SUPERFLUIDS OF CHARGED MESONS

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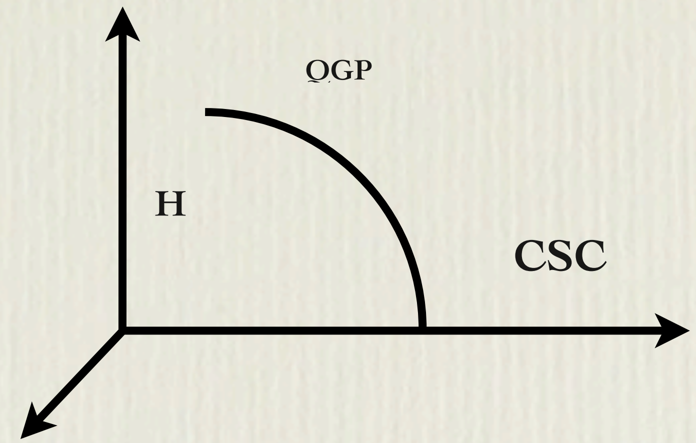
A. Mammarella and M.M. Phys.Rev. D92 (2015) 8, 085025

SMFT Dec 2015

OUTLINE

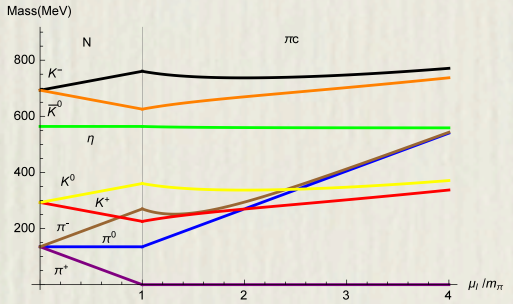
- ***General motivations***



- ***Chiral perturbation theory***

$$\mathcal{L} = \frac{F_0^2}{4}\mathrm{Tr}(D_\nu \Sigma D^\nu \Sigma^\dagger) + \frac{F_0^2}{4}\mathrm{Tr}(X\Sigma^\dagger + \Sigma X^\dagger)$$

- ***Condensation***



- ***Conclusions***

BASIC LITERATURE

- **Shapiro and Teukolsky** ***"Black holes, white dwarfs, and neutron stars"***

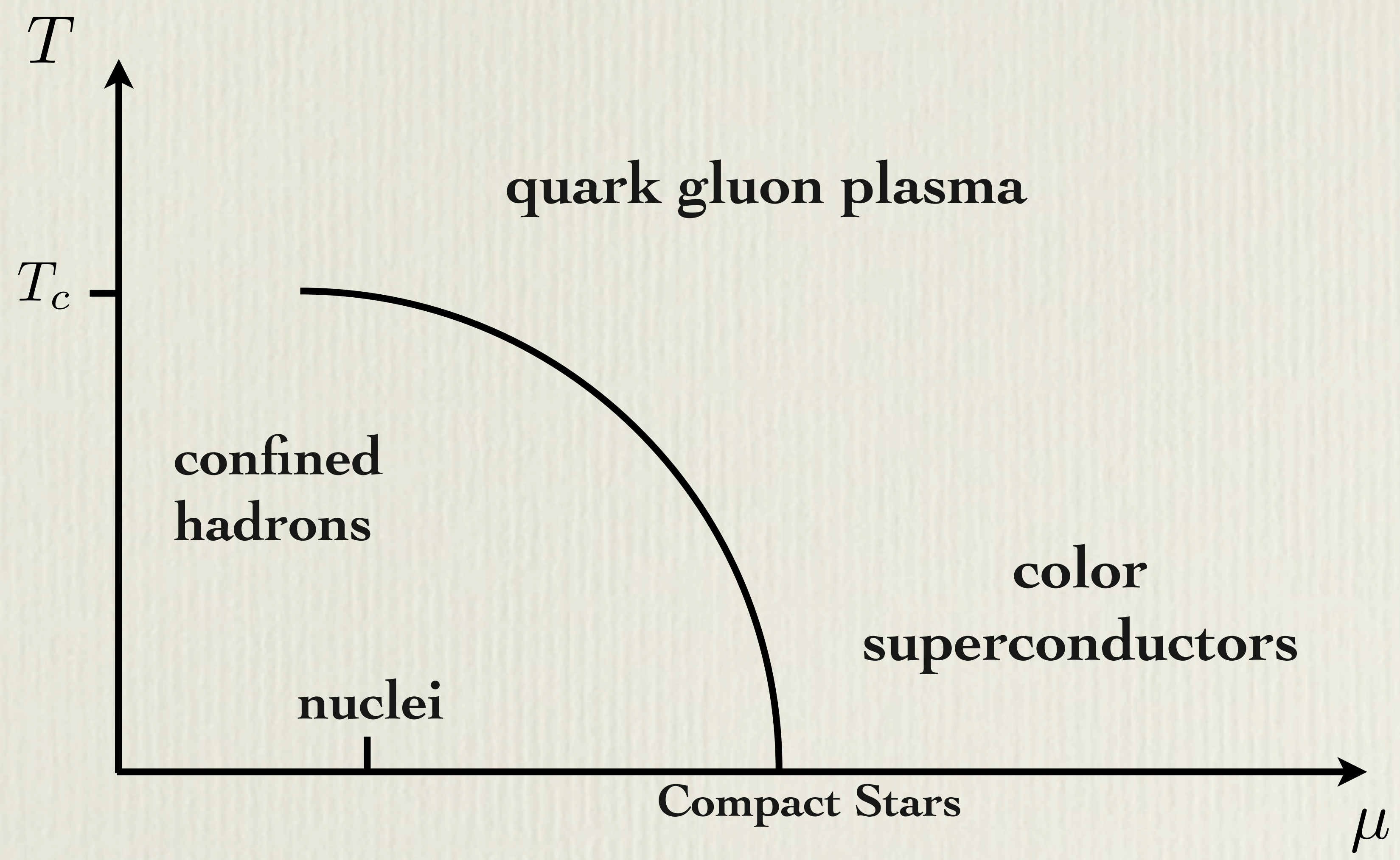
- **Son and Stephanov,** ***``QCD at finite isospin density'' PRL 86, 592 (2001)***

- **Kogut and Toublan,** ***``QCD at small nonzero quark chemical potentials'' PRD 64, 34007, (2001)***

- **Barducci, Casalbuoni, Pettini and Ravagli,** ***"Pion and kaon condensation in a 3-flavor NJL model", PRD 71, 016011, (2005).***

GENERAL MOTIVATIONS

PHASES OF MATTER



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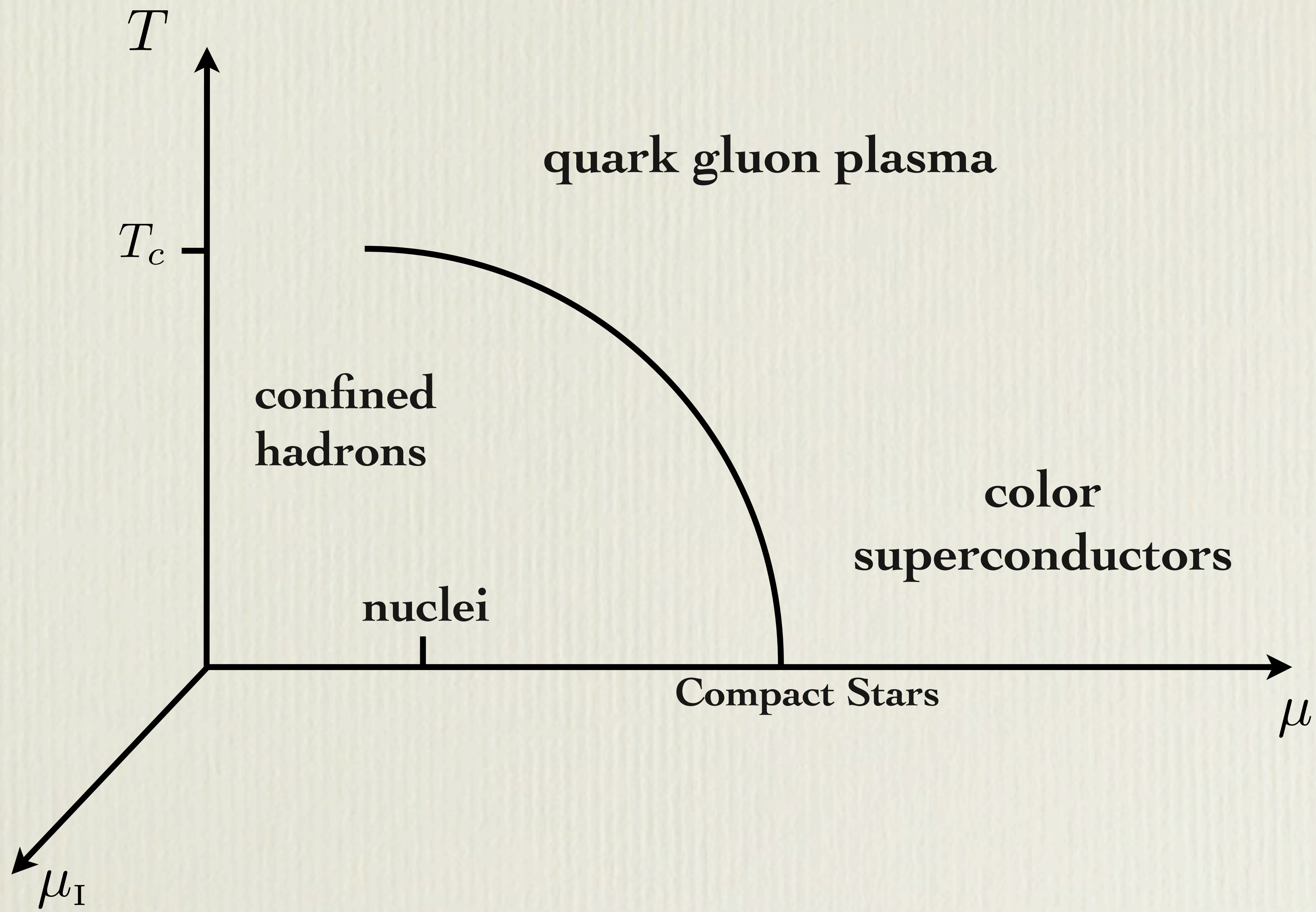

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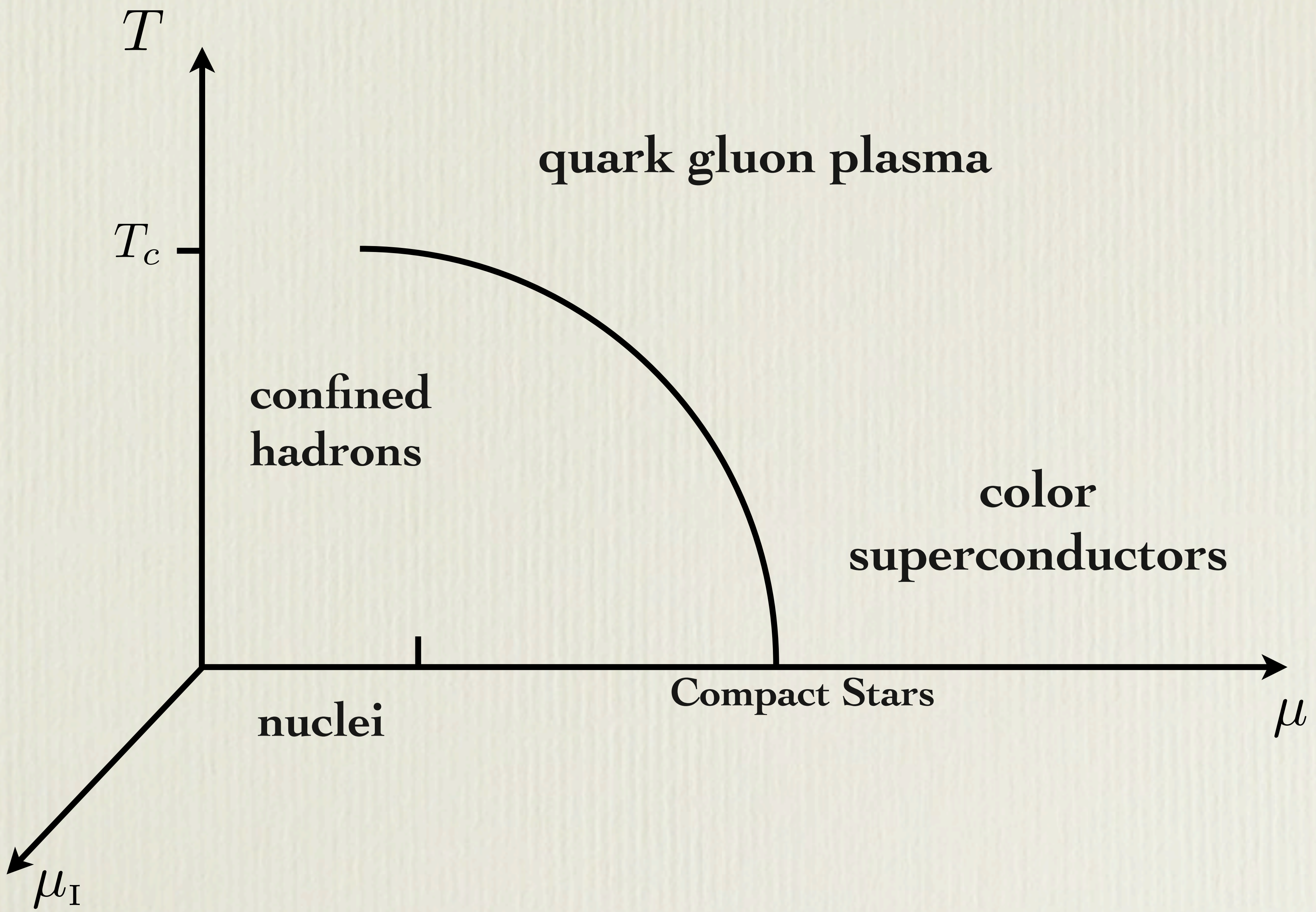

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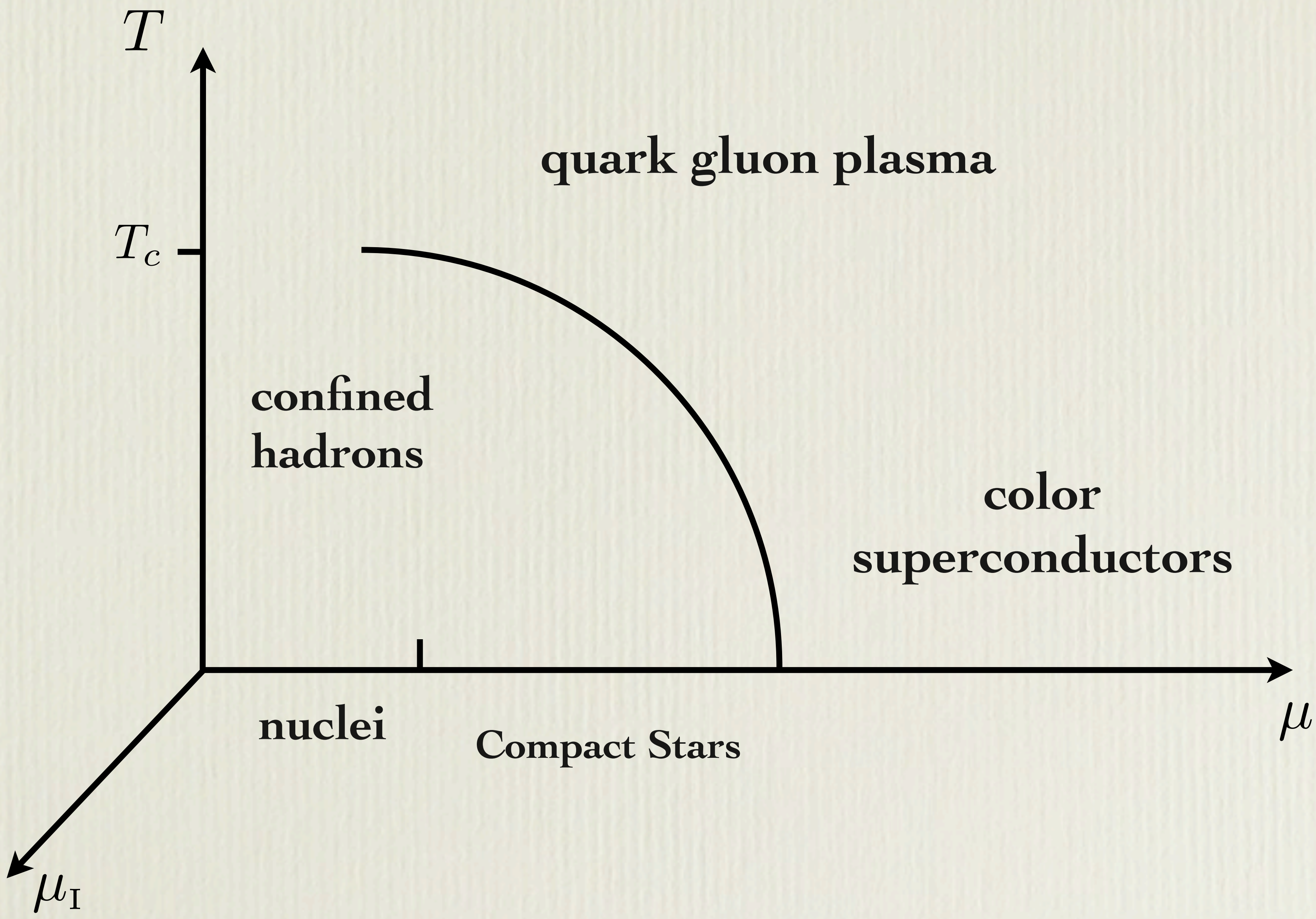

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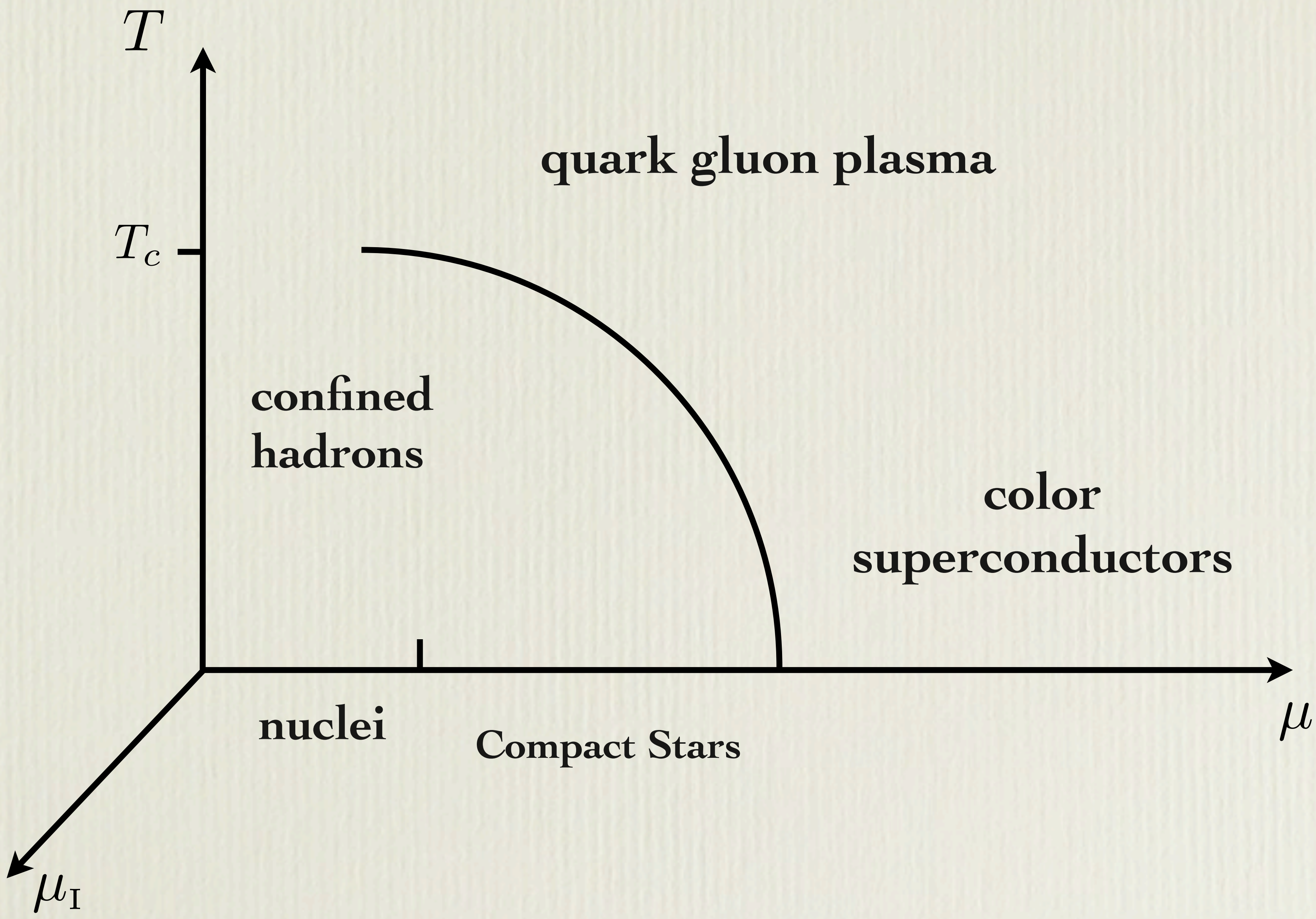

Matter in heavy nuclei and in compact stars is NOT isospin symmetric

TOWARD REALITY

The grandcanonical description of **heavy hadronic matter** requires the inclusion of μ_{I}

Our goal: description of nuclear interactions in an isospin asymmetric environment

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What for?

Decay processes in nuclear matter

Nuclear matter relevant for for compact stars

Chiral perturbation theory (ChPT)

Expanding

At non-asymptotic energy scales QCD is a nonperturbative theory

Chiral perturbation theory is an effective field theory: A realization of hadronic matter at soft energy scales

$$p \ll \Lambda \sim 1\,\mathrm{GeV}$$

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Qualitative picture: We assume to know (or to have a big deal of information about) the nonperturbative vacuum and we "expand" around that vacuum.

Since we are expanding, we do have control parameters

The guiding principles are symmetries. For massless quarks

$$G_{\mathrm{QCD}} = \underbrace{SU(N_f)_L \times SU(N_f)_R \times U(1)_B}_{\supset [U(1)_{\mathrm{e.m.}}]}$$

For describing the mesonic multiplets, we use the fact that some global symmetries of QCD are spontaneously broken

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$$\underbrace{SU(N_f)_L \times SU(N_f)_R \times U(1)_B}_{\supset [U(1)_{\rm e.m.}]} \to \underbrace{SU(N_f)_V \times U(1)_B}_{\supset [U(1)_{\rm e.m.}]}$$

The $N_f^2 - 1$ broken generators correspond to the (pseudo) Nambu-Goldstone bosons identified with the **pseudoscalar mesons**

Expanding

fluctuation

$SU(N_f)$ generators

$$\Sigma = u\bar{\Sigma}u \qquad \text{with} \qquad u = e^{iT\cdot\phi/2}$$

vacuum

mesonic multiplet

The $\mathcal{O}(p^2)$ Lorentz invariant Lagrangian density for pseudoscalar mesons

$$\boxed{\mathcal{L} = \frac{F_0^2}{4}\mathrm{Tr}(D_\nu\Sigma D^\nu\Sigma^\dagger) + \frac{F_0^2}{4}\mathrm{Tr}(X\Sigma^\dagger + \Sigma X^\dagger)}$$

We have introduced the external currents to take into account inmedium propagation, formally preserving the Lorentz invariance

Expanding (Here $N_f = 2$)

$$\phi \sim (\pi_+, \pi_-, \pi_0)$$

$$D_\mu \Sigma = \partial_\mu \Sigma - \frac{i}{2}[v_\mu, \Sigma] - \frac{i}{2}\{a_\mu, \Sigma\} \qquad X = 2B_0(s + ip)$$

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external currents

$$a_\mu = 0, \qquad p = 0, \qquad s \sim M$$

$$v^\mu = -2eQA^\mu - 2\mu\delta^{\mu 0} = -\tilde{A}^\mu_I I - \tilde{A}^\mu_3 \sigma_3$$

$$\mu = \mathrm{diag}(\mu_u, \mu_d) = \frac{\mu_B}{3} + \frac{\mu_I \sigma_3}{2}$$

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$$\tilde{A}_I^\mu = \frac{1}{3}(eA_0 + \mu_B, e\boldsymbol{A})$$

$$\tilde{A}_3^\mu = (eA_0 + \mu_I, e\boldsymbol{A})$$

Does not couple to mesons

Increasing the isospin charge

Mass splitting
proporional to the isospin charge

$$m_{\pi^0} = m_\pi$$
$$m_{\pi^-} = m_\pi + \mu_I$$
$$m_{\pi^+} = m_\pi - \mu_I$$

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Most general vev

$$\bar{\Sigma} = e^{i\boldsymbol{\alpha}\cdot\boldsymbol{\sigma}} = \cos\alpha + i\boldsymbol{n}\cdot\boldsymbol{\sigma}\sin\alpha$$

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Lagrangian at the vev

$$\mathcal{L}_0(\alpha, n_3, \tilde{A}^\mu) = F_0^2 m_\pi^2 \cos\alpha + \frac{F_0^2}{2}\sin^2\alpha \tilde{A}_3^\mu \tilde{A}_{3\mu}(1 - n_3^2)$$

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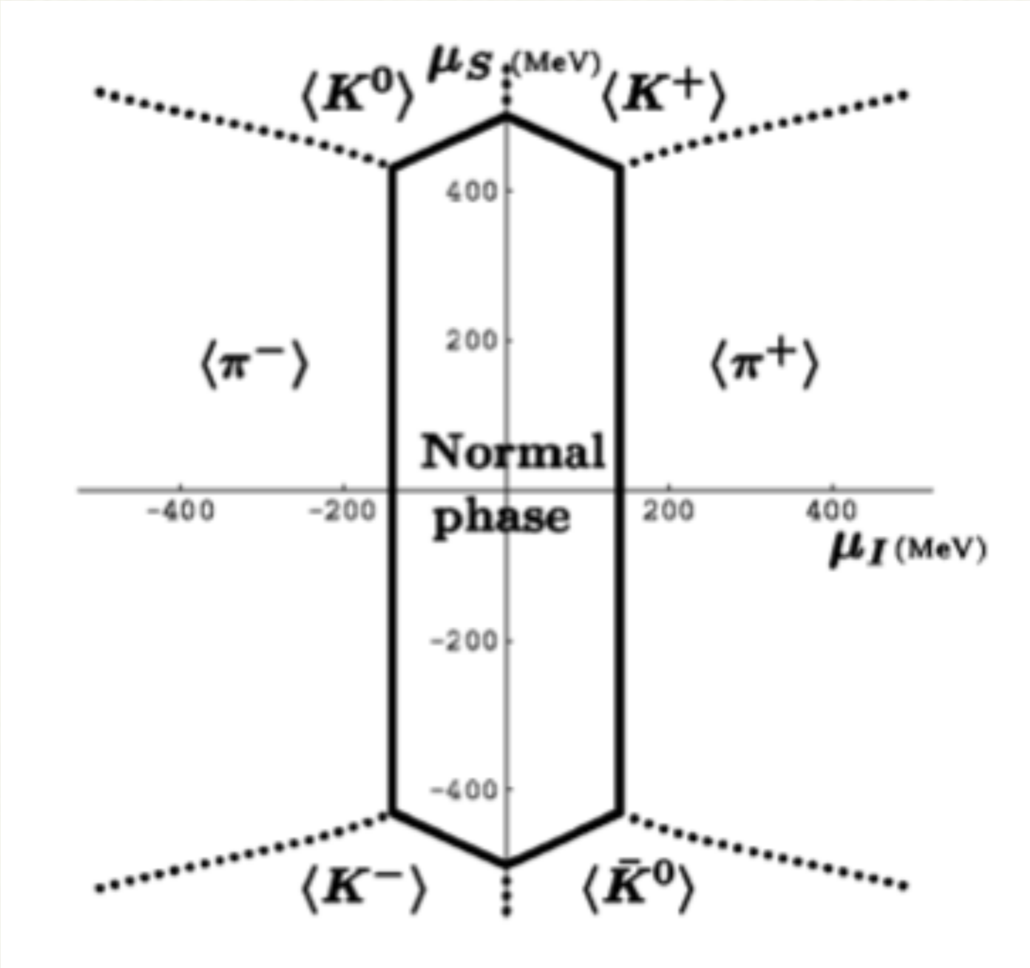
for $\mu_I < m_\pi$ $\quad \cos\alpha = 1$ $\quad$ $\mathcal{L}_0$ independent of $\boldsymbol{n}$

for $\mu_I > m_\pi$ $\quad \cos\alpha_\pi = m_\pi^2/\mu_I^2$ $\quad$ $n_3 = 0$ residual $O(2)$ symmetry

Condensation

Phase diagram

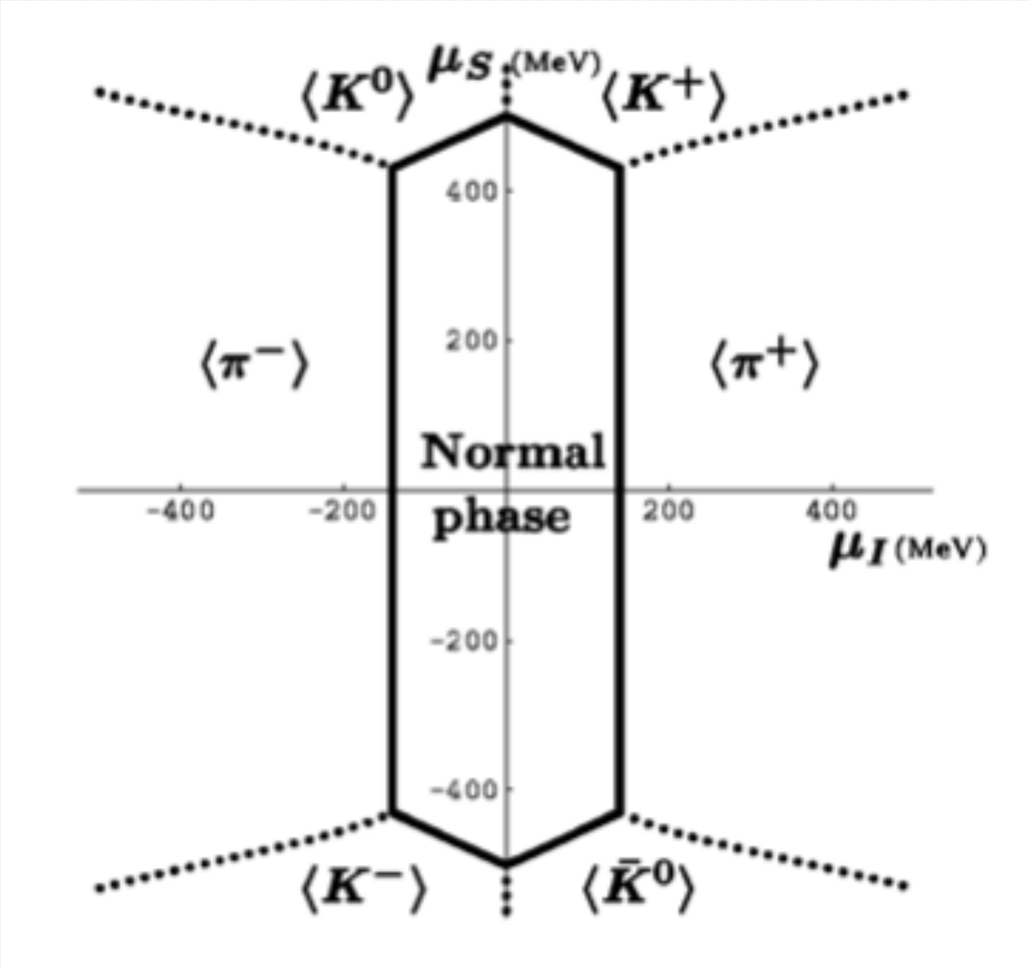


Kogut and Toublan PhysRevD.64.034007

solid line: second order

dotted line: first order

Phase diagram



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Kogut and Toublan PhysRevD.64.034007

In the condensed phases, a superfluid of charged bosons: a superconductor!

$$M_D^2 = M_M^2 = F_0^2 e^2 (\sin\alpha)^2$$

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Mixing and mass splitting

In the conensed phases mesons mix and have nontrivial mass splitting

$$\begin{pmatrix} \tilde{\pi}_+ \\ \tilde{\pi}_- \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix} \begin{pmatrix} \pi_+ \\ \pi_- \end{pmatrix}$$

mass eigenstates

charge eigenstates

$$U_{ij} \equiv U_{ij}(m_\pi, \mu_I, E)$$

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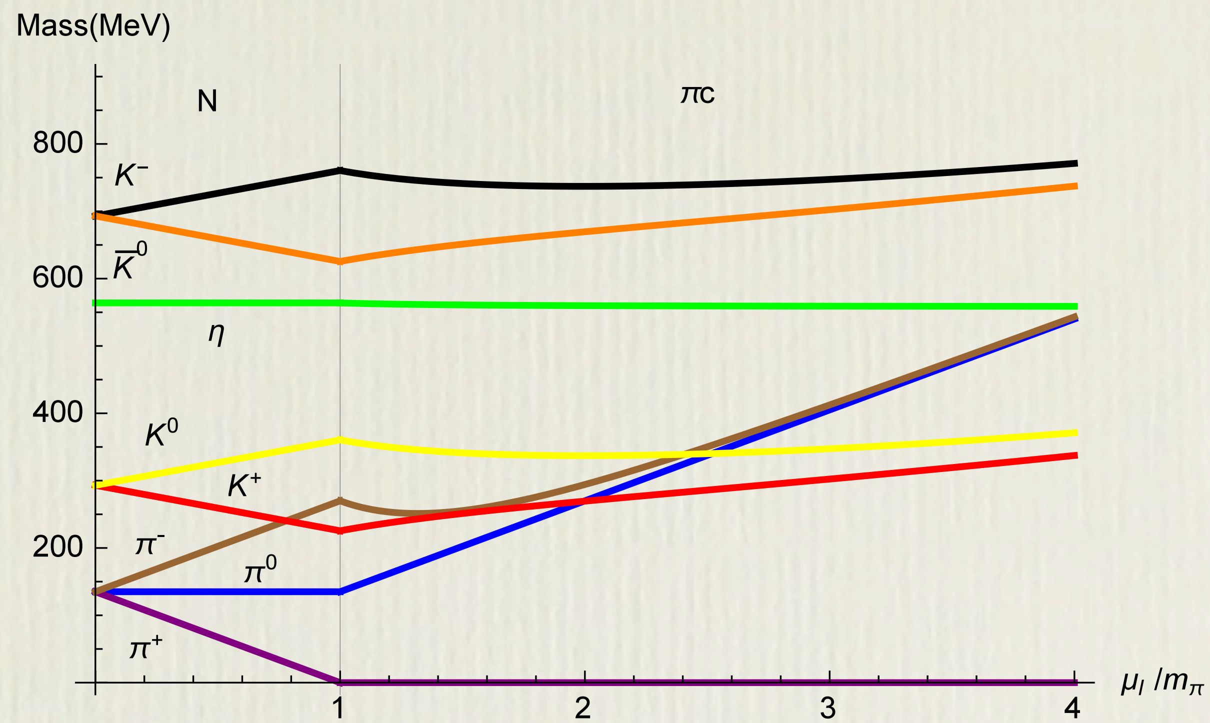

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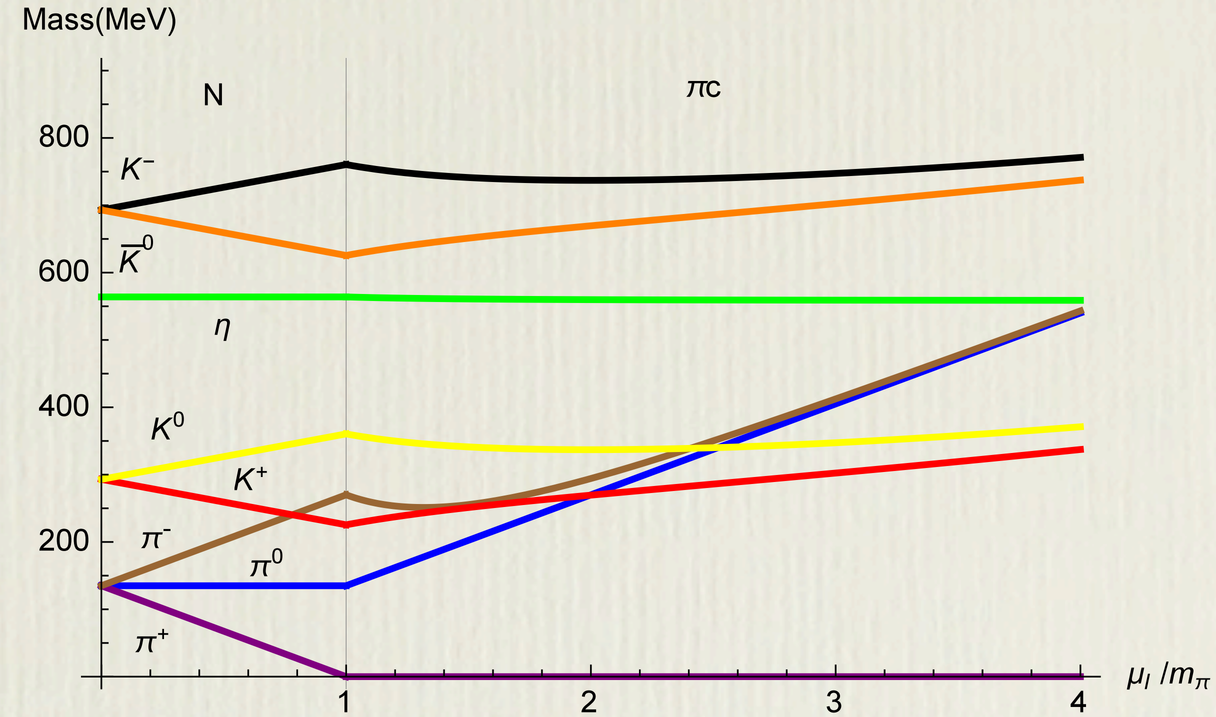

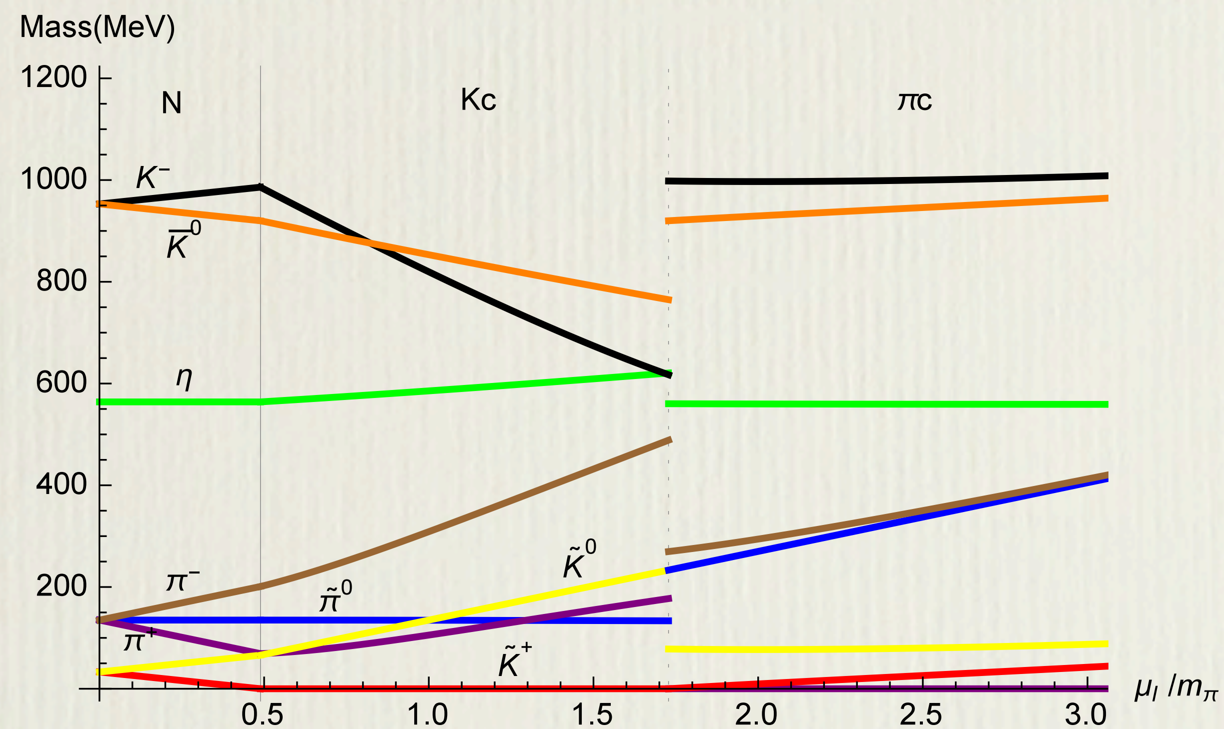

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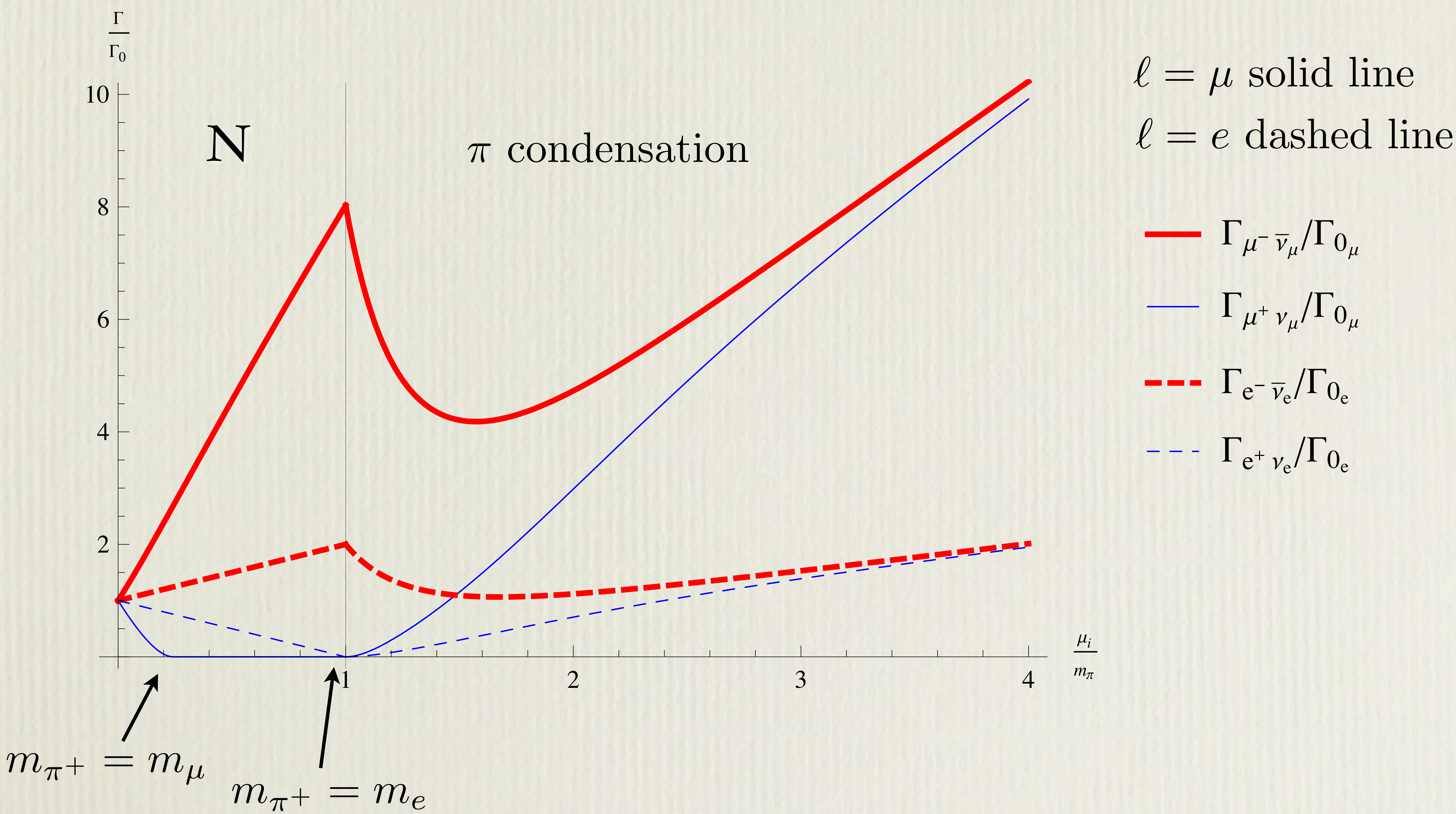
Leptonic decays

Processes $\tilde{\pi}_- \to \ell^{\pm}\nu_\ell$ and $\tilde{\pi}_+ \to \ell^{\pm}\nu_\ell$



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Conclusions

- The realistic conditions in heavy nuclei and in compact stars require a nonvanishing isospin chemical potential

- If isospin is broken nontrivial mass dependence

- In the condensed phase there is mixing and mesons have nontrivial masses and decay patterns

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Outlook

Studying nucleon strong interactions in isospin rich matter

A. Mammarella, M.M. and S. Carignano in preparation

Upcoming conference

Compact Stars in the QCD phase diagram V

23-27 May 2016 LNGS and GSSI L'Aquila, Italy

http://agenda.infn.it/event/compact-stars

- **Equation of state and QCD phase transitions**
- **QCD in astrophysics of compact stellar objects, supernovae and compact stars mergers**
- **Strangeness in Compact Stars**
- **Strange Stars**
- **Hadron production in heavy-ion collisions**
- **Nonequilibrium and transport phenomena in dense matter**

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LNGS and GSSI, L'Aquila (Italy), May 23rd-27th, 2016

Topics

Equation of state and QCD phase transitions, QCD in astrophysics of compact stellar objects, supernovae and compact stars mergers, Strangeness in Compact Stars, Strange Stars, Hadron production in heavy-ion collisions, Nonequilibrium and transport phenomena in dense matter

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Superfluid vs Superconductors

Definitions

Superfluid: frictionless fluid with $v = \nabla\varphi \quad \Rightarrow \quad \nabla\times v = 0$ (irrotational or quantized vorticity)

Superconductor: "screening" of magnetic fields: Meissner effect (almost perfect diamagnet)

Superfluid

Broken global symmetry

Goldstone theorem $\longrightarrow$ "Easy" transport of the quantum numbers of the broken group

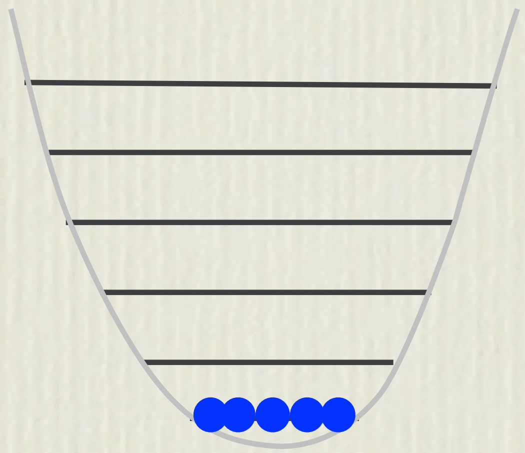
Superconductor

"Broken gauge symmetry"

Higgs mechanism $\longrightarrow$ Gauge fields with mass, M, penetrate for a length $\lambda \propto 1/M$

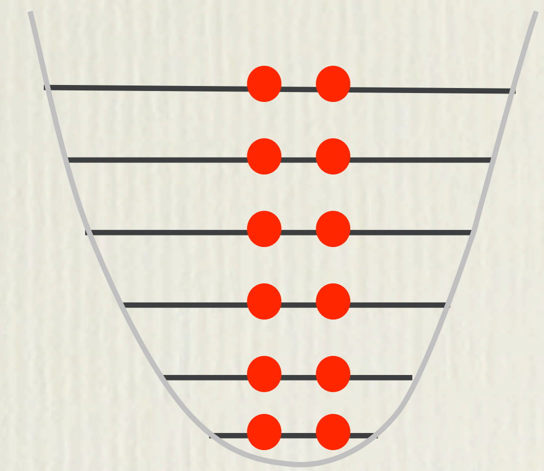
Fermionic and bosonic superfluids at T=0

^{4}He **BOSONS**



^{3}He **FERMIONS**

electrons



Electrically neutral
(really superfluids)

Neutral or charged
(superfluids or superconductos)

Fermionic and bosonic superfluids at T=0

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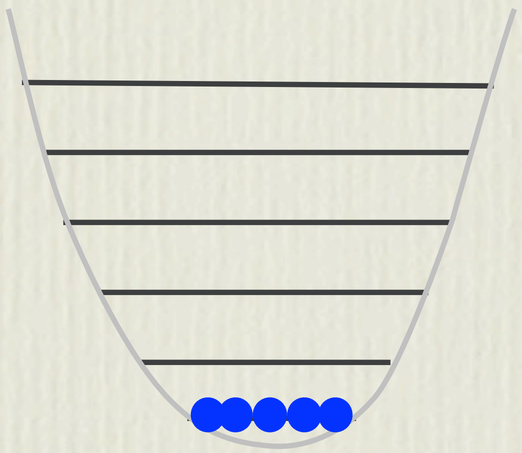

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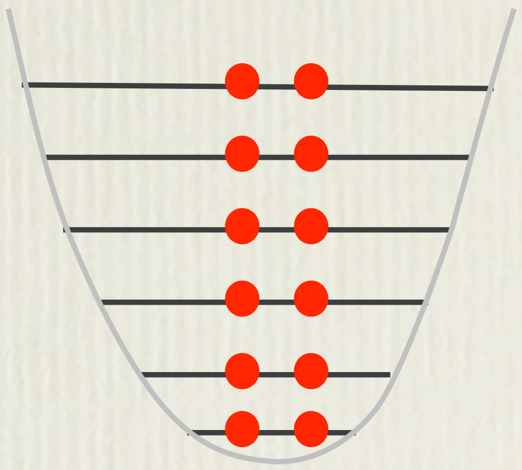

Bosons “like” to move together, no dissipation

^{4}He becomes superfluid at $T_c \simeq 2.17$ K, Kapitsa et al (1938)

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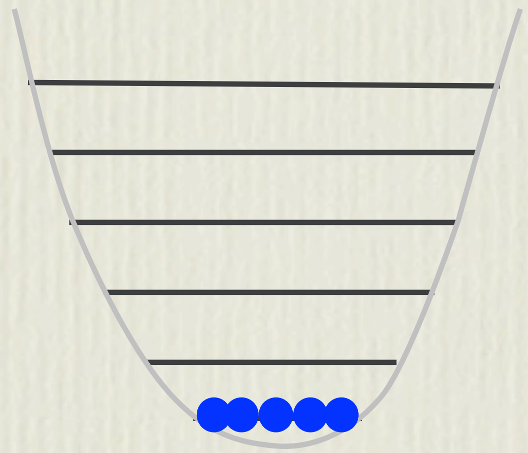

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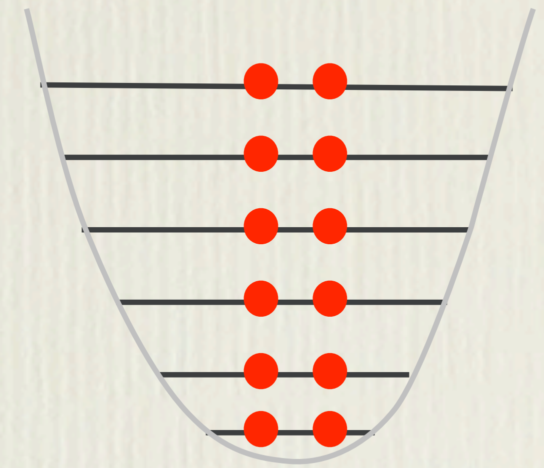

An arbitrary weak interaction leads to the formation of Cooper pairs

^{3}He becomes superfluid at
$T_c \simeq 0.0025$ K, Osheroff (1971)

Neutral or charged
(superfluids or superconductos)