Magnetic and Topological Properties of Strong Interactions

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Many many thanks to:
C. Bonati, M. D'Elia, M. Mariti, M. Mesiti, F. Sanfilippo.

Outline

- Introduction and phenomenological motivations
- QCD $\oplus$ background (electro-)magnetic fields
  - Magnetic Catalysis
    [D'Elia and N, PRD 83 (2011) 114028]
  - Magnetic Susceptibility at finite temperature
    [Bonati, D'Elia, Mariti, N and Sanfilippo, PRL 111 (2013) 182001]
    [Bonati, D'Elia, Mariti, N and Sanfilippo, PRD 89 (2014) 054506]
  - Susceptibility of the QCD vacuum to CP-odd backgrounds
    [D'Elia, Mariti and N, PRL 110 (2013) 082002]
- Phase diagram in the $T - \theta$ plane.
  [D'Elia and N, PRD 88 (2013) 034503] [D'Elia and N, PRL 109 (2012) 072001]
- Conclusions

Intro: the gauge theory of strong interactions

Quantum ChromoDynamics (QCD)

Gauge Quantum Field Theory to describe the way quarks and gluons interact.

$\Lambda_{QCD} \sim 200\ MeV$

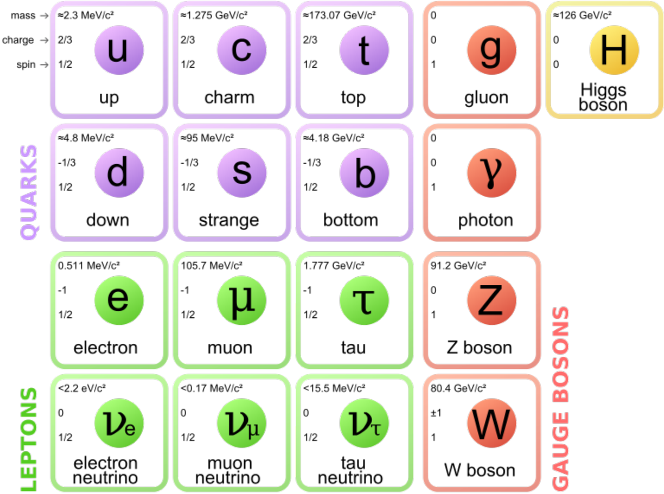
Asymptotic freedom

High Energy

$g_s \ll 1$

Low Energy

$g_s \sim 1$



Intro: the gauge theory of strong interactions

Quantum ChromoDynamics (QCD)

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$$\boxed{\mathcal{L}_{QCD} = \overline{\psi}(i\not{D} - m)\psi - \frac{1}{4} F^{A\mu\nu} F^{A}_{\mu\nu}}$$

$$D_\mu = \partial_\mu + i g_s A^A_\mu T^A$$

Local gauge symmetry $\longrightarrow SU(3)$

Non-perturbative properties

- confinement
- topological activity
- spontaneous breaking of chiral symmetry

$$\begin{cases} \psi \longrightarrow e^{i\alpha_j \sigma_j \gamma_5} \psi \\ \overline{\psi} \longrightarrow \overline{\psi} e^{i\alpha_j \sigma_j \gamma_5} \end{cases} \qquad \langle \overline{\psi} \psi \rangle \neq 0$$

These properties change according to:

T, μ_B, EM Fields, Topological θ term, ...

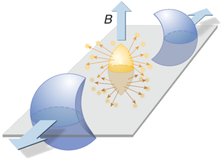
X$\cdots$XL magnetic fields

Electroweak corrections are usually small if compared to the strong interactions. But what happens if there is magnetic field large enough to be comparable with the scale Λ_{QCD}?

- Astrophysics
  Large magnetic fields ($eB \sim 10^{10}$ Tesla) in a class of neutron stars called **magnetars**
  [Duncan and Thompson, '92]

- Cosmology
  Large magnetic fields ($eB \sim 10^{16}$ Tesla, $\sqrt{eB} \sim 1.5$ GeV) may have been produced at the **cosmological electroweak phase transition**
  [Vachaspati, '91; Grasso and Rubinstein, '01]

- Non-central heavy ion collisions (HIC)
  In **non-central HIC** the largest magnetic field ever created in a lab (eB up to 10^{15} Tesla $\sim$ 0.3 GeV^2 $\sim$ $15 m_\pi^2$ at LHC)
  [Skokov, Illarionov and Toneev, '09]



Possible effects of the B field on QCD

The presence of a large B field may have various impacts on QCD. Some examples and possible questions:

- Phase Diagram:
  - Shift of the deconfinement transition $\rightarrow$ T_C (or T_{PC}) decreases with $|B|$
  - New unconventional phases?
- Equation of State:
  - Magnetic contribution to the EoS ✓
  - Is strongly interacting matter paramagnetic of diamagnetic? ✓
- Vacuum Structure:
  - Magnetic catalysis $\rightarrow$ Enhancement of chiral symmetry breaking ✓
  - Influence on the topological properties of QCD? ✓
  - Presence of anisotropies due to the B-field
- The Chiral Magnetic Effect
  [Vilenkin, '80; Kharzeev, McLerran, Warringa, Fukushima, Zhitnitsky, ...]

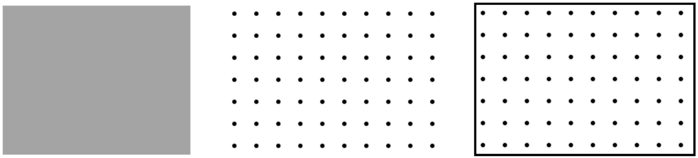
The Non-Perturbative Approach: Lattice QCD

Feynman path integral formulation of the euclidean gauge theory

$$\langle \hat{O} \rangle = \frac{\int \mathcal{D}[\phi] O[\phi] e^{-S_E[\phi]}}{\int \mathcal{D}[\phi] e^{-S_E[\phi]}} = \int \mathcal{D}[\phi] O[\phi] P[\phi]$$

We regularize the integral by introducing a lattice

$\mathbb{R}^4$ UV cut-off $\rightarrow a$ IR cut-off $\rightarrow V$



$$\langle \hat{O} \rangle = \lim_{a \to 0} \lim_{V \to \infty} \int \left(\prod_{n=1}^{N} d\phi(n) \right) O[\{\phi(n)\}] P[\{\phi(n)\}]$$

$$P = P_{Gauge} \cdot P_{Ferm}$$

Lattice QCD & magnetic field

A background QED field enters the Lagrangian by modifying the covariant derivative:

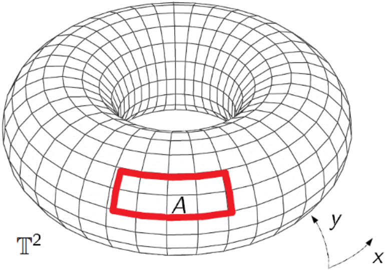
$$D_\mu = \partial_\mu + igA^a_\mu T^a \quad \longrightarrow \quad \partial_\mu + igA^a_\mu T^a + iqa_\mu$$

On the lattice:

- Gluon field $A^a_\mu(x)T^a \longrightarrow U_\mu(n) = \exp(iagA^a_\mu T^a)$, SU(3) (integration link vars)
- Photon field $a_\mu(x) \longrightarrow u_\mu(n) = \exp(iaqa_\mu)$, U(1) (fixed link vars)

The lattice discrete covariant derivative will read:

$$D_\mu\overline{\psi} \longrightarrow \frac{1}{2a}\left(U_\mu(n)u_\mu(n)\psi(n+\hat{\mu}) - U^\dagger_\mu(n-\hat{\mu})u^*_\mu(n-\hat{\mu})\psi(n-\hat{\mu})\right)$$



Quantization of qB [an IR Effect]

Finite volume $\rightarrow$ periodic b.c.
quantization of the B field
$(qB) = 2\pi b/(L_x L_y)$

The same as what happens for the quantization of the magnetic monopole.

Magnetic Catalysis at $T = 0$

Enhancement of chiral symmetry breaking due to B: $\partial_B \langle \overline{\psi}\psi \rangle \neq 0$

$$\langle \overline{\psi}\psi \rangle (m, m', B, B') = \int \mathcal{D}U \, \mathcal{P}[m', B'] \, \mathrm{Tr}(M^{-1}[m, B, q])$$

$\downarrow$ Probability distribution $e^{-S_{Gauge}} \det M[m, B, q]$

$\downarrow$ Observable

Magnetic Catalysis at $T = 0$

Enhancement of chiral symmetry breaking due to B: $\partial_B \langle\overline{\psi}\psi\rangle \neq 0$

- Total Signal:

$$\langle\overline{\psi}\psi\rangle(B) = \int \mathcal{D}U\, \mathcal{P}[m, B]\, \mathrm{Tr}(M^{-1}[m, B, q])$$

- Valence Contribution (or pseudo-quenched):

$$\langle\overline{\psi}\psi\rangle^{\mathrm{val}}(B) = \int \mathcal{D}U\, \mathcal{P}[m, 0]\, \mathrm{Tr}(M^{-1}[m, B, q])$$

- Sea Contribution (or dynamical):

$$\langle\overline{\psi}\psi\rangle^{\mathrm{dyn}}(B) = \int \mathcal{D}U\, \mathcal{P}[m, B]\, \mathrm{Tr}(M^{-1}[m, 0, q])$$

We define $r(B)$ the relative change in the condensate:

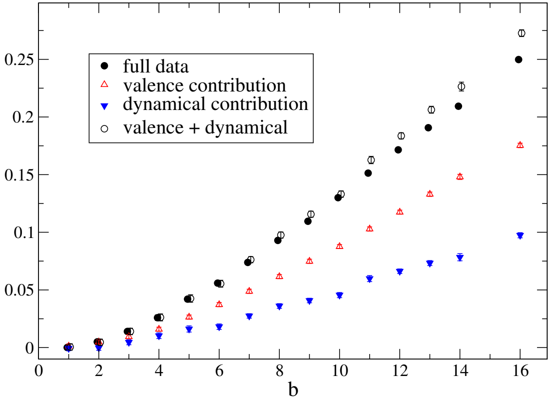
$$r(B) = \frac{1}{\langle\overline{\psi}\psi\rangle(0)} \left(\langle\overline{\psi}\psi\rangle(B) - \langle\overline{\psi}\psi\rangle(0)\right)$$

Assuming that the effect of B on $\mathcal{P}$ and on the observable are small we have:

$$r_{u/d}(B) = r^{\mathrm{val}}_{u/d}(B) + r^{\mathrm{dyn}}_{u/d}(B) + \mathcal{O}(B^4)$$

Valence and Dynamical contributions

Simulation of $N_f = 2$ QCD at higher than physical pion mass.



- $r(B) \simeq r^{\mathrm{dyn}}(B) + r^{\mathrm{val}}(B)$
- $r^{\mathrm{dyn}}(B) \simeq 0.4 \cdot r(B)$

A sizeable part of the signal is due to the modification of the gluon field distribution induced by the magnetic field!

Diamagnet or Paramagnet?

Questions:
Does strongly interacting matter behaves like a paramagnet or like a diamagnet?
What are the magnetic properties of QCD at finite temperature?
Magnetic susceptibility $\chi(T)$: positive or negative? depends on T?

How to answer (?):
in principle one could evaluate directly the magnetization and higher order derivatives

$$M = \chi^{(1)} = \frac{\partial \log \mathcal{Z}}{\partial (eB)}; \quad \chi^{(n)} = \frac{\partial^n \log \mathcal{Z}}{\partial (eB)^n}$$

The free energy density ($f = -(T/V)\log \mathcal{Z}$) can be formally expandend close to $B = 0$

$$f(T,B) = f(T,0) - \frac{T}{V}\sum_{n=1} \frac{\chi^{(n)}|_{eB=0}}{n!}(eB)^n$$

and the magnetic susceptibility would be $\chi^{(2)}|_{eB=0} \equiv \chi$.

But

On the lattice the magnetic field is quantized, hence the derivative $\partial/\partial(eB)$ is not a well defined operation.

Our approach: free energy finite differences

1 – We find a way to compute $\Delta f(T,b) = f(T,b+1) - f(T,b)$.

2 – The determination of Δf can be used to extract χ (*e.g.* by means of suitable fitting procedure).

The approach we devised (based *a posteriori* on a theorem by Jarzynski [Jarzynski, PRL '97]) can be summarized as "differentiate and, then, integrate".

$$f(T,b+1) - f(T,b) = \Delta f = -\frac{T}{V}\int_{(T,b)}^{(T,b+1)} d\vec{p}\, \frac{\partial \log \mathcal{Z}}{\partial \vec{p}}.$$

The efficiency of the method is related to the shape of the path.

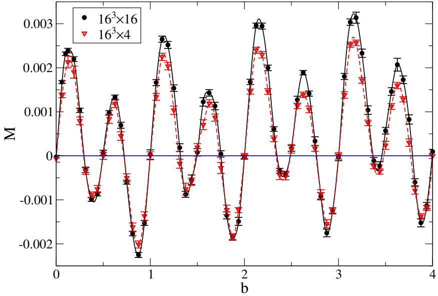
Our choice is to go straight from b to $b+1$ by introducing a real valued magnetic field:

$$\Delta f = -\frac{T}{V}\int_{b}^{b+1} db\, \frac{\partial \log \mathcal{Z}}{\partial b}$$

Warning:
The integrand $\partial log \mathcal{Z}/\partial b$ is not the magnetization, it is the derivative of the interpolating free energy defined at real b.

Our approach: free energy finite differences

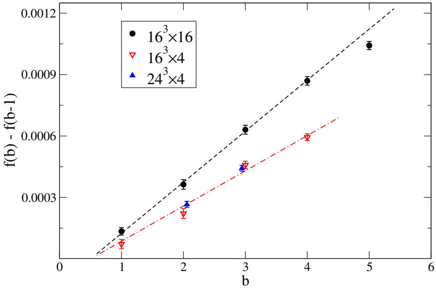
The pseudo–magnetization $M = \partial \log \mathcal{Z} / \partial b$ is an oscillating function.



$$M(b) = 0$$

for all the integer (properly quantized) values of b

$\Longrightarrow$ But its integral is regular!



$$f(b, T) - f(b-1, t) \propto (2b-1)$$

$$f(b, T) \propto b^2$$

The system displays a linear response for small enough b.

So we can compute χ

Renormalization of the Magnetic Susceptibility

Renormalization

Quadratic in B divergences are still present in the finite difference $\Delta f(T,B) = f(T,B) - f(T,0)$: they must properly subtracted.
We are interested in the magnetic properties of the strongly interacting medium.
Hence our renormalization prescription is to subtract the vacuum ($T = 0$) contribution

$$\Delta f_R(T,B) = \Delta f(T,B) - \Delta f(0,B)$$

No further divergences (depending on both B and T) are present.
This procedure has to be carried out at the same UV cutoff.

Numerical Setups

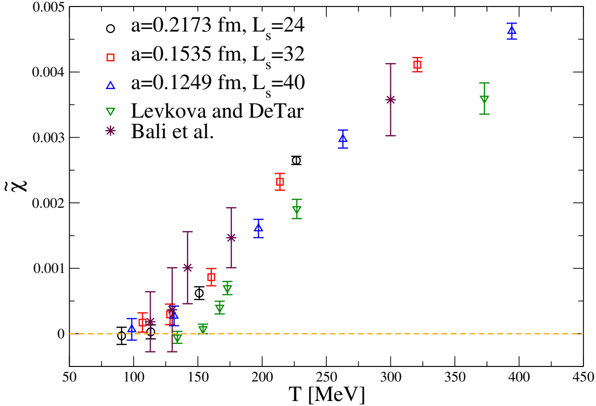
Preliminary Lattice study & discussion of the method
[Bonati, D'Elia, Mariti, N, Sanfilippo, PRL]

- Naive fermionic and gauge discretization
- Higher than physical quarks

Improved Lattice study at the physical point
[Bonati, D'Elia, Mariti, N and Sanfilippo, PRD]

- Stout smeared rooted staggerd fermions
- Tree level Symanzik improved gauge action
- Physical quark masses
- Inclusion of the *strange* quark
- Simulations @ 3 lattice spacings: 0.2173, 0.1535, 0.1249 fm
- Large scale parallel computer: BGQ Fermi - CINECA

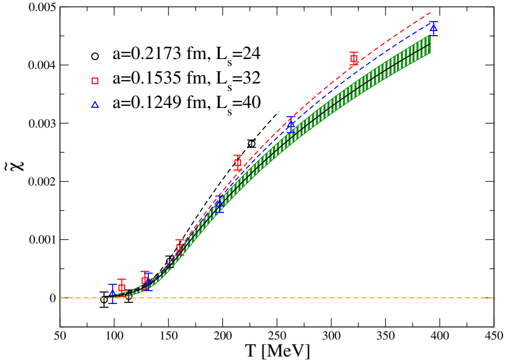
Magnetic Susceptibility - Comparisons



Other methods

- half - half method (or Taylor expansion method) [Levkova and DeTar]
- anisotropy method [Bali, Bruckmann, Endrodi et. al.]
- recently → generalized integral method [Bali, Bruckmann, Endrodi et. al.]

Magnetic Susceptibility - Continuum Limit



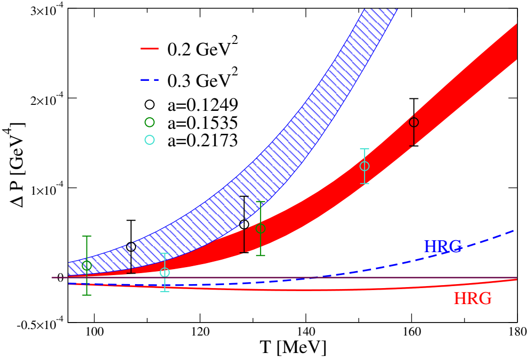
Fit function for the continuum limit:

$$\tilde{\chi}(T) = \begin{cases} A \; \exp(-M/T) & T \leq \tilde{T} \quad \text{inspired by HRG} \\ A' \; \log(T/M') & T > \tilde{T} \quad \text{inspired by Perturbation Theory} \end{cases} \tag{1}$$

Continuous and differentiable matching at $\tilde{T} \rightarrow$ (5-2)=3 parameters
We perform the continuum limit by letting either $A = A_0 + a^2 A_2$ or $M = M_0 + a^2 M_2$.
Remarkably $\tilde{T} = 160(10)$ MeV.

Diamagnetism at low T?

The HRG model predicts a diamagnetic behaviour, due to pions.
We plot at low T the magnetic contribution to the pressure $\Delta P(B,T)$ (both as data and as continuum extrapolation).



Our data are not enough precise to disinguish a possible (small) diamagnetic behaviour.

Recently, (lattice) indications of a possible diamagnetic regime up to $T \sim 120$ MeV. [Bali, Bruckmann, Endrodi et. al., JHEP 2014]

The answer is not yet clear!

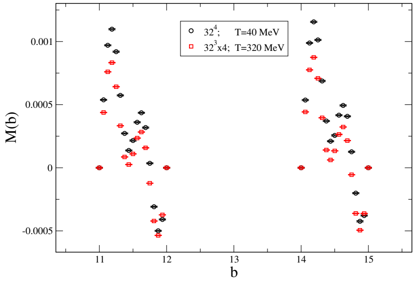
Future Development: in order to give a precise answer regarding this issue, we are adopting another approach, inspired by [DeTar and Levkova]. Work in progress.

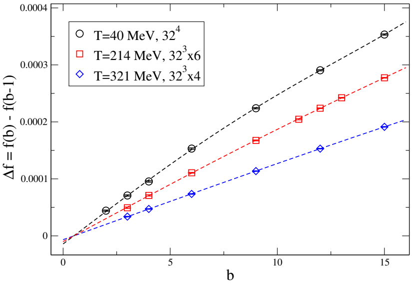
Nonlinearities ↔ Going to large fields

Future Development: determination of higher derivatives of the free energy with respect to the magnetic field.

- We can compute Δf between large quanta.
- Then, we fit according to $f(b) = \frac{c_2}{2} b^2 + \frac{c_4}{4!} b^4 + \frac{c_6}{6!} b^6 + \ldots$

Preliminary results at large magnetic fields at $a = 0.1535$ fm.





QCD-vacuum susceptibility to CP-odd E.M. fields.

We have seen that the magnetic field influences and modifies significantly the gluon field distribution. (in particular Magnetic Catalysis & Anisotropic Potential!)
How does a CP symmetry breaking in the EM sector propagates to the strong sector?

1) we fix an EM background which breaks CP $\rightarrow \ \vec{E} \cdot \vec{B} \neq 0$.
2) this field induces an effective θ_{eff} term:

$$\theta_{eff} \simeq \chi_{CP} e^2 \vec{E} \cdot \vec{B} + \mathcal{O}((\vec{E} \cdot \vec{B})^3)$$

3) the susceptibility χ_{CP} is related to the intensity of the effective pseudoscalar QED - QCD interaction

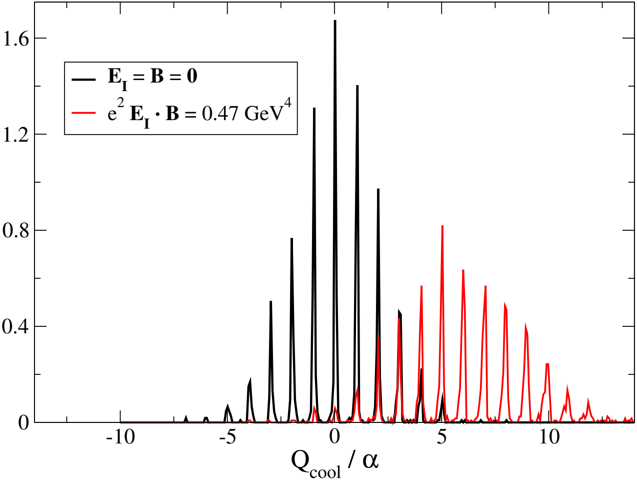
$$\mathcal{L}_{eff} = \chi_{CP} q(x) e^2 \vec{E} \cdot \vec{B} = \kappa \alpha \alpha_s (\vec{E}^a \cdot \vec{B}^a)(\vec{E} \cdot \vec{B})$$

$\Rightarrow$ As a consequence we expect $< Q > \neq 0$!!

We measured χ_{CP} in [D'Elia, Mariti e N, PRL '12]

QCD-vacuum susceptibility to CP-odd E.M. fields.

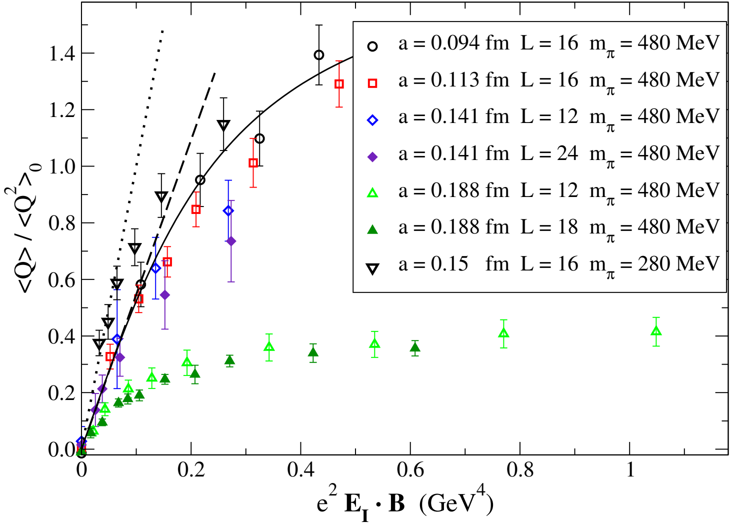
Distribution of the topological charge after cooling for $\vec{E}_I \cdot \vec{B} \neq 0$.



Lattice Spacing $a \simeq 0.12$ fm and $m_\pi = 480$ MeV.
The lattice is 16^4 so to be close to $T \sim 0$.

QCD-vacuum susceptibility to CP-odd E.M. fields.

$$\theta_{I\ eff} \simeq \frac{<Q>_{(\vec{E}_I,\vec{B})}}{<Q^2>_0} \quad \Rightarrow \quad \theta_{I\ eff} \simeq \chi_{CP}\, e^2(\vec{E}_I \cdot \vec{B})$$



QCD-vacuum susceptibility to CP-odd E.M. fields.

Estimate for χ_{CP} and final remarks:

- By fitting in the regime of small $\vec{E} \cdot \vec{B}$ we can extract the value of χ_{CP}

$$\chi_{CP} = (7 \pm 1) GeV^{-4} \text{ at a pion mass of } m_\pi = 480 \text{ MeV}.$$

- Preliminary result (only 1 lattice spacing $a = 0.15$ fm):

$$\chi_{CP} = (10 \pm 1) GeV^{-4} \text{ a } m_\pi = 280 \text{ MeV}.$$

- The phenomenological estimate of [M. Asakawa et al., Phys. Rev. C 81, 064912 (2010)] is based on the effective coupling of the η and η' mesons with 2 photons and 2 gluons:

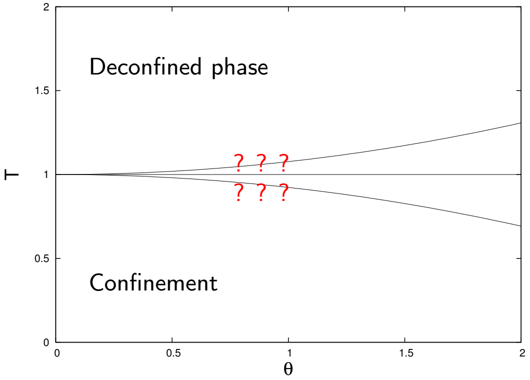
$$\chi_{CP} = 0.73/(\pi^2 f_\eta^2 m_{\eta'}^2) \sim 3 \text{ GeV}^{-4}$$

Limitations of this study:
Higher than physical pions $\oplus$ unimproved discretization

Future development:
Perform the same analysis at the physical pion mass and adopting an improved discretization. We are now running the simulations.

Phase diagram in the $T - \theta$ plane

Aim: SU(3) gauge theory phase diagram in the $T - \theta$ plane.



Does T_c depend on θ? Is it growing or decreasing?

- PNJL model [Mizher, Fraga, Sakai, Kouno et al.]
- semiclassical approximations [Anber, Unsal, Poppitz and Schaefer]
- Lattice Studies [D'Elia and N, PRL '12 & PRD '13]

  + work in progress with Bonati and Capponi.

Topological θ term

We consider the following continuum action in euclidean metric:

$$S = S_{YM} + S_\theta$$

The pure gauge term:

$$S_{YM} = -\frac{1}{4}\int d^4x\ F^a_{\mu\nu}(x)F^a_{\mu\nu}(x)$$

and the topological θ-term:

$$S_\theta = -i\theta\frac{g_0^2}{64\pi^2}\int d^4x\ \epsilon_{\mu\nu\rho\sigma}F^a_{\mu\nu}(x)F^a_{\rho\sigma}(x) \equiv -i\theta\int d^4x\ q(x) \equiv -i\theta Q[A]$$

LGT techniques are based on the possibility to interpret the partition function integrand

$$Z(T,\theta) = \int D[A]\ e^{-S_{YM}+i\theta Q[A]}$$

as a probability distribution for the gauge field configurations.

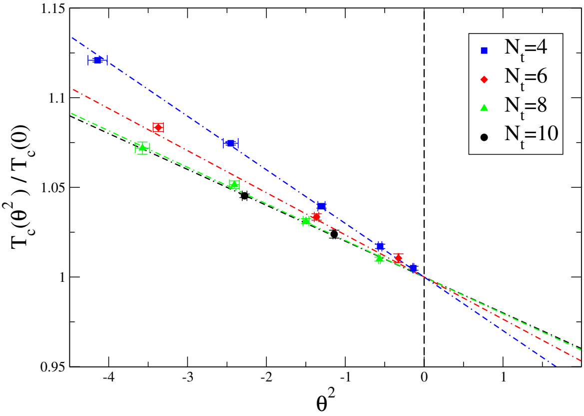
But it is complex! Bad news... sign problem! $\Longrightarrow$ Analytic continuation

Curvature of the critical line.

We fix θ and we search for T_c by monitoring the Polyakov Loop (order parameter) and its susceptibility.

For small θ holds:
$T_c(\theta)/T_c(0) \simeq 1 - R_\theta \theta^2$

We determine R_θ at fixed N_t, *i.e.* at fixed lattice spacing.



$\Downarrow$

Assuming $\mathcal{O}(a^2)$ corrections we get:

$$R_\theta^{\text{cont}} = 0.0178(5)$$

T_c increases for imaginary coupling then, by analytic continuation, it decreases for real θ.

Large N_c estimate

We formulated the following prediction for R_θ in the large N_c limit:

$$R_\theta^{large\ N_c} = \frac{\chi}{2\Delta\epsilon}$$

This equation is based on the assumption that the topological susceptibility χ is non-zero below T_c and then sharply drops to 0. $\Delta\epsilon$ is the latent heat.

Adopting the determination of χ and $\Delta\epsilon$ in the large N_c given in [Lucini, Teper and Wenger, JHEP 2005] we get:

$$R_\theta^{large\ N_c} = \frac{0.253(56)}{N_c^2} + O(\frac{1}{N_c^4})$$

The argument in [Witten, PRL 1998] supports this dependence on N_c.
Large-N_c limit $\rightarrow$ expansion variable $\frac{\theta}{N_c} \rightarrow R_\theta \theta^2 \rightarrow R_\theta \propto \frac{1}{N_c^2}$

Let's recall both our results and compare them in the case $N_c = 3$:

$$\boxed{R_\theta^{cont} = 0.0178(5) \qquad R_\theta^{large\ N_c}(N_c = 3) = 0.028(6)}$$

Future development: compute directly R_θ for $SU(N_c)$ with $N_c > 3$ and test the validity of the large N_c prediction.

Conclusions and perspectives

- Determination of the magnetic susceptibility at $T \neq 0$
  - $\rightarrow$ Diamagnetism below T_c?
- Preliminary results on the nonlinearities
  - $\rightarrow$ Determine higher orders coefficients
- Propagation of CP-breaking: determination of χ_{CP}
  - $\rightarrow$ Study the case at physical pion masses
- Curvature of the critical line in the $T - \theta$ plane
  - $\rightarrow$ Extend the computations to other gauge groups.
  - $\rightarrow$ Check for the validity of the large N_c estimate.