Thermal fluctuations in a kinetic model for multicomponent fluids

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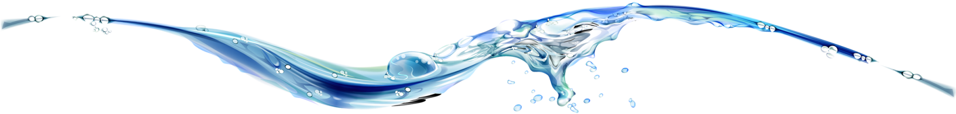
Outline



Kinetic model — Introducing kinetic theory for isothermal binary mixtures

- Binary Boltzmann equation
- Non-ideal effects
- Thermal fluctuations

Technicalities — Sketching how to apply fluctuation-dissipation theorem

- Equilibrium correlations
- Noise correlations

Numerics — Checking theoretics with numerical simulations

- Homogeneous ideal mixture
- Homogeneous interacting mixture
- Non-homogeneous interacting mixture

Conclusions — Summarizing results

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Kinetic model

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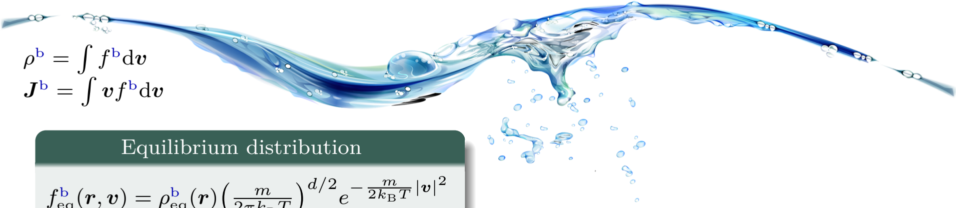
Binary Boltzmann equation

$$\partial_t f^{\mathrm{b}} + \boldsymbol{v} \cdot \boldsymbol{\nabla} f^{\mathrm{b}} = \mathrm{Coll}(f^{\mathrm{b}}, f^{\mathrm{g}})$$

The probability of finding a blue, risp. green, particle at t in a phase space volume $\mathrm{d}\omega = \mathrm{d}\boldsymbol{r}\mathrm{d}\boldsymbol{v}$ around $(\boldsymbol{r}, \boldsymbol{v})$ is $f^{\mathrm{b}}(t, \boldsymbol{r}, \boldsymbol{v})\mathrm{d}\omega$, risp. $f^{\mathrm{g}}(t, \boldsymbol{r}, \boldsymbol{v})\mathrm{d}\omega$.

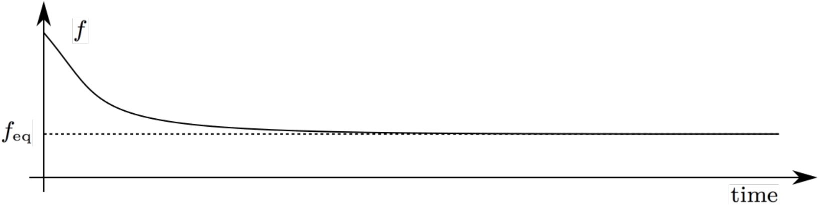


$$\rho^{\mathrm{b}} = \int f^{\mathrm{b}} \mathrm{d}\boldsymbol{v}$$
$$\boldsymbol{J}^{\mathrm{b}} = \int \boldsymbol{v} f^{\mathrm{b}} \mathrm{d}\boldsymbol{v}$$

Equilibrium distribution

$$f^{\mathrm{b}}_{\mathrm{eq}}(\boldsymbol{r}, \boldsymbol{v}) = \rho^{\mathrm{b}}_{\mathrm{eq}}(\boldsymbol{r}) \left(\frac{m}{2\pi k_{\mathrm{B}} T}\right)^{d/2} e^{-\frac{m}{2k_{\mathrm{B}}T}|\boldsymbol{v}|^2}$$

$$f^{\mathrm{b}} \to f^{\mathrm{b}}_{\mathrm{eq}} \implies \boldsymbol{\nabla}\rho^{\mathrm{b}}_{\mathrm{eq}} = \boldsymbol{0}$$



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Non-ideal binary Boltzmann equation

$$\partial_t f^{\mathrm{b}} + \boldsymbol{v} \cdot \nabla f^{\mathrm{b}} = \mathrm{Coll}(f^{\mathrm{b}}, f^{\mathrm{g}}) - \boldsymbol{a}^{\mathrm{b}} \cdot \nabla_{\boldsymbol{v}} f^{\mathrm{b}}$$

The body-force acceleration $\boldsymbol{a}^{\mathrm{b}} = \boldsymbol{a}[\rho^{\mathrm{g}}]$ exerted by a blue particle is assumed to depend on the green mass distribution.

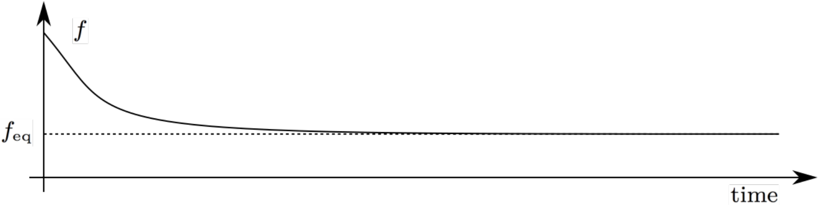
$$\rho^{\mathrm{b}} = \int f^{\mathrm{b}} \mathrm{d}\boldsymbol{v}$$

$$\boldsymbol{J}^{\mathrm{b}} = \int \boldsymbol{v} f^{\mathrm{b}} \mathrm{d}\boldsymbol{v}$$

Equilibrium distribution

$$f^{\mathrm{b}}_{\mathrm{eq}}(\boldsymbol{r}, \boldsymbol{v}) = \rho^{\mathrm{b}}_{\mathrm{eq}}(\boldsymbol{r}) \left(\frac{m}{2\pi k_{\mathrm{B}} T}\right)^{d/2} e^{-\frac{m}{2k_{\mathrm{B}}T}|\boldsymbol{v}|^2}$$

$$f^{\mathrm{b}} \rightarrow f^{\mathrm{b}}_{\mathrm{eq}} \Longrightarrow \nabla \rho^{\mathrm{b}}_{\mathrm{eq}} = \frac{m}{k_{\mathrm{B}}T} \rho^{\mathrm{b}}_{\mathrm{eq}} \boldsymbol{a}[\rho^{\mathrm{g}}_{\mathrm{eq}}]$$



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Non-ideal fluctuating binary Boltzmann equation

$$\partial_t f^{\mathrm{b}} + \boldsymbol{v}\cdot\boldsymbol{\nabla} f^{\mathrm{b}} = \mathrm{Coll}(f^{\mathrm{b}}, f^{\mathrm{g}}) - \boldsymbol{a}^{\mathrm{b}}\cdot\boldsymbol{\nabla}_{\boldsymbol{v}} f^{\mathrm{b}} + \xi^{\mathrm{b}}$$

Averages over all possible trajectories $\langle\cdot\rangle$ are equivalent to canonical ensemble averages.

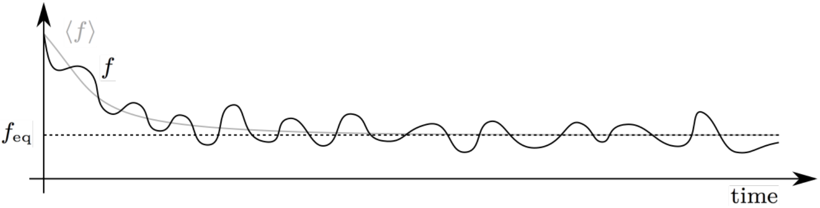
$$\rho^{\mathrm{b}} = \int f^{\mathrm{b}}\mathrm{d}\boldsymbol{v}$$
$$\boldsymbol{J}^{\mathrm{b}} = \int \boldsymbol{v} f^{\mathrm{b}}\mathrm{d}\boldsymbol{v}$$

Equilibrium distribution

$$f^{\mathrm{b}}_{\mathrm{eq}}(\boldsymbol{r},\boldsymbol{v}) = \rho^{\mathrm{b}}_{\mathrm{eq}}(\boldsymbol{r})\left(\frac{m}{2\pi k_{\mathrm{B}}T}\right)^{d/2} e^{-\frac{m}{2k_{\mathrm{B}}T}|\boldsymbol{v}|^2}$$
$$\langle f^{\mathrm{b}}\rangle \to f^{\mathrm{b}}_{\mathrm{eq}} \implies \boldsymbol{\nabla}\rho^{\mathrm{b}}_{\mathrm{eq}} = \frac{m}{k_{\mathrm{B}}T}\rho^{\mathrm{b}}_{\mathrm{eq}}\boldsymbol{a}[\rho^{\mathrm{g}}_{\mathrm{eq}}]$$

Brownian motion

$$\partial_t V = -\lambda V + \xi$$
$$\langle V^2\rangle \to k_{\mathrm{B}}T \implies \langle \xi^2\rangle = 2\lambda k_{\mathrm{B}}T$$



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Multicomponent Boltzmann equation

$$\partial_t f^{\mathrm{S}} + \boldsymbol{v}\cdot\boldsymbol{\nabla} f^{\mathrm{S}} = (\mathrm{Coll} f)^{\mathrm{S}} - \boldsymbol{a}^{\mathrm{S}}\cdot\boldsymbol{\nabla}_{\boldsymbol{v}} f^{\mathrm{S}} + \xi^{\mathrm{S}}$$

The Boltzmann distributions are labeled by the species index $\mathrm{S} = \mathrm{b}, \mathrm{g}, \ldots$ and are collected in $f = (f^{\mathrm{S}})_{\mathrm{S}=\mathrm{b},\mathrm{g},\ldots}$.

$$\rho^{\mathrm{S}} = \int f^{\mathrm{S}} \mathrm{d}\boldsymbol{v}$$

$$\boldsymbol{J}^{\mathrm{S}} = \int \boldsymbol{v} f^{\mathrm{S}} \mathrm{d}\boldsymbol{v}$$

Equilibrium distribution

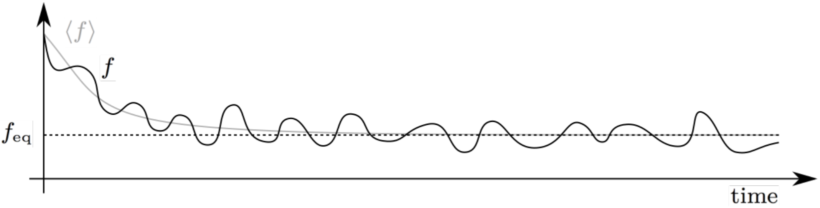
$$f^{\mathrm{S}}_{\mathrm{eq}}(\boldsymbol{r},\boldsymbol{v}) = \rho^{\mathrm{S}}_{\mathrm{eq}}(\boldsymbol{r}) \left(\frac{m}{2\pi k_{\mathrm{B}} T}\right)^{d/2} e^{-\frac{m}{2k_{\mathrm{B}}T}|\boldsymbol{v}|^2}$$

$$\langle f^{\mathrm{S}} \rangle \to f^{\mathrm{S}}_{\mathrm{eq}} \Longrightarrow \boldsymbol{\nabla}\rho^{\mathrm{S}}_{\mathrm{eq}} = \frac{m}{k_{\mathrm{B}}T}\rho^{\mathrm{S}}_{\mathrm{eq}}\boldsymbol{a}[\rho_{\mathrm{eq}}]$$

Brownian motion

$$\partial_t V_i = -(\mathrm{L}V)_i + \xi_i$$

$$\langle V_i V_j^* \rangle \to \mathrm{E}_{ij} \Longrightarrow \langle \xi_i \xi_j^* \rangle = (\mathrm{LE} + \mathrm{EL})_{ij}$$



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Multicomponent Boltzmann equation

$$\partial_t f^{\mathrm{S}} + \boldsymbol{v}\cdot\boldsymbol{\nabla} f^{\mathrm{S}} = (\mathrm{Coll} f)^{\mathrm{S}} - \boldsymbol{a}^{\mathrm{S}}\cdot\boldsymbol{\nabla}_{\boldsymbol{v}} f^{\mathrm{S}} + \xi^{\mathrm{S}}$$

The Boltzmann distributions are labeled by the species index $\mathrm{S} = \mathrm{b}, \mathrm{g}, \ldots$ and are collected in $f = (f^{\mathrm{S}})_{\mathrm{S}=\mathrm{b},\mathrm{g},\ldots}$.

$$\rho^{\mathrm{S}} = \int f^{\mathrm{S}} \mathrm{d}\boldsymbol{v}$$
$$\boldsymbol{J}^{\mathrm{S}} = \int \boldsymbol{v} f^{\mathrm{S}} \mathrm{d}\boldsymbol{v}$$

Equilibrium distribution

$$f^{\mathrm{S}}_{\mathrm{eq}}(\boldsymbol{r},\boldsymbol{v}) = \rho^{\mathrm{S}}_{\mathrm{eq}}(\boldsymbol{r})\left(\frac{m}{2\pi k_{\mathrm{B}}T}\right)^{d/2} e^{-\frac{m}{2k_{\mathrm{B}}T}|\boldsymbol{v}|^2}$$
$$\langle f^{\mathrm{S}}\rangle \to f^{\mathrm{S}}_{\mathrm{eq}} \implies \boldsymbol{\nabla}\rho^{\mathrm{S}}_{\mathrm{eq}} = \frac{m}{k_{\mathrm{B}}T}\rho^{\mathrm{S}}_{\mathrm{eq}}\boldsymbol{a}[\rho_{\mathrm{eq}}]$$

Brownian motion

$$\partial_t V_i = -(\mathrm{L}V)_i + \xi_i$$
$$\langle V_i V_j^*\rangle \to \mathrm{E}_{ij} \implies \langle \xi_i \xi_j^*\rangle = (\mathrm{LE}+\mathrm{EL})_{ij}$$

Linearity: Collisional operator and self-generated forcing are non-linear functional of f $\implies$ Small deviation from equilibrium

ODE: Non-ideal Boltzmann equation involves spatial and velocity gradient of f $\implies$ Velocity moments and Fourier space

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Technicalities

Evolution equation for the velocity moments

$$\partial_t f_a^{\mathrm{S}} + \nabla \cdot (\boldsymbol{v} f^{\mathrm{S}})_a = (\mathrm{Coll}_a f_a)^{\mathrm{S}} - \boldsymbol{a}^{\mathrm{S}} \cdot (\Phi f^{\mathrm{S}})_a + \xi_a^{\mathrm{S}}$$

$\rho^{\mathrm{S}} = \int f^{\mathrm{S}} \mathrm{d}\boldsymbol{v} = f_0^{\mathrm{S}}$

$\boldsymbol{J}^{\mathrm{S}} = \int \boldsymbol{v} f^{\mathrm{S}} \mathrm{d}\boldsymbol{v} = f_{1,\dots,d}^{\mathrm{S}}$

$\dots$

$\xi_\rho^{\mathrm{S}} = \int \xi^{\mathrm{S}} \mathrm{d}\boldsymbol{v} = \xi_0^{\mathrm{S}}$

$\boldsymbol{\xi}_{\boldsymbol{J}}^{\mathrm{S}} = \int \boldsymbol{v} \xi^{\mathrm{S}} \mathrm{d}\boldsymbol{v} = \xi_{1,\dots,d}^{\mathrm{S}}$

$\dots$

Hermite transformation

$$f_a = \int H_a f \mathrm{d}\boldsymbol{v} \qquad H_0 = 1 \quad H_{1,\dots,d} = \boldsymbol{v} \quad \dots$$

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Linear evolution equation for the velocity moments

$$\partial_t \delta f_a^{\mathrm{S}} + \boldsymbol{\nabla}\cdot(\boldsymbol{v}\delta f^{\mathrm{S}})_a = (\lambda_a \delta f_a)^{\mathrm{S}} - \boldsymbol{a}[\rho_{\mathrm{eq}}]\cdot(\Phi\delta f^{\mathrm{S}})_a - \delta\boldsymbol{a}^{\mathrm{S}}\cdot(\Phi f_{\mathrm{eq}}^{\mathrm{S}})_a + \xi_a^{\mathrm{S}}$$

$$\delta\rho^{\mathrm{S}} = \int \delta f^{\mathrm{S}} \mathrm{d}\boldsymbol{v} = \delta f_0^{\mathrm{S}}$$

$$\delta\boldsymbol{J}^{\mathrm{S}} = \int \boldsymbol{v}\delta f^{\mathrm{S}} \mathrm{d}\boldsymbol{v} = \delta f_{1,\dots,d}^{\mathrm{S}}$$

$$\dots$$

$$\xi_\rho^{\mathrm{S}} = \int \xi^{\mathrm{S}} \mathrm{d}\boldsymbol{v} = \xi_0^{\mathrm{S}}$$

$$\boldsymbol{\xi}_{\boldsymbol{J}}^{\mathrm{S}} = \int \boldsymbol{v}\xi^{\mathrm{S}} \mathrm{d}\boldsymbol{v} = \xi_{1,\dots,d}^{\mathrm{S}}$$

$$\dots$$

Hermite transformation

$$f_a = \int H_a f \mathrm{d}\boldsymbol{v} \qquad H_0 = 1 \quad H_{1,\dots,d} = \boldsymbol{v} \quad \dots$$

Small deviation from equilibrium

$$\delta f = f - f_{\mathrm{eq}} \qquad f_{\mathrm{eq}} = \lim_{t\to\infty}\langle f\rangle$$

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Linear evolution equation for the Fourier-transformed velocity moments

$$\partial_t \delta f^{\mathrm{S}}_{a;\boldsymbol{k}} + \mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{v}\delta f^{\mathrm{S}}_{\boldsymbol{k}})_a = (\lambda_a \delta f_{a;\boldsymbol{k}})^{\mathrm{S}} - (\boldsymbol{a}[\rho_{\mathrm{eq}}] * (\Phi \delta f^{\mathrm{S}})_a)_{\boldsymbol{k}} - (\delta \boldsymbol{a}^{\mathrm{S}} * (\Phi f^{\mathrm{S}}_{\mathrm{eq}})_a)_{\boldsymbol{k}} + \xi^{\mathrm{S}}_{a;\boldsymbol{k}}$$

$$\delta\rho^{\mathrm{S}}_{\boldsymbol{k}} = \int \delta f^{\mathrm{S}}_{\boldsymbol{k}} \mathrm{d}\boldsymbol{v} = \delta f^{\mathrm{S}}_{0;\boldsymbol{k}}$$

$$\delta \boldsymbol{J}^{\mathrm{S}}_{\boldsymbol{k}} = \int \boldsymbol{v}\delta f^{\mathrm{S}}_{\boldsymbol{k}} \mathrm{d}\boldsymbol{v} = \delta f^{\mathrm{S}}_{1,\ldots,d;\boldsymbol{k}}$$

$\ldots$

$$\xi^{\mathrm{S}}_{\rho;\boldsymbol{k}} = \int \xi^{\mathrm{S}}_{\boldsymbol{k}} \mathrm{d}\boldsymbol{v} = \xi^{\mathrm{S}}_{0;\boldsymbol{k}}$$

$$\boldsymbol{\xi}^{\mathrm{S}}_{\boldsymbol{J};\boldsymbol{k}} = \int \boldsymbol{v}\xi^{\mathrm{S}}_{\boldsymbol{k}} \mathrm{d}\boldsymbol{v} = \xi^{\mathrm{S}}_{1,\ldots,d;\boldsymbol{k}}$$

$\ldots$

Hermite transformation

$$f_a = \int H_a f \mathrm{d}\boldsymbol{v} \qquad H_0 = 1 \quad H_{1,\ldots,d} = \boldsymbol{v} \quad \ldots$$

Small deviation from equilibrium

$$\delta f = f - f_{\mathrm{eq}} \qquad f_{\mathrm{eq}} = \lim_{t\to\infty}\langle f \rangle$$

Fourier transformation

$$f_{\boldsymbol{k}} = \int e_{\boldsymbol{k}} f \mathrm{d}\boldsymbol{r} \qquad e_{\boldsymbol{k}} = \frac{1}{(2\pi)^{d/2}} e^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}$$

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Linear evolution equation for the Fourier-transformed velocity moments

$$\partial_t \delta f^{\mathrm{b}}_{a;\boldsymbol{k}} + \mathrm{i}\boldsymbol{k}\cdot(\boldsymbol{v}\delta f^{\mathrm{b}}_{\boldsymbol{k}})_a = (\lambda_a \delta f_{a;\boldsymbol{k}})^{\mathrm{b}} - (\boldsymbol{a}[\rho_{\mathrm{eq}}] * (\Phi \delta f^{\mathrm{b}})_a)_{\boldsymbol{k}} - (\delta \boldsymbol{a}^{\mathrm{b}} * (\Phi f^{\mathrm{b}}_{\mathrm{eq}})_a)_{\boldsymbol{k}} + \xi^{\mathrm{b}}_{a;\boldsymbol{k}}$$

$$\delta\rho^{\mathrm{b}}_{\boldsymbol{k}} = \int \delta f^{\mathrm{b}}_{\boldsymbol{k}} \mathrm{d}\boldsymbol{v} = \delta f^{\mathrm{b}}_{0;\boldsymbol{k}} \qquad \xi^{\mathrm{b}}_{\rho} = \int \xi^{\mathrm{b}} \mathrm{d}\boldsymbol{v} = \xi^{\mathrm{b}}_{0}$$

$$\delta\boldsymbol{J}^{\mathrm{b}}_{\boldsymbol{k}} = \int \boldsymbol{v}\delta f^{\mathrm{b}}_{\boldsymbol{k}} \mathrm{d}\boldsymbol{v} = \delta f^{\mathrm{b}}_{1,\ldots,d;\boldsymbol{k}} \qquad \boldsymbol{\xi}^{\mathrm{b}}_{\boldsymbol{J}} = \int \boldsymbol{v}\xi^{\mathrm{b}} \mathrm{d}\boldsymbol{v} = \xi^{\mathrm{b}}_{1,\ldots,d}$$

$\ldots \qquad \ldots$

Equilibrium correlations

$$\langle \delta\rho^{\mathrm{b}}_{\boldsymbol{k}} \delta\rho^{\mathrm{b}}_{-\boldsymbol{k}} \rangle = m \frac{\rho^{\mathrm{b}}}{1-\rho^{\mathrm{b}}\rho^{\mathrm{g}}\alpha^2_{\boldsymbol{k}}} \qquad \langle \delta\rho^{\mathrm{b}}_{\boldsymbol{k}} \delta\rho^{\mathrm{g}}_{-\boldsymbol{k}} \rangle = -m \frac{\rho^{\mathrm{b}}\rho^{\mathrm{g}}\alpha_{\boldsymbol{k}}}{1-\rho^{\mathrm{b}}\rho^{\mathrm{g}}\alpha^2_{\boldsymbol{k}}}$$

$$\langle \delta\boldsymbol{J}_{\boldsymbol{k}} \delta\boldsymbol{J}_{-\boldsymbol{k}} \rangle = k_{\mathrm{B}}T(\rho^{\mathrm{b}} + \rho^{\mathrm{g}})\mathbf{1} \qquad (\boldsymbol{J} = \boldsymbol{J}^{\mathrm{b}} + \boldsymbol{J}^{\mathrm{g}}) \qquad \ldots$$

... self-consistently derived in the theory!

Noise correlations

$$\langle \xi^{\mathrm{b}}_{\rho} \xi^{\mathrm{b}}_{\rho} \rangle = \langle \xi^{\mathrm{b}}_{\rho} \xi^{\mathrm{g}}_{\rho} \rangle = 0 \qquad \langle \boldsymbol{\xi}^{\mathrm{b}}_{\boldsymbol{J}} \boldsymbol{\xi}^{\mathrm{b}}_{\boldsymbol{J}} \rangle = -\langle \boldsymbol{\xi}^{\mathrm{b}}_{\boldsymbol{J}} \boldsymbol{\xi}^{\mathrm{g}}_{\boldsymbol{J}} \rangle = 2\lambda k_{\mathrm{B}}T \frac{\rho^{\mathrm{b}}\rho^{\mathrm{g}}}{\rho^{\mathrm{b}}+\rho^{\mathrm{g}}}\mathbf{1}$$

$\ldots \qquad \Uparrow$ stochastic diffusion fluxes (sdf) $\Uparrow$

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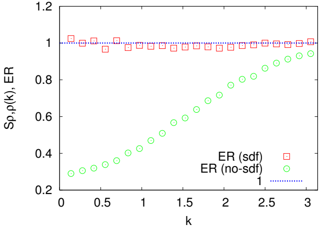
Numerics

Binary lattice Boltzmann equation

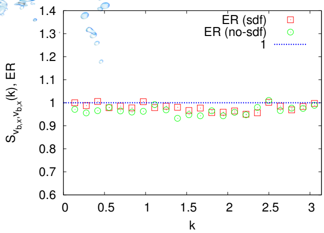
$$f_\ell^{\mathrm{b}}(t+1, \boldsymbol{r}+\boldsymbol{v}_\ell) = f_\ell^{\mathrm{b}}(t, \boldsymbol{r}) + \mathrm{R}_\ell(f^{\mathrm{b}}, f^{\mathrm{g}})$$

The probability of finding a blue, risp. green, particle at t in a unitary lattice cell around $\boldsymbol{r}$ with velocity $\boldsymbol{v}_\ell$ is $f_\ell^{\mathrm{b}}(t, \boldsymbol{r})$, risp. $f_\ell^{\mathrm{g}}(t, \boldsymbol{r})$.

$$\langle \delta\rho_{\boldsymbol{k}}^{\mathrm{b}} \delta\rho_{-\boldsymbol{k}}^{\mathrm{b}} \rangle = m \frac{\rho^{\mathrm{b}}}{1-\rho^{\mathrm{b}}\rho^{\mathrm{g}}\alpha_{\boldsymbol{k}}^2}$$



$$\langle \delta\boldsymbol{J}_{\boldsymbol{k}} \delta\boldsymbol{J}_{-\boldsymbol{k}} \rangle = k_{\mathrm{B}} T (\rho^{\mathrm{b}} + \rho^{\mathrm{g}}) \mathbf{1}$$



Lattice mutual interaction

$$\boldsymbol{a}^{\mathrm{b}}(\boldsymbol{r}) = \boldsymbol{a}[\rho^{\mathrm{g}}](\boldsymbol{r}) = -G \sum_\ell w_\ell \boldsymbol{v}_\ell \rho^{\mathrm{g}}(\boldsymbol{r}+\boldsymbol{v}_\ell) \qquad \alpha_{\boldsymbol{k}} = G(1 - \tfrac{1}{6}|\boldsymbol{k}|^2)$$

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Binary lattice Boltzmann equation

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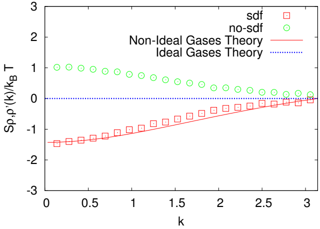
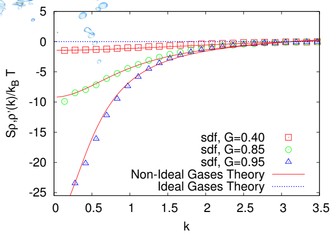
The probability of finding a blue, risp. green, particle at t in a unitary lattice cell around $\boldsymbol{r}$ with velocity $\boldsymbol{v}_\ell$ is $f_\ell^{\mathrm{b}}(t,\boldsymbol{r})$, risp. $f_\ell^{\mathrm{g}}(t,\boldsymbol{r})$.

$$\langle \delta\rho^{\mathrm{b}}_{\boldsymbol{k}}\delta\rho^{\mathrm{g}}_{-\boldsymbol{k}}\rangle = -m\frac{\rho^{\mathrm{b}}\rho^{\mathrm{g}}\alpha_{\boldsymbol{k}}}{1-\rho^{\mathrm{b}}\rho^{\mathrm{g}}\alpha^2_{\boldsymbol{k}}}$$



$$\langle \delta\rho^{\mathrm{b}}_{\boldsymbol{k}}\delta\rho^{\mathrm{g}}_{-\boldsymbol{k}}\rangle = -m\frac{\rho^{\mathrm{b}}\rho^{\mathrm{g}}\alpha_{\boldsymbol{k}}}{1-\rho^{\mathrm{b}}\rho^{\mathrm{g}}\alpha^2_{\boldsymbol{k}}}$$



Lattice mutual interaction

$$\boldsymbol{a}^{\mathrm{b}}(\boldsymbol{r}) = \boldsymbol{a}[\rho^{\mathrm{g}}](\boldsymbol{r}) = -G\sum_\ell w_\ell \boldsymbol{v}_\ell \rho^{\mathrm{g}}(\boldsymbol{r}+\boldsymbol{v}_\ell) \qquad \alpha_{\boldsymbol{k}} = G(1-\tfrac{1}{6}|\boldsymbol{k}|^2)$$

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Binary lattice Boltzmann equation

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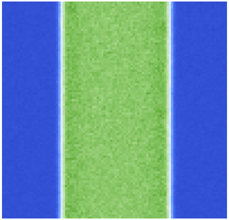
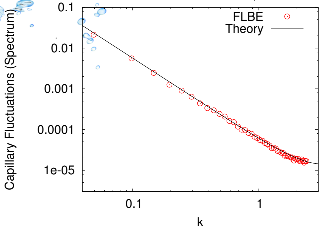
The probability of finding a blue, risp. green, particle at t in a unitary lattice cell around $\boldsymbol{r}$ with velocity $\boldsymbol{v}_\ell$ is $f_\ell^{\mathrm{b}}(t, \boldsymbol{r})$, risp. $f_\ell^{\mathrm{g}}(t, \boldsymbol{r})$.

$$P[h] \propto e^{-\frac{\gamma}{2k_{\mathrm{B}}T}\int (\frac{\mathrm{d}h}{\mathrm{d}x})^2 \mathrm{d}x}$$



$$\langle |h_k|^2 \rangle = \frac{k_{\mathrm{B}}T}{\gamma k^2}$$



Lattice mutual interaction

$$\boldsymbol{a}^{\mathrm{b}}(\boldsymbol{r}) = \boldsymbol{a}[\rho^{\mathrm{g}}](\boldsymbol{r}) = -G \sum_\ell w_\ell \boldsymbol{v}_\ell \rho^{\mathrm{g}}(\boldsymbol{r}+\boldsymbol{v}_\ell) \qquad \alpha_{\boldsymbol{k}} = G(1 - \tfrac{1}{6}|\boldsymbol{k}|^2)$$

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Conclusions

Kinetic model — A non-ideal kinetic model for mixture has been successfully extended to incorporate the effects of thermal fluctuations

Technicalities — Application of fluctuation-dissipation theorem to derive the expression of both equilibrium and noise correlations directly at the kinetic level has allowed to go beyond hydrodynamics, by controlling thermalization of all the kinetic modes (velocity moments)

Numerics — Agreement between numerical simulations and theoretical expectations is very good for all wavevectors investigated in both homogeneous and non-homogeneous (phase separation) cases

Conclusions — Conclusion, thanks for your attention!

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