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TURBULENCE OVER A FRACTAL FOURIER SKELETON

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HOMOGENEOUS AND ISOTROPIC (classical) TURBULENCE

3D Navier-Stokes equations for an incompressible flow

$$\begin{cases} \partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + \nu \Delta \mathbf{v} + \mathbf{f} \\ \nabla \cdot \mathbf{v} = 0 \\ + \quad \text{Boundary conditions} \end{cases}$$

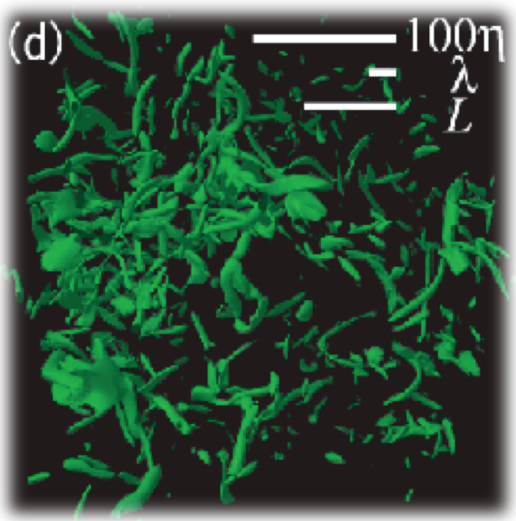
$$Re = \frac{U\,L}{\nu} \rightarrow \infty$$

- out-of-equilibrium, multi-scale interactions
- fully chaotic
- vortex-stretching mechanism amplifies vorticity
- Statistical laws with universal scaling exponents for $< (\delta_r v)> \sim r^{\,z(p)}$ with $z(p) \neq p/2\; z(2)$

L = lenght-scale IR cutoff for the force f

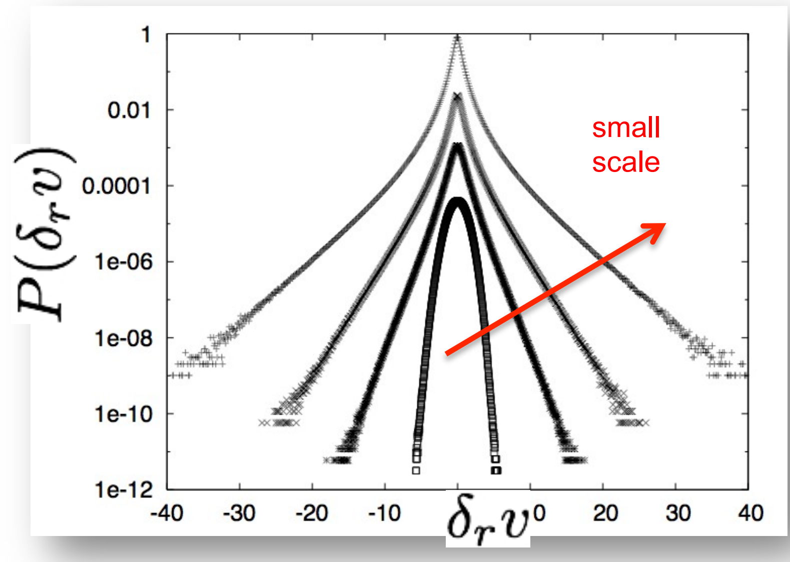
η = lenght-scale UV cutoff for the field

Vorticity filaments



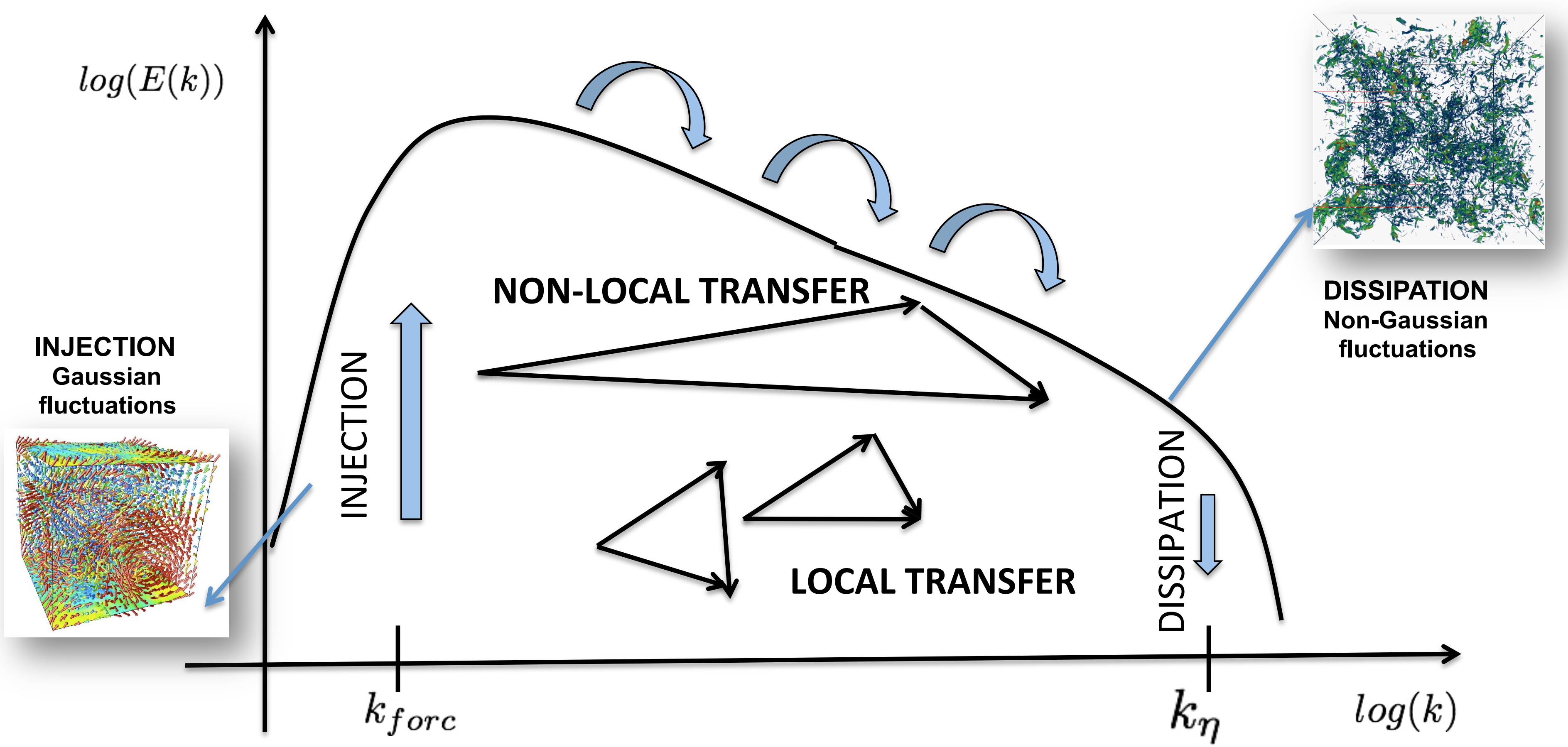
?

PDF of velocity increments



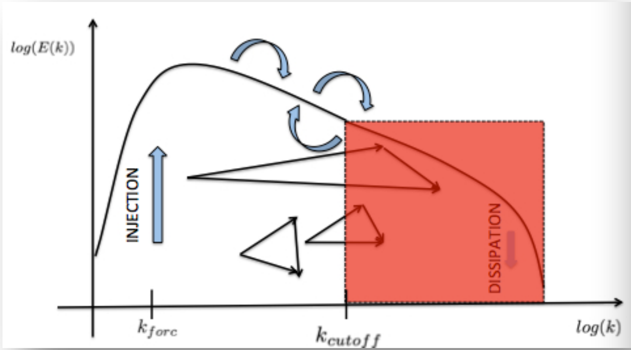
THE PROBLEM: *Intermittency, i.e. scale invariance of statistical laws is broken*

ON THE ORIGIN OF INTERMITTENT CORRECTIONS IN 3D HOMOGENEOUS AND ISOTROPIC TURBULENCE

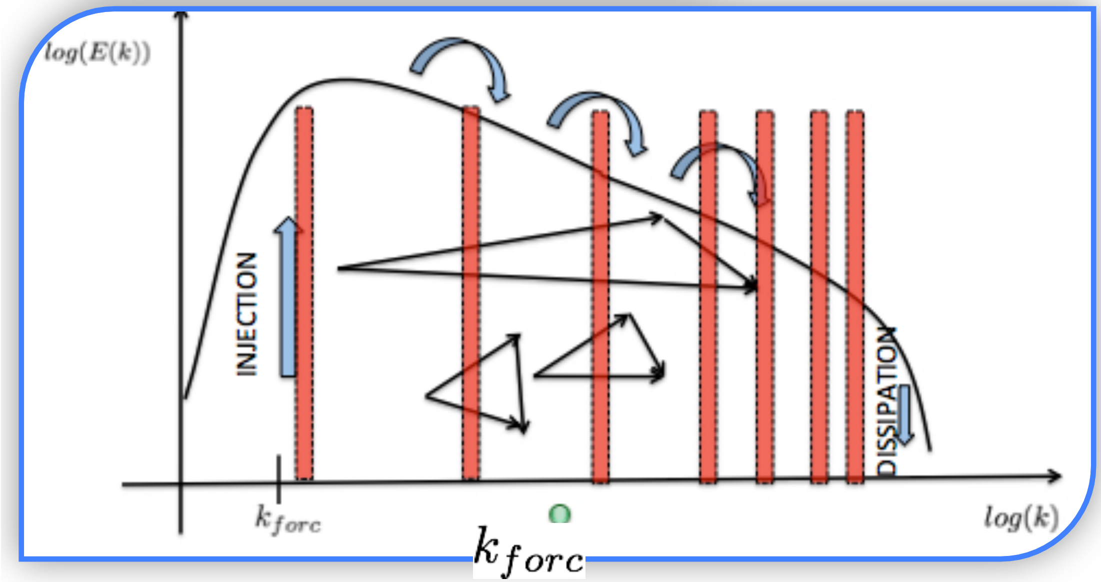


CAN WE HAVE A BETTER UNDERSTANDING OF INTERMITTENCY BY ***DECIMATING*** INTERACTIONS IN THE NON LINEAR TERM ?

DECIMATION TO A FRACTAL SET OF FOURIER MODES



- Truncation of modes |**k**| > k_{cutoff}
  Example : Closures, Large-eddy simulations



- Random ***DECIMATION*** of Fourier modes on a Fractal set of dimension D_F
- Degres of freedom reduction $\#_{dof} \sim k^{DF}$ with a tuning parameter

$$\mathbf{v}(\mathbf{x},t) \to \mathbf{v}^{\mathbf{D_F}}(\mathbf{x},\mathbf{t}) = \mathcal{P}^{\mathbf{D_F}}\mathbf{v}(\mathbf{x},\mathbf{t}) = \sum_{\mathbf{k}\in\mathcal{Z}^3} \mathbf{e}^{\mathbf{ikx}}\gamma_{\mathbf{k}}\hat{\mathbf{v}}(\mathbf{k},\mathbf{t})$$

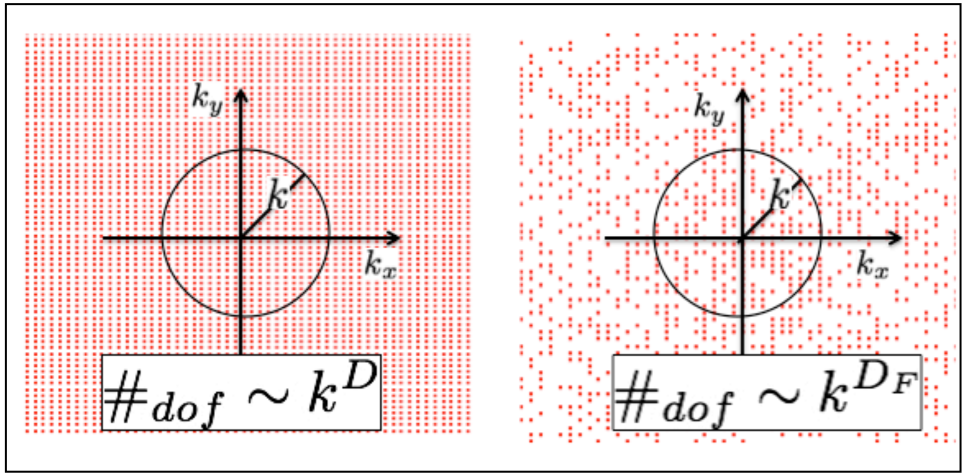
Fourier modes are decimated ($\gamma_k = 0$) with probability ~ $1 - k^{D_F - 3}$

SELF-SIMILAR REDUCTION OF 3D NAVIER-STOKES DYNAMICS

$$\mathbf{u}(\mathbf{x},t) \to \mathbf{v}^{D_F}(\mathbf{x},t) = \mathcal{P}^{D_F}\mathbf{u}(\mathbf{x},t) = \sum_{\mathbf{k}\in\mathcal{Z}^3} e^{i\mathbf{k}\mathbf{x}}\,\gamma_{\mathbf{k}}\,\hat{\mathbf{u}}(\mathbf{k},t)$$

$$\gamma_k = \begin{cases} 1 & \text{with probability } h_k = k^{D_F-3} \\ 0 & \text{with probability } 1-h_k. \end{cases}$$

$$\partial_t \mathbf{v}^{D_F} + \mathcal{P}^{D_F}[\mathbf{v}^{D_F}\cdot\nabla\mathbf{v}^{D_F}] = -\mathcal{P}^{D_F}\nabla p + \nu\Delta\mathbf{v}^{D_F} + \mathcal{P}^{D_F} f$$



SELF-SIMILAR GALERKIN TRUNCATION

- *Decimation is random but frozen in time*
- same inviscid invariants as 3D NS
- *Statistical symmetries preserved*
- *no external scale introduced*

Frisch, Pomyalov, Procaccia, Ray *"Turbulence in non-integer dimensions by fractal Fourier decimation"* PRL 2012
Grossmann, Lohse , Reeh *"Developed Turbulence: From Full Simulations to Full Mode Reductions"* PRL 1996

DNS OF TURBULENCE ON A FRACTAL FOURIER SKELETON

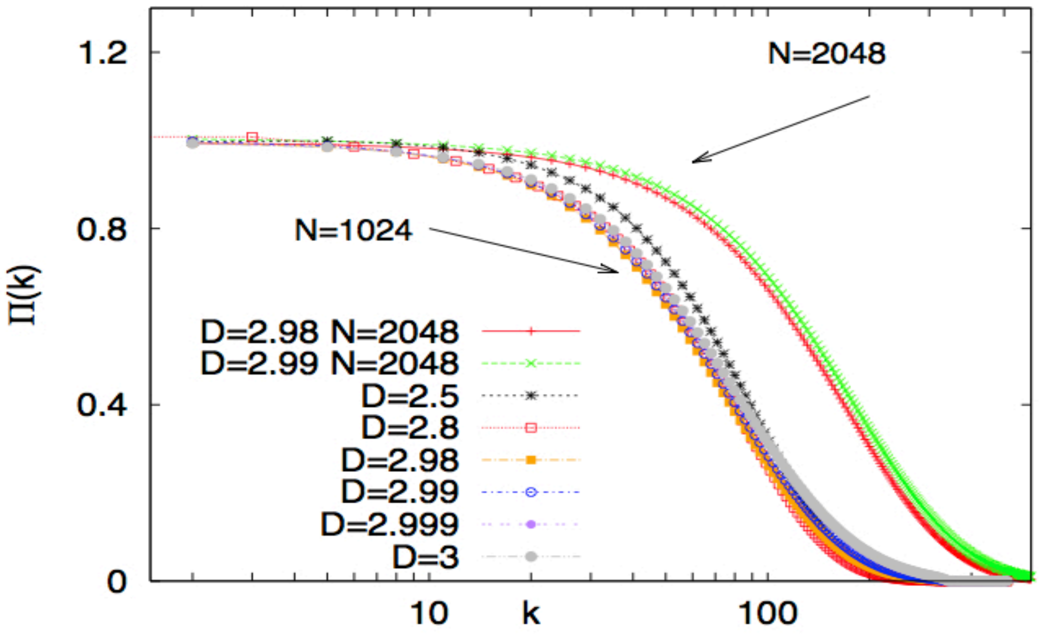
- Periodic, regular grid N^3 points; pseudo-spectral solver
- Homogeneous, isotropic **forcing f** yielding a constant energy injection rate

SUMMARY TABLE

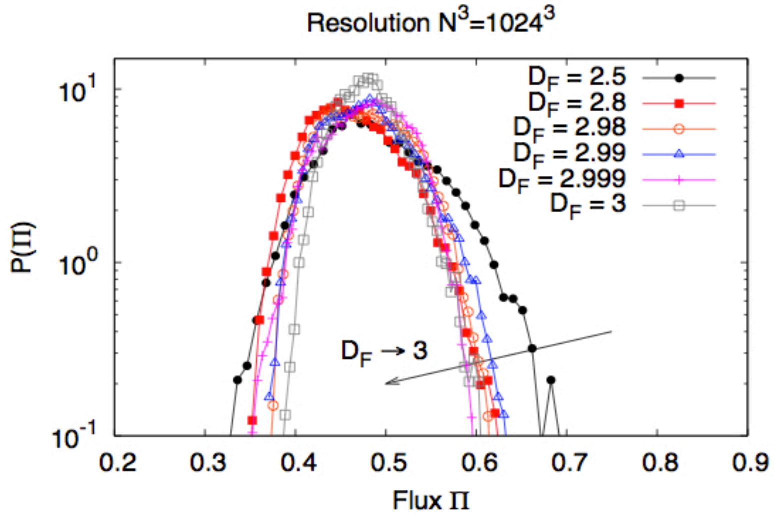
N^3 \ D_F	3.0	2.999	2.99	2.98	2.8	2.5
1024^3	X	X	X	X	X	X
2048^3			X	X		

% of SURVIVING FOURIER MODES $N^3 = 1024$

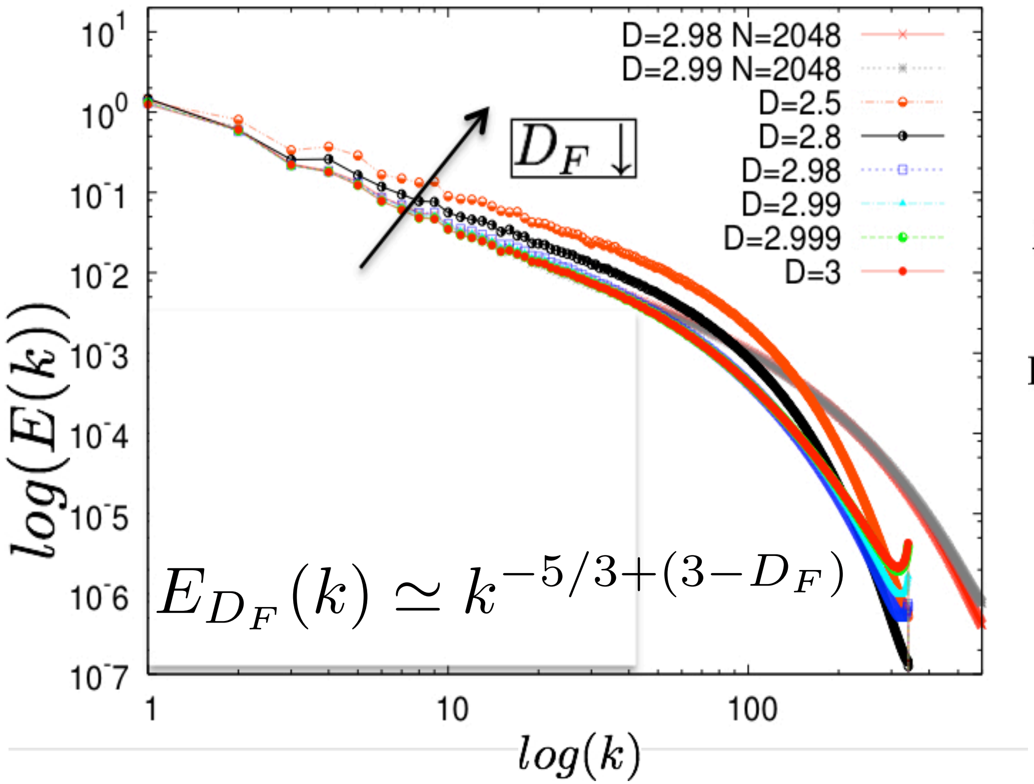
D_F	3.0	2.999	2.99	2.98	2.8	2.5
	100%	99%	93%	87%	25%	3%



Decimated dynamics at $D=D_F$ preserves constant spectral flux



LINEAR CORRECTION TO KOLMOGOROV "-5/3" SPECTRUM EXPONENT



fractal projector $P^D(k)$ (with $k = k_1 = -k_2$)

$$E^{D_F}(k) = \int_{|\mathbf{k}_1|=k} d^3k_1\,\gamma_{\mathbf{k}_1} \int d^3k_2\,\gamma_{\mathbf{k}_2} \langle \hat{\mathbf{u}}(\mathbf{k}_1)\hat{\mathbf{u}}^*(\mathbf{k}_2)\rangle$$

$$\Pi^{D_F}(k) = \int_{|\mathbf{k}_1|<k} d^3k_1\,\gamma_{\mathbf{k}_1} \int d^3k_2\,d^3k_3\,\gamma_{\mathbf{k}_2}\,\gamma_{\mathbf{k}_3}\mathcal{S}(\mathbf{k}_1|\mathbf{k}_2,\mathbf{k}_3)$$

fractal projectors $P^D(k_1)$, $P^D(k_2)$, $P^D(k_3)$

As in Krainchnan 1967, 1971:

$$\hat{u}(k) \simeq k^{-h}$$

SCALING ARGUMENT

$$\Pi_{D_F}(\lambda k) \simeq \lambda^{3D_F+1-3h}\,\Pi_{D_F}(\lambda k)$$

imposing

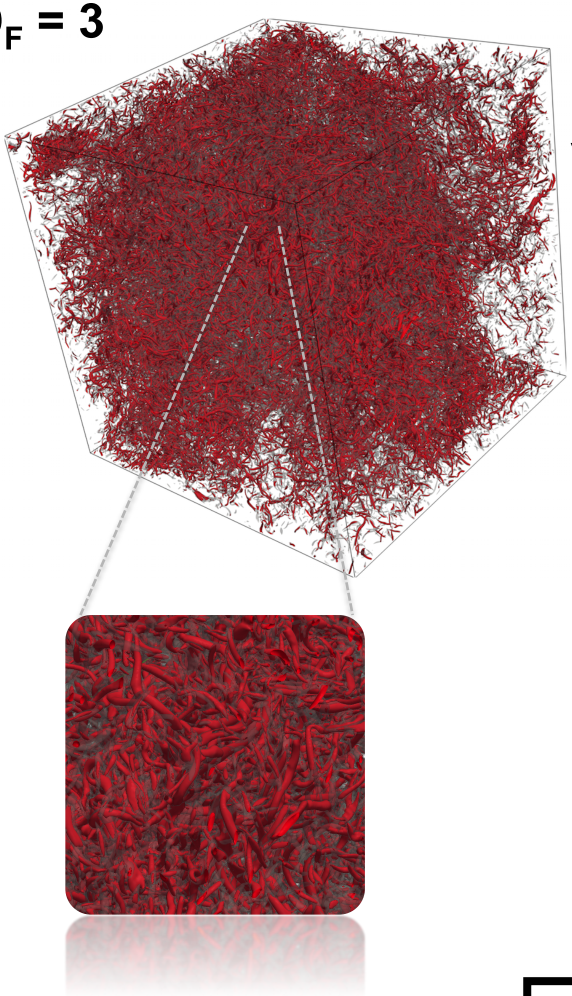
CONSTANT SPECTRAL FLUX

$$h = D_F + \frac{1}{3} \quad \rightarrow \quad E_{D_F}(\lambda k) \simeq \lambda^{-5/3+(3-D_F)}\,E(k)$$

→ *FRACTAL FOURIER DECIMATION MAKES THE* ***SPECTRUM SHALLOWER &*** *THE* ***VELOCITY FIELD ROUGHER***

WHAT ABOUT SMALL SCALES ?

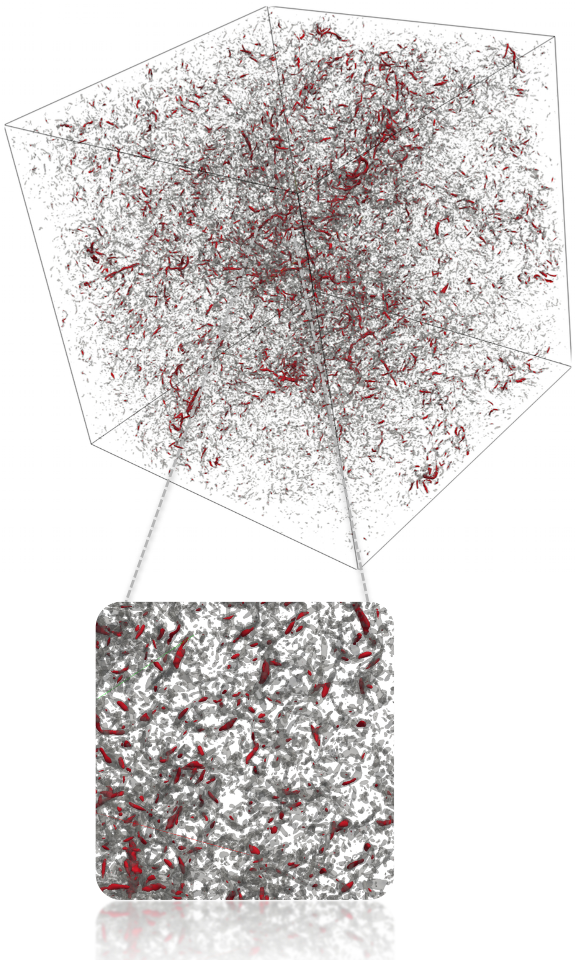
$D_F = 3$



Most Intense
Vortical structures

$D_F = 2.98$

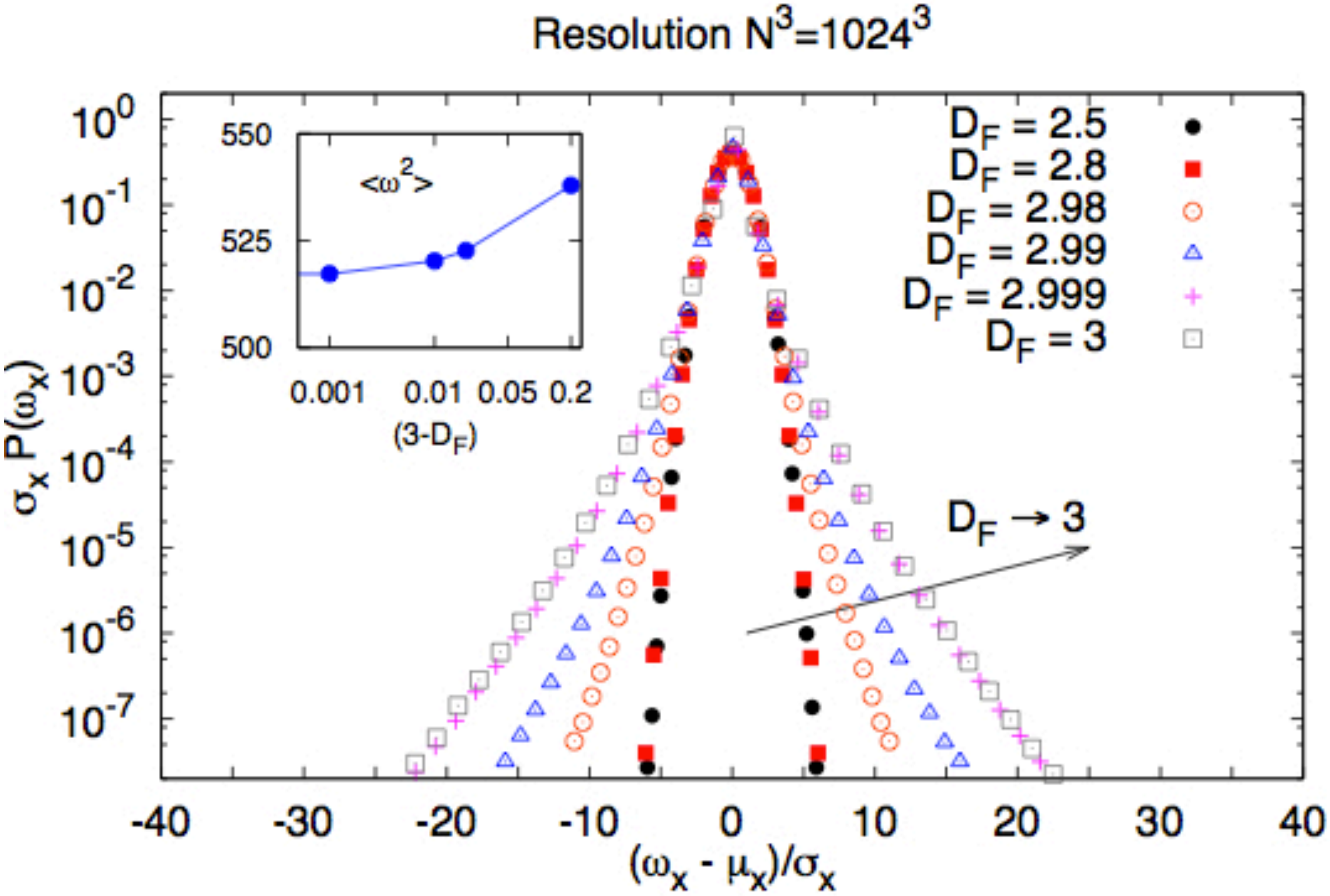
#dof = 87%



HINT:
S. Grossmann, D. Lohse ,
A. Reeh, PRL 1996

In summary, we repeat that in 3D Navier-Stokes turbulence the main origin of intermittency corrections seems to be the proper resolution of the phase space at the scale of interest. Reflections from the VSR seem

VORTICITY PDF AT CHANGING FRACTAL DIMENSION D_F



At D_F=2.98, the fluctuations have decreased their intensity by ~ 30% already.
while mean enstrophy is unchanged

FRACTAL FOURIER DECIMATION : dynamics or geometry ?

- *Fractal Fourier APOSTERIORI Projection on 3D Navier-Stokes realizations*

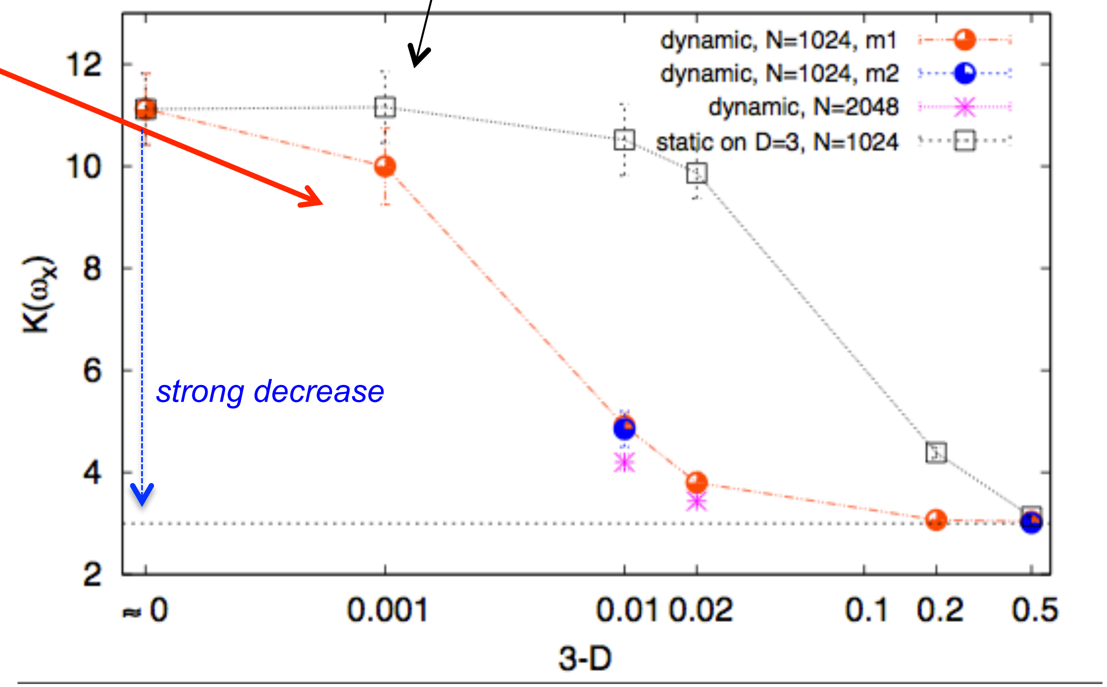
$$\begin{cases} \partial_t \mathbf{u} + B(\mathbf{u},\mathbf{u}) = \nu \Delta \mathbf{u} + \mathbf{f} \\ \mathbf{u} \to \mathcal{P}_{D_F} \mathbf{u} \end{cases}$$

- *Fractally Fourier DYNAMICALLY decimated Navier-Stokes*

$$\partial_t \mathbf{v}^{D_F} + \mathcal{P}_{D_F} B(\mathbf{v}^{D_F}, \mathbf{v}^{D_F}) = \nu \Delta \mathbf{v}^{D_F} + \mathbf{f}^{D_F}$$

Vorticity Kurtosis

$$K_4 = \frac{\langle \omega_x^4 \rangle}{\langle \omega_x^2 \rangle^2}$$



→ *Intermittency decreases does not result from a trivial reduction of degrees of freedom*

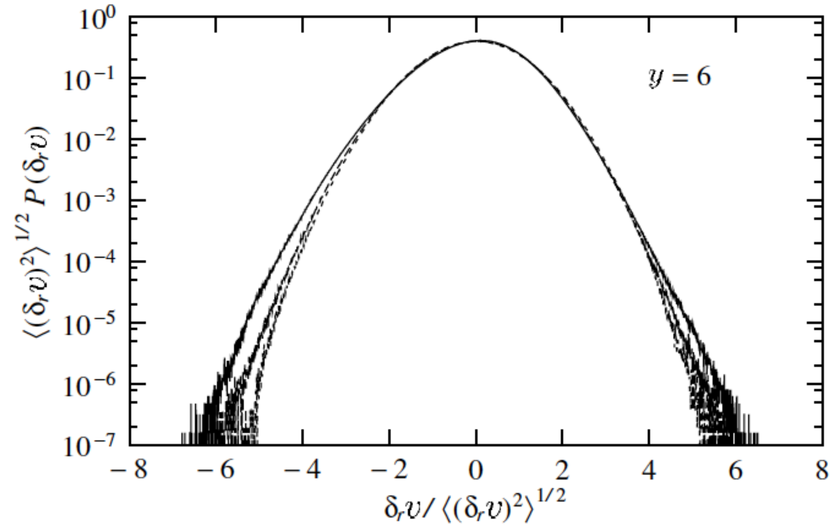
RANDOM NAVIER-STOKES DYNAMICS

$$\begin{cases} \partial_t \mathbf{v} + \mathbf{v}\cdot\nabla\mathbf{v} = -\nabla p + \nu\Delta\mathbf{v} + \mathbf{f} \\ \nabla\cdot\mathbf{v} = 0 \\ \langle f_i(\mathbf{k},t) f_j(\mathbf{k}',t')\rangle \simeq k^{4-D-\epsilon} P_{ij}(\mathbf{k})\,\delta(\mathbf{k}+\mathbf{k}')\,\delta(t-t') \end{cases}$$

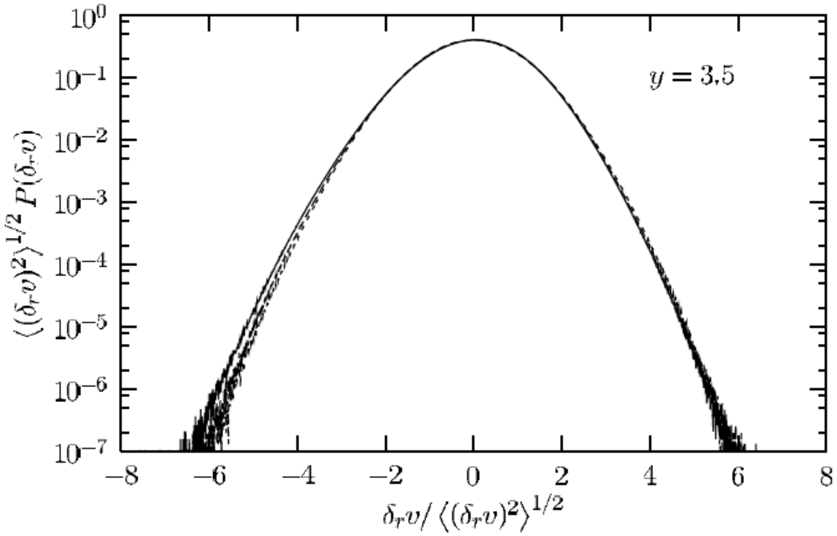
Forster, Nelson, Stephen PRA 1977
Fournier, Frisch PRA 1978

RG-calculation for ε → 0 and D=3 predicts energy spectrum $E(k) \sim k^{1-2/3\epsilon}$
For ε = 4, Kolmogorov spectrum recovered $E(k) = k^{-5/3}$

Intermittent SCALING ε = 6



Normal SCALING ε = 3.5



Numerics suggest a transition towards Gaussian behaviour at $\epsilon_c = 4$

Biferale, Lanotte, Toschi PRL 2004

SUMMARISING

We studied NAVIER-STOKES TURBULENCE in non-integer dimensions D_F= 3 → 2.5

→ The method is a RANDOM REMOVAL OF DEGREES OF FREEDOM

- SAME INVISCID INVARIANTS (kinetic energy and helicity)
- and STATISTICAL SYMMETRIES : HOMOGENEITY & ISOTROPY
- 1 tuning parameter : Fractal Dimension D_F

→ The Fourier Decimated Navier-Stokes eqs. are SELF-SIMILAR, which allows to speculate on the importance of the anomalous vs scale-invariant realizations

→ The goal: how intermittency is modified at changing the **vortex stretching mechanism** and the weight of local and non-local interactions in the NS equations.

- **For the MEAN FLUCTUATIONS, SMALL CORRECTIONS**

  a LINEAR CORRECTION " $-5/3 + (3 - D_F)$" *in the Kinetic Energy Spectrum exponent*

- **For the LARGE FLUCTUATIONS, HUGE CORRECTIONS!!**

i. ***An almost Gaussian** statistics is observed at* $\boldsymbol{D_F=2.98}$ *already.*

   *The system is **Gaussian at*** $\boldsymbol{D_F=2.8}$.

ii. ***Critical dimension** at which intermittency vanishes?*

- **The absence of some Fourier modes modifies the stat of all the others.**

i. Either by killing singular solutions responsible of the intermittent behaviour

ii. Or by modifying the nature of coherent structures governing vortex stretching and turbulent bursts

- **Is INTERMITTENCY a perturbative effect in** $\boldsymbol{(3 - D_F)}$ **?**

  Possible if singular solutions responsible for anomalous scaling survive tiny decimations

Turbulence on a Fractal Fourier set

Lanotte, Benzi, Biferale, Malapaka, Toschi, to appear on Phys. Rev. Lett. (2016)

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