IntraBeam Scattering Calculation

T. Demma, S. Guiducci

SuperB Workshop
LAL, 17 February 09
Calculations procedure

1. Evaluate equilibrium emittances $\varepsilon_i$ and radiation damping times $\tau_i$ at low bunch charge
2. Evaluate the IBS growth rates $1/T_i(\varepsilon_i)$ for the given emittances, averaged around the lattice, using K. Bane approximation (EPAC02)
3. Calculate the "new equilibrium" emittance from:
   \[ \varepsilon_i' = \frac{1}{1 - \tau_i/T_i} \varepsilon_i \]
   - For the vertical emittance use*:
     \[ \varepsilon_y' = (1 - r_{\varepsilon}) \frac{1}{1 - \tau_y/T_y} \varepsilon_y + r_{\varepsilon} \frac{1}{1 - \tau_x/T_x} \varepsilon_x \]
     where $r_{\varepsilon}$ varies from 0 ($\varepsilon_y$ generated from dispersion) to 1 ($\varepsilon_y$ generated from betatron coupling)
4. Iterate from step 2

Bane's approximation


\[
\frac{1}{T_p} \approx \frac{r_0^2 c N(\log)}{16 \gamma^3 \epsilon_x^{3/4} \epsilon_y^{3/4} \sigma_s \sigma_p^3} \left\langle \sigma_H \ g(a/b) \ (\beta_x \beta_y)^{-1/4} \right\rangle
\]

\[
\frac{1}{T_{x,y}} \approx \frac{\sigma_p^2 \langle H_{x,y} \rangle}{\epsilon_{x,y}} \frac{1}{T_p}
\]

\[
\frac{1}{\sigma_H^2} = \frac{1}{\sigma_p^2} + \frac{H_x}{\epsilon_x} + \frac{H_y}{\epsilon_y}
\]

\[(\log)_i = \text{Coulomblog}\]

\[g(\alpha) \approx \alpha^{(0.021 - 0.044 \ln \alpha)} \quad \text{[for } 0.01 < \alpha < 1]\]

\[a = \frac{\sigma_H}{\gamma} \sqrt{\frac{\beta_x}{\epsilon_x}}, \quad b = \frac{\sigma_H}{\gamma} \sqrt{\frac{\beta_y}{\epsilon_y}}\]
Completely-Integrated Modified Piwinski formulae


\[
\frac{1}{T_p} \approx 2\pi^{3/2}A(\log)\left\langle \frac{\sigma_H^2}{\sigma_p^2} \left( \frac{g(b/a)}{a} + \frac{g(a/b)}{b} \right) \right\rangle,
\]

\[
\frac{1}{T_h} \approx 2\pi^{3/2}A(\log)\left\langle -a g\left(\frac{b}{a}\right) + \frac{\mathcal{H}_h \sigma_H^2}{\varepsilon_h} \left( \frac{g(b/a)}{a} + \frac{g(a/b)}{b} \right) \right\rangle.
\]

\[
\frac{1}{T_v} \approx 2\pi^{3/2}A(\log)\left\langle -b g\left(\frac{a}{b}\right) + \frac{\mathcal{H}_v \sigma_H^2}{\varepsilon_v} \left( \frac{g(b/a)}{a} + \frac{g(a/b)}{b} \right) \right\rangle.
\]

\[
g(\omega) = \sqrt{\pi/\omega}\left[ P^0_{-1/2}\left(\frac{\omega^2 + 1}{2\omega}\right) + \frac{3}{2} P^{-1}_{-1/2}\left(\frac{\omega^2 + 1}{2\omega}\right) \right],
\]

\[
\log(\log) \approx \ln\left[ \frac{\gamma^2 \sigma_v \varepsilon_h}{r_0 \beta_h} \right].
\]
### ILC Damping Ring OCS Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy (GeV)</td>
<td>5</td>
</tr>
<tr>
<td>Circumference (m)</td>
<td>6114</td>
</tr>
<tr>
<td>N particles/bunch</td>
<td>2x10^{-10}</td>
</tr>
<tr>
<td>Damping time $\tau_x$ (ms)</td>
<td>22</td>
</tr>
<tr>
<td>Emittance $\gamma\varepsilon_x$ (nm)</td>
<td>5500</td>
</tr>
<tr>
<td>Emittance $\gamma\varepsilon_y$ (nm)</td>
<td>20</td>
</tr>
<tr>
<td>Momentum compaction</td>
<td>1.6x10^{-4}</td>
</tr>
<tr>
<td>Energy spread</td>
<td>1.3x10^{-3}</td>
</tr>
<tr>
<td>Bunch length (mm)</td>
<td>6.0</td>
</tr>
</tbody>
</table>

To check the code the IBS effect calculated for the ILC damping ring OCS lattice has been compared with the results for the configuration options.

### Results for ILC damping ring configuration options

<table>
<thead>
<tr>
<th>N/bunch</th>
<th>$2 \times 10^{10}$</th>
<th>$2 \times 10^{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_x^{ibs}/\varepsilon_x^0$</td>
<td>1.2</td>
<td>1.23</td>
</tr>
<tr>
<td>$\varepsilon_y^{ibs}/\varepsilon_y^0$</td>
<td>1.1</td>
<td>1.12</td>
</tr>
<tr>
<td>$\sigma_p^{ibs}/\sigma_p^0$</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>$\sigma_l^{ibs}/\sigma_l^0$</td>
<td>1.01</td>
<td>1.01</td>
</tr>
</tbody>
</table>
Results for ILC damping ring configuration options

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>0.1e10</th>
<th>1.0e10</th>
<th>2.0e10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_x/\varepsilon_{x0}$</td>
<td>1.01</td>
<td>1.12</td>
<td>1.23</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_y/\varepsilon_{y0}$</td>
<td>1.01</td>
<td>1.06</td>
<td>1.12</td>
<td></td>
</tr>
<tr>
<td>$\sigma_p/\sigma_{p0}$</td>
<td>1.00</td>
<td>1.003</td>
<td>1.006</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_x (\text{nm})$</td>
<td>0.56</td>
<td>0.62</td>
<td>0.68</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon_y (\text{pm})$</td>
<td>2.01</td>
<td>2.12</td>
<td>2.23</td>
<td></td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>0.0013</td>
<td>0.0013</td>
<td>0.0013</td>
<td></td>
</tr>
<tr>
<td>$\sigma_l (\text{mm})$</td>
<td>6.0</td>
<td>6.0</td>
<td>6.1</td>
<td></td>
</tr>
</tbody>
</table>

A. Wolski
# SuperB parameters

<table>
<thead>
<tr>
<th></th>
<th>$E$ [GeV]</th>
<th>$\varepsilon_h$ [nm]</th>
<th>$\varepsilon_v$ [pm]</th>
<th>$\sigma_s$ [mm]</th>
<th>$\sigma_p/p$</th>
<th>$\tau_h/\tau_s$ [ms]</th>
</tr>
</thead>
<tbody>
<tr>
<td>HER</td>
<td>7</td>
<td>1.6</td>
<td>4</td>
<td>5</td>
<td>5.8e-4</td>
<td>40/20</td>
</tr>
<tr>
<td>LER</td>
<td>4</td>
<td>2.8</td>
<td>7</td>
<td>5</td>
<td>8e-4</td>
<td>40/20</td>
</tr>
</tbody>
</table>
IBS momentum spread vs. number of particles/bunch

\[ \frac{\sigma_p}{\sigma_{p_0}} = 1.05, \ @ N=5.5 \cdot 10^{10} \]
IBS horizontal emittance vs. number of particles/bunch

\[ \frac{\varepsilon_h}{\varepsilon_{h0}} = 1.14, \quad @ \quad N = 5.52 \cdot 10^{10} \]

\[ \frac{\varepsilon_h}{\varepsilon_{h0}} = 1.11, \quad @ \quad N = 5.52 \cdot 10^{10} \]
IBS vertical emittance vs. number of particles/bunch for $r_\varepsilon = 0, 0.5, 1$

<table>
<thead>
<tr>
<th>$N = 5.52 \cdot 10^{10}$</th>
<th>$\varepsilon_v/\varepsilon_{v0}$</th>
<th>$r_\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.10</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1.11</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>1.15</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$N = 5.52 \cdot 10^{10}$</th>
<th>$\varepsilon_v/\varepsilon_{v0}$</th>
<th>$r_\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.10</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1.08</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>1.13</td>
<td>1</td>
</tr>
</tbody>
</table>
LER lattice - IBS emittance growth @ N=5.5e10 with and without wigglers

<table>
<thead>
<tr>
<th></th>
<th>No IBS</th>
<th>With IBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{wig}$ (T)</td>
<td>0.</td>
<td>0.85</td>
</tr>
<tr>
<td>$V_{RF}$ (MV)</td>
<td>5.4</td>
<td>6.3</td>
</tr>
<tr>
<td>$\tau_{x,y}$ (ms)</td>
<td>39.0</td>
<td>32.4</td>
</tr>
<tr>
<td>$\tau_p$ (ms)</td>
<td>19.5</td>
<td>16.2</td>
</tr>
<tr>
<td>$\varepsilon_x/\varepsilon_{x0}$</td>
<td>1.15</td>
<td>1.18</td>
</tr>
<tr>
<td>$\varepsilon_y/\varepsilon_{y0}$</td>
<td>1.10</td>
<td>1.12</td>
</tr>
<tr>
<td>$\sigma_p/\sigma_{p0}$</td>
<td>1.06</td>
<td>1.06</td>
</tr>
<tr>
<td>$\varepsilon_x$</td>
<td>2.7e-9</td>
<td>2.3e-9</td>
</tr>
<tr>
<td>$\varepsilon_y$</td>
<td>7.0e-12</td>
<td>6.3e-12</td>
</tr>
<tr>
<td>$\sigma_l$</td>
<td>5.0</td>
<td>4.7</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>8.1e-4</td>
<td>8.2e-4</td>
</tr>
</tbody>
</table>

12 wigglers
$\lambda_{wig} = 0.4\text{m}$
$L_{wig} = 2.45\text{m}$
## LER lattice - IBS emittance growth for different wiggler fields

<table>
<thead>
<tr>
<th>$B_{\text{wig}}$ (T)</th>
<th>0.0</th>
<th>1.0</th>
<th>1.3</th>
<th>1.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{\text{RF}}$ (MV)</td>
<td>5.4</td>
<td>6.6</td>
<td>7.3</td>
<td>8.7</td>
</tr>
<tr>
<td>$\tau_{x,y}$ (ms)</td>
<td>39.0</td>
<td>30.5</td>
<td>26.5</td>
<td>22.8</td>
</tr>
<tr>
<td>$\tau_p$ (ms)</td>
<td>19.5</td>
<td>15.3</td>
<td>13.3</td>
<td>11.4</td>
</tr>
<tr>
<td>$\varepsilon_x/\varepsilon_{x0}$</td>
<td>1.15</td>
<td>1.18</td>
<td>1.18</td>
<td>1.18</td>
</tr>
<tr>
<td>$\varepsilon_y/\varepsilon_{y0}$</td>
<td>1.10</td>
<td>1.12</td>
<td>1.11</td>
<td>1.11</td>
</tr>
<tr>
<td>$\sigma_p/\sigma_{p0}$</td>
<td>1.06</td>
<td>1.06</td>
<td>1.04</td>
<td>1.03</td>
</tr>
<tr>
<td>$\varepsilon_{x0}$</td>
<td>2.7e-9</td>
<td>2.1e-9</td>
<td>1.9e-9</td>
<td>1.7e-9</td>
</tr>
<tr>
<td>$\varepsilon_{y0}$</td>
<td>7.0e-12</td>
<td>6.3e-12</td>
<td>6.3e-12</td>
<td>6.3e-12</td>
</tr>
<tr>
<td>$\sigma_{p0}$</td>
<td>8.1e-4</td>
<td>8.4e-4</td>
<td>9.0e-4</td>
<td>9.9e-4</td>
</tr>
<tr>
<td>$\sigma_{l0}$</td>
<td>5.0</td>
<td>4.7</td>
<td>4.8</td>
<td>4.8</td>
</tr>
<tr>
<td>$\varepsilon_x$</td>
<td>3.1e-9</td>
<td>2.5e-9</td>
<td>2.3e-9</td>
<td>2.0e-9</td>
</tr>
<tr>
<td>$\varepsilon_y$</td>
<td>7.7e-12</td>
<td>7.0e-12</td>
<td>7.0e-12</td>
<td>7.0e-12</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>8.6e-4</td>
<td>8.9e-4</td>
<td>9.4e-4</td>
<td>1.0e-3</td>
</tr>
<tr>
<td>$\sigma_l$</td>
<td>5.3</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
</tr>
</tbody>
</table>

N=5.5e10
Insertion of Wigglers

Damping time $\tau_{x,y}$ and RF voltage $V_{RF}$ vs. wiggler field $B_w$

Relative energy spread $\sigma_p$ and emittance $\varepsilon_x$ vs. wiggler field $B_w$

$N = 5.5e-10$, $\sigma_l = 5$ mm, $\varepsilon_y = 7$ pm
Emittance and sigmap vs. wiggler field with (−) and without (---) IBS

Nominal value
Conclusions

• The effect of IBS on the transverse emittance is reasonably small for both rings.
• It can be compensated by adding 12 low field wigglers to get the nominal transverse emittance at the expenses of an increased energy spread and RF voltage.
• Another possibility to get the nominal emittance is to increase the phase advance in the arc cells.
• An interesting study: develop a MonteCarlo code to study the beam size distribution in the presence of IBS.