On the width of the confining flux tube in the 3D U(1) lattice gauge theory

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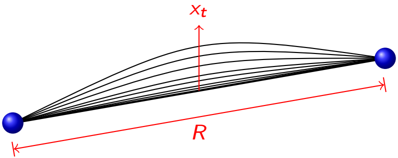
Motivation - 1

The shape of the fluxtube is deeply related to the confining mechanism:



Relevant observables

- The chromoelectric field parallel to the charges axis $E_l(x_t)$, measured in their symmetry plane.
- The fluxtube squared width

$$w^2(R) = \int \mathrm{d}x_t x_t^2 E_l(x_t)$$

Two pictures

- Effective String Theory.
- The Dual superconductor model.

Motivation - 2

The 3D U(1) LGT and the rigid string

Rigid Strings

In the 3D U(1) lattice gauge theory, the static potential can be given an effective description in terms a NambuGoto (NG) + Rigid string[a].

$$S_{\text{eff}} = \underbrace{\sigma \int d^2\xi \sqrt{g}}_{\text{NG}} + \underbrace{\alpha \int d^2\xi \sqrt{g} K^2,}_{\text{Rigidity}},$$

ξ worldsheet coordinates, g induced metric, K extrinsic curvature.

[a](Caselle et al., 2015, Polyakov, 1997)

Consequences for the fluxtube

- Its transverse profile should be exponentially decaying, in contrast to NG (gaussian).
- Its squared width should be constant with R, logarithmic with NG.

Effective String Theory - 1

The fluxtube as a fluctuating string

The effective action

$$Z_R = \int [\mathcal{D}X] \, e^{-S_{\text{eff}}[X]_{\text{RxL}}}$$

R charge separation, L time extent of the system, X_i transverse displacements.

- S_{eff} strongly constrained by Poincaré symmetry: few free parameters.
- Long string approximation,

$$S_{\text{eff}} = \sigma RL + \frac{\sigma}{2} \int \mathrm{d}^2\xi \underbrace{(\partial_\alpha X \cdot \partial^\alpha X)}_{\text{L. O.}} + \cdots , \quad \sigma \text{ string tension},$$

  the profile should be a pure gaussian.
- This theory is valid for $R \gg 1/\sqrt{\sigma}$

Worldsheet observables

$$\langle O \rangle = \frac{1}{Z_R} \int [\mathcal{D}X] \, O(X) e^{-S_{\text{eff}}[X]_{\text{RxL}}}$$

Effective String Theory - 2

Observables

Definitions

$$V(R) = -\frac{1}{L}\log Z_R, \quad w^2(R) = \frac{1}{Z_R}\int [\mathcal{D}X]\, X(R/2)\cdot X(R/2) e^{-S_{\text{eff}}[X]_{\text{RxL}}}$$

$T = 0$

$$V(R) = \sigma R - \frac{(D-2)\pi}{24R}$$

$$w^2(R) = \frac{(D-2)}{2\pi\sigma}\log R/R_c$$

[a]

[a] Luscher et al. (1981)

Finite $T(=1/L)$

$$\sigma(L) = \sigma_0 - \frac{\pi(D-2)}{6L^2}$$

$$w^2(R) \sim \frac{D-2}{4L\sigma}R$$

[a]

[a] Allais and Caselle (2009), Gliozzi et al. (2010a,b)

Comparison with Numerical results

Perfect agreement with the data both for abelian[a] and non abelian models, and at the next to leading order[b] as well, so far...

[a] Caselle (2010), Gliozzi et al. (2011)
[b] Caselle (2010), Gliozzi et al. (2010a,b, 2011)

The dual superconductor model[1]

A phenomenological model of color confinement

The Condensation of (dynamically formed) chromomagnetic monopoles causes Confinement of chromoelectric charges via the (dual) Meissner effect.

The abelian case

$$\mathcal{L} = -\frac{1}{4}\bar{F}^{\mu\nu}\bar{F}_{\mu\nu} + \frac{1}{2}(D_\mu\psi)^\star D^\mu\psi - \frac{1}{2}b(\psi^\star\psi - v^2)^2$$

with $\bar{F}_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \quad D_\mu = \partial_\mu + igB_\mu$, and

- $1/\xi = m_H = 2v\sqrt{b}$ mass of the higgs field (ξ coherence length of the condensate).
- $1/\mu = m_v = gv$ mass of the gauge field (μ London penetration depth of the field).

Tubelike solutions to the EOM

$$E_l(x_t) = \Phi\, m_v^2\, K_0\left(m_v|x_t|\right), \quad \text{for } |x_t| \gg \mu$$

K_0 modified Bessel function, Φ chromoelectric flux.

[1] Mandelstam (1976), Nambu (1974), 't Hooft (1979)

Summarizing...

Effective string theory

- The profile should be a pure Gaussian, at LO.
- The squared width should grow logarithmically with quark separation.

Dual superconductor model

- The profile should be described by a decaying Bessel function.
- The squared width should be constant and close to μ^2.

The $U(1)$ Lattice gauge theory in 3D

Definition

The partition function

On a 3D euclidean spacetime lattice Λ (spacing a),

$$S = \beta \sum_{x\in\Lambda} \sum_{1\leq\mu<\nu\leq 3} \left[1 - \cos\vartheta_{\mu\nu}(x)\right], \quad \beta = \frac{1}{ae^2}, \quad \vartheta_\mu(x) \in (-\pi, \pi],$$

and using discrete forms notation

$$Z_{\mathrm{U}(1)} = \prod_{c_1} \int_{-\pi}^{\pi} \mathrm{d}(\vartheta)\, e^{-\beta \sum_{c_2} (1-\cos \mathrm{d}\vartheta)}, \quad c_i\text{: i simplices}$$

where ϑ is a 1-chain, $\mathrm{d}\vartheta$ is a 2-chain and we get rid of the indices.

The $U(1)$ Lattice gauge theory in 3D

Confinement

The dilute gas description

If $(\beta \gg 1)$, taking the periodicity of S in ϑ into account

$$Z_{\mathrm{U}(1)} = Z_{\mathrm{sw}} Z_{\mathrm{top}} = Z_{\mathrm{sw}} \sum_{\{q\}} e^{-2\pi^2 \beta (q, \Delta^{-1} q)}$$

where Z_{sw} describes spin-waves, Z_{top} describes topological excitations.

The semiclassical approximation[a]

[a](Göpfert and Mack, 1981, Polyakov, 1977)

$$m_0 a = c_0(\simeq 1)\sqrt{8\pi^2\beta}\, e^{-\pi^2 \beta v(0)}, \quad \sigma a^2 \geq \frac{c_\sigma(\simeq 8)}{\sqrt{2\pi^2\beta}} e^{-\pi^2 \beta v(0)}, \quad v(0) = 0.2527$$

- The model is always in the confined phase in $3D$
- The ratio

$$\frac{m_0}{\sqrt{\sigma}} = \frac{2\pi c_0}{\sqrt{c_\sigma}} (2\pi\beta)^{3/4} e^{-\pi^2 v(0)\beta/2},$$

can be tuned at will by an appropriate choice of β, in contrast to the general Yang-Mills case.

The $U(1)$ Lattice gauge theory in 3D

The dual formulation of the model[2]

Each plaquette factor in $Z_{\mathrm{U}(1)}$ is periodic in ϑ

$$e^{-\beta(1-\cos \mathrm{d}\vartheta)} = \sum_{k=-\infty}^{\infty} e^{-\beta} \mathrm{I}_{|k|}(\beta) e^{\imath k \mathrm{d}\vartheta}, \quad I_\alpha \text{ mod. Bessel functions of order } \alpha$$

The integration in $\mathrm{d}(\vartheta)$ yields $\delta k = 0$, which is easily solved by an the integer valued $^\star l$ defined on the dual lattice such that

$$^\star k = \mathrm{d}^\star l.$$

The dual version of $Z_{\mathrm{U}(1)}$

We obtain a globally $\mathbb{Z}$ symmetric spin model

$$Z_{\mathrm{U}(1)} = e^{-\beta N_l} \sum_{\{^\star l=-\infty\}}^{\{\infty\}} \prod_{^\star c_1} \mathrm{I}_{|\mathrm{d}^\star l|}(\beta), \quad ^\star c_1 \text{ dual links.}$$

[2](Savit, 1980)

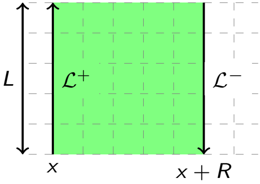
The $U(1)$ Lattice gauge theory in 3D

Sources of field

Static charges can be represented by Polyakov lines

$$Z^R_{\mathrm{U}(1)} = \prod_{c_1} \int_{-\pi}^{\pi} \mathrm{d}\vartheta \, e^{-\beta \sum_{c_2} (1-\cos \mathrm{d}\vartheta)} \prod_{\mathcal{L}^+} e^{\imath\vartheta} \prod_{\mathcal{L}^-} e^{-\imath\vartheta}$$

Perfoming the duality transformation...



$$Z^R_{\mathrm{U}(1)} = e^{-\beta N_l} \sum_{\{^\star l=-\infty\}}^{\{\infty\}} \prod_{^\star c_1} \mathrm{I}_{|\mathrm{d}^\star l + ^\star n|}(\beta)$$

where $^\star n$ is an integer valued dual 1-chain which is nonvanishing only on the links dual to the green surface.

Elements of the EM tensor[a]

[a] (Zach et al., 1998)

$$\langle F_{\mu\nu}(x) \rangle_{q\bar{q}} = \frac{1}{\sqrt{\beta}} \langle \sin \vartheta_{\mu\nu}(x) \rangle_{q\bar{q}} \quad \Rightarrow \quad \langle F(x) \rangle_{q\bar{q}} = \frac{\langle \mathrm{d}^\star l +^\star n \rangle}{\sqrt{\beta}}$$

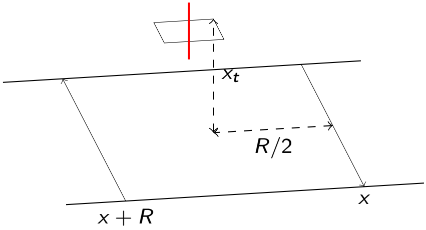
Measurement of the Electric field

Simulation setup

The measured quantity

There is no Schwinger line!

$$E_l(x_t)a^2 = \frac{\langle \mathrm{d}^\star l +^\star n \rangle}{\sqrt{\beta}}$$



Update algorithm

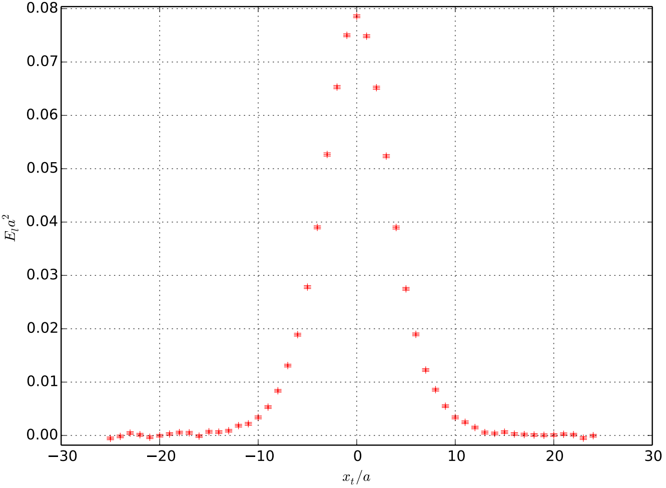
- Local Metropolis update of the dual model.
- Hierarchical update around the local measured quantity.

Lattice setting

- Lattices $(64a)^3$ and $(96a)^3$ respectively at $\beta = 1.7, 2.0$ and $\beta = 2.2, 2.4$
- $T \sim 0$
- Charges separation $R = 4a \div 24a$.

Measurement of the Electric field

Example at $R_{q\bar{q}} = 16a$ and $\beta = 2.0$.



The Squared width

Computation - 1

Two ways of computing w^2

- Fit the profile with some arbitrarily complicated analytically integrable function, then analitically compute w^2: its error will depend on the estimated parameters.
- Use a discretized version of the continuum formula

$$w^2 a^{-2} = \frac{\sum_{x_t} x_t^2 E_l(x_t)}{\sum_{x_t} E_l(x_t)}.$$

For large x_t, $E_l(x_t) \simeq 0$ and the corresponding points only contribute to the (systematical) error of $w^2 a^{-2}$.

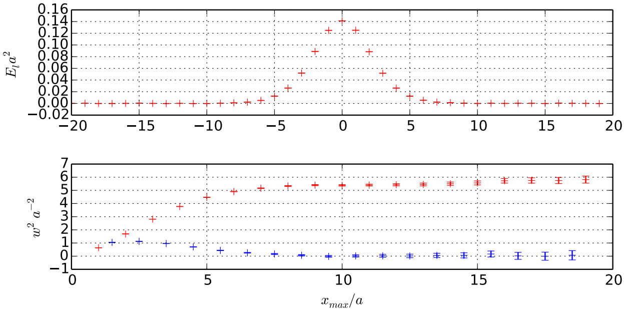
The Squared width

Computation - 2

We used the second way: introducing a cutoff $x_{\max}$ in the sum...

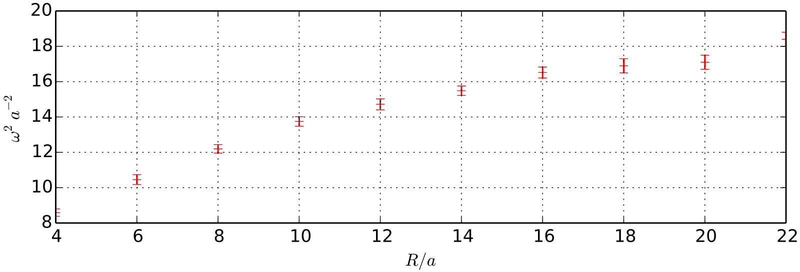
$$w^2 a^{-2} = \frac{\sum^{x_{\max}} x_t^2 E_l(x_t)}{\sum^{x_{\max}} E_l(x_t)}.$$

For $\beta = 1.7$ and $R = 16a$:



The Squared width

$\beta = 1.7$



$w^2 a^{-2}$ grows with R/a! dual superconductor ruled out?

The Squared width

Fit at $\beta = 2.0$

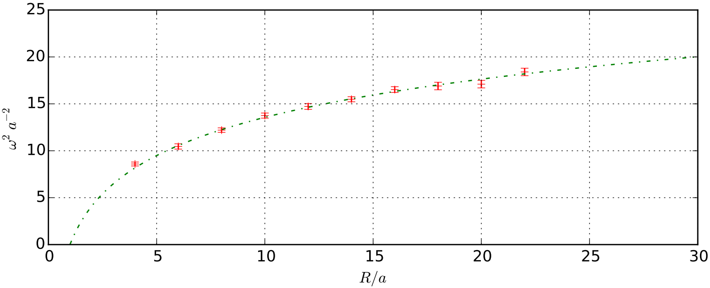
Fit of the prediction of gaussian EST

$$w^2(R) = \frac{1}{2\pi s} \log R/R_c$$

Results of the fit at $\beta = 2.0$

$$sa^2 = 0.0282(8) \quad R_c/a = 0.90(6) \quad \chi_r^2 = 0.58$$

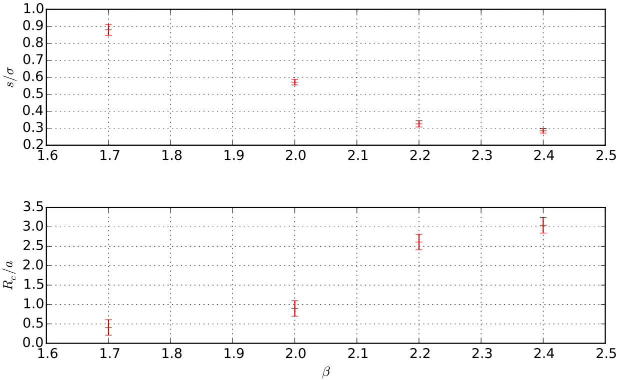
$$1/\sqrt{\sigma} \sim 5a \quad R_{\text{min}}/a = 4 \quad \text{d.o.f.} = 8$$



The Squared width

Value and scaling of the fitted parameters

The logarithmic law describes the data very well, but the coefficient s is not compatible with σ!



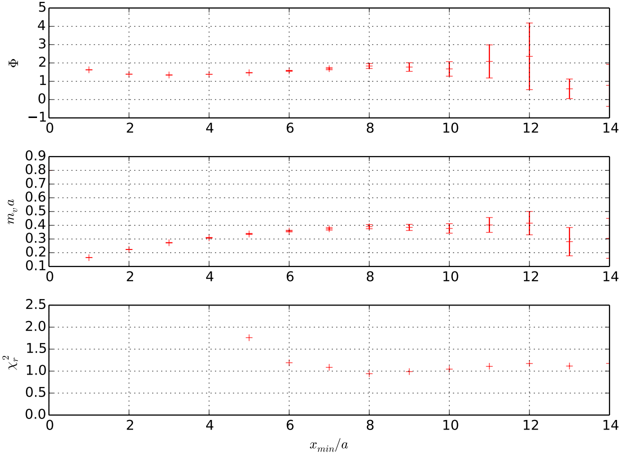
The transverse profile of the flux tube

Predictions of the dual superconductor model

The solution of the EOM of the dual Abelian higgs model

$$E_l(x_t) = \Phi m_v^2 K_0\left(m_v|x_t|\right)$$

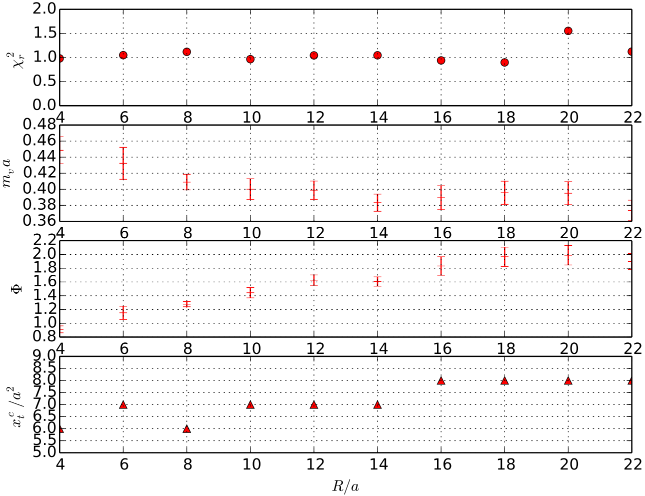
should predict the profile outside of the vortex core: for $x_t > x_{\text{min}}$...



The transverse profile of the flux tube

fits for the whole range of R at $\beta = 2.0$.

..and $m_v a$ should be independent of R (not so for Φ and x_t^c!!)



Results - 1

β	m_v/m_0
1.7	0.941(15)
2.0	0.985(27)
2.2	1.02(5)
2.4	1.03(8)

To within the errors $m_v = m_0$, and

$$w^2 = \frac{A}{m_0^2} \log R/R_c$$

Now if $m_0 = c\sqrt{\sigma}$ and A const. with β, then there are two distinct cases

The 3D U(1) LGT

$c = c(\beta)$,

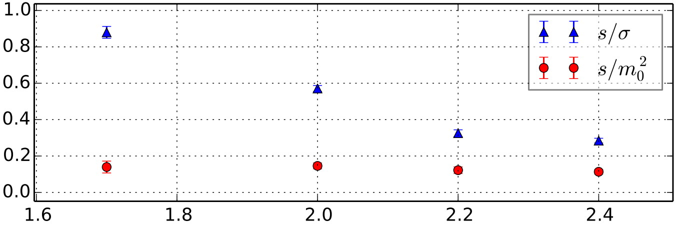
$$w^2(R) = \frac{A}{c^2(\beta)\sigma} \log R/R_c$$

Other LGT's

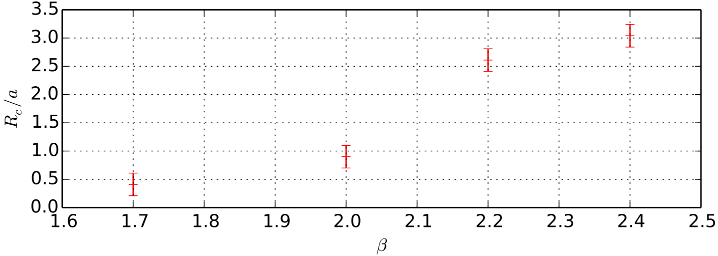
i.e. YM in 4D where c is a constant,

$$w^2(R) = \frac{A}{c^2\sigma} \log R/R_c = \frac{1}{2\pi\sigma} \log R/R_c$$

Results - 2



1.0
0.8
0.6
0.4
0.2
0.0
1.6
1.8
2.0
2.2
2.4
s/σ
s/m_0^2



R_c/a
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3
2.4
2.5
β

Conclusions

- The squared width scales with m_0^2,

$$w^2 = \frac{A}{m_0^2} \log R/R_c$$

  with $A =$ const. with β.

- The thickness of the fluxtube is the sum of two contributions:
  1. one, coming from the tails of the fluxtube, is equal to the London penetration depth of the model: constant with quark separation.
  2. the other, coming the core of the fluxtube, has a bell shaped appearance: grows with quark separation.

Future steps:

- Try to fit other solutions to the Abelian higgs model to the profiles[3].
- Probe the value of w^2 under the scale $1/\sqrt{\sigma}$ of validity of "standard"(NG) effective string theory and compare with predictions that include rigidity.
- Work at finite T.

[3](Cea et al., 2014)

Bibliography I

A. Allais and M. Caselle. On the linear increase of the flux tube thickness near the deconfinement transition. *JHEP*, 0901:073, 2009. doi: 10.1088/1126-6708/2009/01/073.

M. Caselle. Flux tube delocalization at the deconfinement point. *JHEP*, 08:063, 2010. doi: 10.1007/JHEP08(2010)063.

Michele Caselle, Marco Panero, Roberto Pellegrini, and Davide Vadacchino. A different kind of string. *JHEP*, 1501:105, 2015. doi: 10.1007/JHEP01(2015)105.

Paolo Cea, Leonardo Cosmai, Francesca Cuteri, and Alessandro Papa. Flux tubes in the SU(3) vacuum: London penetration depth and coherence length. *Phys. Rev.*, D89(9):094505, 2014. doi: 10.1103/PhysRevD.89.094505.

F. Gliozzi, M. Pepe, and U. J. Wiese. The Width of the Color Flux Tube at 2-Loop Order. *JHEP*, 11:053, 2010a. doi: 10.1007/JHEP11(2010)053.

F. Gliozzi, M. Pepe, and U. J. Wiese. The Width of the Confining String in Yang-Mills Theory. *Phys. Rev. Lett.*, 104:232001, 2010b. doi: 10.1103/PhysRevLett.104.232001.

F. Gliozzi, M. Pepe, and U. J. Wiese. Linear Broadening of the Confining String in Yang-Mills Theory at Low Temperature. *JHEP*, 01:057, 2011. doi: 10.1007/JHEP01(2011)057.

Markus Göpfert and Gerhard Mack. Proof of Confinement of Static Quarks in Three-Dimensional U(1) Lattice Gauge Theory for All Values of the Coupling Constant. *Commun. Math. Phys.*, 82:545, 1981. doi: 10.1007/BF01961240.

M. Luscher, G. Munster, and P. Weisz. How Thick Are Chromoelectric Flux Tubes? *Nucl. Phys.*, B180:1, 1981. doi: 10.1016/0550-3213(81)90151-6.

Bibliography II

S. Mandelstam. Vortices and Quark Confinement in Nonabelian Gauge Theories. *Phys.Rept.*, 23: 245–249, 1976. doi: 10.1016/0370-1573(76)90043-0.

Yoichiro Nambu. Strings, Monopoles and Gauge Fields. *Phys. Rev.*, D10:4262, 1974. doi: 10.1103/PhysRevD.10.4262.

Alexander M. Polyakov. Quark Confinement and Topology of Gauge Groups. *Nucl. Phys.*, B120: 429–458, 1977. doi: 10.1016/0550-3213(77)90086-4.

Alexander M. Polyakov. Confining strings. *Nucl. Phys.*, B486:23–33, 1997. doi: 10.1016/S0550-3213(96)00601-3.

Robert Savit. Duality in field theory and statistical systems. *Rev. Mod. Phys.*, 52:453–487, Apr 1980. doi: 10.1103/RevModPhys.52.453. URL http://link.aps.org/doi/10.1103/RevModPhys.52.453.

Gerard 't Hooft. A Property of Electric and Magnetic Flux in Nonabelian Gauge Theories. *Nucl. Phys.*, B153:141, 1979. doi: 10.1016/0550-3213(79)90595-9.

Martin Zach, Manfried Faber, and Peter Skala. Investigating confinement in dually transformed U(1) lattice gauge theory. *Phys. Rev.*, D57:123–131, 1998. doi: 10.1103/PhysRevD.57.123.