Quantum Quench Dynamics in 2d Supersymmetric Models

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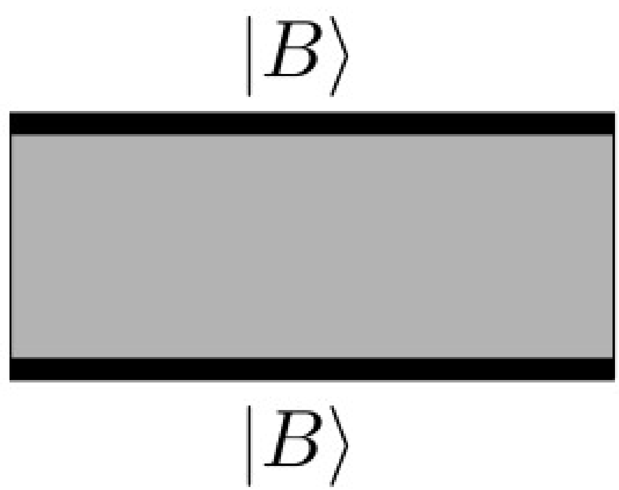
Quantum Quench

$$H = \begin{cases} H_0, & t < 0 \\ H', & t > 0 \end{cases}$$

$|B(0)\rangle$ Is the ground state of the pre-quench Hamiltonian, becomes the Boundary state of the post-quench Hamiltonian.



$$U = e^{iH't}$$

Quantum quench + long time = equilibrium?

**Time evolution remembers too much for integrable theories

Generalized Gibbs Ensemble

$$\rho_{\mathrm{GGE}} = \frac{1}{\mathcal{Z}} \exp\left(-\sum_{j=1}^{\infty} \beta_j I_j \right)$$

$$\langle B | I_j | B \rangle = \mathrm{tr}\left(I_j \rho_{\mathrm{GGE}} \right)$$

Integrable boundary states
(Gloshal and Zamolodchikov)

Translation invariance is broken by the presence of a boundary at *t=0,* Boundary state has nonzero “energy”.

We can keep half of the translation invariance (boundary state can be required to have zero “momentum”.

$$(P_s - \bar{P}_s)|B\rangle = 0, \qquad s \in \text{ Integers}$$

Boundary state composed of “Cooper pairs”

$$|B\rangle = \exp\left(\int d\theta K(\theta) A^\dagger(-\theta) A^\dagger(\theta) \right) |0\rangle$$

What about even more symmetry?

Thermal SUSY breaking

Many, many examples of finite temperature in 3+1d and 2+1d:

See Ashok Das's book, *Finite Temperature Field Theory,* for example.

Quantum Quench in 2+1d:

Hung, Smolkin, Sorkin,
(Non) supersymmetric quantum quenches,
10.1007/JHEP12(2013)022

Supersymmetric QFT

There exists a Fermionic conserved charge,

$$Q^2 = P \qquad \bar{Q}^2 = \bar{P}$$

$$Q|Boson\rangle = |Fermion\rangle, \quad Q|Fermion\rangle = |Boson\rangle$$

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$$H = Q^2 + \bar{Q}^2$$

Unbroken SUSY

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$$\langle 0|H|0\rangle = \langle 0|Q^2 + \bar{Q}^2|0\rangle = 0$$

Ground state energy is a commonly used order parameter

Thermal SUSY breaking

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A worse problem is that thermodynamics distinguishes bosons and fermions:

$$n_{B,F}(E) = \frac{1}{e^{\beta E} \pm 1}$$

Particle statistics are a dynamical property in 1+1 d

Zamolodchikov algebra:

$$A^{\dagger}(\theta_1)A^{\dagger}(\theta_2) = S(\theta_1 - \theta_2)A^{\dagger}(\theta_2)A^{\dagger}(\theta_1)$$

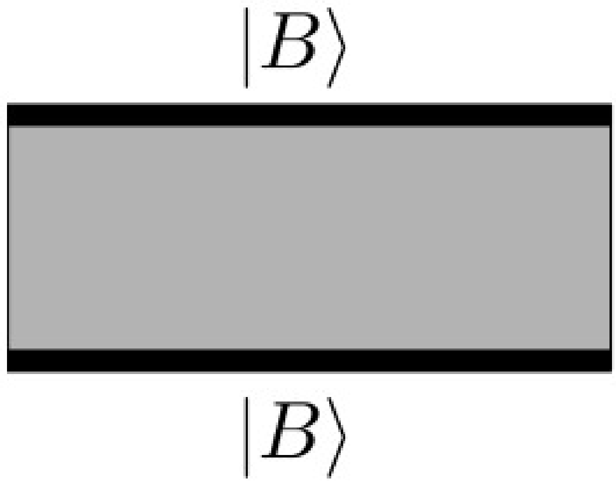
For all known *INTERACTING* integrable theories:

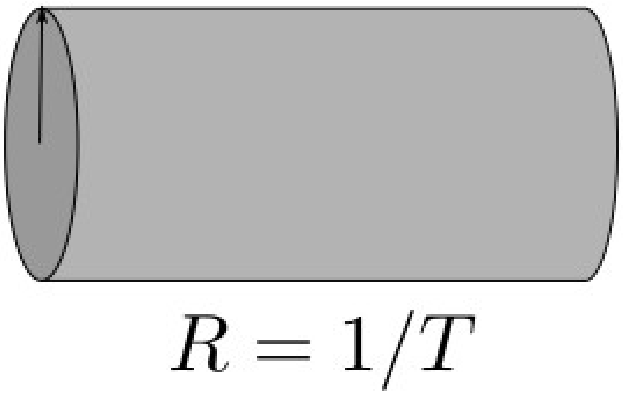
$$S(0) = -1$$

Pauli exclusion principle!

Both bosons and fermions obey Fermi-Dirac statistics

Finite temperature vs. quench





The bulk is supersymmetric in the practical sense that: $n_B = n_F$

Is it possible to design a boundary state that in some way preserves SUSY?

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The “half SUSY” condition

Corresponds to the “square root” of the spin-1 condition:

$$(P_1 - \bar{P}_1)|B\rangle = (Q \pm i\bar{Q})(Q \pm i\bar{Q})|B\rangle = 0$$

A SUSY boundary state satisfies the stronger condition

$$(Q \pm i\bar{Q})|B\rangle = 0$$

What does this condition implies for the Cooper-pairs boundary state?

SUSY Cooper pairs

Assume the Cooper pairs are of the form

$$K_{ij}(\theta)|A_i(-\theta)A_j(\theta)\rangle = a(\theta)|bb\rangle + b(\theta)|ff\rangle + c(\theta)|bf\rangle + d(\theta)|fb\rangle$$

Does SUSY impose any extra conditions conditions on the functions, *a,b,c,d* ?

Action of supercharge on two-particle state

$$Q|A_i(\theta)A_j(-\theta)\rangle = \sqrt{m}(x\mathcal{Q}\otimes\mathbf{1} + x^{-1}\mathcal{Q}_L\otimes\mathcal{Q})|A_i(\theta)A_j(-\theta)\rangle$$

$$\bar{Q}|A_i(\theta)A_j(-\theta)\rangle = \sqrt{m}(x^{-1}\bar{\mathcal{Q}}\otimes\mathbf{1} + x\mathcal{Q}_L\otimes\bar{\mathcal{Q}})|A_i(\theta)A_j(-\theta)\rangle$$

$$x = \exp(\theta/2)$$

$$\mathcal{Q} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \bar{\mathcal{Q}} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \mathcal{Q}_L = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$a(\theta) = \pm \left(\coth \frac{\theta}{2} \right)^{\pm 1} b(\theta)$$

$$c(\theta) = \pm d(\theta)$$

SUSY boundary state

$$ {}_{\text{out}}\langle A_i(-\theta)A_j(\theta)|Q|A_i(-\theta)A_j(\theta)\rangle_{\text{in}} = 0 $$

If the conditions between *a,b,c,d* are satisfied.

Therefore (exponentiate) $\langle B|Q|B\rangle = 0$

Correlations of superfields

$$\Phi(x,t,\theta,\bar{\theta}) = \phi(x,t) + \bar{\theta}\psi(x,t) + \frac{1}{2}\bar{\theta}\theta F(x,t)$$

Invariant under "half SUSY" infinitesimal transformation

$$U = \exp i\epsilon(Q \pm i\bar{Q})$$

$$\langle B|\Phi|B\rangle = \langle B|U\Phi U^{\dagger}|B\rangle$$

Strong constraints on correlations of components, example:

$$\langle B|\psi_2|B\rangle = \mp i\langle B|\psi_1|B\rangle$$

$$\langle B|i\partial\!\!\!/\phi|B\rangle = \langle B|F|B\rangle$$

Some two-point functions

$$\langle B(t)|\,[\psi_+(x_1) \mp i\psi_-(x_1)]\,[\psi_+(x_2) \mp i\psi_-(x_2)]\,|B(t)\rangle = -2i\langle B(t)|\partial_{x_1^0}\phi(x_1)\,\phi(x_2)|B(t)\rangle,$$

$$\langle B(t)|\,[\psi_+(x_1) \pm i\psi_-(x_1)]\,[\psi_+(x_2) \mp i\psi_-(x_2)]\,|B(t)\rangle = \pm 2\langle B(t)|[-i\partial_{x_1^1}\phi(x_1) + F(x_1)]\phi(x_2)|B(t)\rangle$$

$$\langle B(t)|\psi_+(x_1)\,[\partial_+\psi_+(x_2) \mp i\partial_-\psi_-(x_2)]\,|B(t)\rangle = -i\langle B(t)|\,(\mp i\partial_-\phi(x_1) + F(x_1))\,F(x_2)|B(t)\rangle,$$

$$\langle B(t)|\psi_-(x_1)\,[\partial_+\psi_+(x_2) \mp i\partial_-\psi_-(x_2)]\,|B(t)\rangle = \pm\langle B(t)|\,(\pm i\partial_+\phi(x_1) + F(x_1))\,F(x_2)|B(t)\rangle,$$

Sine Gordon at $g^2 = 16\pi/3$

$$S(\theta) = R(\theta) \begin{pmatrix} -1/\cosh(\theta/2) & i\tanh(\theta/2) & 0 & 0 \\ i\tanh(\theta/2) & -1/\cosh(\theta/2) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Actually invariant under a SUSY transformation, by shifting basis (Similarity transformation)

$$\{|S\bar{S}\rangle, |\bar{S}S\rangle, |SS\rangle, |\bar{S}\bar{S}\rangle\} \to \{|bb\rangle, |ff\rangle, |bf\rangle, |fb\rangle\}$$

Gloshal and Zamolodchikov's “fixed” boundary condition satisfies our SUSY constraint, corresponding to $\pm = +$

Tricritical Ising model

$\Phi_{1,3}$ Deformation. Three adjacent vacua. Spectrum consists on four kinks connecting them.

SUSY is a transformation between kinks.

Higher-dimensional representation of SUSY:

$$\{|A_{-1,0}\rangle, |A_{1,0}\rangle, |A_{0,-1}\rangle, |A_{0,1}\rangle\}$$

$$\mathcal{Q} = \begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \bar{\mathcal{Q}} = \begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \mathcal{Q}_L = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Cooper pairs

Topologically allowed pairs:

$$|AA\rangle = a|A_{-1,0}, A_{0,1}\rangle + b|A_{-1,0}, A_{0,-1}\rangle + c|A_{1,0}, A_{0,-1}\rangle$$
$$+d|A_{1,0}A_{0,1}\rangle + e|A_{0,-1}A_{-1,0}\rangle + f|A_{0,1}, A_{1,0}\rangle$$

For $\pm = + : \quad b, d = 0, \;\; e = f, \;\; a, c = \text{any values}$

For $\pm = - : \quad a, c = 0, \;\; e = -f, \;\; b, d = \text{any value}$

Topologically charged boundary

$$\{Q, \bar{Q}\} = Z \qquad Z|B\rangle = z|B\rangle$$

The SUSY condition is no longer the “square root” of the spin-1 condition, has to be modified

$$(Q \pm i\bar{Q} + \beta\Gamma)|B\rangle = 0 \qquad \beta^2 = \mp iz$$

$$\Gamma^2 = 1, \; \{\Gamma, Q\} = \{\Gamma, \bar{Q}\} = 0$$

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Only possible solution is

$$a = b = c = d = e = f = 0$$

Topologically charged states not allowed by SUSY

Some very few words about SGGE

Q is a conserved charge in the sense that $[H, Q] = 0$

Do I need to include it in the GGE then?

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Do I need to include it in the GGE then?

Not necessary if the boundary state satisfies the SUSY condition, since

$$\langle B|Q|B\rangle = 0$$

Inclusion of supercharge is only necessary in principle if the boundary state is not supersymmetric, but the Hamiltonian evolution is.

Some very few words about SGGE

Some problems:

Q is not simultaneously diagonalizable with other charges,
It changes the state it acts on, not of the particle-counting form

$$\int d\theta q(\theta) A^\dagger(\theta) A(\theta)$$

Worse problem, fermionic charge is not extensive.

Possible solution by Kapusta, Pratt, Visnjic, (1983)
Introduce abstract Clifford-algebra fields, and build bosonic charges with same information as Q

$$\{c(p), c(q)\} = 2\delta(p-q), \quad c(p)^\dagger = c(p)$$

$$\tilde{Q} = i \int d^2p \, c(p) Q(p)$$

Fin