Thin metal film theory

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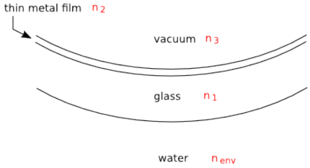
[1] AstroParticules et Cosmologie, Paris

Simulation workshop, Genova, April 2015

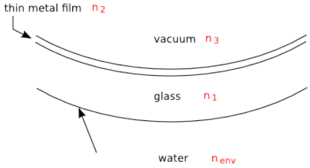
Concept

- 3 layers plus the environment



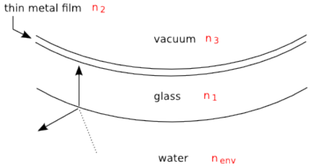
Concept

- 3 layers plus the environment
- incoming photon



Concept

- 3 layers plus the environment
- incoming photon
- reflexion/refraction



Concept

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- incoming photon
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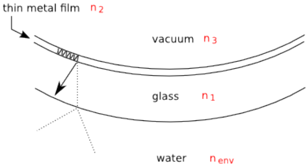
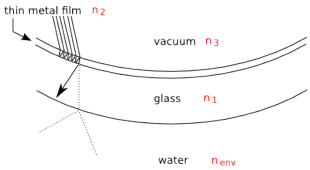


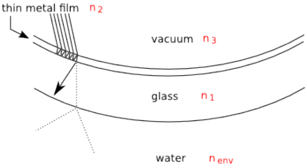
Concept

- 3 layers plus the environment
- incoming photon
- reflexion/refraction
- reflexion/refraction
- multiple reflexions



Concept

- 3 layers plus the environment
- incoming photon
- reflexion/refraction
- reflexion/refraction
- multiple reflexions
- constructive waves



Generalities

Two crucial points

- Wave:
  due to the thickness of the film and to the multiple reflexions, the path length in the film is longer in average than the intrinsic thickness. Exiting waves can also add coherently.
- Particle:
  the photon get absorbed in the metal. The probability depends on the path length in the metal

Formalisms

- Physicist convention:

$$\vec{E}(\vec{r}, t) = \vec{E}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

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- Crucial when applying Maxwell equations and Snell-Descartes law for an absorbing media.

Complex index of refraction

From Maxwell equations

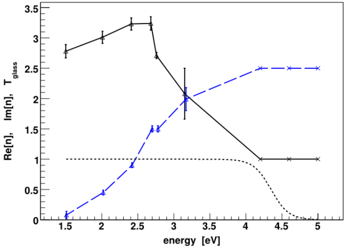
Physicist convention

$$\tilde{n} = \eta + i\eta\kappa$$

Engineer convention

$$\tilde{n} = \eta - i\eta\kappa$$

ref. Born, Wolf, principles of Optics, 7th edition, Cambridge University Press, 1999.



η and κ both positive

ref. M.E. Moorhead, N.W. Tanner, Nucl. Instr. and Meth. A 378 (1996) 162

Fresnel equations

From the continuity of the tangential component of $\vec{E}$ and $\vec{B}$

P component

$$r_{12}^{p} = \frac{\tilde{n}_2 \cos\theta_1 - n_1 \cos\theta_2}{\tilde{n}_2 \cos\theta_1 + n_1 \cos\theta_2}$$

$$t_{12}^{p} = \frac{2 n_1 \cos\theta_1}{\tilde{n}_2 \cos\theta_1 + n_1 \cos\theta_2}$$

S component

$$r_{12}^{s} = \frac{n_1 \cos\theta_1 - \tilde{n}_2 \cos\theta_2}{n_1 \cos\theta_1 + \tilde{n}_2 \cos\theta_2}$$

$$t_{12}^{s} = \frac{2 n_1 \cos\theta_1}{n_1 \cos\theta_1 + \tilde{n}_2 \cos\theta_2}$$

for both formalisms.

The same equations can be written between media 2 and 3.

ref. Born, Wolf, principles of Optics, 7th edition, Cambridge University Press, 1999.

ref. H.A. Macleod, Thin-Film Optical Filters, third edition, Institute of Physics Publishing, Bristol, 2001.

Snell-Descartes law

In order to solve Fresnel equations, we get

- $\tilde{\theta}_2$ from $\tilde{n}_2 \sin \tilde{\theta}_2 = n_1 \sin \theta_1$
- $\tilde{\theta}_3$ from $n_3 \sin \tilde{\theta}_3 = \tilde{n}_2 \sin \tilde{\theta}_2$

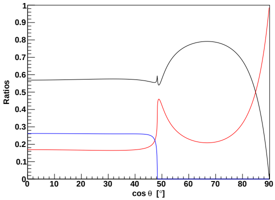
Physicist convention	Engineer convention
$\text{Im}[\tilde{\theta}_3] < 0$	$\text{Im}[\tilde{\theta}_3] > 0$

θ_3 can not be extracted from $n_1 \sin \theta_1 = n_3 \sin \theta_3$.

Theoritical results

$$n_{\text{env}} = 1.34$$
$$n_1 = 1.49$$
$$\tilde{n}_2 = 2.7 + i1.56$$
$$n_3 = 1.00$$
$$h = 20\,\mathrm{nm}$$

R,T,A



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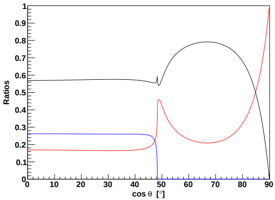
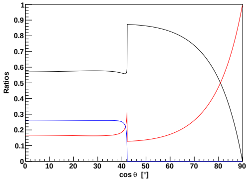
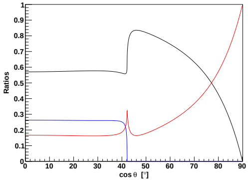




$\mathrm{Im}[\theta_3] = 0.$



$-\mathrm{Im}[\theta_3].$

Conclusions

- theory well understood
- code implementation done (among other characteristics)
- comparison with the OM calibration to perform