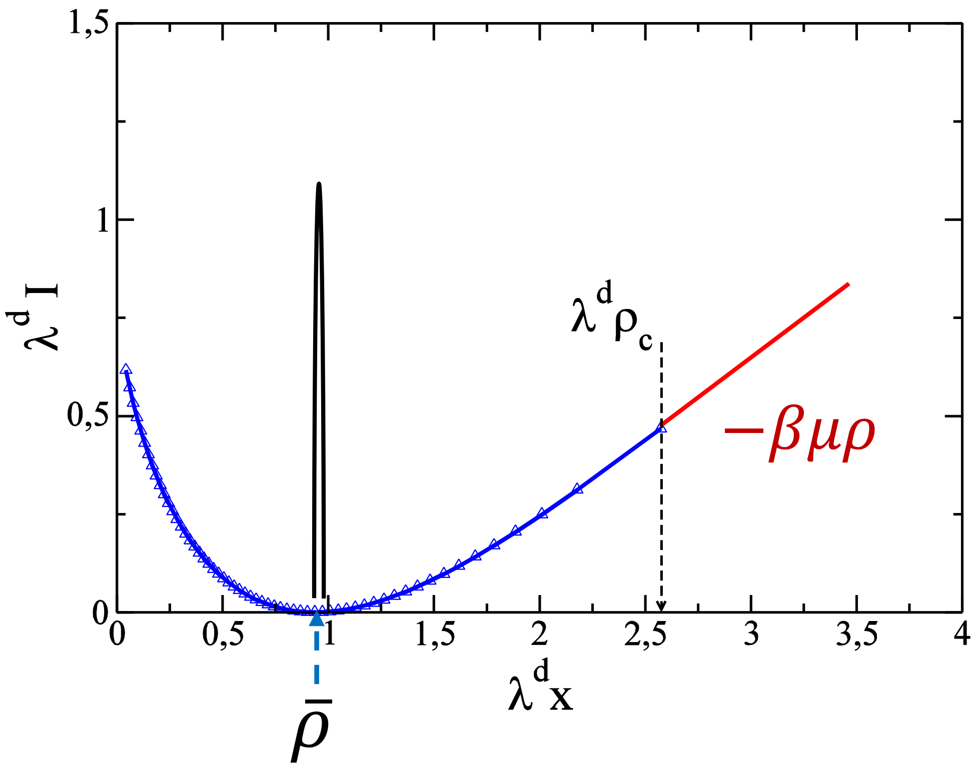
<u>macrofluctuations</u> exceeding $\bar{\rho}_C$ occur only through the <u>single</u> microvariable n_0



$$\lim_{V\to\infty} P_{gc}(\rho) = \delta(\rho - \bar{\rho})$$

condensation of fluctuations

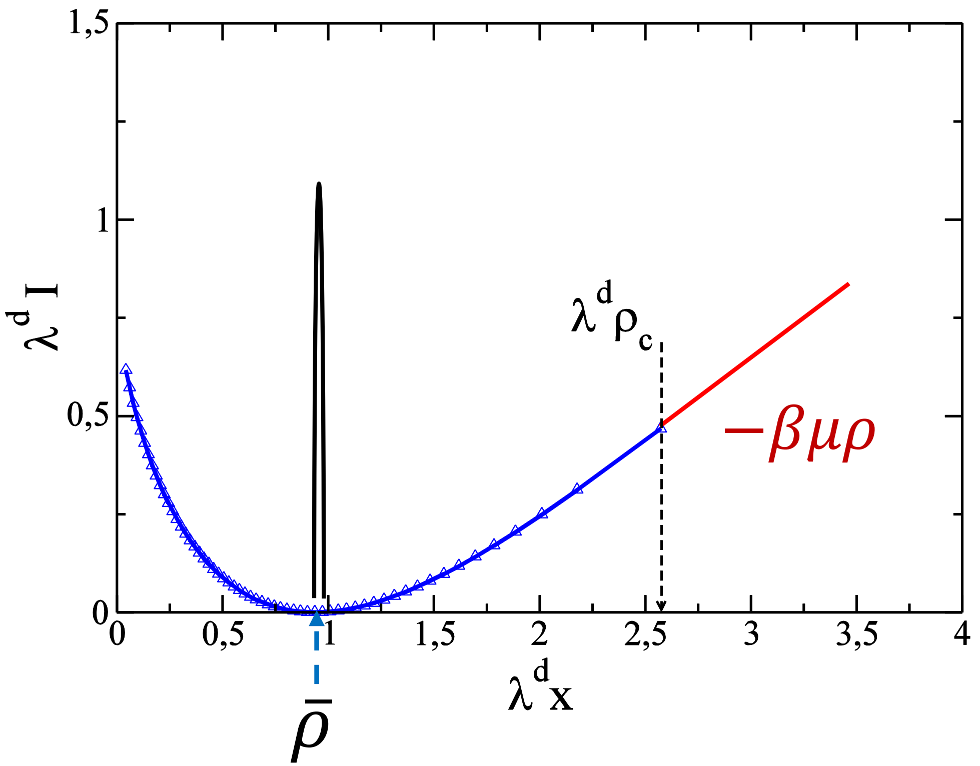
- *rare event*
- ***how to observe it?***



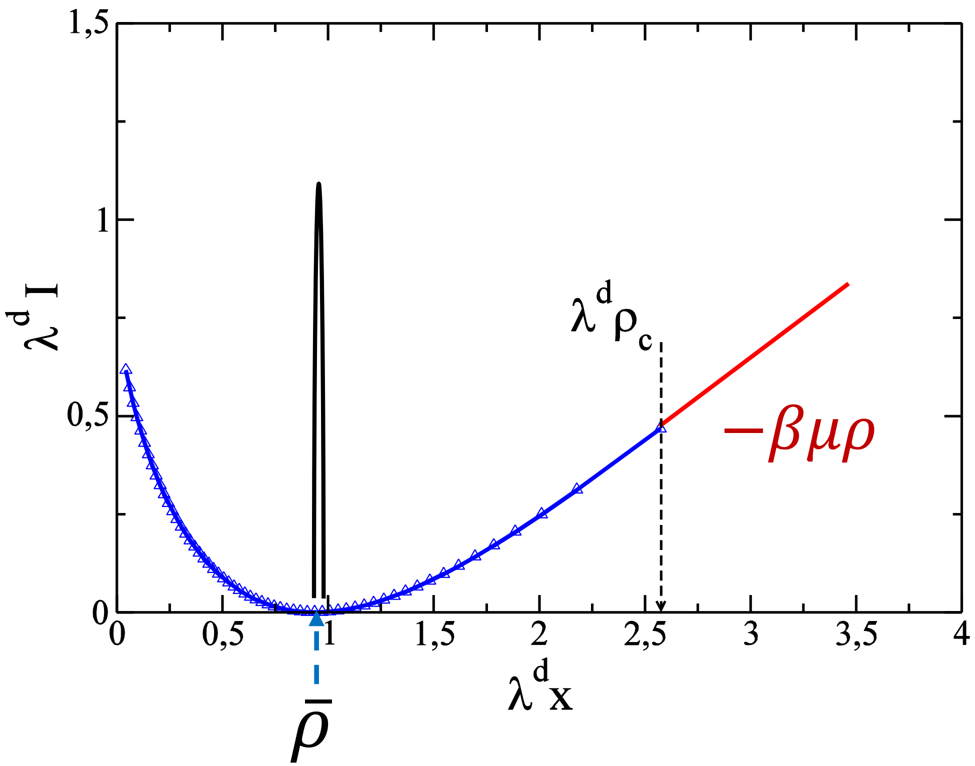
$$\lim_{V\to\infty} P_{gc}(\rho) = \delta(\rho - \bar{\rho})$$

condensation of fluctuations

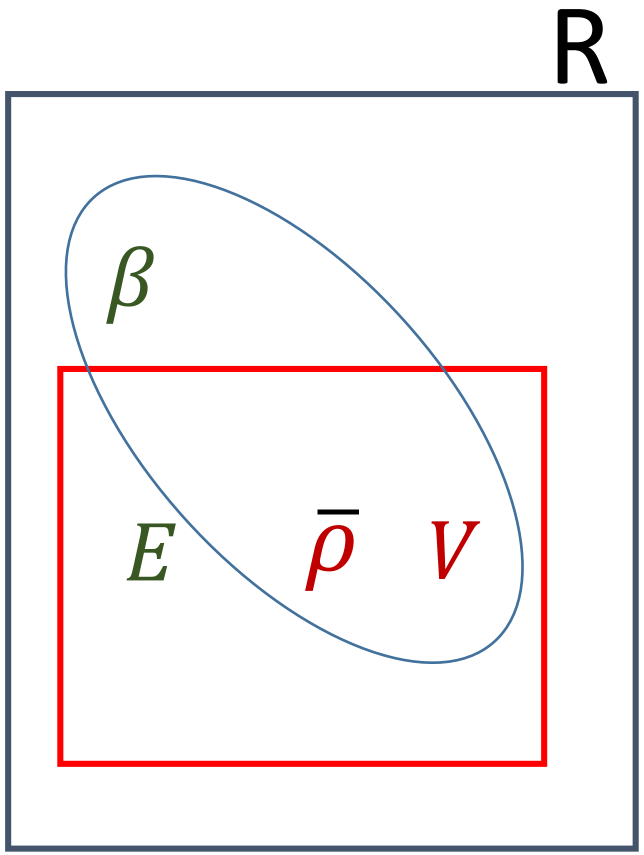
- *rare event*
- ***how to observe it?***
- *μ controls the slope*



invert $\bar{\rho}(\beta, z, V)$ *and use*
$\bar{\rho}$ *as control parameter:*
no restrictions on its value!

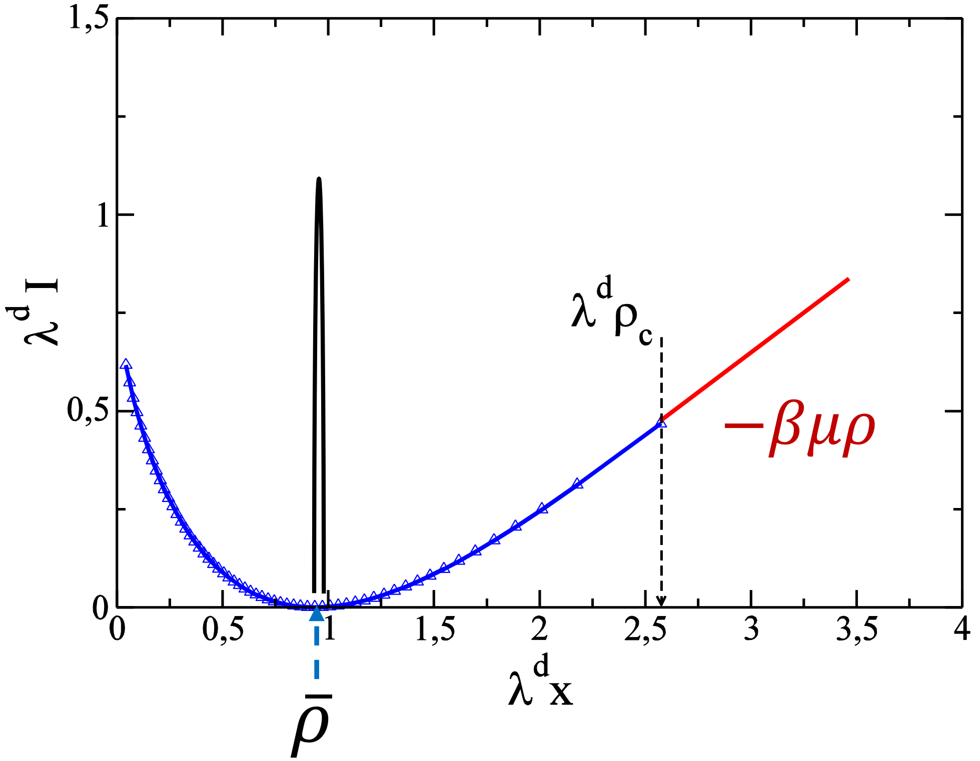


invert $\bar{\rho}(\beta, z, V)$ *and use*
$\bar{\rho}$ *as control parameter:*
no restrictions on its value!



Beware! Ensemble is changed to
MEAN CANONICAL ENSEMBLE

$$J = (\beta, z, V) \rightarrow (\beta, \bar{\rho}, V)$$

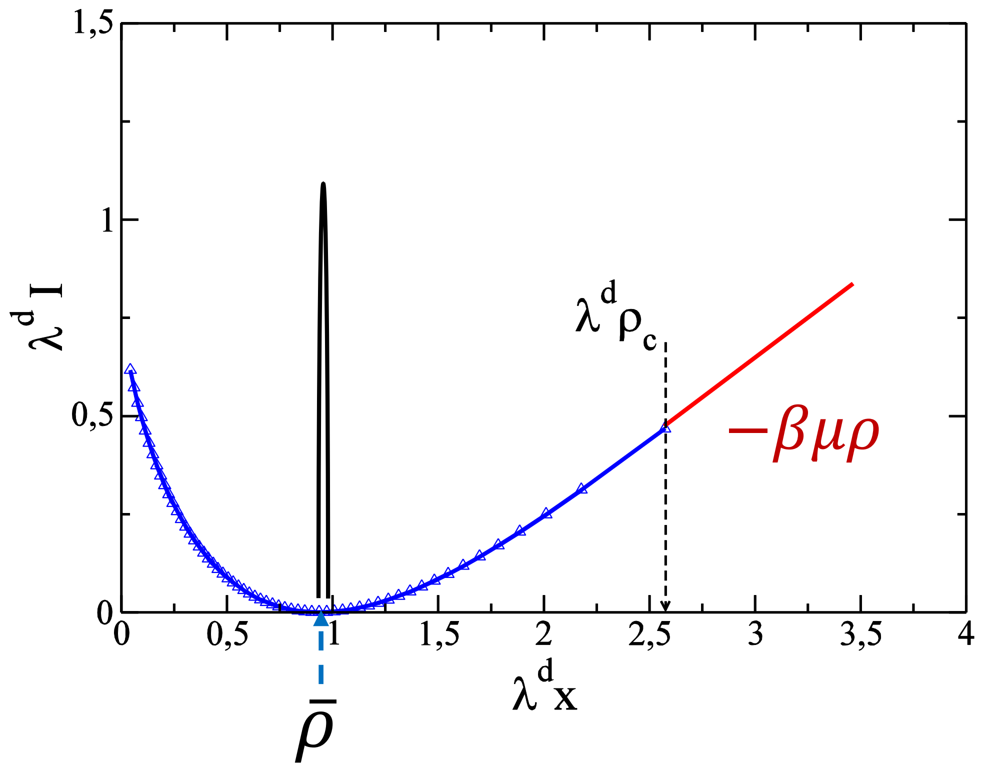


invert $\bar{\rho}(\beta, z, V)$ *and use*
$\bar{\rho}$ *as control parameter:*
no restrictions on its value!

$$\mu(\bar{\rho}) \sim const, \qquad \bar{\rho} < \bar{\rho}_C$$

$$\mu(\bar{\rho}) \sim -V^{-2/d}, \qquad \bar{\rho} = \bar{\rho}_C$$

$$\mu(\bar{\rho}) \sim -1/V[\bar{\rho} - \bar{\rho}_C], \qquad \bar{\rho} > \bar{\rho}_C$$



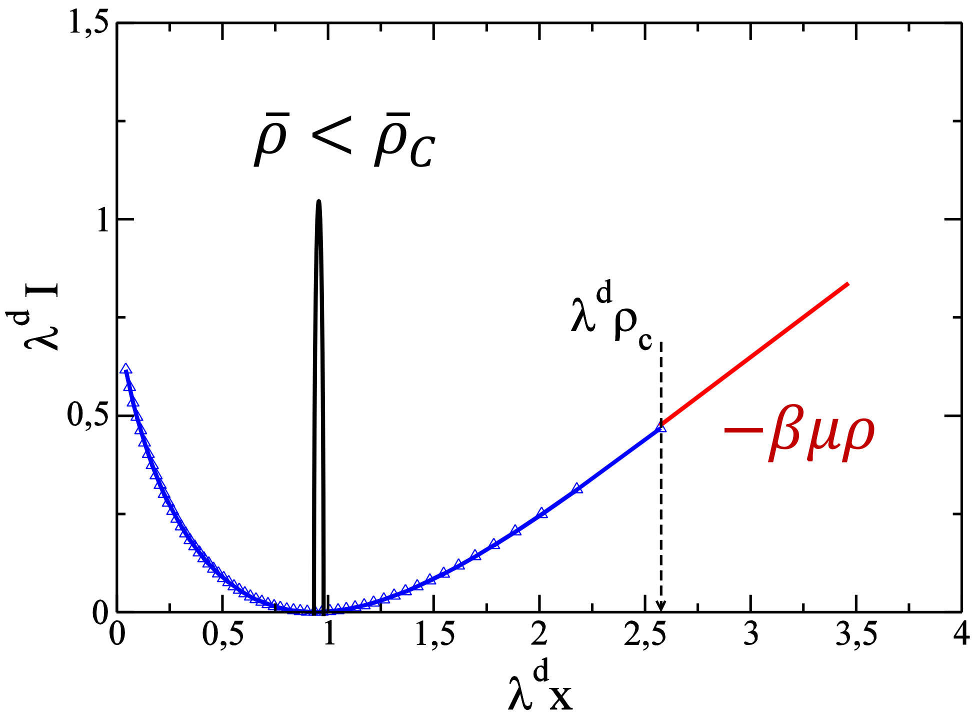
invert $\bar{\rho}(\beta, z, V)$ *and use*
$\bar{\rho}$ *as control parameter:*
no restrictions on its value!

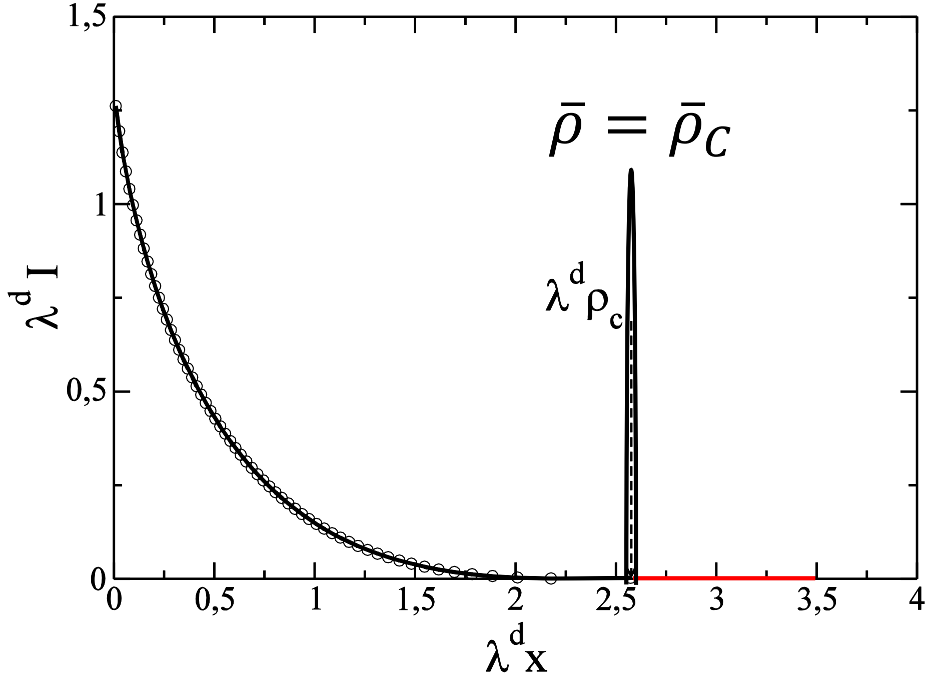
$$\mu(\bar{\rho}) \sim const, \qquad \bar{\rho} < \bar{\rho}_C$$

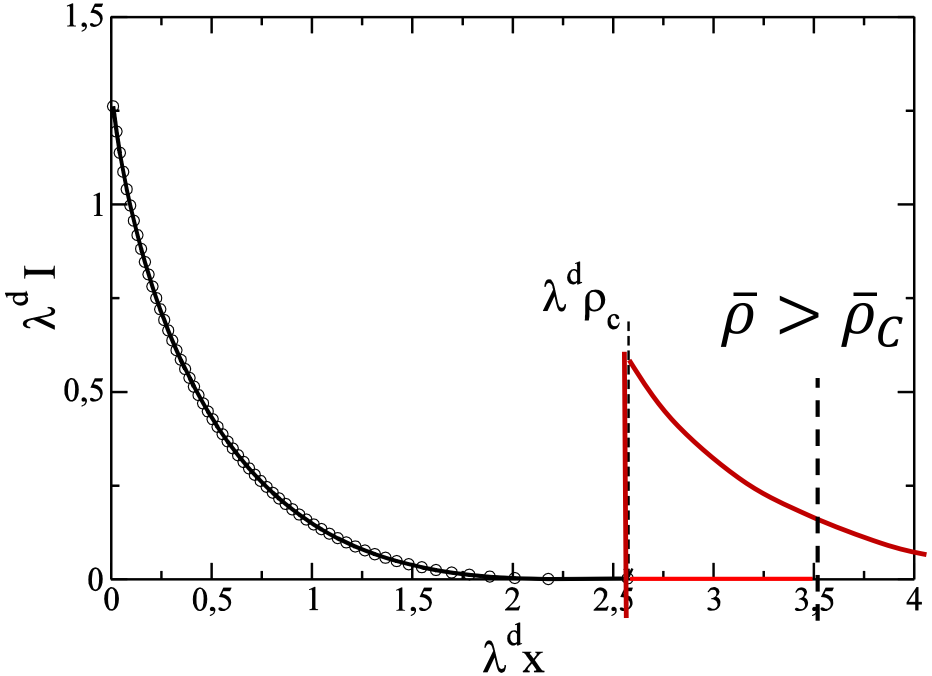
$$\mu(\bar{\rho}) \sim -V^{-2/d}, \quad \bar{\rho} = \bar{\rho}_C$$

$$\mu(\bar{\rho}) \sim -1/V[\bar{\rho} - \bar{\rho}_C], \quad \bar{\rho} > \bar{\rho}_C$$

$$P_{gc}(N|\beta, z, V) \sim e^{-V I_{gc}(\rho|\beta, z)}$$







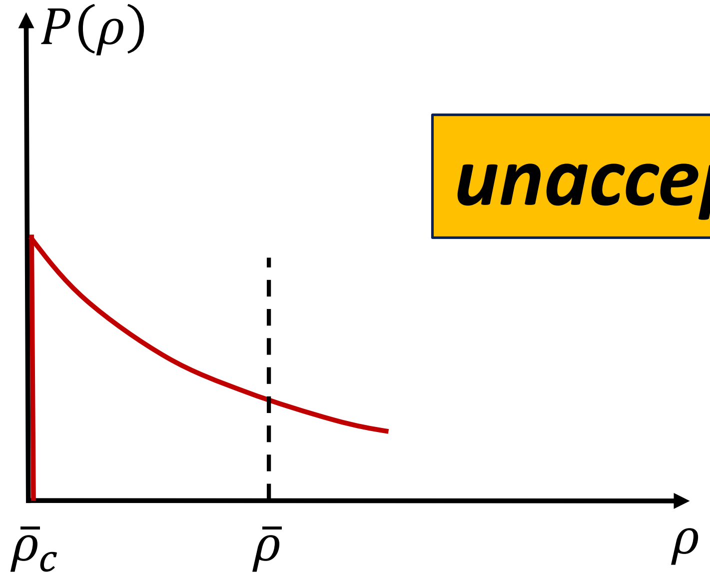
$$P_0(\rho) = \frac{exp\left(-\frac{\rho - \bar{\rho}_c}{\bar{\rho} - \bar{\rho}_c}\right)}{\bar{\rho} - \bar{\rho}_c}$$

overdoing it?

Problem: **fluctuations are HUGE**

$$\langle \delta n^2 \rangle \sim V^2$$

worst: persist down to $T = 0$*!*



unacceptable!

GRAND CANONICAL CATASTROPHE

$$\langle \delta n^2 \rangle \sim V^2$$

I. Fujiwara et al. «exact but unphysical» (J.Stat.Phys. 1970)

M. Holthaus et al. «serious failure of GCE…unacceptable; when all particles occupy the ground state, the fluctuations has to die out», (Ann. of Phys. 1998)

M. Wilkens et al. «..the vagueries of number fluctuations…it is the grand canonical ensemble which is flawed..»

L.Pitaevskii and S.Stringari «...pathological feature of the GCE..» (Oxford University Press 2003)

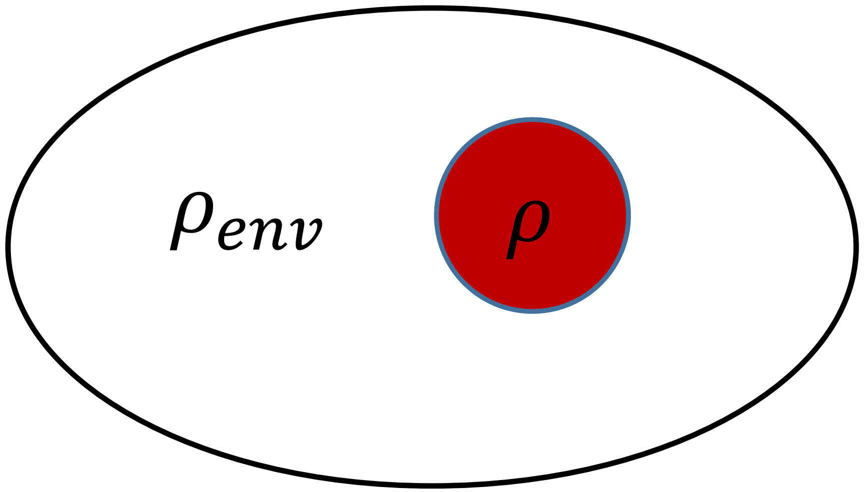
GRAND CANONICAL CATASTROPHE

$$\langle \delta n^2 \rangle \sim V^2$$

I. Fujiwara et al. «exact but unphysical» (J.Stat.Phys. 1970)

It means that there is nothing wrong with the derivation, but the physical conditions for the MCE are unrealizable

R. M. Ziff, G. E. Ulhenbeck and M. Kac,
Physics Reports **4**, 169 (1977)



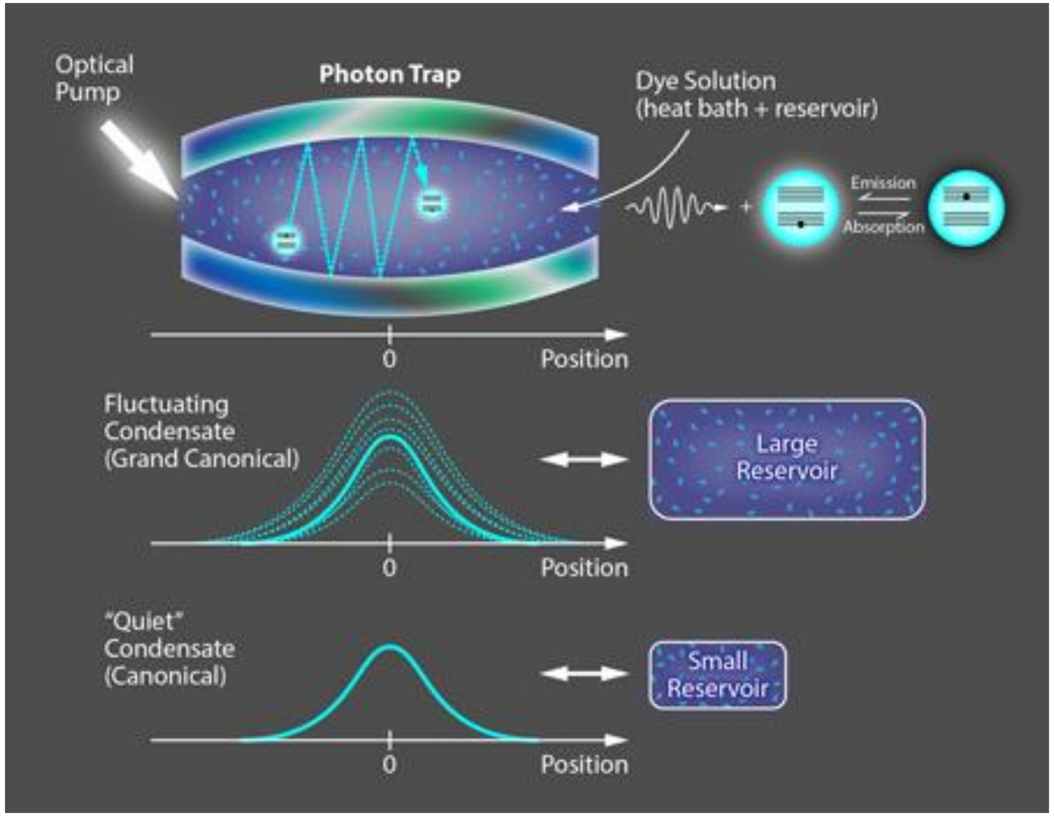
no grand canonical catastrophe!

$$P(\rho|\rho_{env}) = \delta(\rho - \rho_{env})$$

mean canonical conditions in the lab

Klaers et al., *Bose-Einstein condensation of photons in an optical microcavity* (Nature 2010)

Schmitt et al., *Observation of the Grand Canonical Number Statistics in a Photon Bose-Einstein Condensate*, (PRL 2014)



Selected for a Viewpoint in *Physics*
PRL **112,** 030401 (2014) PHYSICAL REVIEW LETTERS week ending 24 JANUARY 2014

Observation of Grand-Canonical Number Statistics in a Photon Bose-Einstein Condensate

Julian Schmitt,* Tobias Damm, David Dung, Frank Vewinger, Jan Klaers,† and Martin Weitz
Institut für Angewandte Physik, Universität Bonn, Wegelerstraße 8, 53115 Bonn, Germany
(Received 25 October 2013; published 21 January 2014; publisher error corrected 22 January 2014)

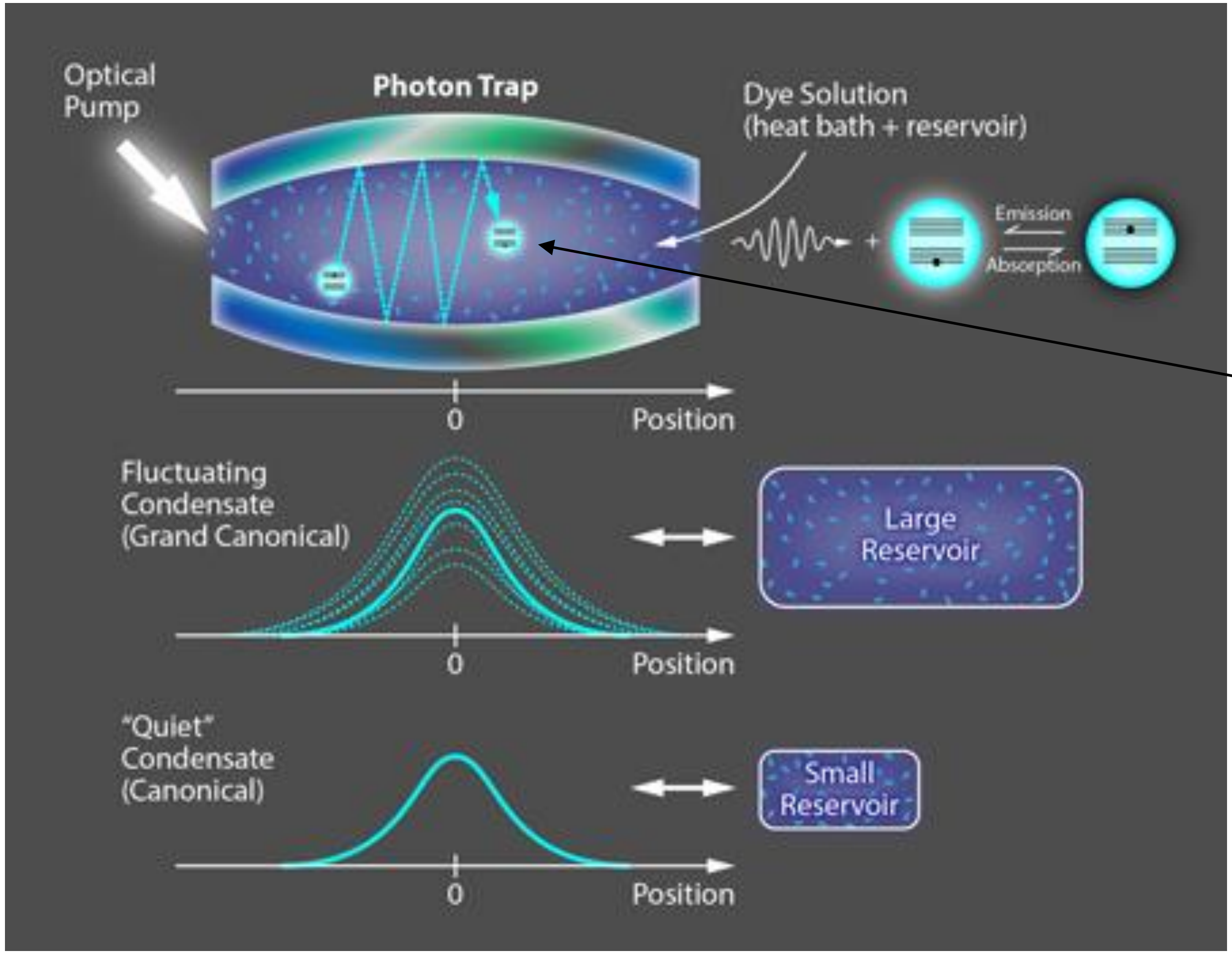
We report measurements of particle number correlations and fluctuations of a photon Bose-Einstein condensate in a dye microcavity using a Hanbury Brown-Twiss experiment. The photon gas is coupled to a reservoir of molecular excitations, which serve as both heat bath and particle reservoir to realize grand-canonical conditions. For large reservoirs, we observe strong number fluctuations of the order of the total particle number extending deep into the condensed phase. Our results demonstrate that Bose-Einstein condensation under grand-canonical ensemble conditions does not imply second-order coherence.

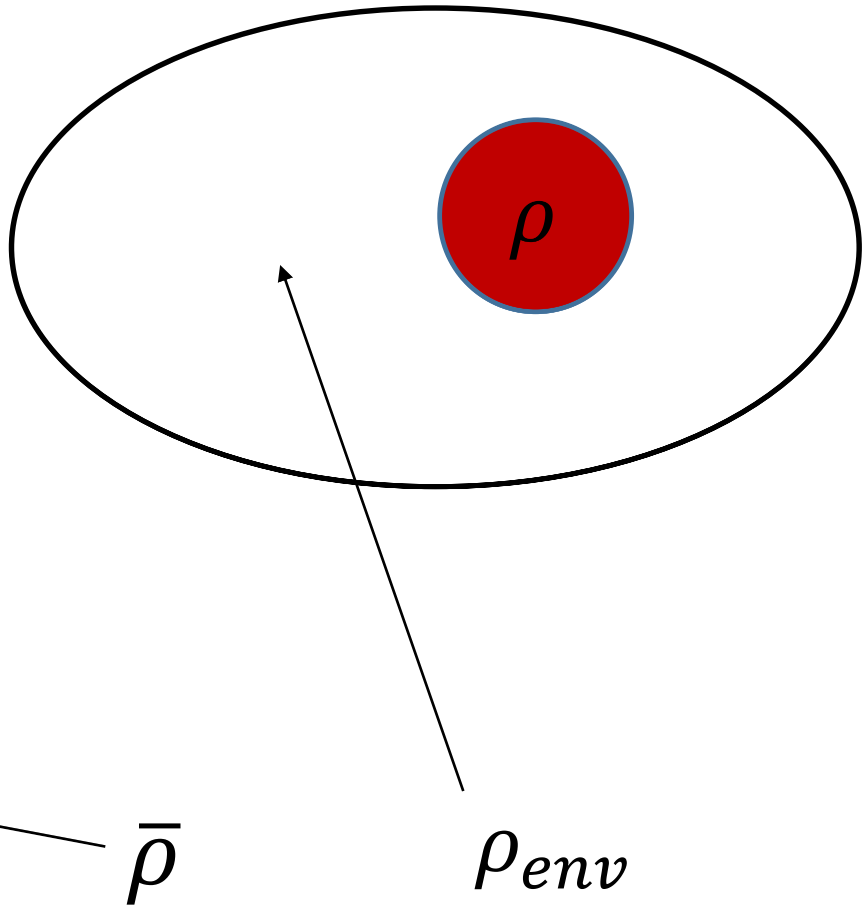
DOI: 10.1103/PhysRevLett.112.030401 PACS numbers: 03.75.Hh, 42.50.Ar, 67.85.Hj

Large statistical number fluctuations are a fundamental

with energy and particle number strictly fixed, while in the

R. M. Ziff, G. E. Ulhenbeck and M. Kac,
Physics Reports **4**, 169 (1977)

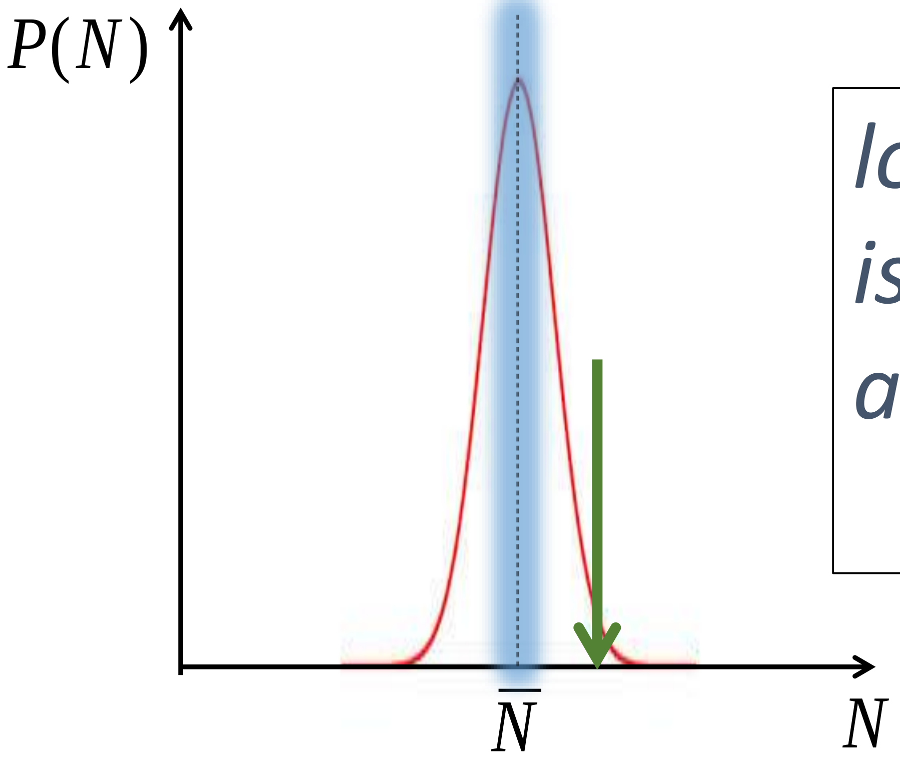




different control parameters
$\Rightarrow$ *different ensembles*

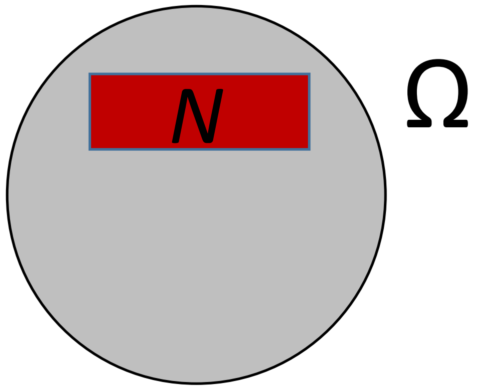
CONCLUSIONS

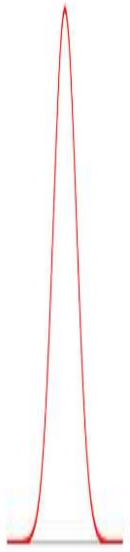
- *as a consequence of duality fluctuations may show non trivial behavior even in non interacting system*
- *the grand canonical catastrophe is condensation of fluctuations made observable*
- *The lab realization of the MCE reveals extreme sensitivity of the statistical ensemble to control parameters*

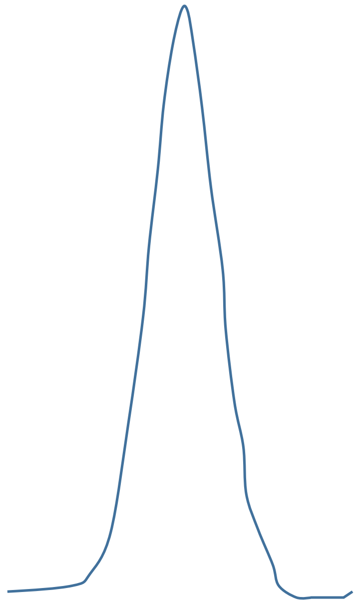


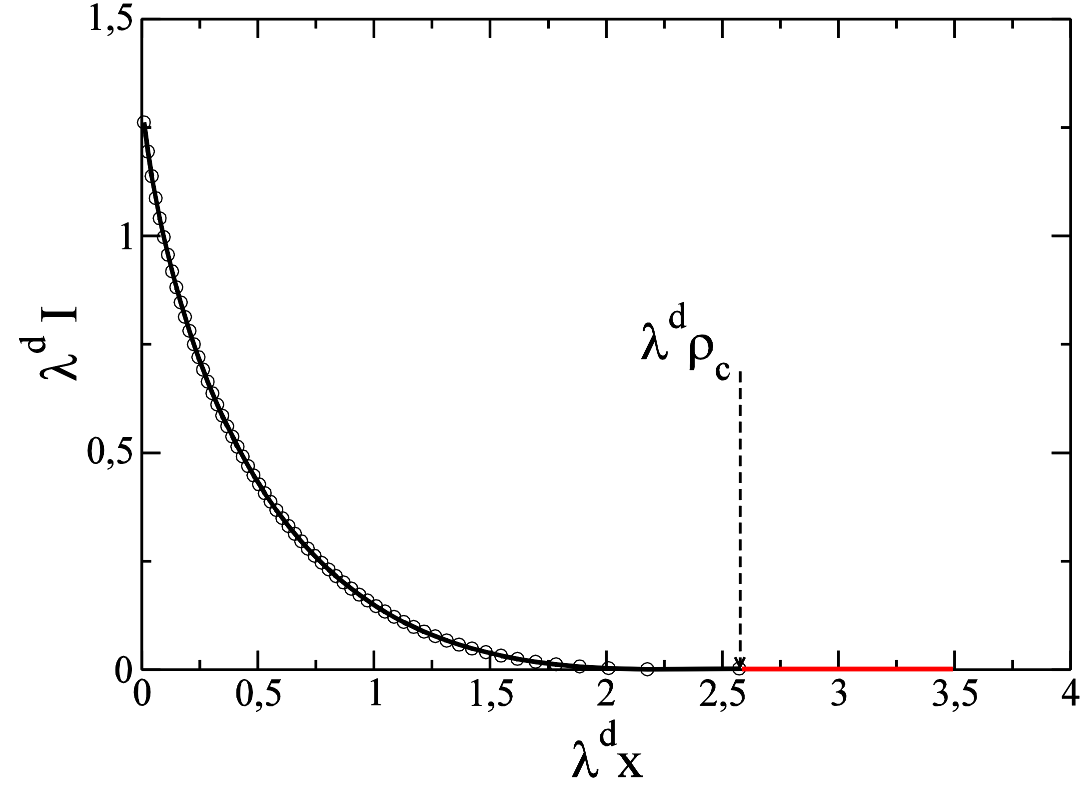
looking to a fluctuation
is ***equivalent*** *to put*
a constraint on the system

$$\mathcal{N}(\omega) = N$$



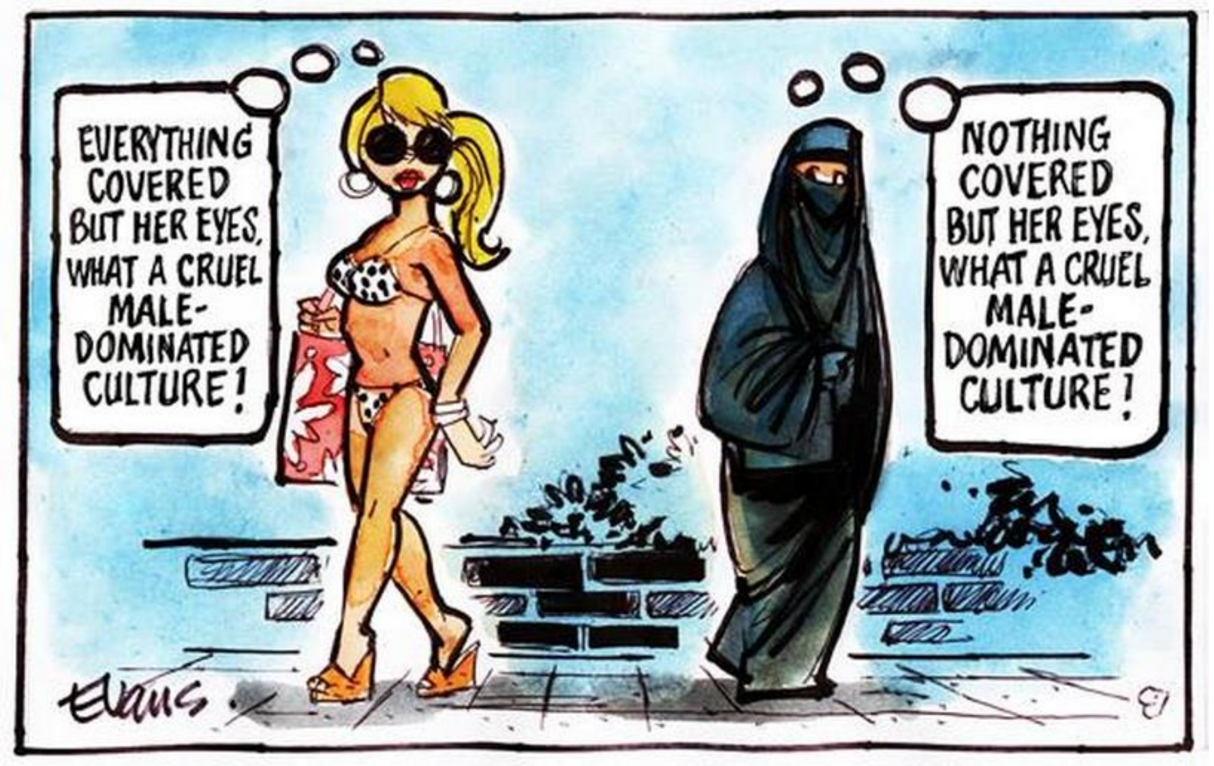






1,5
1
0,5
0
0
0,5
1
1,5
2
2,5
3
3,5
4
$\lambda^d I$
$\lambda^d x$
$\lambda^d \rho_c$

oddity and normality are «cultural» concepts



What is strange for someone is normal for someone else