Pseudocritical line from Lattice QCD and comparison with freeze-out curves

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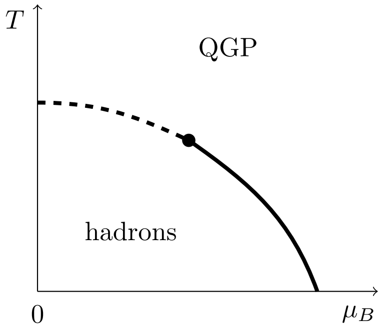
The expected phase diagram

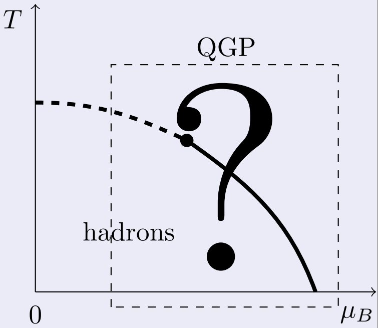


Main features:

- analytic crossover for $\mu = 0$ (no known symmetries to break, it would be a real transition for massless light quarks)
- first order transition for $T = 0$ (simple argument based on light particles counting)
- a second order transition somewhere in the middle

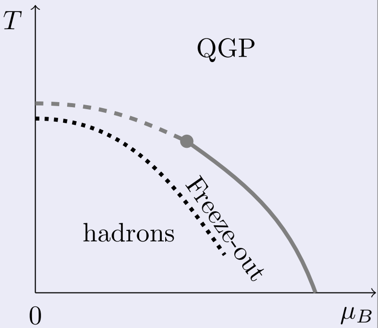
The phase diagram

From Lattice QCD



- equilibrium physics
- $\mu \neq 0$ problematic

From experiments



- we see the freeze-out
- equilibrium physics?

Parametrization of the pseudocritical line

The critical temperature at $\mu_B = 0$ can be determined "easily" by LQCD:

$T_c\|_{\mathrm{BW}} = 152(5)\,\mathrm{MeV}$	Aoki et al. Phys. Lett. B **643**, 46 (2006)
$T_c\|_{\mathrm{hotQCD}} = 154(9)\,\mathrm{MeV}$	Bazavov et al. Phys. Rev. D **85** 054503 (2012)

Caution: since no real phase transition is present it is not possible to unambiguously define T_c. These values refer to chiral observables.

A convenient parametrization for $T_c(\mu_B)$ is

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c(0)} \right)^2 + \cdots$$

where only even power of μ_B appear because of $Z(\mu) = Z(-\mu)$.
κ is the curvature of the pseudocritical line.

Determinations of κ

From experiments

κ is extracted from experimental data by fitting abundances with statistical hadronization models

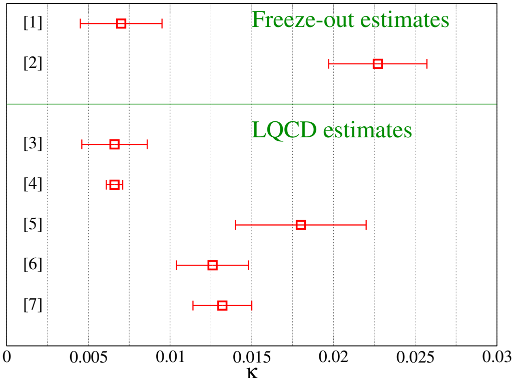
From LQCD

- simulations at $\mu = 0$ extrapolated to $\mu \neq 0$ using reweighting or Taylor expansion (need to define a prescription to locate the transition at $\mu \neq 0$)
- simulations at $\mu^2 < 0$ (just look e.g. at the position of the susceptibility maximum)

Possible systematics to check:

- thermodynamical/continuum limit
- value of the strange quark chemical potential

The curvature estimates



1. Becattini et al. Phys. Rev. Lett. **111**, 082302 (2013)
2. Cleymans et al. Phys. Rev. C **73**, 034905 (2006)
3. Endrodi et al. JHEP **1104**, 001 (2011)
4. Kaczmarek et al. Phys. Rev. D **83**, 014504 (2011) (see also Laermann et al. 1304.3247)
5. Cea et al. Phys. Rev. D **89** 074512 (2014)
6. Bonati et al. Phys Rev D **90**, 114025 (2014) (from $\chi_{\bar{\psi}\psi}$)
7. Bonati et al. Phys Rev D **90**, 114025 (2014) (from $\bar{\psi}\psi$)

LQCD assisted freeze-out determination

Are there LQCD observables that can be used as "thermometer" to determine T, μ_B at freeze-out?

Proposal (Bazavov et al. Phys. Rev. Lett. **109**, 192302 (2012)): use ratios of conserved charges susceptibilities:

$$\frac{\chi_{lmn}^{BSQ}}{T^{l+m+n}} = \frac{\partial^{l+m+n}(p/T^4)}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}$$

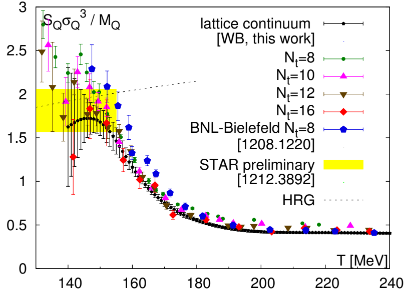
Ratios are related to the momenta of the conserved charge fluctuations:

$$R_{12}^X = \chi_1^X/\chi_2^X = M_X/\sigma_X^2; \qquad R_{31}^X = \chi_3^X/\chi_1^X = S_X\sigma_X^3/M_X$$

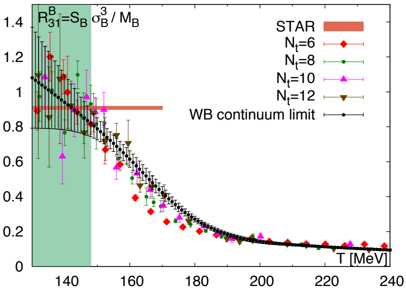
and the leading order behaviours are

$$R_{12}^X = \mathcal{O}(\mu_X); \qquad R_{31}^X = R_{31}^X(T, \mu_X = 0) + \mathcal{O}(\mu_X^2)$$

LQCD assisted freeze-out determination (2)



Borsányi et al. Phys. Rev. Lett. **111**, 062005 (2013)



Borsányi et al. Phys. Rev. Lett. **113**, 052301 (2014)

Problems:

- Poor precision for $T \lesssim 155\,\mathrm{MeV}$
- Fluctuations of B difficult to measure (neutrons)

Conclusions

- Theoretical determinations of $T_c(\mu_B = 0)$ and κ have by now systematical errors under control
- LQCD is starting to be used to determine the freeze-out conditions
- More precise data (both theoretical and experimental) will enable to cross validate the various results. In particular to verify/disprove that matter at freeze-out is in thermal equilibrium.
- By now there is no theoretically sound (i.e. without large systematics) prediction for the critical end-point position. Maybe experimentalists will find it out before than theorists.

Thank you!

Backup slides

The quark chemical potentials (1)

The conserved charges

$$B = (N_u + N_d + N_s)/3$$
$$Q = (2N_u - N_d - N_s)/3$$
$$S = -N_s$$

The quark chemical potentials

$$\mu_u = \mu_B/3 + 2\mu_Q/3$$
$$\mu_d = \mu_B/3 - \mu_Q/3$$
$$\mu_s = \mu_B/3 - \mu_Q/3 - \mu_S$$

The quark chemical potentials (2)

In all current simulations $\mu_Q = 0$ and two cases are used:

$$\begin{aligned} &1) \quad \mu_u = \mu_d = \mu_B/3; \quad \mu_s = 0 \\ &2) \quad \mu_u = \mu_d = \mu_s = \mu_B/3 \end{aligned}$$

that correspond to

$$\begin{aligned} &1) \quad \mu_S = \mu_B/3 \\ &2) \quad \mu_S = 0 \end{aligned}$$

From the vanishing of the average strangeness one obtains

$$0 = \frac{\partial \log Z(\mu_B, \mu_S)}{\partial \mu_S} \simeq \left.\frac{\partial \log Z}{\partial \mu_S \partial \mu_S}\right|_{\mu=0} \mu_S + \left.\frac{\partial \log Z}{\partial \mu_S \partial \mu_B}\right|_{\mu=0} \mu_B + \cdots$$

that at $T \approx T_c$ gives $\mu_S \simeq \mu_B/4$ and thus $\mu_s \simeq \mu_B/12 = \mu_u/4$.

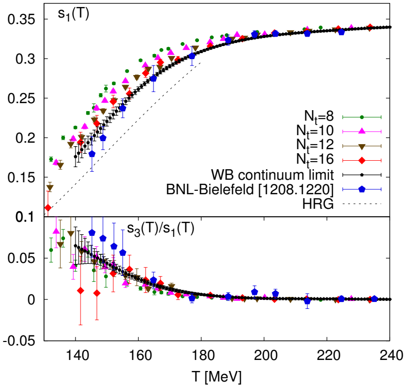
The μ_S and μ_Q chemical potentials

Imposing $M_S = 0$ one gets

$$\frac{\mu_S}{T} = s_1(T)\frac{\mu_B}{T} + s_3(T)\left(\frac{\mu_B}{T}\right)^3 + \cdots$$

In a similar way μ_Q can be fixed by imposing $M_Q = rM_B$, where $r = Z/A$:

$$\frac{\mu_Q}{T} = q_1(T)\frac{\mu_B}{T} + q_3(T)\left(\frac{\mu_B}{T}\right)^3 + \cdots$$



Borsányi et al. Phys. Rev. Lett. **111**, 062005 (2013)

$T_c(\mu)$ determination from $\mu = 0$ simulations

Kaczmarek et al. Phys. Rev. D **83**, 014504 (2011)

If the transiton with $m_l = 0$ is second order then we can define for $m_l \approx 0$ the scaling variables

$$t \simeq \frac{1}{t_0}\left(\frac{T - T_c(0)}{T_c(0)} + \kappa\left(\frac{\mu_B}{T_c(0)}\right)^2\right) \qquad h \simeq \frac{1}{h_0}\frac{m_l}{m_s}$$

and thermodynamical observables have the scaling form $\phi = \phi(t, h)$, thus

$$\kappa = T_c(0)\left.\frac{\partial\phi/\partial\mu_B^2}{\partial\phi/\partial T}\right|_{\substack{\mu_B=0\\ T=T_c}} = \frac{t_0}{\partial_t\phi|_{\substack{\mu_B=0\\ t=0}}}\left.\frac{\partial\phi}{\partial(\mu_B/T)^2}\right|_{\substack{\mu_B=0\\ t=0}}$$

To numerically estimate $\partial_t\phi$ the scaling form of the $O(4)$ model was used.

Problems: valid if the transition at $m_l = 0$ is second order $O(4)$ *and* if $m_l \approx 0$.

$T_c(\mu)$ determination from $\mu = 0$ simulations (2)

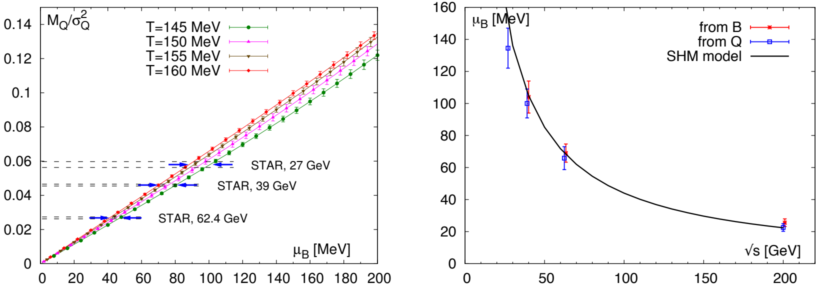
Endrodi et al. JHEP **1104**, 001 (2011)

Take your preferred observable $\phi(T, \mu_B)$ that is monotone at $\mu_B = 0$ and let ϕ_0 be its value at the $\mu_B = 0$ transition. Then define $T_c(\mu_B)$ by $\phi(T_c(\mu_B), \mu_B) \equiv \phi_0$. Then

$$\kappa \equiv -T_c(0) \left. \frac{\mathrm{d} T_c(\mu_B)}{\mathrm{d}\mu_B^2} \right|_{\mu_B=0} = T_c(0) \left. \frac{\partial\phi/\partial\mu_B^2}{\partial\phi/\partial T} \right|_{\substack{\mu_B=0 \\ T=T_c}}$$

Problems: why should it work? Partial solution: when the method by Kaczmarek et al. works, the method by Endrodi et. al also works (and it gives the same answer, compare equations).

LQCD determination of μ_B given T



Borsányi et al. Phys. Rev. Lett. **113**, 052301 (2014)

In this analysis $T_c(\mu_B)$ was assumed to be $\approx T_c(\mu_B = 0)$.

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