

# Study of Lepton Number Conserving and Non-Conserving Processes Using GERDA Phase I Data

Sabine Hemmer  
*INFN Padova  
Università degli Studi di Padova*

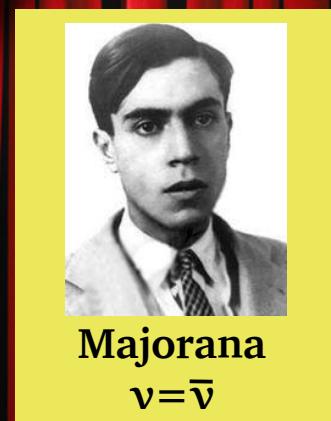


Frascati, 24 June 2015

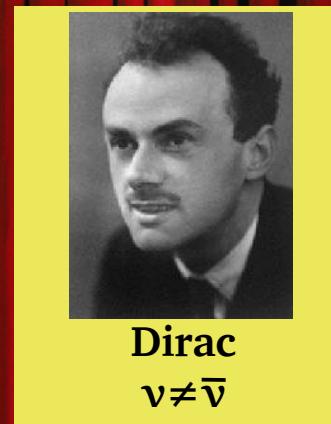
# Outline

- Neutrinos and Double Beta Decay
- The GERDA experiment
- Monte Carlo Simulation of the GERDA Experiment
- Background Model for the GERDA Phase I Data
- $2\nu\beta\beta$  and  $0\nu\beta\beta\chi(\chi)$  in GERDA Phase I
- $0\nu\beta\beta$  in GERDA Phase I

# Unveil the nature of the neutrino



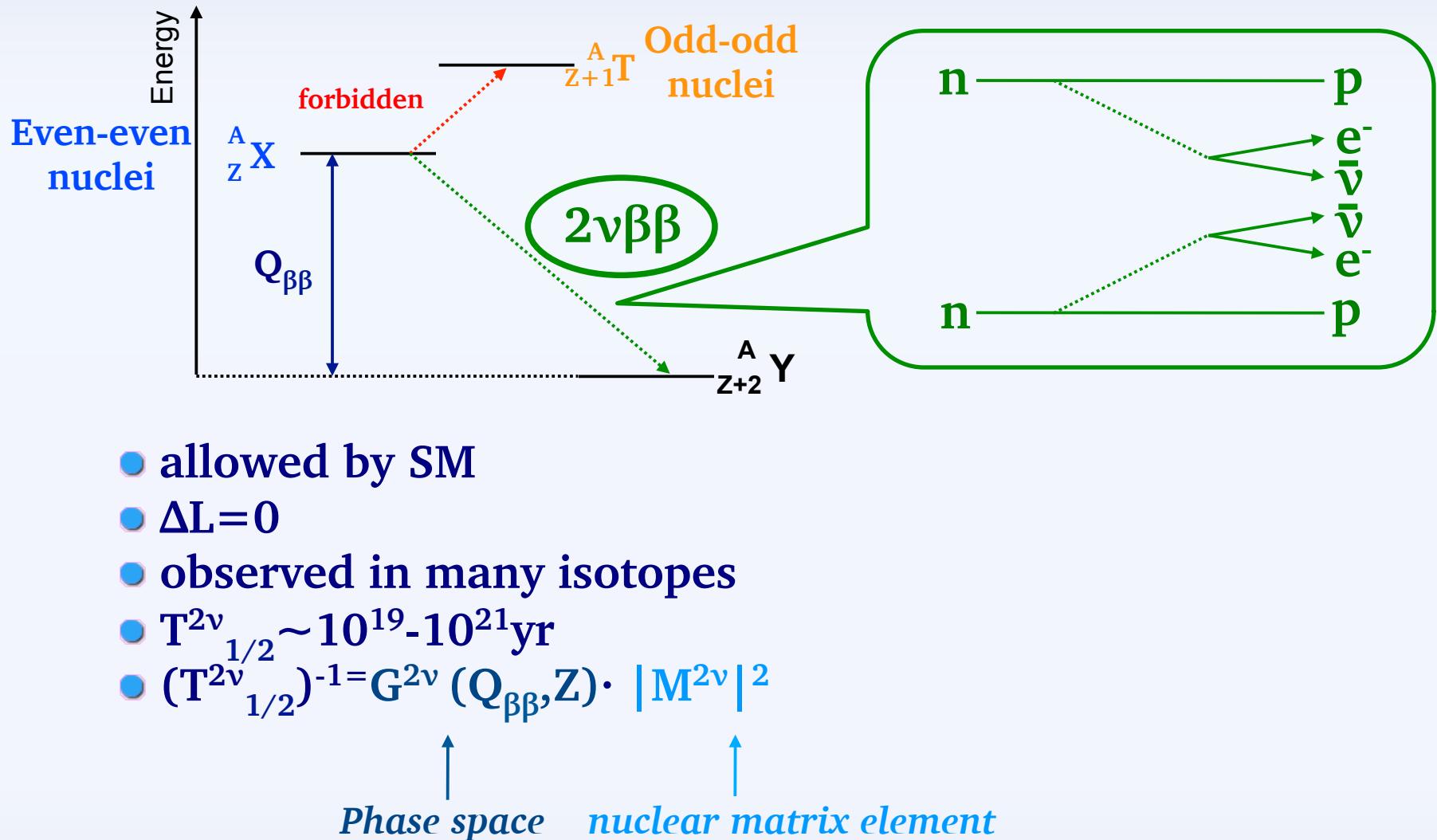
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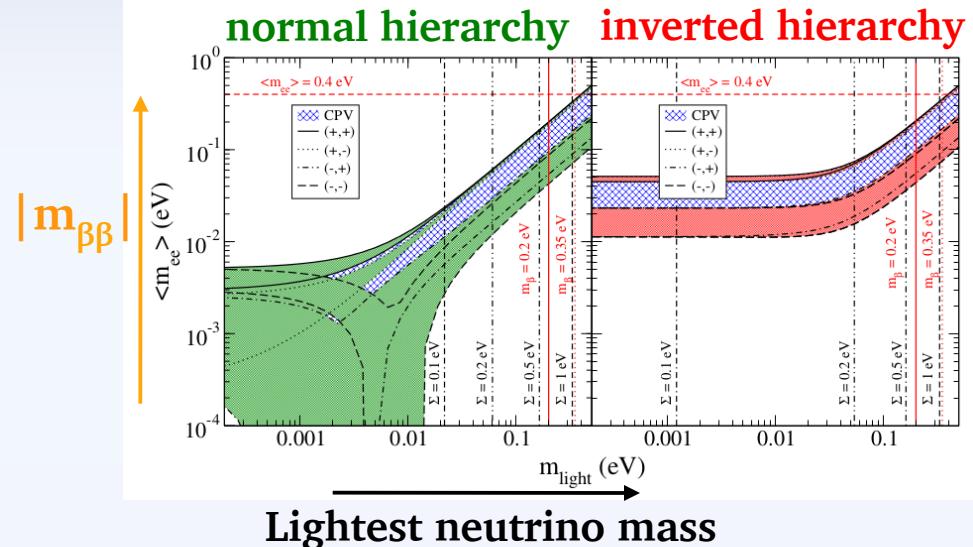
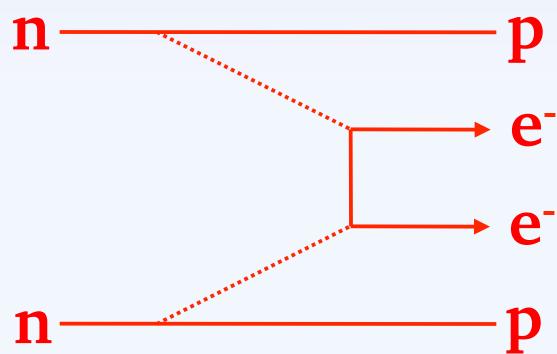


If  $0\nu\beta\beta$  is observed:

- Lepton number violation  $\Delta L=2$
- Neutrino has a Majorana mass term
- Sheds light on absolute neutrino mass scale
- Sheds light on neutrino mass hierarchy

# $2\nu\beta\beta$





- Forbidden process in SM, needs Majorana neutrino
- $\Delta L=2$
- $(T^{0\nu}_{1/2})^{-1} = G^{0\nu}(Q_{\beta\beta}, Z) \cdot |M^{0\nu}|^2 \cdot \langle m_{\beta\beta} \rangle^2$ \*

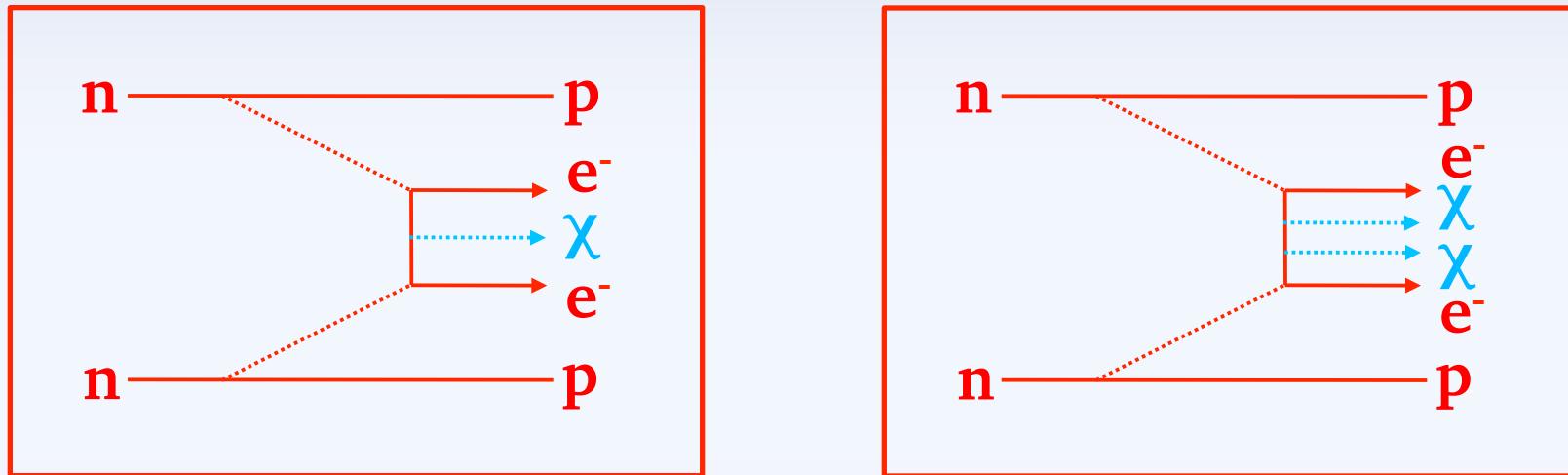
*Phase space ( $\sim Q_{\beta\beta}^5$ )*

*nuclear matrix element*

$\langle m_{\beta\beta} \rangle^2 = |\sum_i U_{ei}^2 m_i|^2$   
*Majorana neutrino mass*

\* Assuming exchange of light Majorana neutrino

# $0\nu\beta\beta\chi(\chi)$



- Majoron  $\chi$ : Additional particle emitted in  $0\nu\beta\beta$

- $\Delta L=2$  or  $\Delta L=0$

- $(T^{0\nu\chi}_{1/2})^{-1} = | \langle g \rangle |^2 \cdot G^{0\nu\chi}(Q_{\beta\beta}, Z) \cdot | M^{0\nu\chi} |^2$

- $(T^{0\nu\chi\chi}_{1/2})^{-1} = | \langle g \rangle |^4 \cdot G^{0\nu\chi\chi}(Q_{\beta\beta}, Z) \cdot | M^{0\nu\chi\chi} |^2$

effective coupling constant

Phase space  
 $\sim (Q_{\beta\beta} - E)^n$

Spectral index n

nuclear matrix element

# ov $\beta\beta\chi(\chi)$

Lepton  
number  
violating

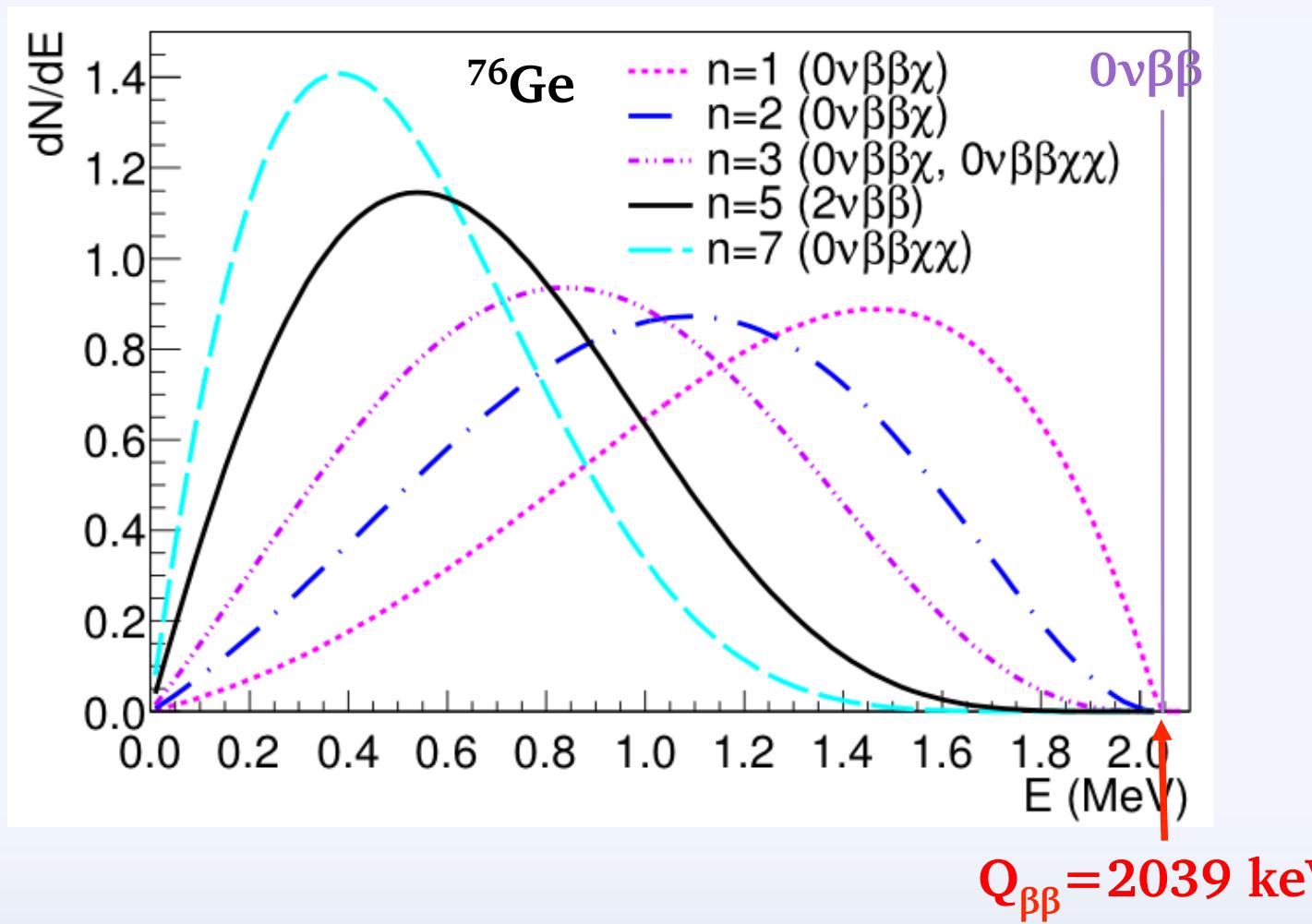
Lepton  
number  
conserving

**Number of  $\chi$**       **Leptonic charge of  $\chi$**       **Spectral index**

Model	Mode	Goldstone boson	$L$	$n$	Matrix element
IB	$\chi$	no	0	1	$M_F - M_{GT}$
IC	$\chi$	yes	0	1	$M_F - M_{GT}$
ID	$\chi\chi$	no	0	3	$M_{Fw^2} - M_{GTw^2}$
IE	$\chi\chi$	yes	0	3	$M_{Fw^2} - M_{GTw^2}$
IF (bulk)	$\chi$	bulk field	0	2	-
<hr/>					
IIB	$\chi$	no	-2	1	$M_F - M_{GT}$
IIC	$\chi$	yes	-2	3	$M_{CR}$
IID	$\chi\chi$	no	-1	3	$M_{Fw^2} - M_{GTw^2}$
IIIE	$\chi\chi$	yes	-1	7	$M_{Fw^2} - M_{GTw^2}$
IIIF	$\chi$	gauge boson	-2	3	$M_{CR}$

# Experimental signatures

→ Measure combined electron energy spectrum E:



# Previous measurements

Isotope	$T^{2\nu}_{1/2} (10^{19} \text{ yr})$	$T^{0\nu}_{1/2} (\text{yr}) \text{ (90\% C.L.)}$	$T^{0\nu X}_{1/2} (10^{21} \text{ yr}) \text{ (90\% C.L.)}$ (n=1)
$^{76}\text{Ge}$	$184^{+14}_{-10} [1]$	$2.1 \cdot 10^{25} [2]$ $1.9 \cdot 10^{25} [3]$ $1.6 \cdot 10^{25} [4]$	$64 [3]$
$^{100}\text{Mo}$	$0.71 \pm 0.04 [5]$	$1.1 \cdot 10^{24} [6]$	$27 [7]$
$^{130}\text{Te}$	$68 \pm 12 [5]$	$4.0 \cdot 10^{24} [8]$	$16 [9]$
$^{136}\text{Xe}$	$216.5 \pm 6.1 [10]$ $230 \pm 12 [13]$	$1.1 \cdot 10^{25} [11]$ $1.9 \cdot 10^{25} [14]$	$1.2 \cdot 10^3 [12]$ $2.6 \cdot 10^3 [15]$

Claim of signal for  $0\nu\beta\beta$ :

$$T^{0\nu}_{1/2} (^{76}\text{Ge}) = 1.19^{+0.37}_{-0.23} \cdot 10^{25} \text{ yr} [14]$$

- [1] J. Phys. G: Nucl. Part. Phys. 40 (2013), 035110
- [2] Phys. Rev. Lett. 111 (2013), 122503
- [3] Eur. Phys. J. A12 (2001), 147-154
- [4] Phys. Rev. D 65 (2002), 092007
- [5] Phys. Atom. Nucl. 74 (2011), 603-613
- [6] Phys. Atom. Nucl. 74 (2011), 312-317
- [7] Nucl. Phys. A 765 (2006), 483-494
- [8] arXiv:1504.02454

- [9] Phys. Rev. Lett. 107 (2011), 062504
- [10] Phys. Rev. C 89 (2014), 015502
- [11] Nature 510 (2014), 229
- [12] Phys. Rev. D 90 (2014), 092004
- [13] Phys. Rev. C 86 (2012), 021601
- [14] Phys. Rev. Lett. 110 (2013), 062502
- [15] Phys. Lett. B 586 (2004), 198-212

# Searching in $^{76}\text{Ge}$

$$S \sim \epsilon \cdot f \cdot \sqrt{\frac{M \cdot t_{\text{run}}}{BI \cdot \Delta E}}$$

S: sensitivity

$\epsilon$ : efficiency

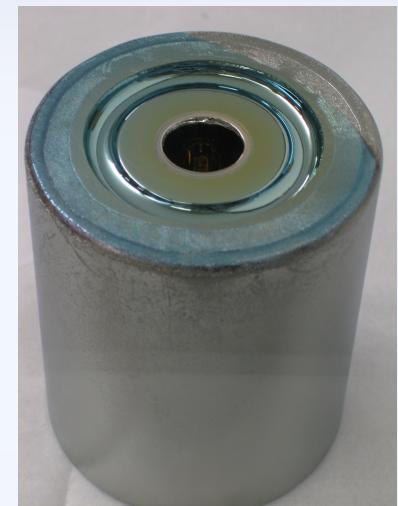
f: abundance of  $0\nu\beta\beta$  isotope

M: detector mass

$t_{\text{run}}$ : measurement time

BI: background index

$\Delta E$ : energy resolution at  $Q_{\beta\beta}$



Germanium detector

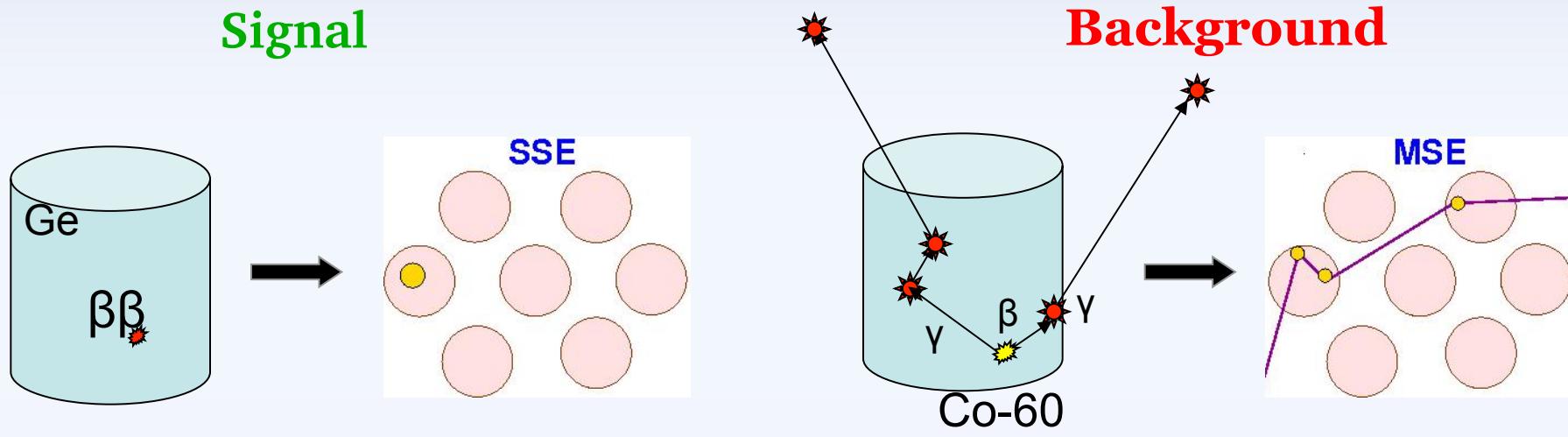
## Advantages of Germanium:

- High  $\epsilon$ : Source = Detector
- Small intrinsic BI: High purity Ge
- Excellent  $\Delta E$ : FWHM  $\sim (0.1-0.2)\%$
- Well-established technology

## Disadvantages of Germanium:

- High external BI:  $Q_{\beta\beta} = 2039\text{keV}$
- Small f of  $^{76}\text{Ge}$ :  
 $7.8\% \rightarrow$  Enrichment needed!
- Limited sources of crystal & detector manufacturers
- Small  $G^{0\nu}$  ( $Q_{\beta\beta}, Z$ )

# Additional background reduction



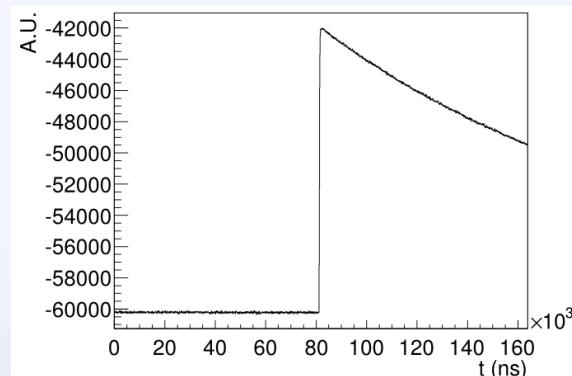
Point-like (single-site) energy deposition inside one HP-Ge diode (Range:  $\sim 1$  mm)

Multi-site energy deposition inside HP-Ge diode (Compton scattering)

## Signal analysis:

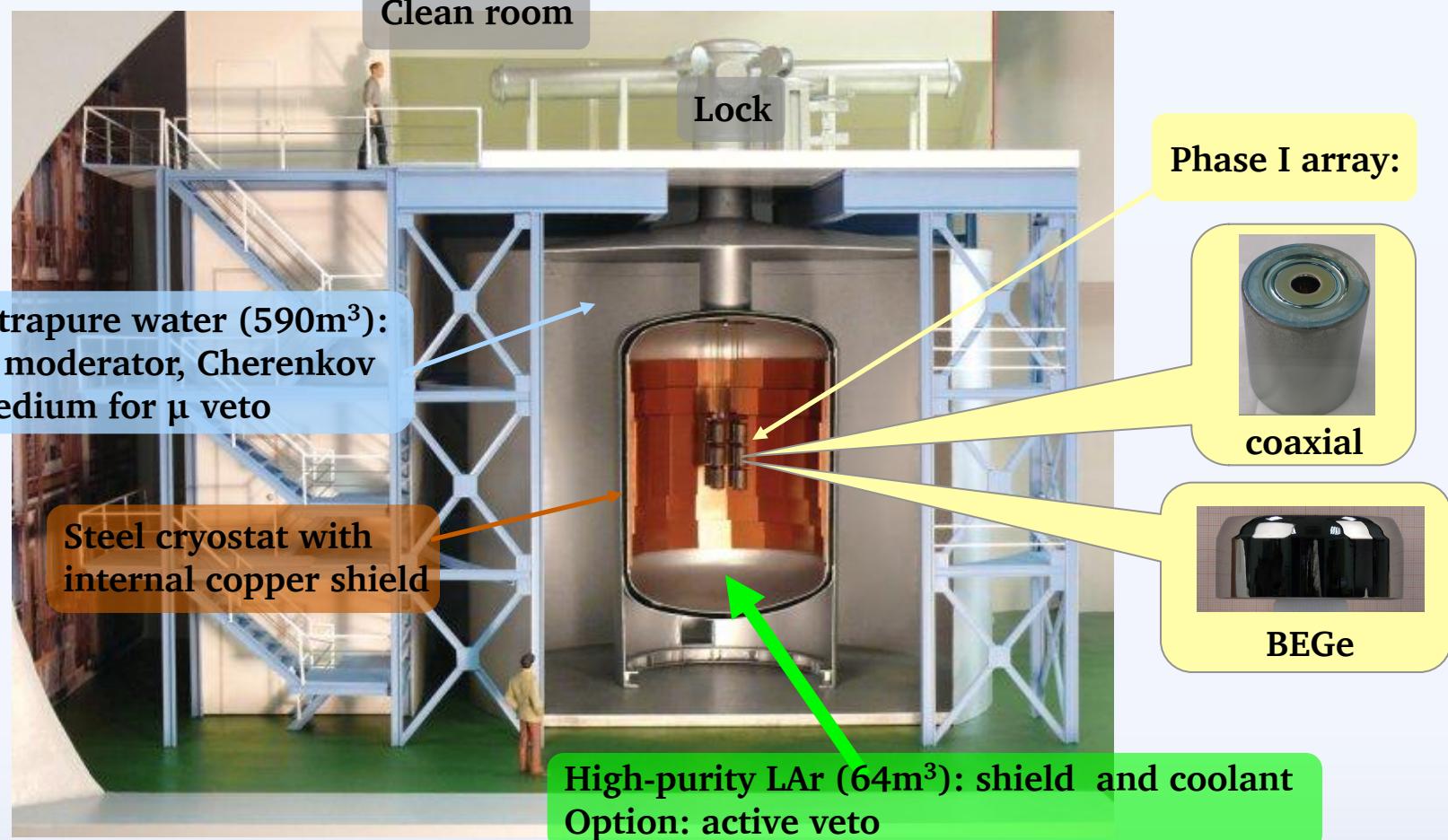
- anti-coincidence between detectors
- pulse shape analysis (PSA)

Example of charge pulse



# The GERDA experiment

- Situated in LNGS underground laboratories (3500 m w.e. shielding)
- Graded shielding against ambient radiation
- Rigorous material selection, avoid exposure above ground for detectors



# The GERDA experiment



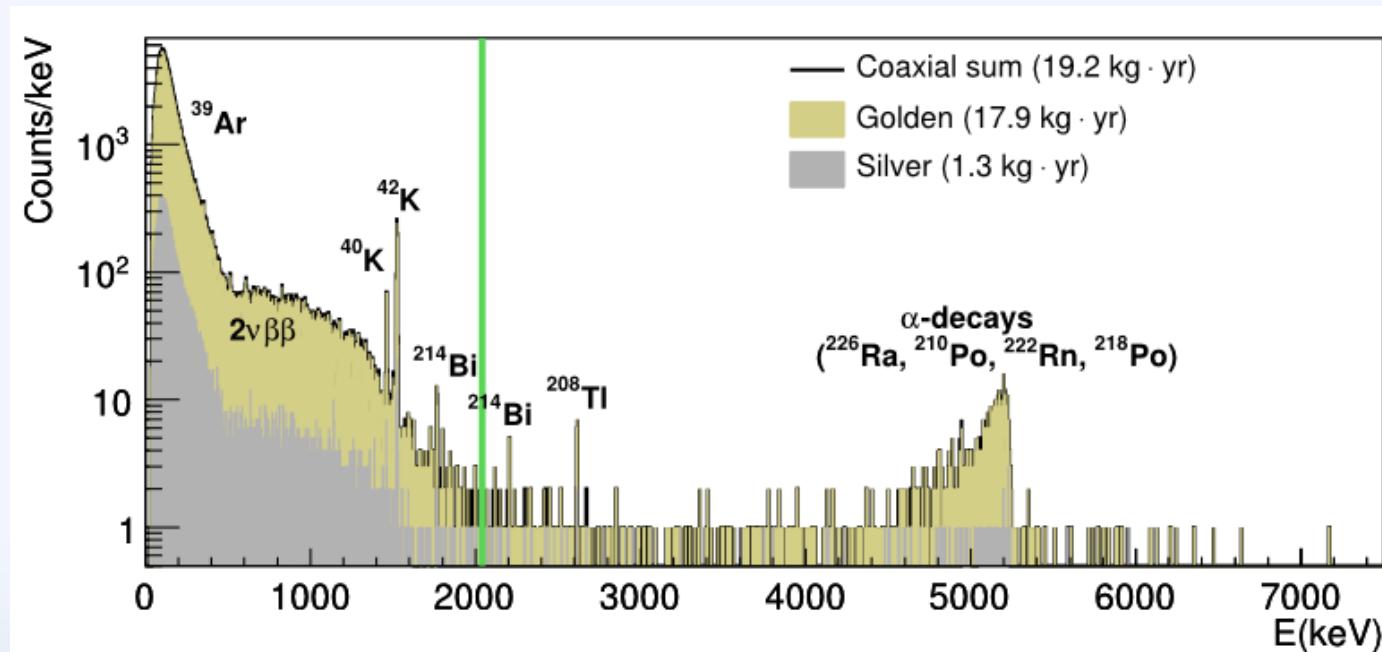
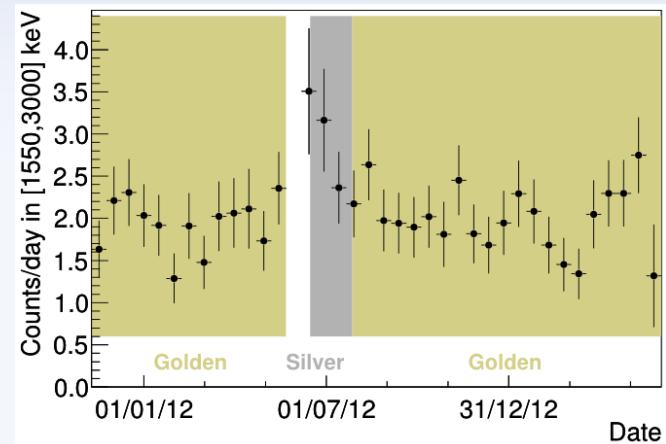
# GERDA Phase I data

**Golden data set:**

**17.9 kg·yr with 6 coaxial detectors**  
**FWHM at  $Q_{\beta\beta} = (4.83 \pm 0.19)$  keV**

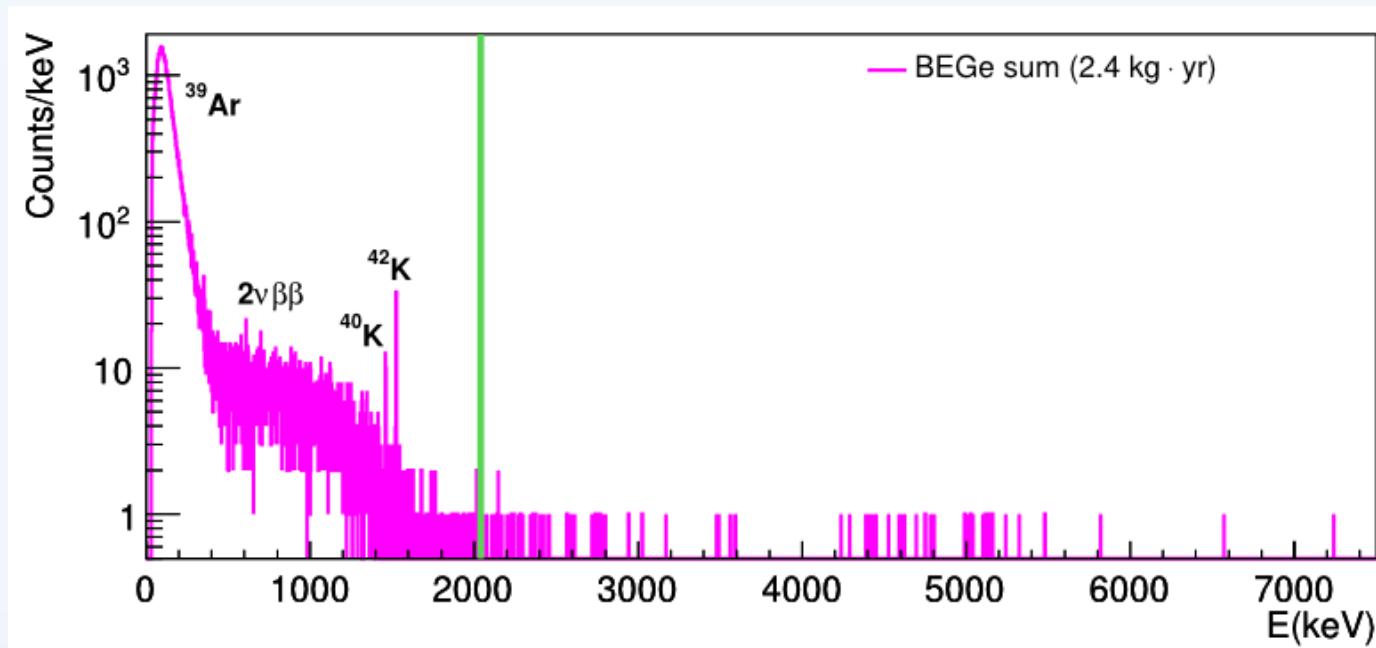
**Silver data set:**

**1.3 kg·yr with 6 coaxial detectors**  
**FWHM at  $Q_{\beta\beta} = (4.63 \pm 0.24)$  keV**



# GERDA Phase I data

BEGe sum data set:  
2.4 kg·yr with 4 BEGe detectors  
FWHM at  $Q_{\beta\beta} = (3.24 \pm 0.17)$  keV



# Background in GERDA Phase I data

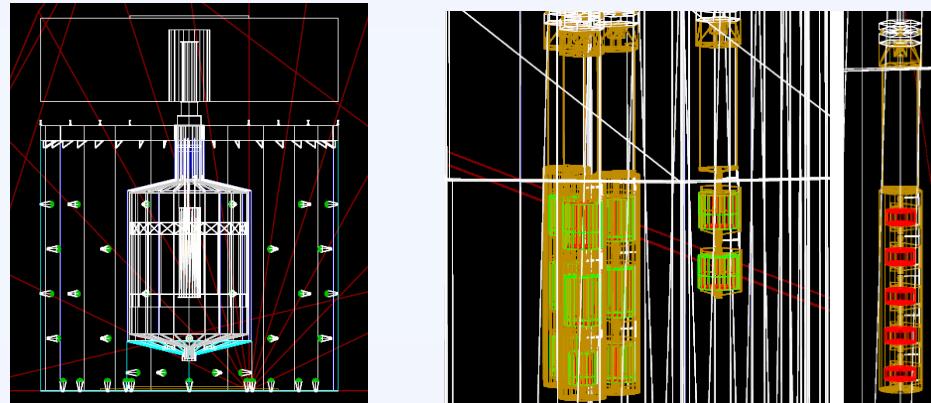
## Backgrounds assessed from photon line intensities and screening measurements:

Isotope	Energy (keV)	Coaxial sum		Golden		Silver		BEGe sum	
		$r_S$	cts/(kg·yr)	$r_S$	cts/(kg·yr)	$r_S$	cts/(kg·yr)	$r_S$	cts/(kg·yr)
$^{42}\text{K}$	1524.7	60.6	[58.8,62.6]	59.6	[57.6,61.5]	75.1	[67.7,83.5]	46.6	[41.7,51.2]
$^{40}\text{K}$	1460.8	14.1	[12.9,15.2]	14.5	[13.2,15.6]	14.5	[13.2,15.6]	12.7	[9.6,15.9]
$^{60}\text{Co}$	1173.2	2.9	[1.3,4.4]	2.5	[0.9,4.0]	5.1	[0.8,9.9]	<8.6	
	1332.5	<1.9		<1.8		5.2	[1.5,9.1]	<6.3	
$^{228}\text{Ac}$	911.2	3.1	[1.1,4.9]	4.0	[1.8,5.9]	<8.9		<8.0	
	969.0	6.7	[4.6,8.5]	5.6	[3.5,7.6]	19.7	[12.2,27.5]	<8.2	
$^{212}\text{Bi}$	727.3	2.3	[0.5,4.1]	<4.7		17.8	[8.7,24.9]	<6.7	
$^{208}\text{Tl}$	583.2	4.0	[1.9,6.2]	3.0	[1.0,5.0]	14.0	[5.9,21.8]	<11.0	
	860.6	<3.1		<3.6		<11.9		<7.0	
	2614.5	1.5	[1.1,1.7]	1.5	[1.1,1.7]	1.2	[0.2,2.4]	0.6	[0.1,1.3]
$^{214}\text{Pb}$	351.9	9.6	[4.3,14.1]	10.1	[5.0,15.2]	<34.8		13.5	[5.6,22.7]
$^{214}\text{Bi}$	609.3	8.1	[5.6,10.3]	8.6	[5.9,10.8]	<16.1		12.0	[6.7,18.2]
	768.4	4.3	[2.1,6.3]	3.6	[1.4,5.5]	16.9	[8.1,23.9]	6.5	[1.6,10.7]
	1120.3	<2.9		<3.1		<8.9		6.7	[2.5,10.7]
	1238.1	<2.8		<2.8		<10.7		<6.6	
	1764.5	3.2	[2.7,3.7]	3.3	[2.7,3.7]	2.2	[0.2,4.0]	<2.5	
	2204.2	0.9	[0.6,1.2]	0.9	[0.6,1.2]	<3.3		1.0	[0.3,1.8]
$^{137}\text{Cs}$	661.7	<4.6		<5.0		<14.7		7.0	[2.3,11.8]
$e^+e^-$									
$\& ^{208}\text{Tl}$	511.0	10.4	[7.8,12.8]	10.6	[7.9,13.1]	<20.0		16.5	[10.4,21.9]

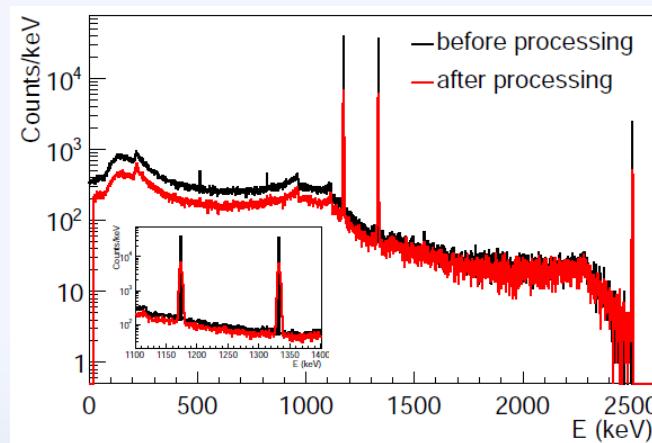
Component	units	$^{40}\text{K}$	$^{214}\text{Bi}/^{226}\text{Ra}$	$^{228}\text{Th}$	$^{60}\text{Co}$	$^{222}\text{Rn}$	BI
Close sources: up to 2 cm from detectors							
Copper det. support	$\mu\text{Bq}/\text{det}$	<7	<1.3	<1.5			<0.2
PTFE det. support	$\mu\text{Bq}/\text{det}$	6.0 (11)	0.25 (9)	0.31 (14)			0.1
PTFE in array	$\mu\text{Bq}/\text{det}$	6.5 (16)	0.9 (2)				0.1
minishroud	$\mu\text{Bq}/\text{det}$		22 (7)				2.8
Li salt ( $n^+$ -contact)	$\text{mBq}/\text{kg}$		17 (5)				$\approx 0.003$
Medium distant sources: 2 cm to 50 cm from detectors							
CC2 preamps	$\mu\text{Bq}/\text{det}$	600 (100)	95 (9)	50 (8)			0.8
cables and suspension	$\text{mBq}/\text{m}$	1.40 (25)	0.4 (2)	0.9 (2)	76 (16)		0.2
Distant sources: further than 50 cm from detectors							
cryostat	$\text{mBq}$					54.7 (35)	<0.7
copper of cryostat	$\text{mBq}$	<784	264 (80)	216 (80)	288 (72)		<0.05
steel of cryostat	$\text{kBq}$	<72	<30	<30	475		
lock system	$\text{mBq}$					2.4 (3)	<0.03
$^{228}\text{Th}$ calib. source	$\text{kBq}$		20				<1.0

# Monte Carlo Simulation

- Detailed description of geometry in GEANT4 based MaGe framework



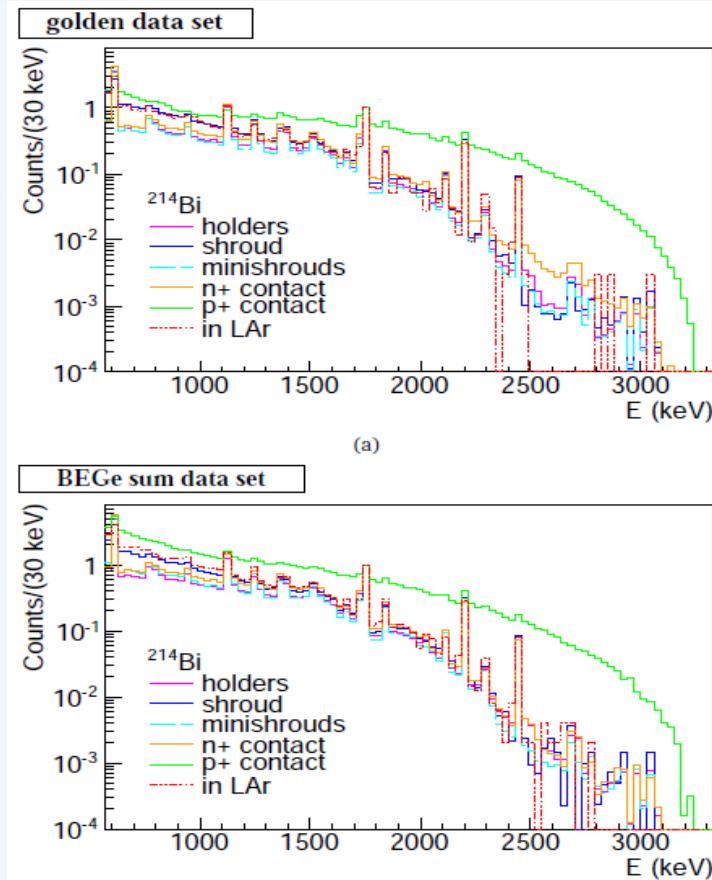
- Account for energy resolution and data selection cuts



# Monte Carlo Simulation

Simulate all relevant contributions to the energy spectra

- $^{76}\text{Ge}$  (2ν $\beta\beta$  and 0ν $\beta\beta\chi(\chi)$ )  
intrinsic
- $^{214}\text{Bi}$  sub-chain  
holders, shroud, p<sup>+</sup>-contact, minishrouds,  
n<sup>+</sup>-contact, in LAr
- $^{228}\text{Ac}$   
holders, shroud
- $^{228}\text{Th}$  sub-chain  
holders, shroud, heat exchanger
- $^{60}\text{Co}$   
holders, intrinsic
- $^{68}\text{Ga}$  (only for BEGe data set)  
intrinsic
- $^{40}\text{K}$   
holders
- $^{42}\text{K}$   
in LAr, p<sup>+</sup>-contact, n<sup>+</sup>-contact
- $\alpha$ -decays from  $^{226}\text{Ra}$  sub-chain and  $^{210}\text{Po}$   
p<sup>+</sup>-contact, LAr close to p<sup>+</sup>-contact



# Background model

Binned maximum likelihood approach using BAT framework  
→ posterior probability distribution for **model  $\lambda$**  given **data spectrum  $n$** :

$$P(\boldsymbol{\lambda}|\mathbf{n}) = \frac{P(\mathbf{n}|\boldsymbol{\lambda}) P_0(\boldsymbol{\lambda})}{\int P(\mathbf{n}|\boldsymbol{\lambda}) P_0(\boldsymbol{\lambda}) d\boldsymbol{\lambda}}$$

derived with Bayes' theorem from likelihood

$$P(\mathbf{n}|\boldsymbol{\lambda}) = \prod_i P(n_i|\lambda_i) = \prod_i \frac{e^{-\lambda_i} \lambda_i^{n_i}}{n_i!}$$

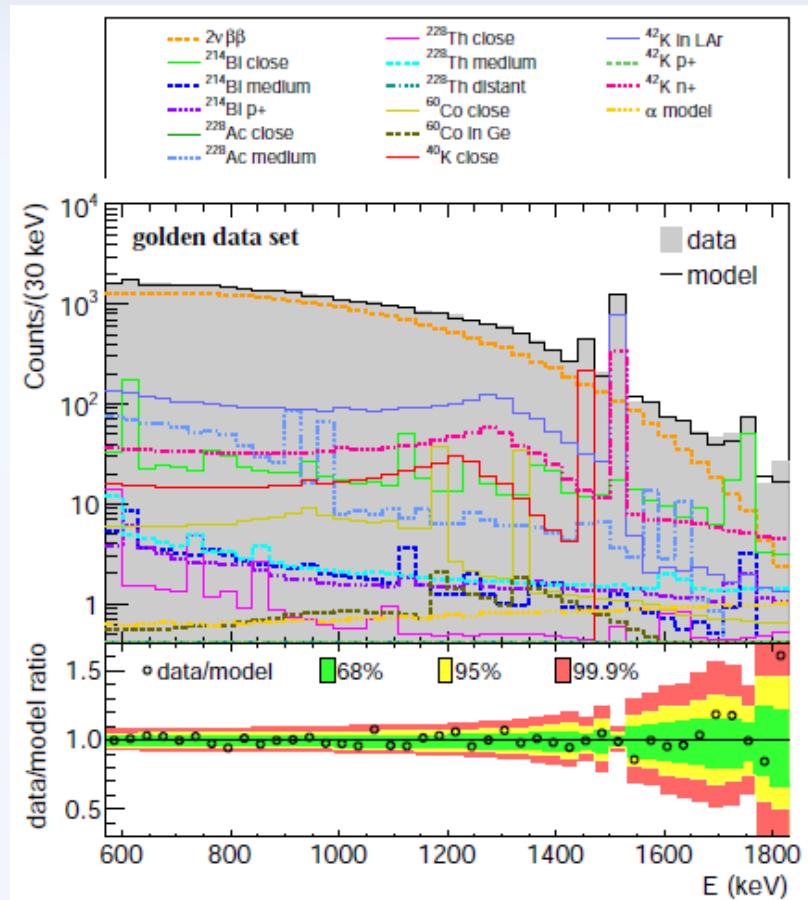
$\lambda_i = \sum_c \lambda_i^c$ : sum of contributions from single components,  $c$ , in  $i$ -th bin

$n_i$ : number of events in  $i$ -th bin of data spectrum

- ➡ one parameter for each model contribution!
- ➡ best-fit results and marginalized results!

# Background model

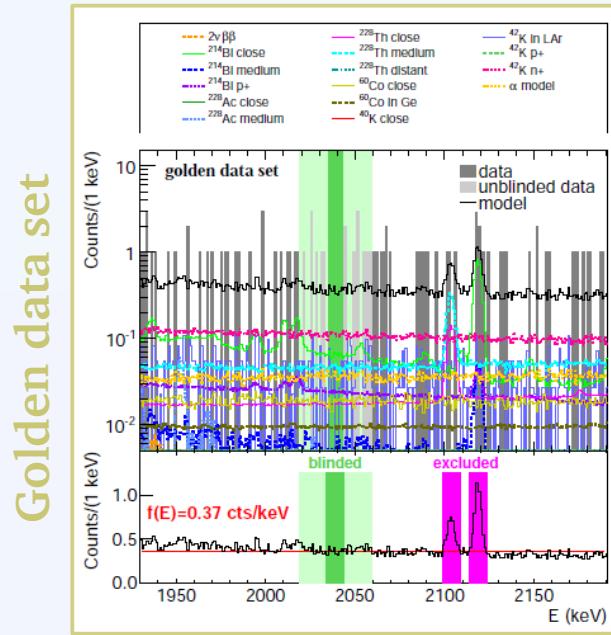
- Build separate models for golden and BEGe sum data set
- Alpha models:
  - fit to high energy region with alpha components  
→ [3500; 7500] keV in 50 keV bins
- Full background models:
  - fit to full energy range  
→ [570; 7500] keV in 30 keV bins
  - alpha model contributes as one component
  - $^{214}\text{Bi}$  on  $\text{p}^+$ -surface, intrinsic  $^{60}\text{Co}$  and  $^{68}\text{Ga}$ ,  $^{214}\text{Bi}$  sub-chain on holders and shroud,  $^{228}\text{Th}$  sub-chain on holders and shroud informative prior based on prior knowledge
  - region between 2019 keV and 2059 keV blinded



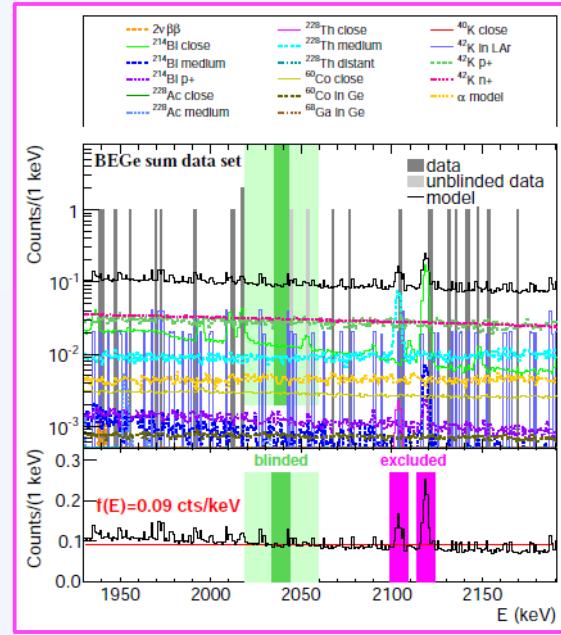
# Background model around $Q_{\beta\beta}$

BI ( $10^{-3}$  cts/(keV·kg·yr)):

Model:  $21.0 [19.8, 22.2]$   
 Data:  $18.5^{+2.3}_{-2.2}$



Model:  $40.6 [36.5, 45.2]$   
 Data:  $41.3^{+10.4}_{-8.4}$



- BI in [1930; 2190] keV can be approximated as flat
- Dominant contributions:  
 $^{214}\text{Bi}$  and  $^{228}\text{Th}$  sub-chain decays,  $^{42}\text{K}$  decays, and  $\alpha$ -decays
- BI predictions around  $Q_{\beta\beta}$  stable for variations within uncertainties of priors, active volume fractions, enrichment fractions, as well as for different source positions and binning

# $2\nu\beta\beta$ - Analysis

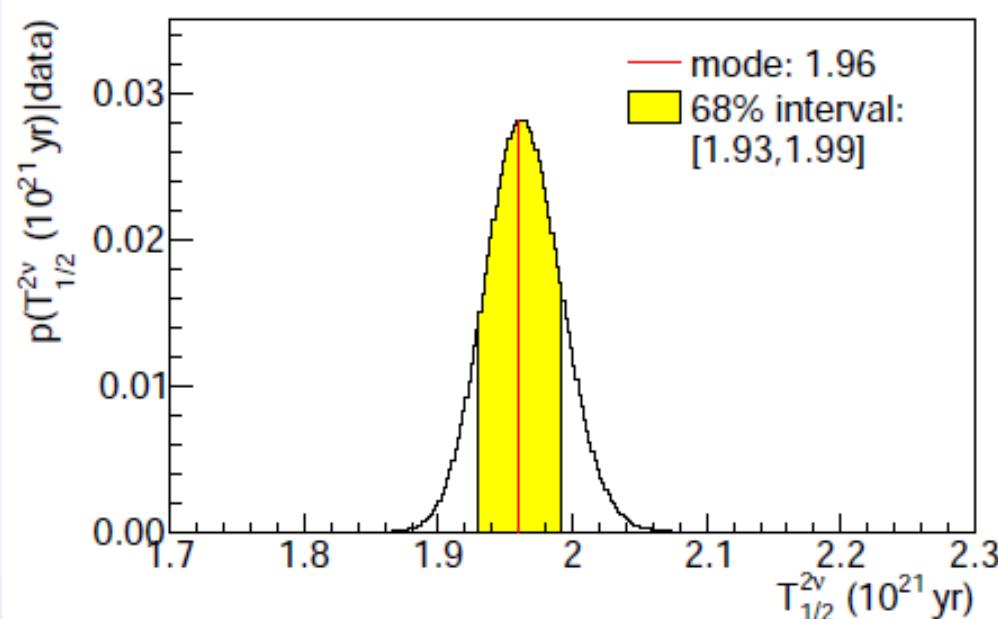
Measure  $T^{2\nu}_{1/2} \rightarrow$  combine information from golden and BEGe sum data set:

$$P(n^{\text{gold}}, n^{\text{BEGe}} | \lambda^{\text{gold}}, \lambda^{\text{BEGe}}) = \prod_i P(n_i^{\text{gold}} | \lambda_i^{\text{gold}}) \cdot \prod_j P(n_j^{\text{BEGe}} | \lambda_j^{\text{BEGe}})$$

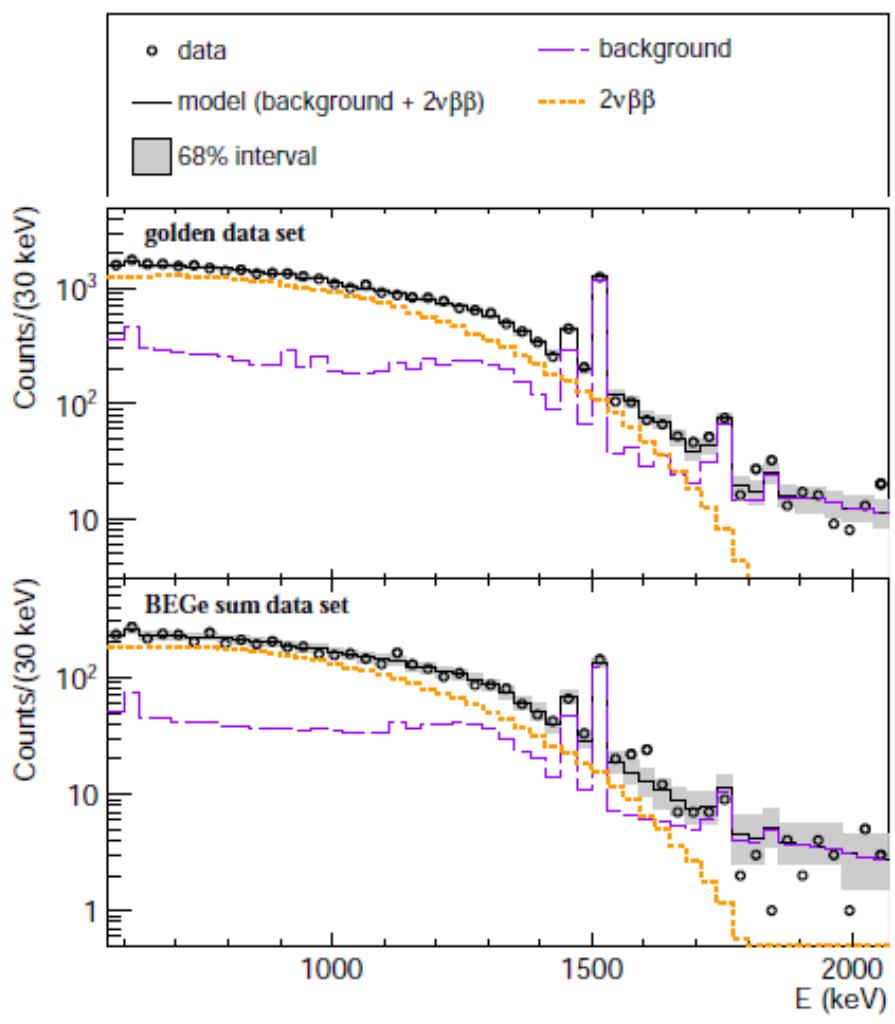
→  $T^{2\nu}_{1/2}$  common parameter for both data sets

$$T^{2\nu}_{1/2} = 1.96 [1.93, 1.99] \cdot 10^{21} \text{ yr}$$

(systematic uncertainty  
not accounted for)



# $2\nu\beta\beta$ - Analysis



Signal to background:  
3:1

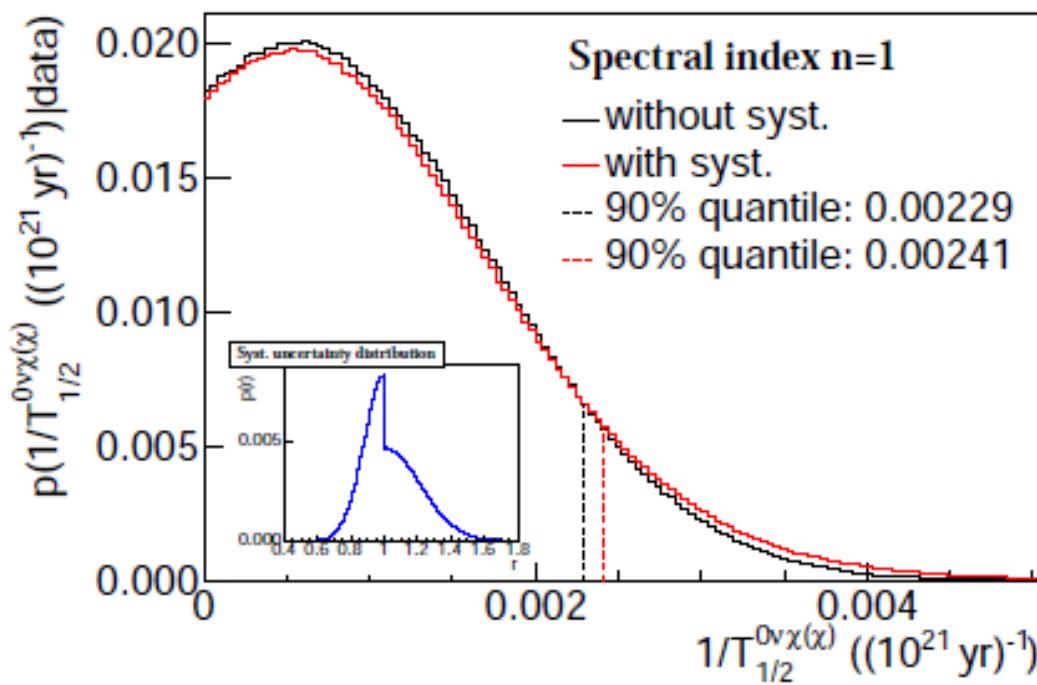
Golden:  
Data: 35868 events  
Best-fit model: 35902.3 events

BEGe sum:  
Data: 5035 events  
Best-fit model BEGe sum: 5062.1 events

# $\text{Ov}\beta\beta\chi(x)$ - Analysis

Add also contributions from Majoron mode ( $n=1, 2, 3$ , or  $7$ ) to model

$\rightarrow 1/T^{0\nu\chi(x)}_{1/2}$  common parameter for both data sets



$T^{0\nu\chi(x)}_{1/2}$  (90% quantile)

$n=1 > 4.36 \cdot 10^{23} \text{ yr}$

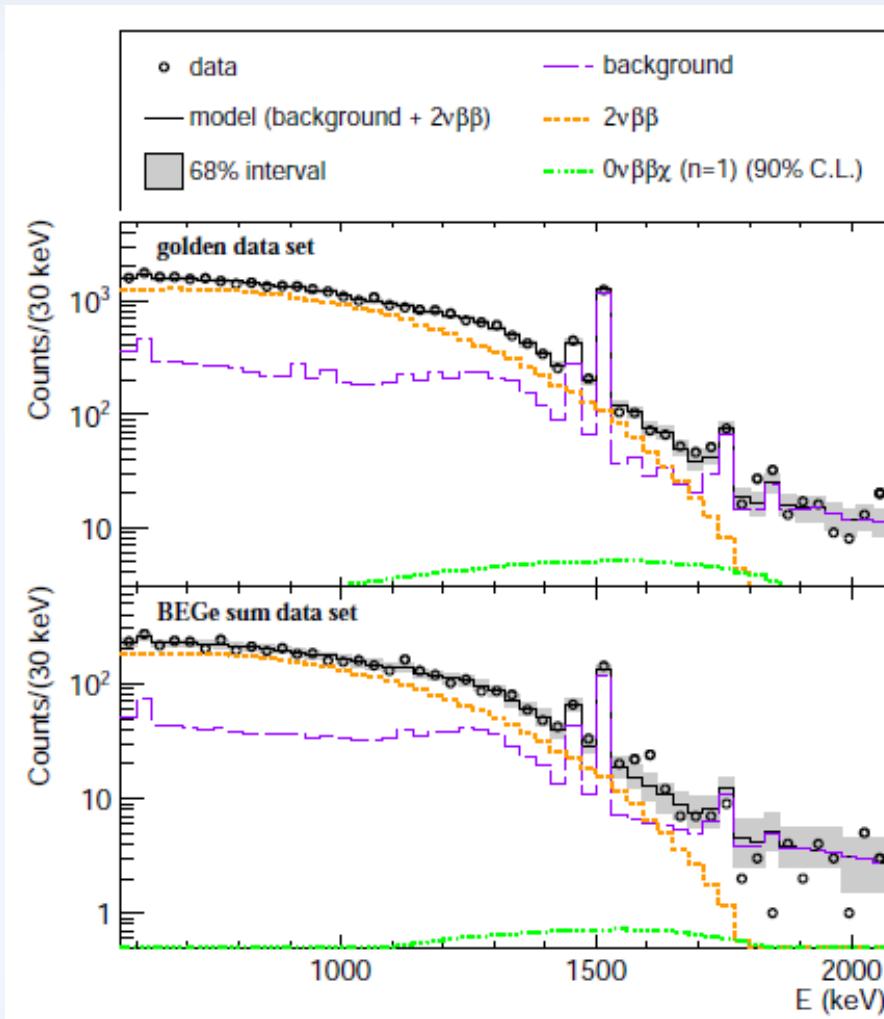
$n=2 > 1.91 \cdot 10^{23} \text{ yr}$

$n=3 > 0.94 \cdot 10^{23} \text{ yr}$

$n=7 > 0.35 \cdot 10^{23} \text{ yr}$

(systematic uncertainty not accounted for)

# $0\nu\beta\beta\chi(\chi)$ - Analysis



- ★ For all spectral modes good agreement of best-fit model with data
- ★ Result for  $T^{2\nu}_{1/2}$  in all cases in agreement with previous result

# Systematic uncertainties

Systematic uncertainties estimated by cross-checks changing conditions in fit or input parameters (case n=1 chosen as representative)

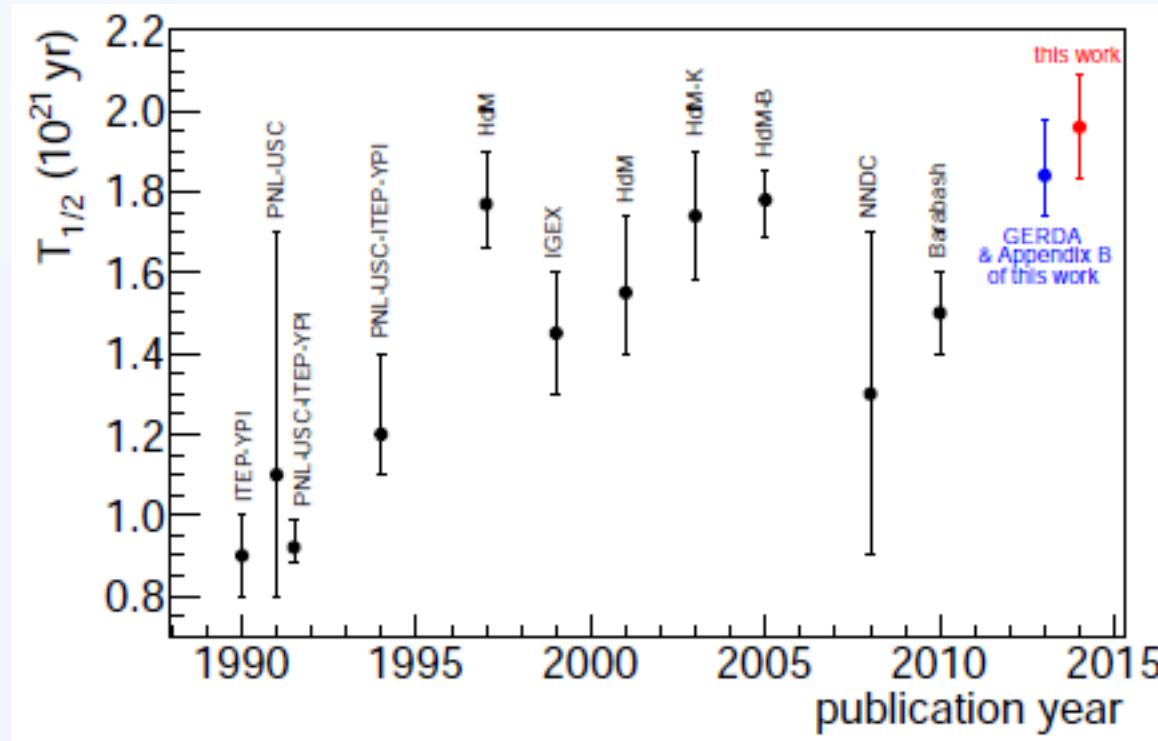
Item	Uncertainty on $T_{1/2}^{2\nu}$ (%)	Uncertainty on $1/T_{1/2}^{0\nu\chi(x)}$ (% on 90% quantile)
Screening results	–	+0.5 –1.3
Binning	±0.5	+16.1
Active volume fraction	±5.6	+5.7 –8.0
Enrichment fraction	±2.0	±1.7
Source positions	–	+12.6 –8.3
Transition layer	±0.5	±0.5
<b>Total fit model</b>	<b>±6.0</b>	<b>+21.3 –11.7</b>
MC geometry	±1.0	±1.0
MC tracking	±2.0	±2.0
<b>Total MC simulation</b>	<b>±2.2</b>	<b>±2.2</b>
Data acquisition and selection	±0.5	±0.5
<b>Total systematic uncertainty</b>	<b>±6.4</b>	<b>+21.4 –12.0</b>

Add individual contributions in quadrature

For limits on  $1/T_{1/2}^{0\nu\chi}$ : Fold systematic uncertainty in posterior distribution

# $2\nu\beta\beta$ – Final result

$$T_{1/2}^{2\nu} = (1.96 \pm 0.03_{\text{fit}} \pm 0.13_{\text{syst}}) \cdot 10^{21} \text{ yr} = (1.96 \pm 0.13) \cdot 10^{21} \text{ yr}$$



Good agreement with previous measurement using GERDA data

# $\text{ov}\beta\beta\chi(x)$ – Final results

Model	n	Mode	Goldstone boson	L	$T_{1/2}^{0\nu\chi}$ [ $10^{23}$ yr]	$\mathcal{M}^{0\nu\chi}$	$G^{0\nu\chi}$ [yr $^{-1}$ ]	$\langle g \rangle$
IB	1	$\chi$	no	0	> 4.2	(2.30 – 5.82)	$5.86 \cdot 10^{-17}$	$< (3.4 - 8.7) \cdot 10^{-5}$
IC	1	$\chi$	yes	0	> 4.2	(2.30 – 5.82)	$5.86 \cdot 10^{-17}$	$< (3.4 - 8.7) \cdot 10^{-5}$
ID	3	$\chi\chi$	no	0	> 0.8	$10^{-3 \pm 1}$	$6.32 \cdot 10^{-19}$	$< 2.1^{+4.5}_{-1.4}$
IE	3	$\chi\chi$	yes	0	> 0.8	$10^{-3 \pm 1}$	$6.32 \cdot 10^{-19}$	$< 2.1^{+4.5}_{-1.4}$
IF	2	$\chi$	bulk field	0	> 1.8	–	–	–
IIB	1	$\chi$	no	-2	> 4.2	(2.30 – 5.82)	$5.86 \cdot 10^{-17}$	$< (3.4 - 8.7) \cdot 10^{-5}$
IIC	3	$\chi$	yes	-2	> 0.8	0.16	$2.07 \cdot 10^{-19}$	$< 4.7 \cdot 10^{-2}$
IID	3	$\chi\chi$	no	-1	> 0.8	$10^{-3 \pm 1}$	$6.32 \cdot 10^{-19}$	$< 2.1^{+4.5}_{-1.4}$
IIE	7	$\chi\chi$	yes	-1	> 0.3	$10^{-3 \pm 1}$	$1.21 \cdot 10^{-18}$	$< 2.2^{+4.9}_{-1.4}$
IIF	3	$\chi$	gauge boson	-2	> 0.8	0.16	$2.07 \cdot 10^{-19}$	$< 4.7 \cdot 10^{-2}$

Most stringent limits obtained to date for Majoron accompanied decay of  ${}^{76}\text{Ge}$

# $\text{Ov}\beta\beta$ - Analysis

## Blinded analysis

Use Pulse Shape Analysis (PSA):

Event selection based on charge and current pulse characteristics

Combine information from **golden**, **silver**, and **BEGe sum** data set:

$$\text{Total likelihood } L = L_{golden} \cdot L_{silver} \cdot L_{BEGe}$$

Fit function: constant + gaussian peak centered at  $Q_{\beta\beta}$

→ 4 parameters (background of three data sets and  $T^{0\nu}_{1/2}$ )

Systematic uncertainties folded in with Monte Carlo approach:

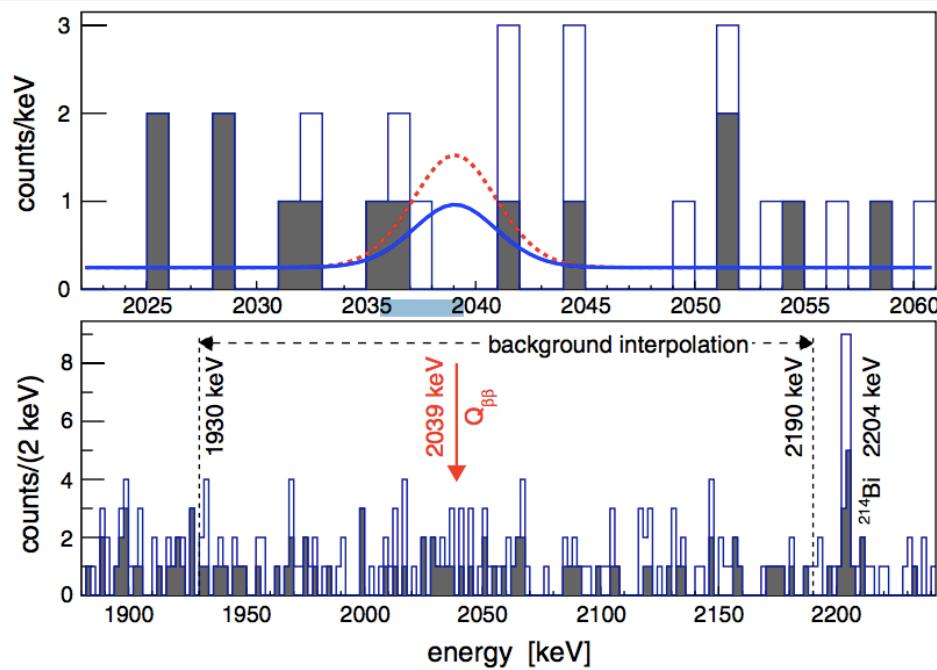
Probability distribution for each source of uncertainty

- Detection efficiency ( $\langle \epsilon \rangle = \langle f_{76} \cdot f_{act} \cdot f_{FEP} \rangle$ )
- Energy resolution
- Signal acceptance after PSA
- Position of the signal peak

# $0\nu\beta\beta$ – Final Results

Expectation in ROI (after PSA):  $2.0 \pm 0.3$  events

Observation in ROI (after PSA): 3 events (none in  $\pm\sigma$  around  $Q_{\beta\beta}$ )



Final GERDA result \*:

$$T^{0\nu}_{1/2} > 2.1 \cdot 10^{25} \text{ yr (90% C.L.)}$$

Combine with information from energy spectra of HDM and IGEX:

$$T^{0\nu}_{1/2} > 3.0 \cdot 10^{25} \text{ yr (90% C.L.)}$$

Assuming exchange of light Majorana neutrino as dominating mechanism for  $0\nu\beta\beta$ :

$$\langle m_{\beta\beta} \rangle < (0.2-0.4) \text{ eV}$$

\*frequentist approach, with Bayesian approach:  $T^{0\nu}_{1/2} > 1.9 \cdot 10^{25} \text{ yr (90% C.I.)}$

Signal claim of  $T^{0\nu}_{1/2} = 1.19^{+0.37}_{-0.23} \cdot 10^{25} \text{ yr [1]}$  not supported

[1] H.V. Klapdor-Kleingrothaus et al., Phys. Lett. B 586 (2004) 198-212

# Conclusions

Analysis of GERDA Phase I data comprising:

- ◆ Description of all relevant background sources with MC simulation
- ◆ Development of complete background model
- ◆ Measurement of half-life of  $2\nu\beta\beta$  of  $^{76}\text{Ge}$  with unprecedented precision:  $T_{1/2}^{2\nu} = (1.96 \pm 0.13) \cdot 10^{21} \text{ yr}$
- ◆ Search for  $0\nu\beta\beta\chi(\chi)$  modes of  $^{76}\text{Ge}$  leading to improved lower limits for all four modes
- ◆ Search for  $0\nu\beta\beta$  of  $^{76}\text{Ge}$  resulting in  $T_{1/2}^{0\nu} > 2.1 \cdot 10^{25} \text{ yr}$

# Additional material

# Previous measurements and claim

# Previous ov $\beta\beta$ experiments

	HdM	IGEX
<b>Location</b>	LNGS	Homestake, Baksan, Canfranc
<b>Exposure [kg·yr]</b>	71.1	8.8
<b>Bg [cts/(keV·kg·yr)]</b>	$\geq 0.11$	0.17
<b>T<sub>1/2</sub> limit (90% CL) [yr]</b>	$1.9 \cdot 10^{25}$ [1]	$1.6 \cdot 10^{25}$ [2]

[1] *Eur. Phys. J. A12*, 147-154 (2001)

[2] *Phys. Rev. D* 65, 092007 (2002)

**Claim of signal from part of HdM:**

$$T_{1/2} ({}^{76}\text{Ge}) = 1.19^{+0.37}_{-0.23} \cdot 10^{25} \text{ yr} \quad \textit{Phys. Lett. B} 586, 198-212 (2004)$$

# HdM claim

Data acquisition and analysis of the  ${}^{76}\text{Ge}$  double beta experiment in Gran Sasso 1990–2003

H.V. Klapdor-Kleingrothaus<sup>a,1</sup>, A. Dietz, I.V. Krivosheina<sup>2</sup>, O. Chkvorets

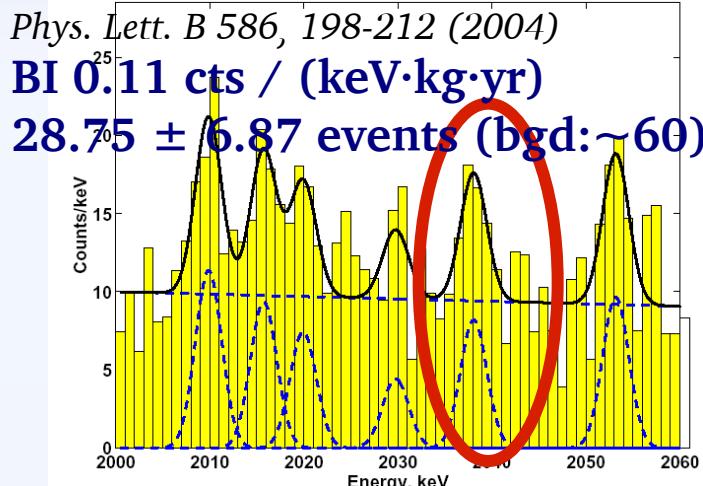


Fig. 17. The total sum spectrum of all five detectors (in total 10.96 kg enriched in  ${}^{76}\text{Ge}$ ), for the period November 1990–May 2003 (71.7 kg year) in the range 2000–2060 keV and its fit (see Section 3.2).

- Nov 1990 - May 2003
- 71.7 kg·yr
- 4.2 $\sigma$ /6 $\sigma$  evidence for  $0\nu\beta\beta$

$(0.69 - 4.18) \cdot 10^{25}$  yr (3 $\sigma$ )  
Best fit:  $1.19 \cdot 10^{25}$  yr

Phys. Lett. B 586, 198-212 (2004)

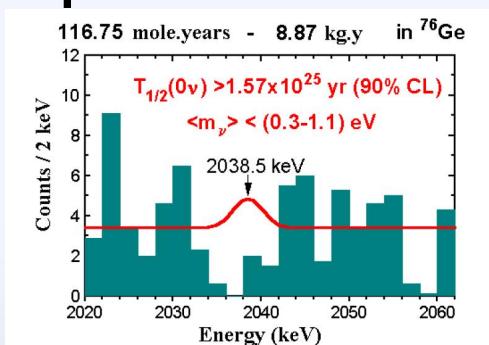
$2.23^{+0.44}_{-0.31} \cdot 10^{25}$  yr

Mod. Phys. Lett. A 21, 1547-1566 (2006)

Criticism in Ann. Phys. 525, 269-280 (2013)

- $m_{\beta\beta} = (0.24-0.58)$  eV (best fit 0.44 eV)  
 $/ 0.32 \pm 0.03$  eV

Note: statistical significance  
depends on background model!



# Refusal of signal claim

Assuming  $T_{1/2}^{0\nu} = 1.19^{+0.37}_{-0.23} \cdot 10^{25}$  yr: expect  **$5.9 \pm 1.4$  signal events** in ROI

Expectation from background (after PSA):  **$2.0 \pm 0.3$  events** in ROI

Observation: **3 events**

From analysis:

**$BF << 1$  for both frequentist and Bayesian approaches**

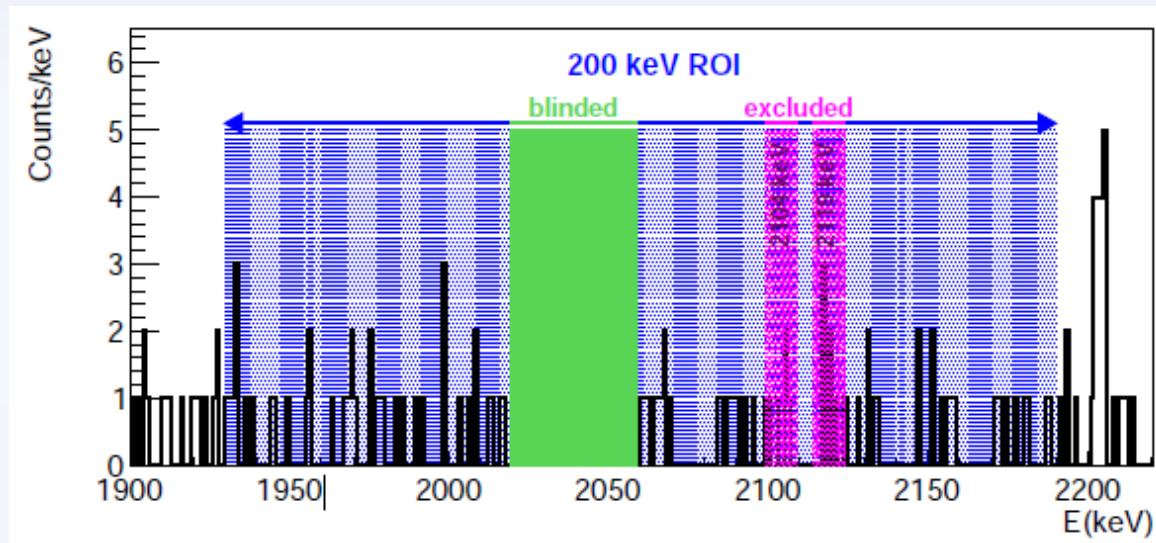
**Best fit value:  $N^{0\nu}=0$  for both approaches**

**$P(N^{0\nu}=0 | T_{1/2}^{0\nu} = 1.19 \cdot 10^{25} \text{ yr}) = 0.01$  for frequentist approach**

→ **Claim of observation of  $0\nu\beta\beta$  not supported by GERDA Phase I data**

# BI definition

# Definition of BI

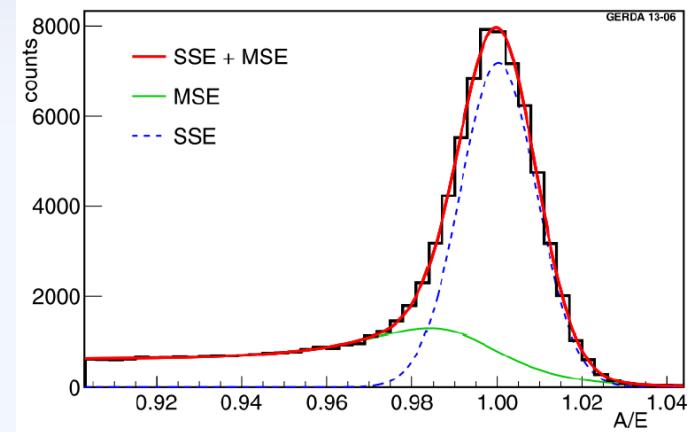
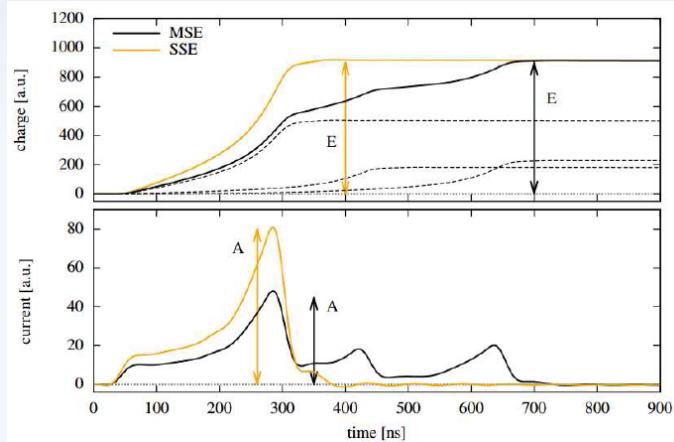


BI ( $10^{-3}$  cts/(keV·kg·yr)) after partial unblinding:

	Before PSA	After PSA
Golden:	$18.5^{+2.3}_{-2.2}$	$10.9^{+1.9}_{-1.6}$
Silver:	$63.4^{+18.0}_{-14.3}$	$30.1^{+13.7}_{-9.8}$
BEGe sum:	$41.3^{+10.4}_{-8.4}$	$5.4^{+5.2}_{-2.9}$

# Pulse Shape Analysis

# Pulse shape analysis – BEGe detectors



Cut based on  $A/E \rightarrow$  keep events with  $2\sigma_{A/E} < A/E < 4\sigma_{A/E}$   
(tuned using DEP, compton continuum and  $2\nu\beta\beta$  events)

$$\epsilon_{PSA} = 0.92 \pm 0.02 \text{ for SSE}$$

BI ( $10^{-3}$  cts/(keV·kg·yr)) after partial unblinding:

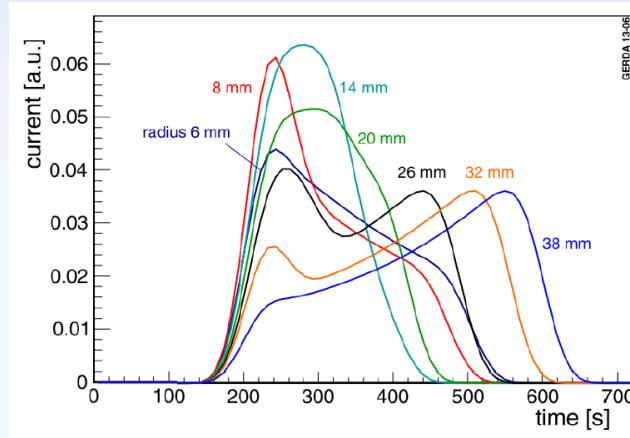
BEGe sum:

Before PSA  
 $41.3^{+10.4}_{-8.4}$

After PSA  
 $5.4^{+5.2}_{-2.9}$

In ROI: 1 event detected  $\rightarrow$  rejected by PSA

# Pulse shape analysis – coaxial detectors



Cut based on ANN using rising part of charge pulse  
(tuned using DEP,  $2\nu\beta\beta$ , compton edge events)

$$\varepsilon_{\text{PSA}} = 0.90^{+0.05}_{-0.09} \text{ for SSE}$$

BI ( $10^{-3}$  cts/(keV·kg·yr)) after partial unblinding:

	Before PSA	After PSA
Golden:	$18.5^{+2.3}_{-2.2}$	$10.9^{+1.9}_{-1.6}$
Silver:	$63.4^{+18.0}_{-14.3}$	$30.1^{+13.7}_{-9.8}$

In ROI (golden/silver): 5/1 event detected  $\rightarrow$  3/0 rejected by PSA

Good agreement with alternative PSA methods

# Pulse shape analysis in ROI

Data set	Detector	E (keV)	Date	Passed PSA
golden	ANG5	2041.8	Nov 18, 2011 22:52	–
silver	ANG5	2036.9	Jun 23, 2012 23:02	✓
golden	RG2	2041.3	Dec 16, 2012 00:09	✓
BEGe	GD32B	2036.6	Dec 28, 2012 09:50	–
golden	RG1	2035.5	Jan 29, 2013 03:35	✓
golden	ANG3	2037.4	Mar 2, 2013 08:08	–
golden	RG1	2041.7	Apr 27, 2013 22:21	–

# Decays

# Decay chains

## $^{238}U$ chain

Nuclide	mode	$T_{1/2}$	Q-value (keV)	decay product	$E_\gamma$ (keV)
$^{238}\text{U}$	$\alpha$	$4.5 \cdot 10^9$ yr	4270.0	$^{234}\text{Th}$	–
$^{234}\text{Th}$	$\beta$	24.1 d	273.0	$^{234m}\text{Pa}$	–
$^{234m}\text{Pa}$	$\beta$	1.2 min	2195.0	$^{234}\text{U}$	–
$^{234}\text{U}$	$\alpha$	$2.5 \cdot 10^5$ yr	4858.5	$^{230}\text{Th}$	–
$^{230}\text{Th}$	$\alpha$	$7.5 \cdot 10^4$ yr	2770.0	$^{226}\text{Ra}$	–
$^{226}\text{Ra}$	$\alpha$	$1.6 \cdot 10^3$ yr	4870.6	$^{222}\text{Rn}$	–
$^{222}\text{Rn}$	$\alpha$	3.8 d	5590.3	$^{218}\text{Po}$	–
$^{218}\text{Po}$	$\alpha$	3.1 min	6114.7	$^{214}\text{Pb}$	–
$^{214}\text{Pb}$	$\beta$	26.8 min	1024.0	$^{214}\text{Bi}$	351.9
$^{214}\text{Bi}$	$\beta$	19.9 min	3272.0	$^{214}\text{Po}$	609.3
					768.4
					1120.3
					1238.1
					1764.5
					2204.2
$^{214}\text{Po}$	$\alpha$	$164.3\ \mu\text{s}$	7833.5	$^{210}\text{Pb}$	–
$^{210}\text{Pb}$	$\beta$	22.3 yr	63.5	$^{210}\text{Bi}$	–
$^{210}\text{Bi}$	$\beta$	5.0 d	1162.1	$^{210}\text{Po}$	–
$^{210}\text{Po}$	$\alpha$	138.4 d	5407.5	$^{206}\text{Pb}$	–

## $^{232}\text{Th}$ chain

Nuclide	mode	$T_{1/2}$	Q-value (keV)	decay product	$E_\gamma$ (keV)
$^{232}\text{Th}$	$\alpha$	$1.4 \cdot 10^{10}$ yr	4082.8	$^{228}\text{Ra}$	–
$^{228}\text{Ra}$	$\beta$	5.8 yr	45.9	$^{228}\text{Ac}$	–
$^{228}\text{Ac}$	$\beta$	6.2 h	2127.0	$^{228}\text{Th}$	911.2
					969.0
$^{228}\text{Th}$	$\alpha$	1.9 yr	5520.1	$^{224}\text{Ra}$	–
$^{224}\text{Ra}$	$\alpha$	3.7 d	5788.9	$^{220}\text{Rn}$	–
$^{220}\text{Rn}$	$\alpha$	55.6 s	6404.7	$^{216}\text{Po}$	–
$^{216}\text{Po}$	$\alpha$	0.1 s	6906.5	$^{212}\text{Pb}$	–
$^{212}\text{Pb}$	$\beta$	10.6 h	573.8	$^{212}\text{Bi}$	–
$^{212}\text{Bi}$	$\beta$	60.6 min	2254.0	$^{212}\text{Po}$	727.3
	$\alpha$		6207.1	$^{208}\text{Tl}$	
$^{212}\text{Po}$	$\alpha$	$0.3\ \mu\text{s}$	8954.1	$^{208}\text{Pb}$	–
$^{208}\text{Tl}$	$\beta$	3.1 min	5001.0	$^{208}\text{Pb}$	510.8
					583.2
					860.6
					2614.5

$^{42}\text{Ar}$ :  $\beta$  (599 keV, 33 yr)  $\rightarrow$   $^{42}\text{K}$ :  $\beta$  (3525.4 keV, 1524.7 keV photon, 12 h)

$^{40}\text{K}$ :  $\beta/\beta^+ + \text{ec}$  (1311.1/1504.9 keV, 1460.8 keV photon,  $1.3 \cdot 10^9$  yr)

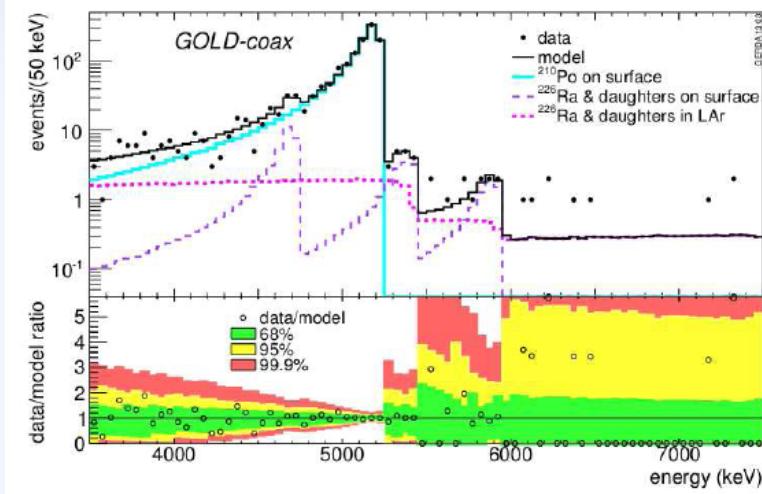
$^{60}\text{Co}$ :  $\beta$  (2823.9 keV, 1173.3 keV & 1332.5 keV photon, 5.3 yr)

$^{68}\text{Ge}$ :  $\text{ec}$  (106.0 keV, 270 d)  $\rightarrow$   $^{68}\text{Ga}$ :  $\text{ec} + \beta^+$  (2921.1 keV, 1077.4 keV photon, 67.6 min)

# Details on Background Models

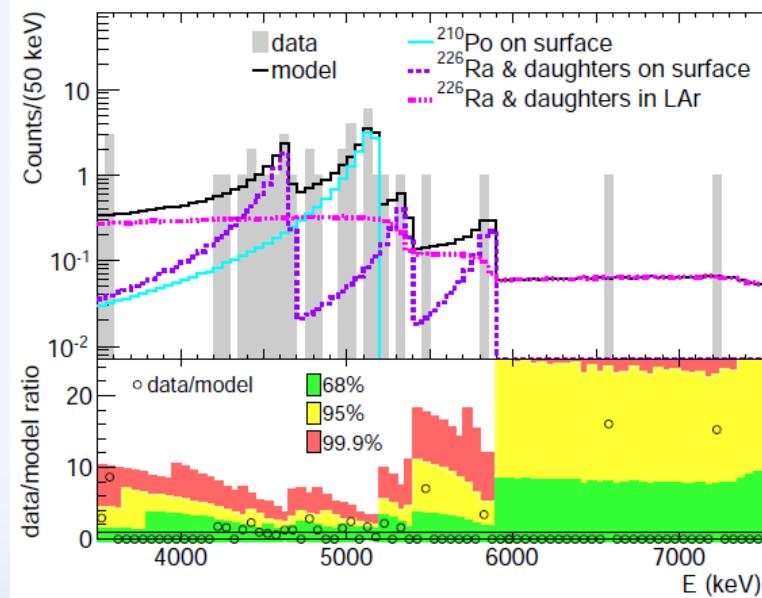
# Alpha models

## Golden data set



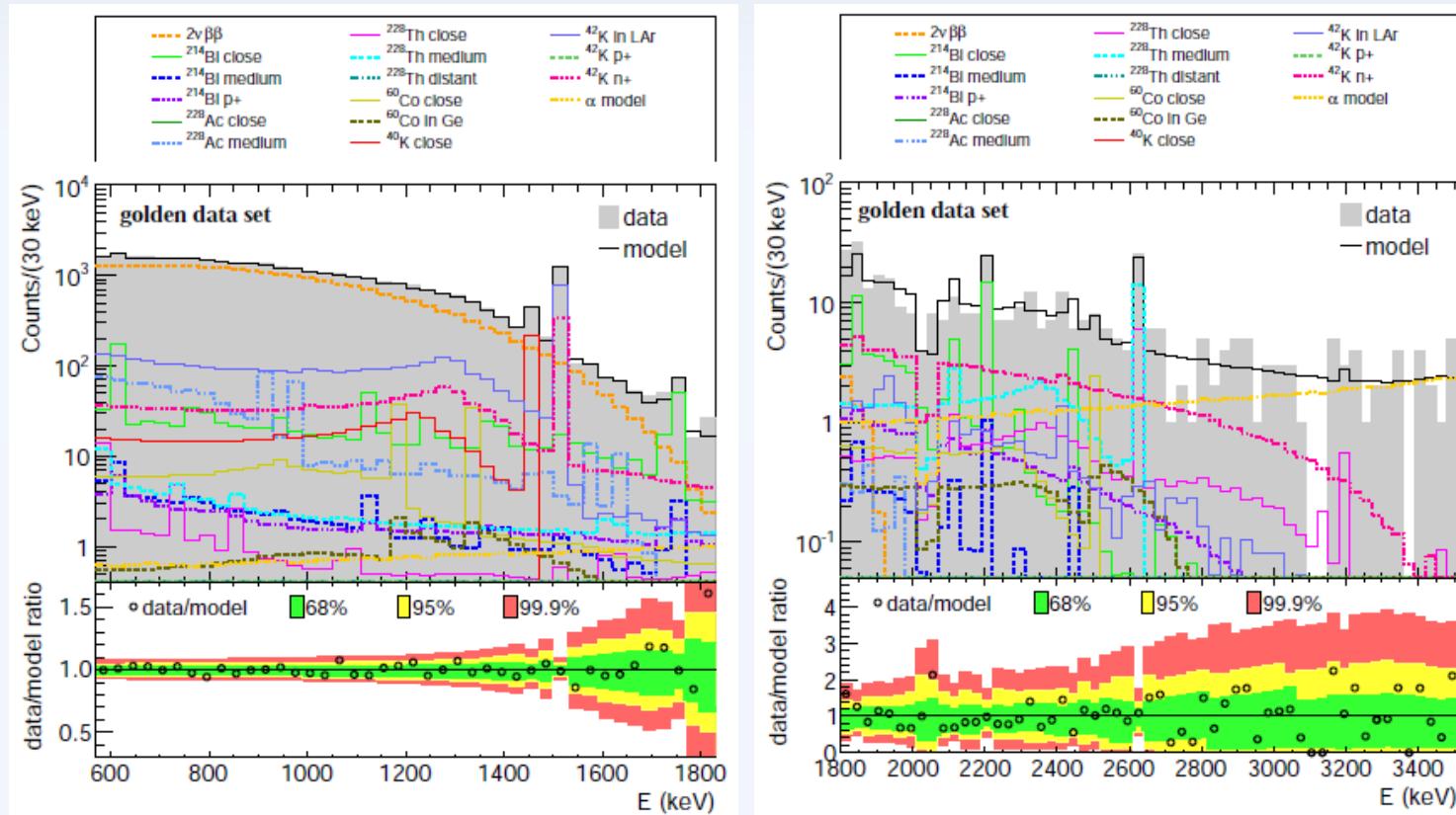
Isotope	Location	Number of counts
$^{210}\text{Po}$	$p^+$ -surface	1355 [1310,1400]
$^{226}\text{Ra}$	$p^+$ -surface	51 [36,65]
$^{222}\text{Rn}$	$p^+$ -surface	25 [18,33]
$^{218}\text{Po}$	$p^+$ -surface	14 [9,19]
$^{214}\text{Po}$	$p^+$ -surface	<10
$^{226}\text{Ra}$	LAr	<159
$^{222}\text{Rn}$	LAr	<64
$^{218}\text{Po}$	LAr	<30
$^{214}\text{Po}$	LAr	20 [10,29]

## BEGe data set



Isotope	Location	Number of counts
$^{210}\text{Po}$	$p^+$ -surface	14.0 [9.4,19.4]
$^{226}\text{Ra}$	$p^+$ -surface	6.1 [2.2,10.2]
$^{222}\text{Rn}$	$p^+$ -surface	<7.1
$^{218}\text{Po}$	$p^+$ -surface	<4.8
$^{214}\text{Po}$	$p^+$ -surface	<4.3
$^{226}\text{Ra}$	LAr	<51.2
$^{222}\text{Rn}$	LAr	<28.6
$^{218}\text{Po}$	LAr	<20.4
$^{214}\text{Po}$	LAr	<12.0

# Background model fits golden data

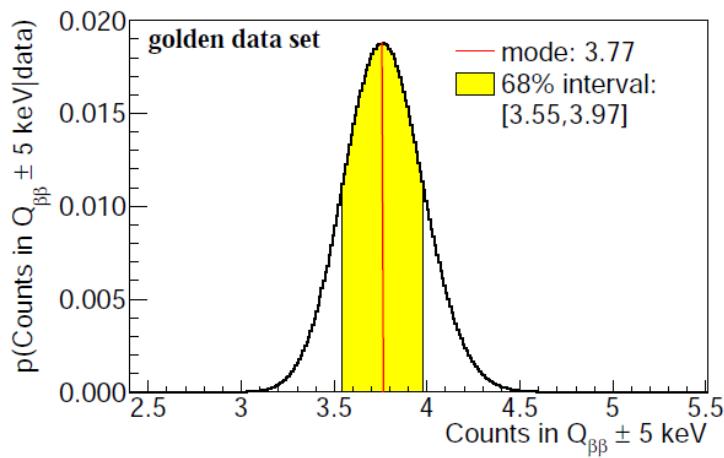


p-value: 0.07

35847 events in data and best-fit model

$$T_{2\nu}^{1/2} = 1.94 [1.91, 1.97] \cdot 10^{21} \text{ yr}$$

# Background model around $Q_{\beta\beta}$



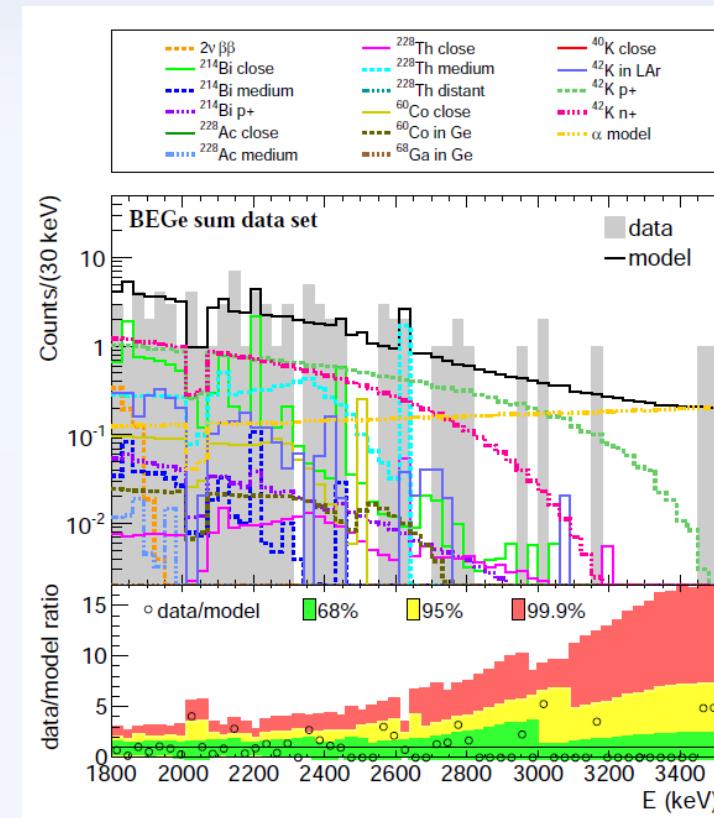
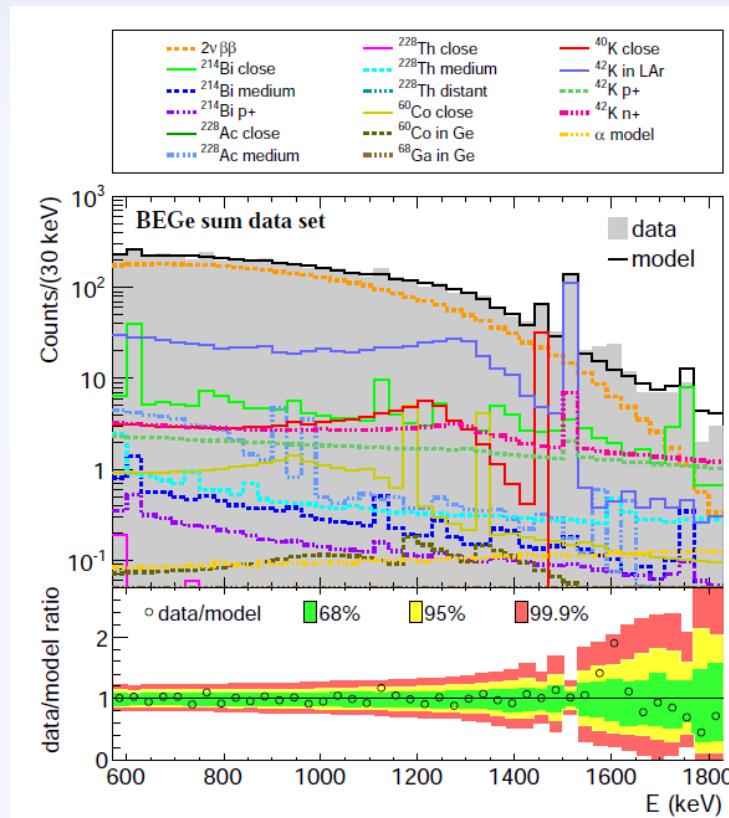
Item	$\Delta BI_{\text{global}}$	$\Delta BI_{\text{marg}}$	Model composition varied	
	global	marg.	global	marg.
Screening results	-	-	-	-
Active volume fraction				
for internal sources	-	-	-	-
for $^{42}\text{K}$ on $n^+$ -surface	+0.5	+0.4	✓	✓
Enrichment fraction	-	-	-	-
Source positions	$^{+1.6}_{-0.2}$	$^{+0.2}_{-0.1}$	✓	✓
Binning	-0.1	-0.1	✓	✓

Table 9.2: Estimates of the  $BI$  from the background model in a 10 keV-window around  $Q_{\beta\beta}$ . Listed are the results for the total model and all individual model components whose spectrum extends up to the window. The global modes, as well as the modes and smallest 68% interval of the marginalized distributions are given. In case the 68 % interval contains zero, the 90 % quantile of the marginalized distribution is shown. All values are given in units of  $10^{-3}$  cts/(keV · kg · yr).

Isotope	Source position	Global mode	Marg. mode	68 % interval/ 90 % quantile
alpha model	$p^+$ -surface	2.1	2.1	[2.0,2.1]
$^{214}\text{Bi}$ chain	close	3.6	2.9	[2.6,3.4]
$^{214}\text{Bi}$ chain	medium distant	0.3	0.3	[0.3,1.2]
$^{214}\text{Bi}$ chain	$p^+$ -surface	1.3	1.3	[0.9,1.7]
$^{228}\text{Th}$ chain	close	1.0	0.8	[0.1,1.4]
$^{228}\text{Th}$ chain	medium distant	2.6	0.8	[0.8,2.3]
$^{228}\text{Th}$ chain	distant	-		<3.3
$^{60}\text{Co}$	close	1.0	1.0	[0.5,1.5]
$^{60}\text{Co}$	in Ge	0.5		<0.5
$^{42}\text{K}$	LAr	1.7	2.2	[1.9,2.4]
$^{42}\text{K}$	$p^+$ -surface	-		<4.9
$^{42}\text{K}$	$n^+$ -surface	6.2		<5.5
<b>Total</b>		20.3	21.0	[19.8,22.2]

**Data (in background window):**  
 **$BI = 18.5^{+2.3}_{-2.2} \cdot 10^{-3}$  cts/(keV·kg·yr)**

# Background model fits BEGe data

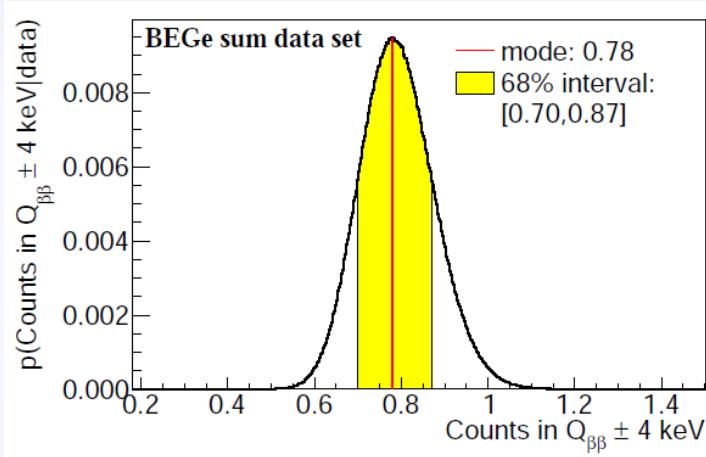


p-value: 0.42

5032 events in data and best-fit model

$$T_{2\nu}^{1/2} = 2.10 [2.02, 2.19] \cdot 10^{21} \text{ yr}$$

# Background model around $Q_{\beta\beta}$



Item	$\Delta BI_{\text{global}}$	$\Delta BI_{\text{marg}}$	Model composition varied	
	global	marg.		
Screening results	-	-	-	✓
Active volume fraction				
for internal sources	-	+0.3	-	✓
for $^{42}\text{K}$ on $n^+$ -surface	-0.4	+0.5	✓	✓
Enrichment fraction	-	-	-	✓
Source positions	$^{+1.1}_{-0.5}$	+1.1	✓	✓
Binning	$^{+1.3}_{-0.5}$	$^{+1.3}_{-1.2}$	✓	✓

Table 9.5: Estimates of the  $BI$  from the background model in a 8 keV-window around  $Q_{\beta\beta}$ . Listed are the results for the total model and all individual model components whose spectrum extends up to the window. The global modes, as well as the modes and smallest 68% interval of the marginalized distributions are given. In case the 68% interval contains zero, the 90% quantile of the marginalized distribution is shown. All values are given in units of  $10^{-3} \text{ cts}/(\text{keV} \cdot \text{kg} \cdot \text{yr})$ .

Isotope	Source position	Global mode	Marg. mode	68% interval/ 90 % quantile
alpha model	$p^+$ -surface	1.8	1.8	[1.5,2.1]
$^{214}\text{Bi}$ chain	close	5.4	3.2	[2.7,5.1]
$^{214}\text{Bi}$ chain	medium distant	0.3	0.3	[0.3,2.7]
$^{214}\text{Bi}$ chain	$p^+$ -surface	0.5	0.5	[0.2,0.8]
$^{228}\text{Th}$ chain	close	0.1	0.1	[0.1,3.0]
$^{228}\text{Th}$ chain	medium distant	3.7	0.8	[0.8,4.2]
$^{228}\text{Th}$ chain	distant	-		<8.4
$^{60}\text{Co}$	close	1.2		<4.2
$^{60}\text{Co}$	in Ge	0.3		<0.3
$^{68}\text{Ga}$	in Ge	-		<3.6
$^{42}\text{K}$	LAr	2.0	2.1	[1.9,2.3]
$^{42}\text{K}$	$p^+$ -surface	11.8	5.1	[0.5,10.5]
$^{42}\text{K}$	$n^+$ -surface	12.9		<16.4
Total		40.0	40.6	[36.5,45.2]

Data (in background window):  
 $BI = 41.3^{+10.4}_{-8.4} \cdot 10^{-3} \text{ cts}/(\text{keV} \cdot \text{kg} \cdot \text{yr})$

# Background model prior knowledge

## Prior information for single background components:

- $^{214}\text{Bi}$  on p<sup>+</sup>-surface gaussian prior derived from  $^{226}\text{Ra}$  activity determined with alpha model
- $^{60}\text{Co}$  in germanium and  $^{68}\text{Ga}$  flat prior with maximum number of events deduced from activation calculations

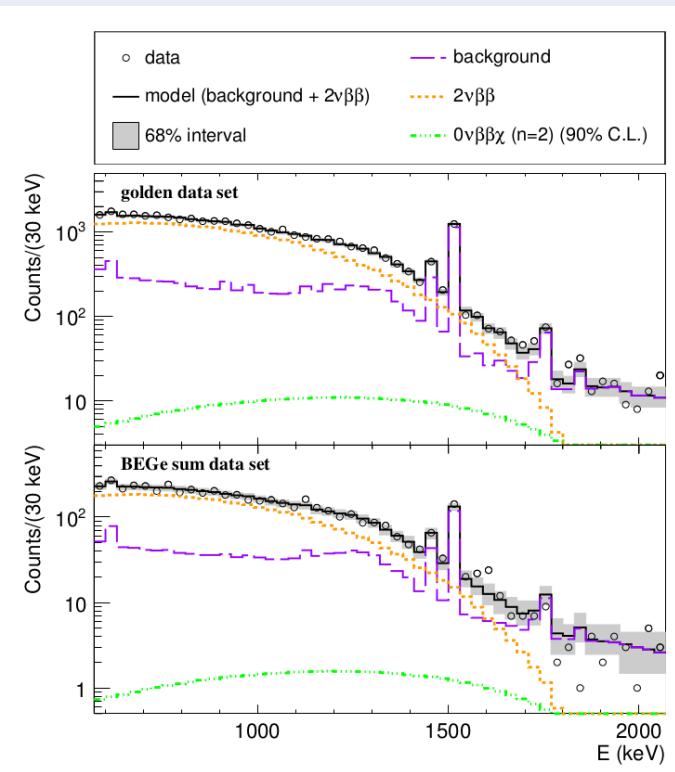
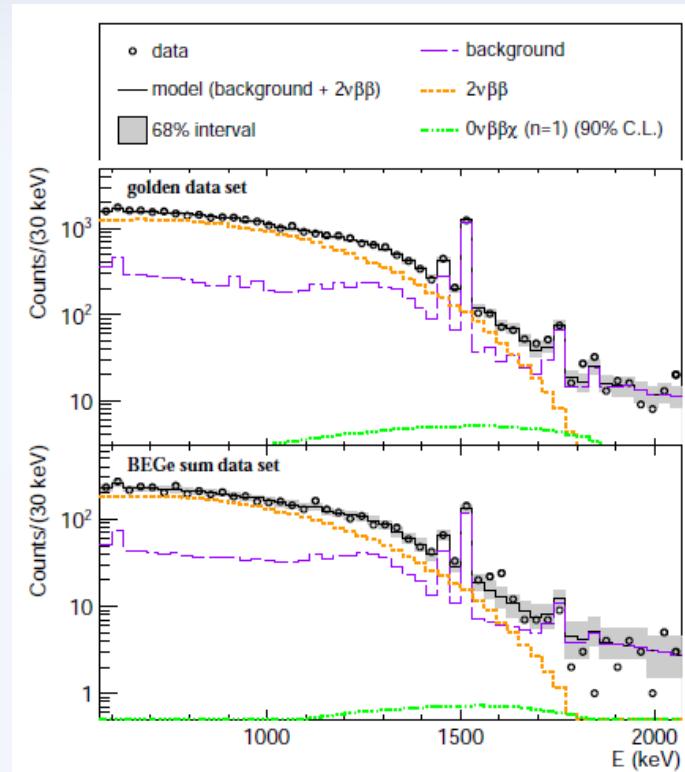
- Flat prior with minimum number of events for  $^{214}\text{Bi}$  on holders and shroud and  $^{228}\text{Th}$  on holders and shroud derived from screening measurements

- All remaining background components:  
Flat priors without restrictions

Component	units	$^{40}\text{K}$	$^{214}\text{Bi}/^{226}\text{Ra}$	$^{228}\text{Th}$	$^{60}\text{Co}$	$^{222}\text{Rn}$	BI
Close sources: up to 2 cm from detectors							
Copper det. support	$\mu\text{Bq}/\text{det}$	<7	<1.3	<1.5			<0.2
PTFE det. support	$\mu\text{Bq}/\text{det}$	6.0 (11)	0.25 (9)	0.31 (14)			0.1
PTFE in array	$\mu\text{Bq}/\text{det}$	6.5 (16)	0.9 (2)				0.1
mini-shroud	$\mu\text{Bq}/\text{det}$		22 (7)				2.8
Li salt	$\text{mBq}/\text{kg}$		17 (5)				$\approx 0.003$
Medium distance sources: 2 cm to 50 cm from detectors							
CC2 preamps	$\mu\text{Bq}/\text{det}$	600 (100)	95 (9)	50 (8)			0.8
cables and suspension	$\text{mBq}/\text{m}$	1.40 (25)	0.4 (2)	0.9 (2)	76 (16)		0.2
Distant sources: further than 50 cm from detectors							
cryostat	$\text{mBq}$				54.7 (35)		<0.7
copper of cryostat	$\text{mBq}$	<784	264 (80)	216 (80)	288 (72)		<0.05
steel of cryostat	$\text{kBq}$	<72	<30	<30	475		
lock system	$\text{mBq}$					2.4 (3)	<0.03
$^{228}\text{Th}$ calib. source	$\text{kBq}$		20				<1.0

# Details on Majoron Analysis

# Majoron results

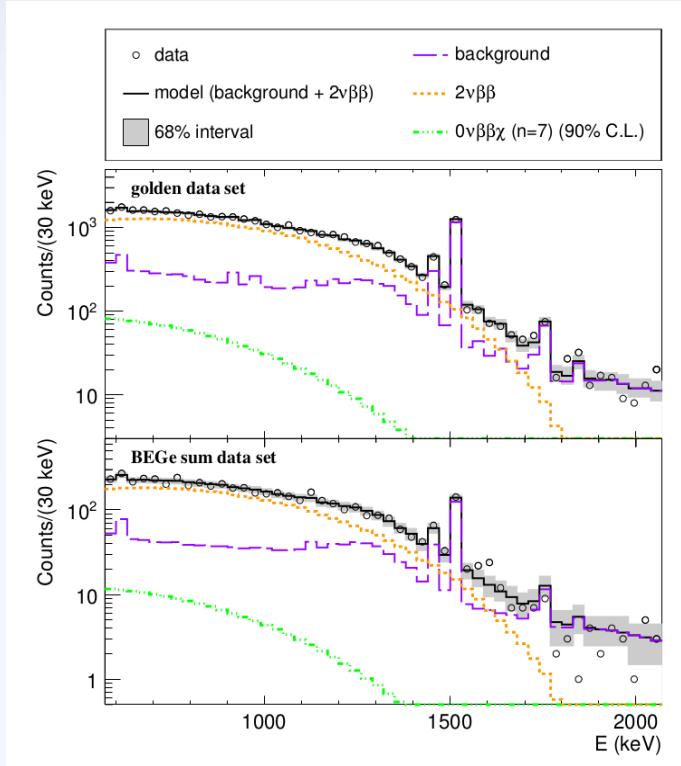
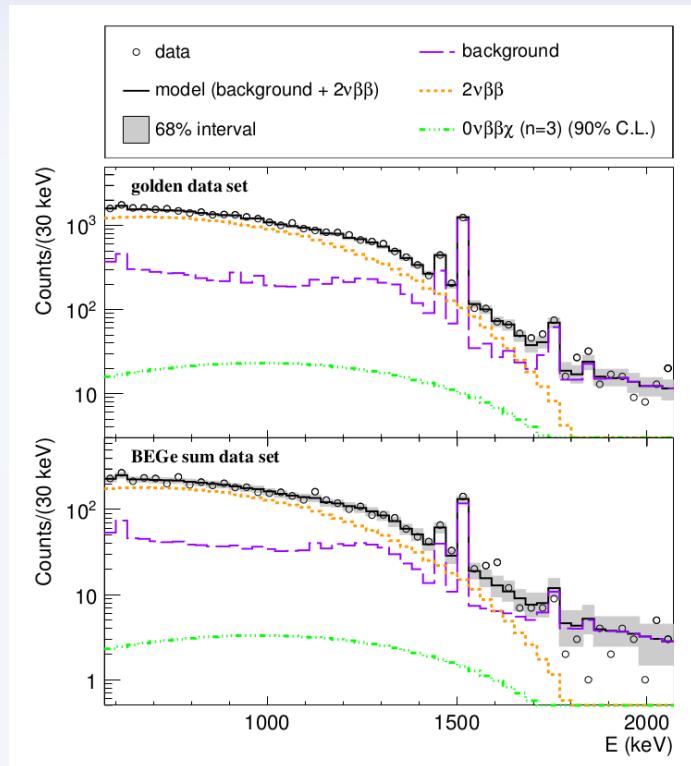


Data: 35868 events in golden, 5035 events in BEGe sum data set

n=1: p-value=0.13  
35834.0/5081.4 events  
54.5/7.8 Majoron  
 $T^{2\nu}_{1/2} = 1.96 \cdot 10^{21}$  yr

n=2: p-value=0.14  
35794.7/5061.8 events  
234.0/33.6 Majoron  
 $T^{2\nu}_{1/2} = 1.97 \cdot 10^{21}$  yr

# Majoron results



Data: 35868 events in golden, 5035 events in BEGe sum data set

$n=3:$      $p\text{-value}=0.12$   
35841.5/5046.4 events  
384.9/55.1 Majoron  
 $T^{2\nu}_{1/2}=1.98 \cdot 10^{21} \text{ yr}$

$n=7:$      $p\text{-value}=0.13$   
35795.8/5049.6 events  
9.7/1.4 Majoron  
 $T^{2\nu}_{1/2}=1.99 \cdot 10^{21} \text{ yr}$

# Previous results

Half-life  $T_{1/2}^{0\nu\chi(x)} (10^{23} \text{ yr})$

This analysis		KamLAND-Zen <sup>1)</sup>	EXO-200 <sup>2)</sup>	HdM <sup>3)</sup>
<b>n=1</b>	>4.2	> 26	> 12	> 0.64
<b>n=2</b>	> 1.8	> 10	> 2.5	-
<b>n=3</b>	> 0.8	> 4.5	> 0.27	>0.14
<b>n=7</b>	> 0.3	> 0.11	> 0.06	>0.07

<sup>1)</sup> A. Gando et al., *Phys. Rev. C* 86 (2012)

<sup>2)</sup> *Phys. Rev. D* 90 (2014), 092004

<sup>3)</sup> H. V. Klapdor-Kleingrothaus et al., *Eur. Phys. J. A* 12 (2001),  
M. Günther, et al., *Phys. Rev. D* 54 (1996), 3641–3644

# Previous results

Coupling constant       $\langle g_\alpha \rangle$

	This analysis	KamLAND-Zen <sup>1)</sup>	EXO-200 <sup>2)</sup>	HdM <sup>3)</sup>
$n=1, \chi$ (IB, IC, IIB)	$< (3.4\text{-}8.7) \cdot 10^{-5}$	$< (0.8\text{-}1.6) \cdot 10^{-5}$	$< (0.8\text{-}1.7) \cdot 10^{-5}$	$< 8.1 \cdot 10^{-5}$
$n=3, \chi\chi$ (ID, IE, IID)	$< 2.1^{+4.5}_{-1.4}$	$< 0.68$	$< (0.6\text{-}5.5)$	$< 3.3$
$n=3, \chi$ (IIC, IIF)	$< 0.047$	$< 0.013$	$< 0.06$	$< 0.12$
$n=7, \chi\chi$ (IIE)	$< 2.2^{+4.9}_{-1.4}$	$< 1.2$	$< (0.5\text{-}4.7)$	$< 3.3$

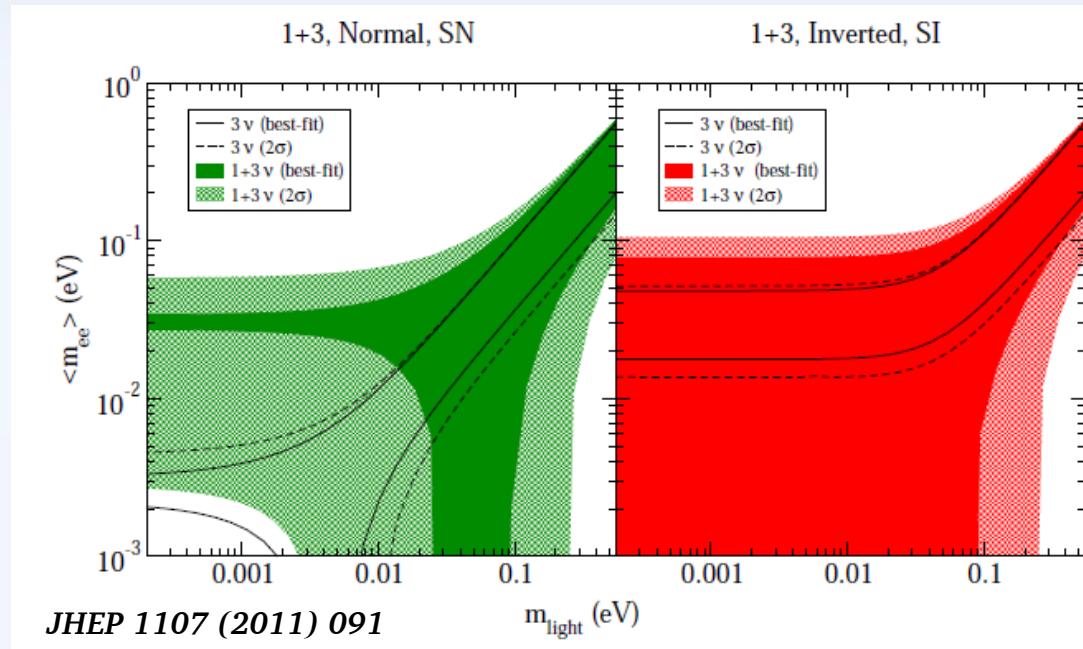
<sup>1)</sup> A. Gando *et al.*, Phys. Rev. C 86 (2012) )

<sup>2)</sup> Phys. Rev. D 90 (2014), 092004

<sup>3)</sup> H. V. Klapdor-Kleingrothaus *et al.*, Eur. Phys. J. A 12 (2001),  
M. Günther, *et al.*, Phys. Rev. D 54 (1996), 3641–3644

# Sterile Neutrinos

# Sterile neutrinos



Case of one additional sterile neutrino with  $m_{\text{st}} \gg m_{1,2,3}$ :

- $\langle m_{\beta\beta} \rangle = \langle m_{\beta\beta} \rangle^{\text{act}} + \langle m_{\beta\beta} \rangle^{\text{st}}$
- If  $m_{\text{light}} < 0.01$  eV  $\rightarrow \langle m_{\beta\beta} \rangle$  cannot vanish for NH
- $\langle m_{\beta\beta} \rangle$  can vanish for IH also for small active neutrino masses

# References for Phase Spaces and Matrix Elements

# References G and M

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## M<sup>0ν</sup> & M<sup>0νχ</sup> (n=1):

- F. Šimkovic, V. Rodin, A. Faessler, and P. Vogel, Phys. Rev. C 87 (2013), 045501
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- T. R. Rodriguez and G. Martinez-Pinedo, Phys. Rev. Lett. 105 (2010), 252503
- J. Menéndez, A. Poves, E. Caurier, and F. Nowacki, Nucl. Phys. A 818 (2009), 139–151
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- J. Suhonen and O. Civitarese, Nucl. Phys. A 847 (2010), 207 – 232
- A. Meroni, S. Petcov, and F. Šimkovic, J. High Energy Phys. 2013 (2013), 1–29

## G<sup>0νχ</sup> (n=1) :

J. Suhonen and O. Civitarese, Phys. Rep. 300 (1998), 123 – 214

## G<sup>0νχ(x)</sup> & M<sup>0νχ(x)</sup> (n=2, 3, 7) :

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