Computing hadronic vacuum polarisation from first principles

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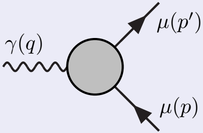
The magnetic moment of the lepton: a_l

- Intrinsic magnetic moment of any spinning particle

$$\vec{\mu} = g_l \frac{e\hbar}{2m_l c} \vec{S}$$

- For leptons, $s = \frac{1}{2}$, the giromagnetic from Dirac theory: $g_l = 2$



$$= (-ie)\bar{u}(p')\left[\gamma^\mu F_E(q^2) + \frac{\sigma^{\mu\nu} q_\nu}{2m_l} F_M(q^2)\right] u(p)$$

- $q = p' - p \quad q^2 = 0$: $F_E(0) = 1, F_M(0) = a_l = \frac{g_l - 2}{2}$, $l = e, \mu, \tau$

- quantum fluctuations due to heavier particles or contributions from higher energy scales

$$\frac{\delta a_l}{a_l} \propto \frac{m_l^2}{M^2}$$

M - mass of heavier SM/BSM particle, or scale of new physics ...

a_μ as a stringent test of the SM

- quantum fluctuations due to heavier particles or contributions from higher energy scales

$$\boxed{\frac{\delta a_l}{a_l} \propto \frac{m_l^2}{M^2};} \qquad (m_\mu/m_e)^2 \sim 4 \times 10^4$$

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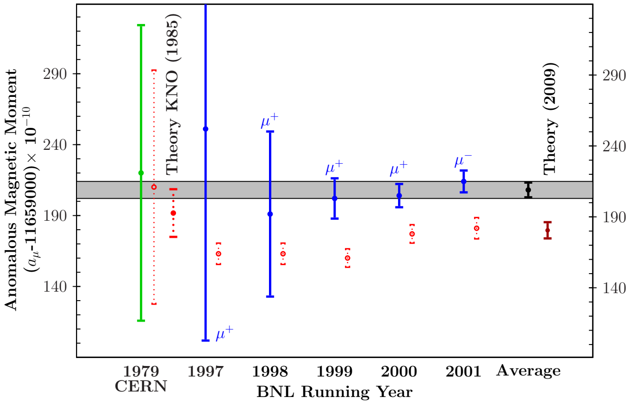
M - mass of heavier SM/BSM particle, or scale of new physics ...

- $a_\mu^{exp} = 11659208.0(6.3) \times 10^{-10}$ (0.54ppm) [BNL, 2006-2008]
- Current theoretical and experimental estimates:
  - $2.9\sigma/3.6\sigma$ discrepancy (e^+e-/τ data)
  - $a_\mu^{exp} - a_\mu^{th,SM} = 287(63)(51) \times 10^{-11}$
- New experiments (J-PARC, Fermilab) expected to perform $4\times$ more precise measurement
- Improved precision of the theoretical estimates with dominating uncertainty required

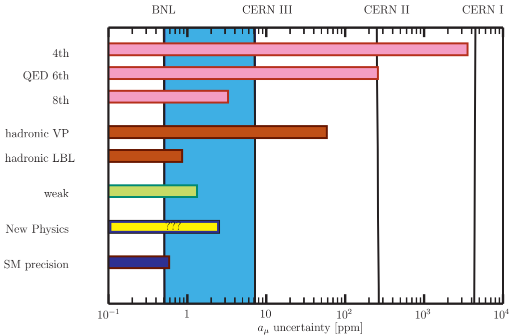
a_μ as a stringent test of the SM

- Evolution of the $(th - exp)$ tension [Jegerlehner, Nyffeler 0902.3360]



a_μ as a stringent test of the SM

- Sensitivity of different g-2 experiments to various contributions [Jegerlehner arxiv:0703125]
  "New Physics": $\delta a_\mu = a_\mu^{exp} - a_\mu^{th,SM} = 287(91) \times 10^{-11} \to 3.2\sigma$

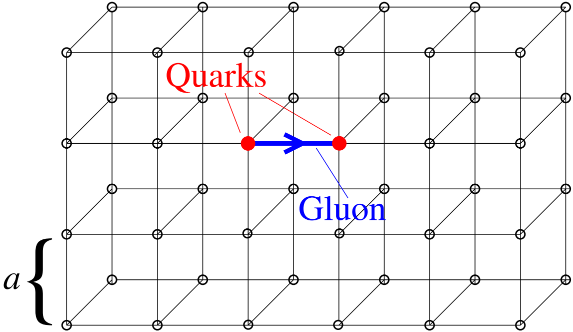


Theoretical uncertainties:

- HVP ($O(10^{-10})$)
- HLbL ($O(10^{-10})$)
- other contributions (unceirt. $O(10^{-11})$ or less)

- Lattice provides the model-independent setup for the computation of hadronic contribution(s)

Lattice QCD computation

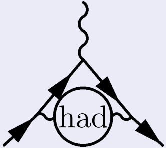


Quarks $\sim \overline{\psi}(x), \psi(x)$

Gluons $\sim U_\mu(x) = e^{iagA_\mu}$

- Generate ensembles of field configurations using Monte Carlo
- Average over a set of gauge configurations
- Typically compute correlation function of fields, extract Euclidean matrix element or amplitude
- Computational cost is dominated by quarks: inverse of large, sparse matrix
- Extrapolate to continuum, infinite volume, physical quark masses (now directly accessible)

Can be computed in Euclidean space-time [Lautrup, de Rafael '69, Blum '02]



- $a_\mu^{HLO} = (\frac{\alpha}{\pi})^2 \int_0^\infty dQ^2 f(Q^2) \times \hat{\Pi}(Q^2)$
- $f(Q^2) = m_{\mu^2} Q^2 Z^3(Q^2) \frac{1 - Q^2 Z(Q^2)}{1 + m_\mu^2 Q^2 Z^2(Q^2)}$
- $Z(Q^2) = (\sqrt{(Q^2)^2 + 4m_\mu^2 Q^2} - Q^2)/(2m_\mu^2 Q^2)$

Hadronic vacuum polarisation

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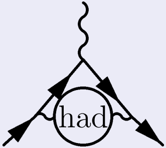

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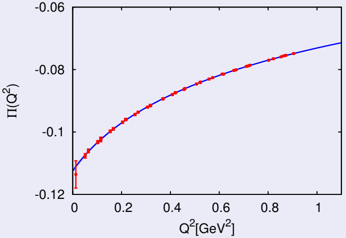
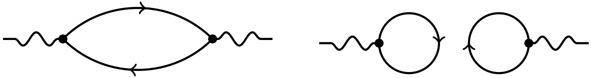


- $\boxed{\hat{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0)}$
- $\Pi_{\mu\nu}(Q) = a^4 \sum_x e^{iQx} \langle J_\mu^{em}(x) J_\nu^{em}(0) \rangle$
- $\Pi_{\mu\nu}(Q) = (Q^2 \delta_{\mu\nu} - Q_\mu Q_\nu) \Pi(Q^2)$

- Connected and disconnected contribution to the HVP



- Disconnected:
  - Computationaly very demanding
  - ChPT estimate $\propto$ 10% [Della Morte, Juettner '10]
  - Direct estimates from the lattice in progress [Guelpers et al. '14]

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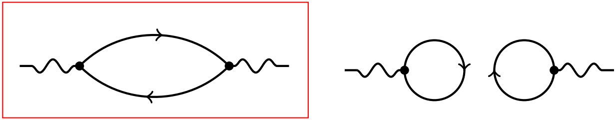
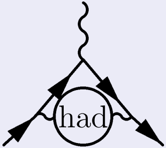


- Disconnected:
  - Computationaly very demanding
  - ChPT estimate $\propto 10\%$ [Della Morte, Juettner '10]
  - Direct estimates from the lattice in progress [Guelpers et al. '14]
- In the following we will discuss only the connected part

Hadronic vacuum polarisation

Can be computed in Euclidean space-time [Lautrup, de Rafael '69, Blum '02]



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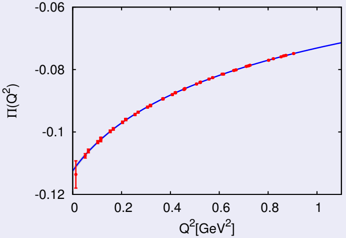

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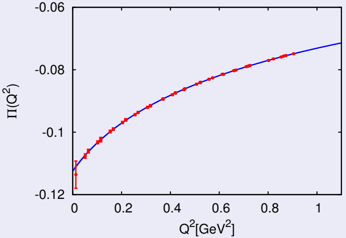

- $a_\mu^{HLO} = (\frac{\alpha}{\pi})^2 \int_0^\infty dQ^2 f(Q^2) \times (\Pi(Q^2) - \Pi(0))$
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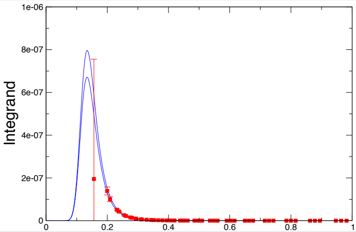


- Transverse projection: $Q_\mu = 0$
- Take only diagonal components $\Pi_{\mu\mu}$
- $\Pi(Q^2) = -\frac{\Pi_{\mu\mu}(Q^2)}{Q^2}$

Systematic uncertainties to be controlled - general

1. Simulations at physical m_π
2. Controlled continuum limit, FV effects
3. Disconnected diagrams
4. Obtaining a real world result: charm quark, isospin effects ...

Systematic uncertainties to be controlled - HVP related

- Conventional simulations do not allow access to sufficiently low Fourier momenta
- Integral is dominated in the region where relative errors are enhanced
- Structure of HVP tensor is such that $\Pi(0)$ is not directly accessible
- Systematic uncertainty introduced by extrapolation

Choosing the appropriate fit functions

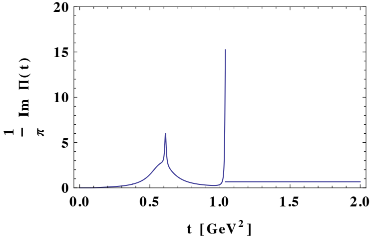
- First attempts to obtain a_μ^{HLO} from the lattice:
  - asumed functional forms for $\Pi(Q^2)$ based on VMD
  - model-dependent, not physicaly motivated, introduces bias ...
- [Aubin,Blum,Golterman,Peris 1205.3695] recently proposed:
  - use a series of Padé approximants

$$\Pi(Q^2) = \Pi(0) + Q^2(a_0^2 + \sum_{n=1}^{N} \frac{a_n^2}{b_n^2+Q^2} + \dots)$$

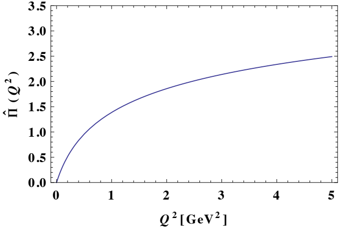
- A convergence theorem for $N \to \infty$ exists:
  - $\frac{\Pi(Q^2)-\Pi(0)}{Q^2}$ bound by $[N, N]$ and $[N+1, N]$ PA's
  - [N+1,N]: $a_0 \neq 0$, [N,N]: $a_0 \neq 0$
    * different from the notation in the ref.

Phenomenological model of HVP [Bernecker, Meyer, 1107.4388]

- Phenomenological R ratio ($e^+e^- \to hadrons$)
- $\Pi(Q^2) - \Pi(0) = Q^2 \int_0^\infty ds \frac{\rho(s)}{s(s+Q^2)}$



$\longrightarrow$



[L.Lellouch, Talk at MITP $g_\mu - 2$ workshop, Mainz, 1-5 April 2014]

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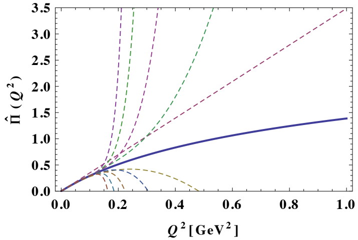
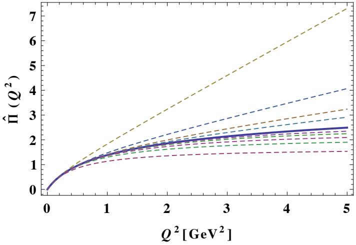


Taylor expansions for $N = 1, \cdots, 10$

$\longrightarrow$



$[N, N-1]$ & $[N, N]$ Padé's: $[1, 1] \to [5, 5]$

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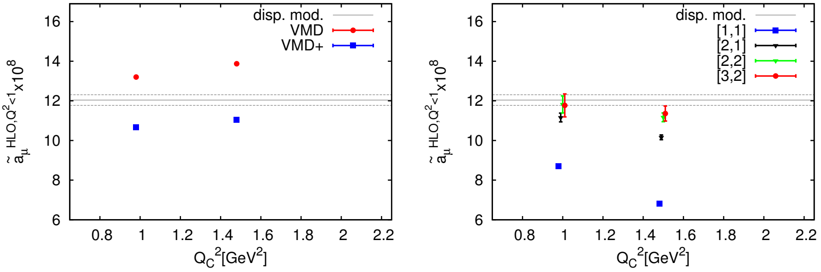
Phenomenological model of HVP [Golterman, Maltman, Peris 1309.2153]

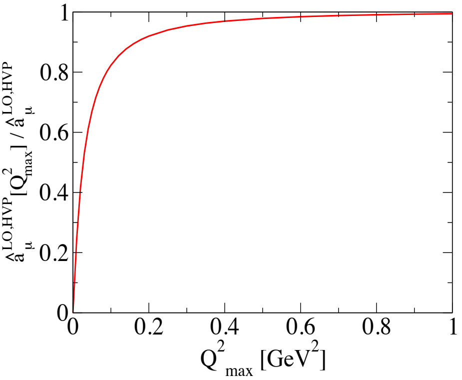
- A method to quantitatively examine the systematics of lattice computations
- Dispersive τ-based $I = 1$ model: $\hat{\Pi}^{I=1}(Q^2) = Q^2 \int_{4m_\pi^2}^{\infty} ds \frac{\rho^{I=1}(s)}{s(s+Q^2)}$
- Fake lattice data for $\Pi(Q^2) - \Pi(0)$ & compared with true answer from model



- Outcome:
  - Fitting until high Q^2 dangerous, unless higher order Padés used
  - Better focus on low-Q^2 region needed

Improving the systematics of connected HVP

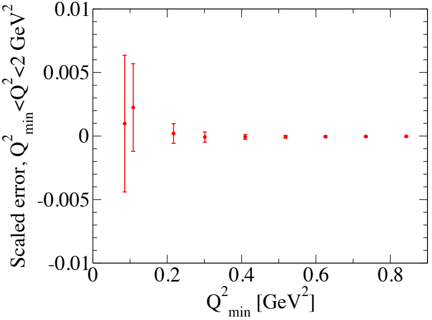
- A "Hybrid strategy" [Golterman, Maltman, Peris 1405.2389]



- More than 80% of a_μ^{HLO} is accumulated below $Q^2_{max} = 0.1\mathrm{GeV}^2$
- More than 90% below $Q^2_{max} = 0.2\mathrm{GeV}^2$

Improving the systematics of connected HVP

- A "Hybrid strategy" [Golterman, Maltman, Peris 1405.2389]
  - low-Q^2 contributions by fitting low-Q^2 region only $[0, Q^2_{min}]$
  - numericaly integrate $[Q^2_{min}, Q^2_{max}]$
  - apply PT for $[Q^2_{max}, \infty]$

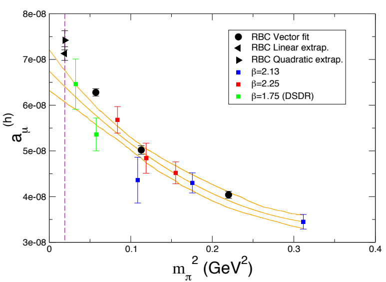


- Statistical, systematic (trapezoid rule) and errors on $\Pi(0)$
- Investigated using fake data from $I = 1$ dispersive model

Previous RBC-UKQCD computation of a_μ^{HLO} [Boyle et al'11]

Non physical m_π, $a^{-1} \approx 1.3, 1.7, 2.3$ GeV

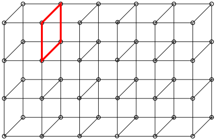
- Domain Wall Fermion (DWF)
- Fitting Q^2- dependence of $\Pi(Q^2)$ up to $Q_C^2 \approx 2.5 - 9$ GeV^2
- Local current at source, conserved at sink

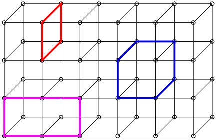


- Strong m_π dependence
- Eliminate the systematics of chiral extrapolation: computing HVP at m_π^{phys}

Cost of the fermions on the lattice

- $\langle O[\psi, \overline{\psi}, U] \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\overline{\psi} e^{-S_G[U] - S_f[U,\psi,\overline{\psi}]} O[\psi, \overline{\psi}, U]$
- S_G Wilson Luscher-Weisz





- S_F

$$\int \mathcal{D}\ \psi \mathcal{D}\ \overline{\psi}\ e^{-\overline{\psi}\ (\gamma_\mu D_\mu + m_q)\ \psi} \quad \approx \quad det\ (\gamma_\mu D_\mu + m_q)$$

- Non-local object on the lattice → impossible to compute exactly!
- Solving:

$$\chi = (\gamma_\mu D_\mu + m_q)^{-1}\ \Phi$$

very expensive for: small quark mass m, large $\frac{L}{a}$.

Cost of the fermions on the lattice

- $\langle O[U] \rangle = \frac{1}{Z} \int DU\ e^{-S_G[U]}\ [det\ (\gamma_\mu D_\mu + m_q)]^{N_f}\ O[U]$
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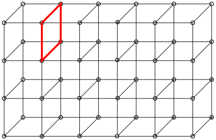

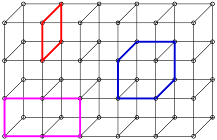

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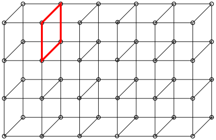

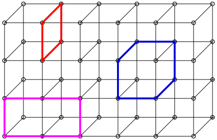

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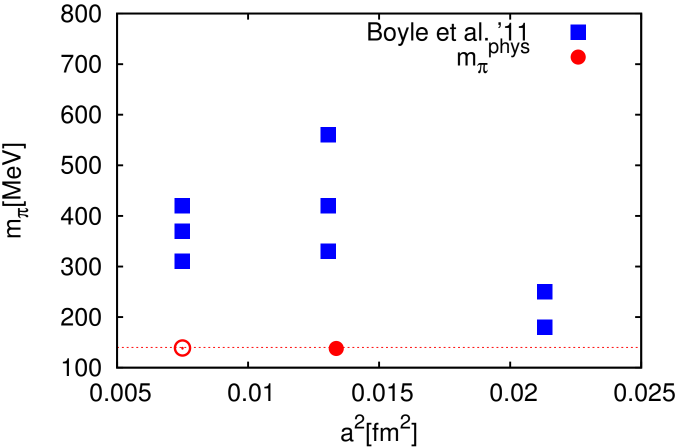
RBC-UKQCD $N_f = 2 + 1$ Domain Wall ensembles



- a_μ^{HLO} from DWF for non-physical m_π [Boyle et al '11]
- physical point HVP (•) recently measured → preliminary fits

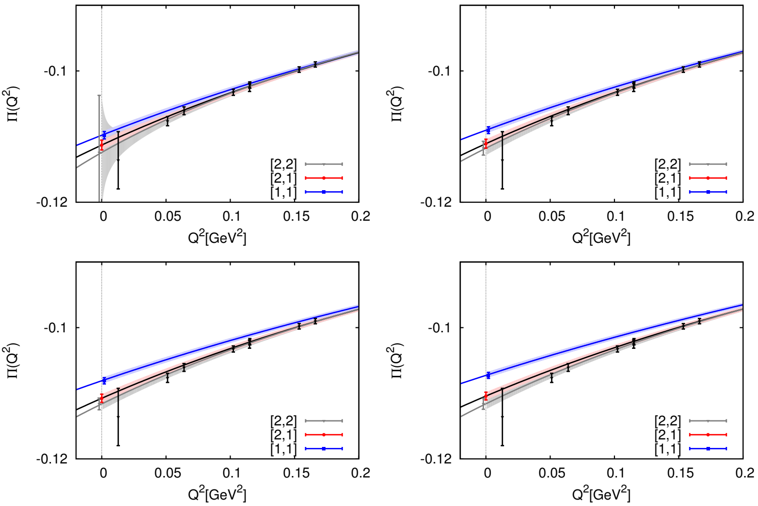
Physical point HVP from $N_f = 2 + 1$ DWF

Physical point data:

- $L/a = 48^3 \times 94 \times 24, \quad a^{-1} = 1.73 GeV$
- $\Pi(Q^2)$ convergent sequence of PAs[Aubin et al,'13]
  - VMD is unreliable
- Padé approximants [N,D]

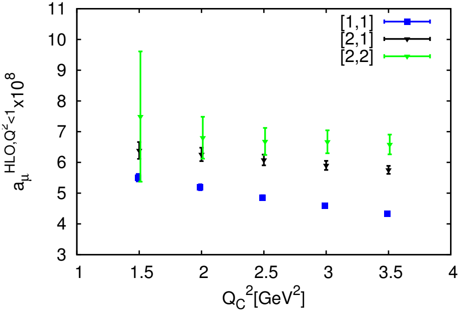
$$\Pi_{[N,D]}(Q^2) = \frac{\sum_{n=0}^{N-1} a_n Q^{2n}}{1+\sum_{m=1}^{D} b_m Q^{2m}}$$

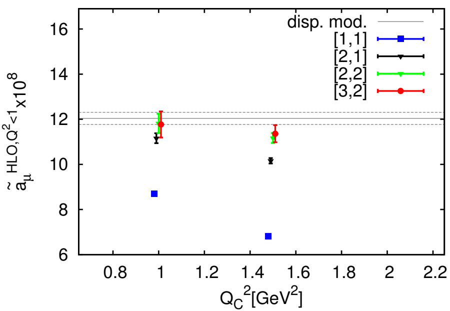
Physical point HVP from $N_f = 2 + 1$ DWF



- $L/a = 48, a^{-1} = 1.73\ \mathrm{GeV},\ m_\pi = 138\ \mathrm{MeV}$
- $Q_C^2 = 1.5; 2.0; 2.5; 3.0\ \mathrm{GeV}^2$

Physical point HVP from $N_f = 2 + 1$ DWF

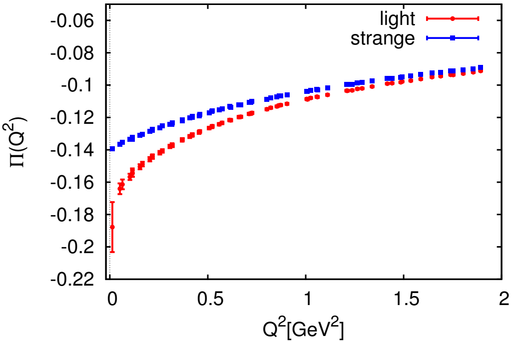




- **Left**: Physical point data (Möbius DWF)
- **Right:** Dispersive model study [Golterman et al. '13]
- Same qualitative behaviour - Padé [2,1] looks acceptable
- Nevertheless, even for Padé [2,1]
  - Removing correlations
  - Results for different choice of Q_C^2 not compatible
- Quoting the value for a_μ^{HLO} would be premature

Physical point HVP from $N_f = 2 + 1$ DWF

Light and strange contributions separated



Limited statistics with physical m_π already gives:

- $\frac{\delta a_\mu^{stat.}}{a_\mu}$ for light contribution is $O(10)$ larger than for strange HVP

Promising methods I have not discussed . . .

- HVP at $Q^2 = 0$
  - New way for direct extraction of zero momentum form factors on the lattice [de Divitiis, R. Petronzio, N. Tantalo 1208.5914]

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  - New way for direct extraction of zero momentum form factors on the lattice [de Divitiis, R. Petronzio, N. Tantalo 1208.5914]
  - For off-diagonal elements, $\mu \neq \nu$:
    - $\Pi_{\mu\nu}(Q) = -Q_\mu Q_\nu \Pi(Q^2)$
    - $\frac{\partial^2}{\partial Q_\mu \partial Q_\nu} \Pi_{\mu\nu}(Q)|_{Q^2=0}$

      $= -\frac{\partial^2}{\partial Q_\mu \partial (Q_\nu} (Q_\mu Q_\nu \Pi(Q^2))|_{Q^2=0}$

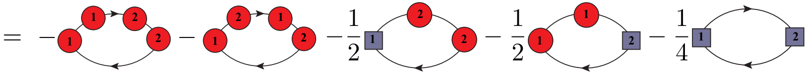
      $= -\Pi(0)$
  - <u>Works for the connected contribution</u>

Promising methods I have not discussed ...

- HVP at $Q^2 = 0$
  - New way for direct extraction of zero momentum form factors on the lattice [de Divitiis, R. Petronzio, N. Tantalo 1208.5914]
  - $\Pi_{12}(Q) = \sum_x \langle Tr\{S[y,x;U]\Gamma_V^1(x,\vec{q})S[x,y;U,\lambda^p]\Gamma_V^2(y,\vec{0})\}\rangle.$
  - $\Pi(0) = -\frac{\partial \Pi_{12}(Q)}{\partial Q_1 \partial Q_2}|_{Qs=0}$

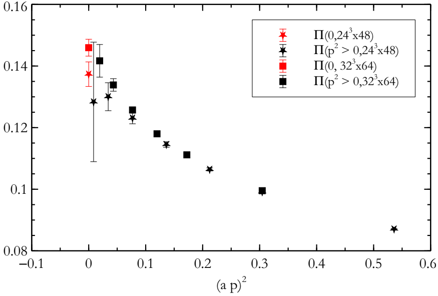
$$= -\frac{1}{(TL^3)^2}\sum_{x,y}\langle Tr\left[S\Gamma_V^1\frac{\partial^2 S}{\partial Q_1\partial Q_2}\Gamma_V^2\right] - \frac{i}{2}Tr\left[S\Gamma_T^1\frac{\partial S}{\partial Q_2}\Gamma_V^2\right]$$

$$-\frac{i}{2}Tr\left[S\Gamma_V^1\frac{\partial S}{\partial Q_1}\Gamma_T^2\right] - \frac{1}{4}Tr\left[S\Gamma_T^1 S\Gamma_T^2\right]\rangle$$



Promising methods I have not discussed ...

- HVP at $Q^2 = 0$
  - New way for direct extraction of zero momentum form factors on the lattice [de Divitiis, R. Petronzio, N. Tantalo 1208.5914]
  - Gain in statistics and stabilizing the fits at the cost of the evaluation of 3pt and 4pt functions



Promising methods I have not discussed ...

- HVP at $Q^2 = 0$
  - New way for direct extraction of zero momentum form factors on the lattice [de Divitiis, R. Petronzio, N. Tantalo 1208.5914]

- HPQCD time moments
  - Strange and charm quark contributions to the anomalous magnetic moment of the muon
    [Chakraborty, Davies, Donald, Dowdall, Koponen, Lepage, Teubner 1403.1778]

- Its extensions involving different definitions of discrete moments

- We are looking into it

RBC-UKQCD Collaboration members

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Hantao Yin (Columbia)
Jianglei Yu (Columbia)
Daiqian Zhang (Columbia)

HVP, together with: J.Hudspith, R. Lewis, K. Maltman (York University)

Summary

- a_μ good for constraining new physics
- Exp. precision $0.54 p.p.m.$ $\longrightarrow$ improvement $4\times$ expected (J-PARC, Fermilab)
- Lattice gives an independent theory prediction of HVP

- Significant increase signal/noise ratio near $Q^2 = 0$ coming from the light sector
- Large systematics with conventional procedure anticipated
- Current status with DWF
- Refinements in progress
  (higher statistics, hybrid strategy, moments methods, ...)
- Ultimate goal: a_μ^{HLO} with full control over syst. and stat. uncertainties ($< 1\%$)
- Still un(not enough)tackled challenges: isospin breaking effects, disconnected contribution, HLbL ...

Thank you !

Acknowledgements

- RBC-UKQCD collab. members & DWF-GM2 working group, for useful discussions

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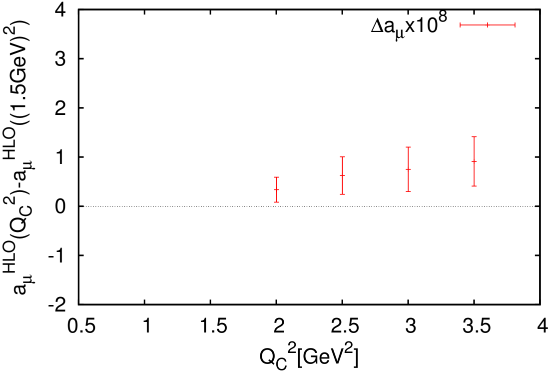
- The calculations reported here have been done on DIRAC Bluegene/Q computer at the University of Edinburgh's Advanced Computing Facility

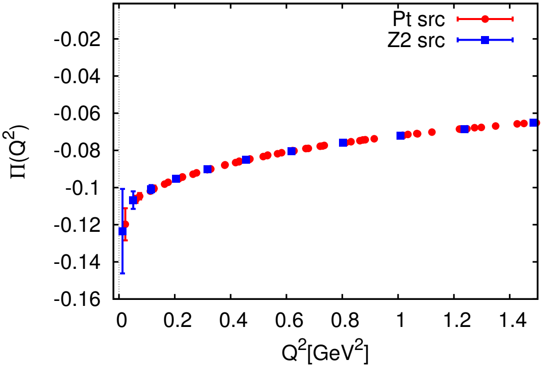
Physical point HVP



- $[2,2]$ Padé fits for different Q_C^2
- Take correlations into account
- Reference $a_\mu^{HLO}(Q_{C\ ref}^2)$ subtracted under bootstrap $[Q_{C\ ref}^2 = 1.5GeV^2]$
- Results for different choice of Q_C^2 not combatible $\rightarrow$ uncontrolled systematics

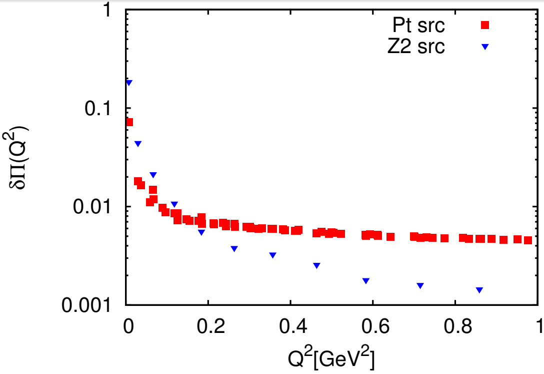
Point vs. stochastic source

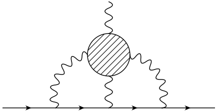
- Point source, 12 source positions
- Z(2) wall source, 48 source positions
- (one-end trick) [McNeile et al. '06]

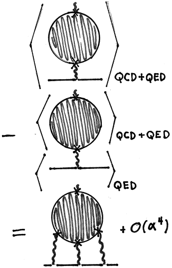


Point vs. stochastic source

- Point source, 12 source positions
- Z(2) wall source, 48 source positions
- (one-end trick) [McNeile et al. '06]
- Comparison (12 src. positions each, log scale on y-axis)
- Point src. better in low-Q^2 region ($Q^2 <\sim 0.2\ GeV^2$)







[Blum, Chowdhury, Hayakawa, Izubuchi, 1407.2923]