Leptonic Decays

Headaches with soft photons
Leptonic decays: simplest probe to learn on flavor physics in and beyond the Standard Model

\[ \Gamma(P \rightarrow \ell \nu) = \frac{G_F^2 m_P^3}{8\pi} |V_{UD}|^2 f_P^2 \left( \frac{m_\ell}{m_P} \right)^2 \left[ 1 - \left( \frac{m_\ell}{m_P} \right)^2 \right] \]

Perspectives:
theory (precision determination by LQCD)
experiment (check on the systematics)
**Leptonic decays**: simplest probe to learn on flavor physics in and beyond the Standard Model

**Perspectives**: theory (precision determination by LQCD)

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<th>$N_f=0$</th>
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<td>1.156(29) Wilson: $a=0.05\text{fm}\rightarrow 0$, $m_\pi&gt;500$ MeV</td>
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<td>1.214(21) TMQCD: $m_\pi&gt;300$ MeV</td>
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<td>1.210(14) Stag. $a=0.06\text{fm}\rightarrow 0$</td>
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<td>1.197_{-13}^{+7} Stag. $m_\pi&gt;280$ MeV, $L_m&gt;5.0$</td>
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**Leptonic decays:** simplest probe to learn on flavor physics in and beyond the Standard Model

**Perspectives:**
- **theory** (precision determination by LQCD)
- **experiment** (check on the systematics)

**Errors:**
- **theory** (don’t extrapolate the errors - once you touch the precision under 5%, many hidden skeletons come out of the cupboard)
- **experiment** (early 2000 - experts said - No way to see $B \rightarrow \tau \bar{\nu}$; $B$-factories detected those events and now extrapolate to potentially visible $B \rightarrow \mu \bar{\nu}$) Is systematics under control?
Soft photons

Experimental event selection of $B \rightarrow \mu \nu$

$\mu$ with energy $\sim m_B/2 \pm 200$ MeV in the $B$ rest frame

N.B. Photons with energy less than 500 MeV in B-factories are invisible! CLEO-c cuts about 300 MeV
Soft photons

Experimental event selection of $B \rightarrow \mu \nu$

$\mu$ with energy $\sim \frac{m_B}{2} \pm 200$ MeV in the $B$ rest frame

Radiative leptonic decays are suppressed by $\alpha_{\text{em}} \approx 1/137$!

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Soft photons

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Radiative leptonic decays are suppressed by $\alpha_{\text{em}} \approx \frac{1}{137}$!

They are NOT helicity suppressed $m_B/m_\mu \approx 2500$!
Soft photons

Experimental event selection of $B \rightarrow \mu \nu$

$\mu$ with energy $\approx m_B/2 \pm 200$ MeV in the $B$ rest frame

How many events in the sample are fake?
Decay distribution

\[ B^+(p) \rightarrow l^+(p_l) \nu_l(p_\nu) \gamma(k) \]

\[ M(B^+ \rightarrow l^+ \nu_l \gamma) = M_{IB} + M_{SD} \]

\[ M_{IB} = i e \frac{G_F}{\sqrt{2}} V_{ub} f_B m_l \epsilon^*_\mu L^\mu \]

\[ M_{SD} = -i \frac{G_F}{\sqrt{2}} V_{ub} f_B m_l \epsilon^*_\mu \tilde{H}^{\mu \nu} l_\nu \]

\[ L^\mu = m_l \bar{u}(p_\nu) (1 + \gamma_5) \left( \frac{2p_\mu}{2p \cdot k} - \frac{2p_\mu^l + k \gamma^\mu}{2p_l \cdot k} \right) v(p_l, s_l) \]

\[ l^\mu = \bar{u}(p_\nu) \gamma^\mu (1 + \gamma_5) v(p_l, s_l) \]

\[ \tilde{H}^{\mu \nu} = i F_V(q^2) \epsilon^{\mu \nu \alpha \beta} k_\alpha p_\beta - F_A(q^2) (p \cdot k g^{\mu \nu} - p^\mu k^\nu) \]
Decay distribution

\[ x = \frac{2p \cdot k}{M_B^2} = \frac{2E_\gamma}{M_B} \]

\[ y = \frac{2p \cdot p_l}{M_B^2} = \frac{2E_l}{M_B} \]

\[ 0 \leq x \leq 1 - r_l \]

\[ 1 - x + \frac{r_l}{1 - x} \leq y \leq 1 + r_l \]

\[ r_l = \frac{m_l^2}{M_B^2} \]
N.B. Photons with energy less than 500 MeV in B-factories are invisible! CLEO-c cuts about 300 MeV
Structure dependent part

\[ B^+(p) \rightarrow l^+(p_l) \nu_l(p_\nu) \gamma(k) \]

\[ M(B^+ \rightarrow l^+ \nu_\gamma) = M_{IB} + M_{SD} \]

Due to the nearness of \( M_{B^*} \) wrt \( M_B \), the form factor \( F_V \) is dominant and in the small \( x \)-region

\[ F_V(x) \approx \frac{f_{B^*} M_{B^*} g_{B^*} B_\gamma}{M_{B^*}^2 - q^2} \bigg|_{q^2 = M_B^2 (1-x)} \]

\[ L^\mu = m_l \bar{u}(p_\nu) (1 + \gamma_5) \left( \frac{2p^\mu}{2p \cdot k} - \frac{2p_l^\mu + k \gamma^\mu}{2p_l \cdot k} \right) v(p_l, s_l) \]

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Structure dependent part

Due to the nearness of $M_B^*$ wrt $M_B$ the form factor $F_V$ is dominant and in the small $x$-region

$$B^+(p) \rightarrow l^+(p_l)\nu_l(p_\nu)\gamma(k)$$

$$F_V(x) \approx \frac{f_{B^*} M_{B^*} g_{B^* B \gamma}}{M_{B^*}^2 - q^2} \bigg|_{q^2 = M_B^2 (1-x)}$$

$$\langle \gamma(p_\gamma, \epsilon) B(p_B) | B^*(\eta) \rangle = -i \varepsilon_{\mu\nu\alpha\beta} p^\mu_{\gamma} \eta^\nu \gamma^\alpha \epsilon^\beta g_{B^* B \gamma}$$

$$g_{B^* B \gamma} = e M_B \left( \frac{Q_b}{m_b} + Q_q \beta \right)$$

light quark contribution to the magnetic moment of the vector meson

Various models $1.7 \leq |g_{B^* B^+ \gamma}| \leq 3.0$ [LCSR 2.7, Aliev et al, 2001]
Due to the nearness of $M_{B^*}$ wrt $M_B$ the form factor $F_V$ is dominant and in the small $x$-region

$$F_V(x) \approx \frac{f_{B^*} M_{B^*} g_{B^* \gamma}}{M_{B^*}^2 - q^2} \bigg|_{q^2 = M_B^2 (1-x)}$$

$$\left( \vec{\alpha} \cdot \vec{p} + m_\beta + V(r) - E \right) \Psi(r) = 0$$

$$V(r) = -\frac{\kappa}{r} + \beta (ar + c)$$

**Dirac Model which was successfull in $D^*D\pi$ coupling**

$$\beta = \frac{2}{3} \int_0^\infty \left\{ f_{1/2}^{(-1)^*} g_{1/2}^{(-1)} + g_{1/2}^{(-1)^*} f_{1/2}^{(-1)} \right\} r^3 dr$$
Structure dependent part

Due to the nearness of $M_{B^*}$ wrt $M_B$ the form factor $F_V$ is dominant and in the small $x$-region

$$B^+(p) \rightarrow l^+(p_l)\nu_l(p_\nu)\gamma(k)$$

$$F_V^{\text{pole}}(x) = \frac{C_V^b}{x - 1 + \Delta_b}$$

$$C_V^b = \frac{f_{B^*} m_{B^*} g_{B^*} B^{-\gamma}}{m_B^2 \sqrt{4\pi\alpha_{\text{em}}}}; \quad \Delta_b = \frac{m_{B^*}^2}{m_B^2}$$

**Dirac Model which was successfull in $D^*D\pi$ coupling**

$$\beta = \frac{2}{3} \int_0^\infty \left\{ f_1^{(-1)*} g_1^{(-1)} + g_1^{(-1)*} f_1^{(-1)} \right\} r^3 dr$$

$$\beta = 1.5 \pm 0.2 \text{ GeV}^{-1}$$
As a result...

\[ B^+(p) \rightarrow l^+(p_l) \nu_l(p_{\nu}) \gamma(k) \]

- **$B \rightarrow \mu \nu$ signal region**
  \[ E_\mu = E_{\text{miss}} = \frac{M_B}{2} \simeq 2.6 \text{ GeV} \]

- **Cross-talk region (invisible photon)**
  \[ E_\mu > 2.4 \text{ GeV}, \quad E_\gamma < 0.5 \text{ GeV} \]

\[ x = \frac{2E_\gamma}{M_B}, \quad y = \frac{2E_\mu}{M_B} \]
In the case of D-decays the effect is -of course- smaller: We just finished computing $D^*D\gamma$-coupling on the lattice (first ever!) ➔ D-leptonic decays will be under better control

In the case of B-decays needs to have excellent photon resolution: computing $B^*B\gamma$-coupling on the lattice is difficult ➔ ideas to do it in the static limit

Leptonic B-decay to $\tau$ (tau) is difficult but is theoretically cleaner

What can be [really] done in Super-B?
List of References and detailed discussion of what I just announced here will soon appear in our -Orsay- papers

- Discussion of the soft photon traps in B-leptonic decay: paper with B.Haas and E.Kou

- In the case of D-decays the effect is -of course- smaller: D*Dγ-coupling together with D*Dπ ➔ paper with B.Haas to appear

- D-meson decay constant (and Ds) with the discussion of the soft photons ➔ paper with B.Haas and E.Kou to appear

- Figuring out the way to compute B*By in the static limit ➔ with E.Chang