ECHO-QGP: a new resource for the study of the QGP

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May 26th, 2015

ECHO-QGP Collaboration

The ECHO-QGP collaboration involves the Universities and the INFN sections of Ferrara, Firenze and Torino; since 2015 also the FIAS (Frankfurt am Main) joined the collaboration.

ECHO-QGP

Del Zanna, V. Chandra, G. Inghirami, V. Rolando, A. Beraudo, A. De Pace, G. Pagliara, A. Drago, and F. Becattini,
Relativistic viscous hydrodynamics for heavy-ion collisions with ECHO-QGP,
Eur.Phys.J. C73 (2013) 2524, 1305.7052
arXiv(nucl-th):1305.7052

What is ECHO-QGP

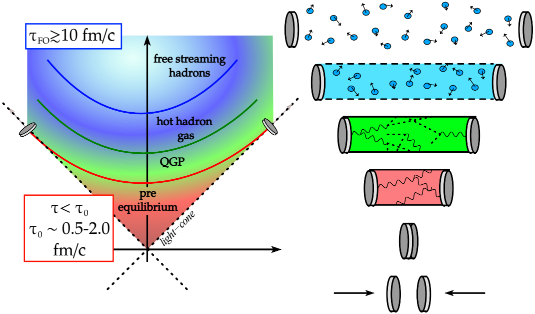
The code has been built on top of the Eulerian Conservative High-Order astrophysical code for general relativistic magnetohydrodynamics:

ECHO

L. Del Zanna, O. Zanotti, N. Bucciantini, and P. Londrillo, *ECHO: an Eulerian Conservative High Order scheme for general relativistic magnetohydrodynamics and magnetodynamics*
Astron.Astrophys. 528 (2011) A101, 1010.3532
arXiv(astro-ph):0704.3206

The original ECHO code can handle non-vanishing conserved-number currents as well as electromagnetic fields, which are essential for the astrophysical computations, in any (3+1)-D metric of General Relativity.

Heavy ion collisions



freeze-out

unknown

equation of state

unknown

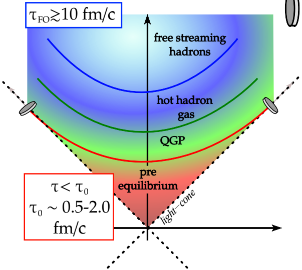
hydrodynamics

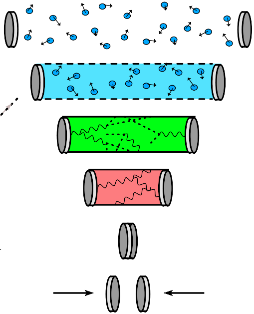
assumption

initial conditions

unknown

Heavy ion collisions





freeze-out
Phenomenological models
equation of state
To be tested
hydrodynamics
assumption
initial conditions
Phenomenological models

Relativistic Ideal Hydrodynamics

$$N^\mu = n u^\mu$$
$$T^{\mu\nu} = e u^\mu u^\nu + P \Delta^{\mu\nu}$$

Orthogonal projector

$$\Delta^{\mu\nu} \equiv g^{\mu\nu} + u^\mu u^\nu$$

Covariant derivative

$$d_\mu = \underbrace{-u_\mu D}_{D \equiv u^\alpha d_\alpha} + \underbrace{\nabla_\mu}_{\nabla_\mu \equiv \Delta_\mu^\alpha d_\alpha}$$

Set of equations

$$\begin{cases} d_\mu N^\mu = 0, \\ d_\mu T^{\mu\nu} = 0 \\ EoS \end{cases}$$

Conservative form

$$\partial_0 U + \partial_k F^k = S$$

Relativistic Viscous Hydrodynamics

Israel-Stewart theory[1]

$$N^\mu = nu^\mu + V^\mu$$
$$T^{\mu\nu} = eu^\mu u^\nu + (P+\Pi)\Delta^{\mu\nu} + \pi^{\mu\nu} + u^\mu w^\nu + u^\nu w^\mu$$

$$\pi^{\mu\nu} = [\tfrac{1}{2}(\Delta^\mu_{[\alpha}\Delta^\nu_{\beta]}) - \tfrac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}]T^{\alpha\beta}$$
$$V^\mu = \Delta^\mu_\alpha N^\alpha$$
$$P + \Pi = \frac{1}{3}\Delta_{\mu\nu}T^{\mu\nu}$$
$$w^\mu = -\Delta^\mu_\alpha T^{\alpha\beta}u_\beta$$

Set of equations

$$\begin{cases} d_\mu N^\mu = 0, \\ d_\mu T^{\mu\nu} = 0 \\ \pi^{\mu\nu} \text{ evol.} \\ \Pi \text{ evol.} \\ EoS \end{cases}$$

[1] W. Israel and J. Stewart, Annals of Physics 118 (1979) 341

Relativistic Viscous Hydrodynamics

Israel-Stewart theory

- Landau Frame $\rightarrow$ $w^\mu = 0$

$$u^\mu = \frac{u_\nu T^{\mu\nu}}{\sqrt{u_\alpha T^{\alpha\beta} T_{\beta\gamma} u^\gamma}}$$

- Vanishing net baryon density $\rightarrow$ $n = 0$

$$\cancel{N^\mu = n u^\mu + V^\mu}$$

$$T^{\mu\nu} = e u^\mu u^\nu + (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu} + \cancel{u^\mu w^\nu} + \cancel{u^\nu w^\mu}$$

Set of equations

$$\begin{cases} d_\mu N^\mu = 0, \\ d_\mu T^{\mu\nu} = 0 \\ \pi^{\mu\nu} \text{ evol.} \\ \Pi \text{ evol.} \\ EoS \end{cases}$$

Conservative form

$$\partial_0 U + \partial_k F^k = S$$

Relativistic Viscous Hydrodynamics

$$T^{\mu\nu} = eu^\mu u^\nu + (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

Conservative form

$$\partial_0 U + \partial_k F^k = S$$

Set of equations

$$\begin{cases} d_\mu N^\mu = 0, \\ d_\mu T^{\mu\nu} = 0 \\ \pi^{\mu\nu} \text{ evol.} \\ \Pi \text{ evol.} \\ EoS \end{cases}$$

Conservative form of equations

$$\partial_0 \mathbf{U} + \partial_k \mathbf{F}^k = \mathbf{S},$$

where

$$\mathbf{U} = |g|^{\frac{1}{2}} \begin{pmatrix} N \equiv N^0 \\ S_i \equiv T^0_i \\ E \equiv -T^0_0 \\ N\Pi \\ N\pi^{ij} \end{pmatrix}, \quad \mathbf{F}^k = |g|^{\frac{1}{2}} \begin{pmatrix} N^k \\ T^k_i \\ -T^k_0 \\ N^k\Pi \\ N^k\pi^{ij} \end{pmatrix}$$

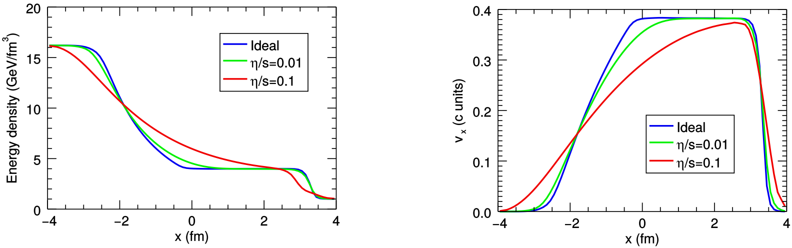
$$\mathbf{S} = |g|^{\frac{1}{2}} \begin{pmatrix} 0 \\ \frac{1}{2}T^{\mu\nu}\partial_i g_{\mu\nu} \\ -\frac{1}{2}T^{\mu\nu}\partial_0 g_{\mu\nu} \\ n[-\frac{1}{\tau_\pi}(\Pi + \zeta\theta) - \frac{4}{3}\Pi\theta] \\ n[-\frac{1}{\tau_\pi}(\pi^{ij} + 2\eta\sigma^{ij}) - \frac{4}{3}\pi^{ij}\theta + \mathcal{I}^{ij}_0 + \mathcal{I}^{ij}_1 + \mathcal{I}^{ij}_2] \end{pmatrix}.$$

Numerical implementation

$\forall\, \tau$:

- $\mathbf{P} = \{n, v^i, P, \Pi, \pi^{ij}\}$ - We start from *Primitive* variables, evaluated at cell centers.
- $\mathbf{U}$ and $\mathbf{F}$ - *Conservative* variables and their fluxes are calculated at cell interfaces for each direction.
  Several reconstruction algorithms available, default is MPE5.
  Riemann solver: by default HLL (Harten - Lax - van Leer) upwind two-wave
- $\mathbf{S}$ "source terms" are added.
- Runge-Kutta (RK2) method is employed to update the evolution.
- $\mathbf{P}$ are computed from the updated set of corresponding $\mathbf{U}$

2D shock tubes



$T^L = 0.4$ GeV $\quad$ $T^R = 0.2$ GeV

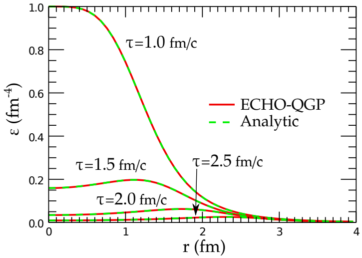
$(P^L = 5.40$ GeV/fm$^3)$ $\quad$ $(P^R = 0.34$ GeV/fm$^3)$

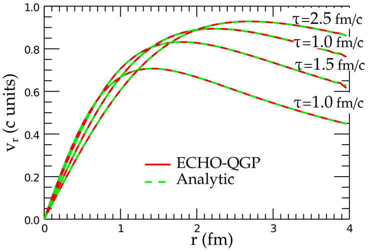
The Gubser test

cold plasma limit[2,3]

2+1D IS Boost invariant and azhymuthal symmetry:

$SO(3)_q \otimes SO(1,1) \otimes Z_2$



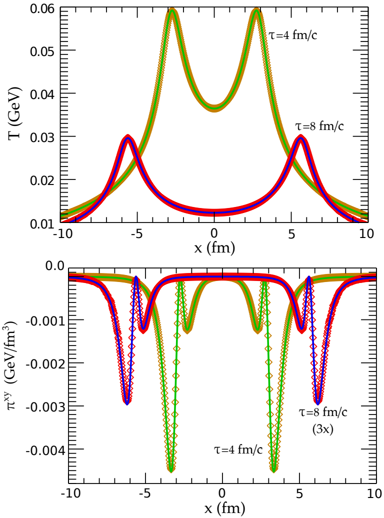


[2] S.S. Gubser, Phys.Rev. D82 (2010) 085027, 1006.0006,

[3] H. Marrochio et al., (2013), 1307.6130

The Gubser test

semi-analytic results

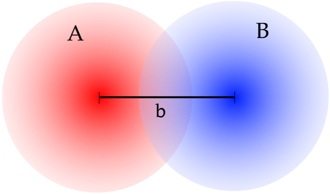


Initial Conditions

The Glauber Model

$$T(\mathbf{x}) = \int_{-\infty}^{\infty} \frac{\rho_0 \, dz}{1 + e^{(\sqrt{\mathbf{x}^2+z^2}-R)/\delta}}$$

$$T_{A\pm} = T_A\left(\mathbf{x} \pm \mathbf{b}/2\right)$$



$$n_{\rm coll}(\mathbf{x}; b) = \sigma^{NN} T_{A+} T_{B-}$$

$$n_{\rm part}(\mathbf{x}; b) = n^A_{\rm part} + n^B_{\rm part}$$

$$n^A_{\rm part} = T_{A+}\left[1 - \left(1 - \frac{\sigma^{NN}}{B} T_{B-}\right)^B\right],$$

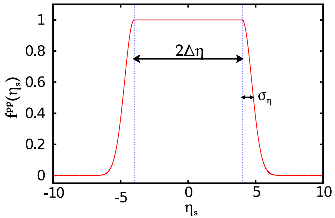
$$n^B_{\rm part} = T_{B-}\left[1 - \left(1 - \frac{\sigma^{NN}}{A} T_{A+}\right)^A\right],$$

2D

$$e(\tau_0, \mathbf{x}; b) = e_0 \frac{(1-\alpha)\, n_{\rm part}(\mathbf{x}; b) + \alpha\, n_{\rm coll}(\mathbf{x}; b)}{(1-\alpha)\, n_{\rm part}(\mathbf{0}; 0) + \alpha\, n_{\rm coll}(\mathbf{0}; 0)}$$

Initial Conditions

The Glauber Model



$$f^{\mathrm{pp}}(\eta_s) = \exp\left[-\theta(|\eta_s| - \Delta\eta)\frac{(|\eta_s| - \Delta\eta)^2}{4\sigma_\eta^2}\right]$$

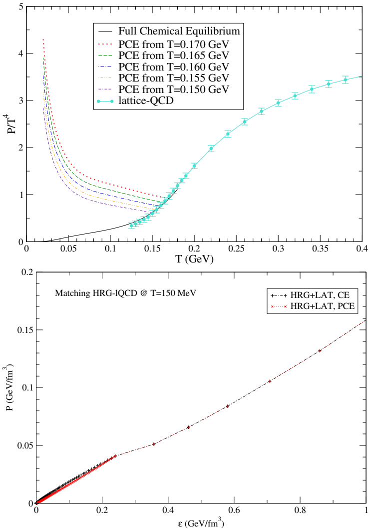
$$\tilde{n}_{\mathrm{part}} = \left(\frac{Y_b - \eta_s}{Y_b} n_{\mathrm{part}}^A + \frac{Y_b + \eta_s}{Y_b} n_{\mathrm{part}}^B\right)$$

3D

$$e = \tilde{e}_0\,\theta(Y_b - |\eta_s|)\, f^{\mathrm{pp}}(\eta_s)\frac{(1-\alpha)\,\tilde{n}_{\mathrm{part}}(\mathbf{x};b) + \alpha\, n_{\mathrm{coll}}(\mathbf{x};b)}{(1-\alpha)\,\tilde{n}_{\mathrm{part}}(\mathbf{0};0) + \alpha\, n_{\mathrm{coll}}(\mathbf{0};0)}$$

EoS

- Tabulated
- $P = \frac{e}{3} = \frac{37\,\pi^2}{90} T^4$ (non-interacting QGP with 2 light flavors)
- Weak-coupling QCD calculation with realistic quark masses derived in: M. Laine and Y. Schroder, Phys.Rev. D73 (2006) 085009, hep-ph/0603048
- Match of HRG-EoS with continuum-extrapolated l-QCD results: M. Bluhm et al., Nucl.Phys. A929 (2014) 157, 1306.6188
  - EoS-CE
  - EoS-PCE



Decoupling Stage

The Cooper-Frye prescription [4]

$$\frac{1}{\partial u} = \tau_{exp} \backsimeq \tau_{scatt} = \frac{1}{\langle v\sigma \rangle n}$$

strong dependence on T $\Longrightarrow$ Σ in an isothermal hypersurface

$$E\frac{d^3 N_i}{dp^3} = \frac{g_i}{(2\pi)^3}\int_\Sigma -f_i(x,p)\, p^\mu \, d^3\Sigma_\mu$$

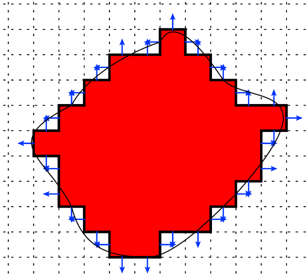
$$f_i(x,p) = \left[e^{-\frac{1}{T}(u^\nu p_\nu + \mu_i)} \pm 1\right]^{-1}$$

[4] F. Cooper and G. Frye, Phys.Rev. D10 (1974) 186

Decoupling Stage

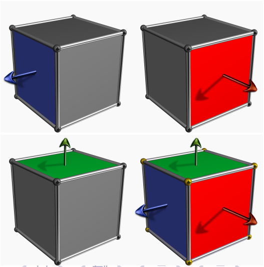
The Cooper-Frye prescription

$$E\frac{d^3N_i}{dp^3} = \frac{g_i}{(2\pi)^3}\int_{\Sigma} -f_i(x,p)\, p^\mu\, d^3\Sigma_\mu$$



$$d^3\Sigma_\mu = \begin{pmatrix} dV^{\perp\tau} \\ dV^{\perp x} \\ dV^{\perp y} \\ dV^{\perp\eta} \end{pmatrix} = \begin{pmatrix} \tau\ \Delta x \Delta y \Delta\eta_s\ s^\tau \\ \tau\ \Delta y \Delta\eta_s \Delta\tau\ s^x \\ \tau\ \Delta\eta_s \Delta\tau \Delta x\ s^y \\ \frac{1}{\tau}\ \Delta\tau\Delta x\Delta y\ s^\eta \end{pmatrix}$$

$$s^\mu = -\mathrm{sign}\left(\frac{\partial T}{\partial x^\mu}\right)$$



Observables

Particle spectra

$$E\frac{dN_i}{d^3p} = \frac{dN}{p_T dp_T dy d\phi} = \frac{1}{(2\pi\,\hbar c)^3}\int_\Sigma p^\mu \mathrm{d}\Sigma_\mu\, f(x,p)$$

Elliptic flow

$$v_2(\mathrm{y}, p_T) = \frac{\int_0^{2\pi} d\phi\ \cos(2\phi) E\frac{dN_i}{d^3p}}{\int_0^{2\pi} d\phi\ E\frac{dN_i}{d^3p}}$$

Rapidity spectra

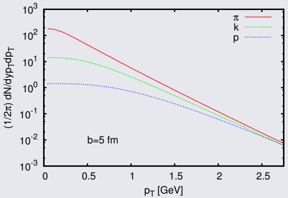
$$\frac{dN}{dy} = \int_0^\infty p_T dp_T \int_0^{2\pi} d\phi\ E\frac{dN_i}{d^3p}$$

Directed flow

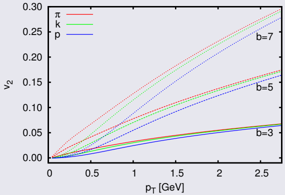
$$v_1(\mathrm{y}) = \frac{\int_0^{2\pi} d\phi \cos(\phi)\ E\frac{dN}{d^3p}}{\frac{dN}{dy}}$$

Observables

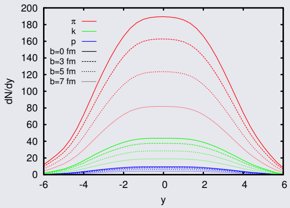
Particle spectra



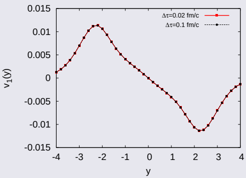
Elliptic flow



Rapidity spectra



Directed flow

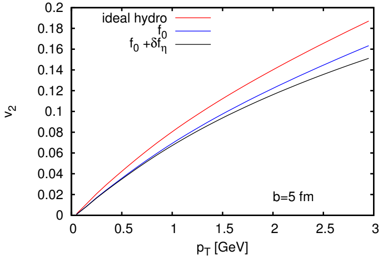


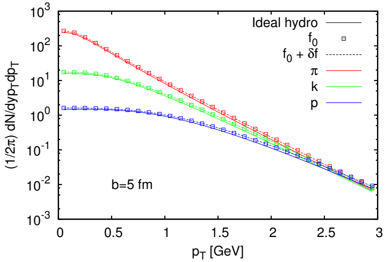
Decoupling Stage

The effect of viscosity[5]

$$f(x,p) \Longrightarrow f(x,p) = f_0(x,p) + \delta f(x,p)$$

$$\delta f(x,p) = f_0(1 \pm f_0)\frac{p^\alpha p^\beta \pi_{\alpha\beta}}{2T^2(e+p)}$$





[5] P. Romatschke, Int.J.Mod.Phys. E19 (2010) 1, 0902.3663

The param.dat file

Most options are contained inside the `param.dat`, which is read *at runtime*:

```
! kind of initialization
INIT_TYPE=0                  !0=Geometric Glauber,...
! kind of simulation
COORD....=2                  !system coordinates: ...
VISCOUS..=1                  !it takes into accoun...
BULK.....=1                  !if 0 it cuts off bul...
NS.......=0                  !if 0 it uses I-S sec...
CUT_TEMP.=0.08               !if > 0 it fixes the ...
```

The param.dat file - Grid setup

Grid options:

```
! grid parameters
NX.......=101                  !number of cells alon...
NY.......=101                  !number of cells alon...
NZ.......=101                  !number of cells alon...
XMIN.....=-15.                 !minimum value for x ...
XMAX.....=15.                  !maximum value for x ...
YMIN.....=-15.                 !minimum value for y ...
YMAX.....=15.                  !maximum value for y ...
ZMIN.....=-15.                 !minimum value for z ...
ZMAX.....=15.                  !maximum value for z ...
```

Other examples:

```
! time parameters
TSTART...=1.0                  !start simulation pro...
TSTOP....=15.                  !stop simulation prop...
TEMP_END.=0.135                !simulation ends when...
```

```
! beam parameters
NUCLEUS..=Au                   !symbol of the collid...
RADS.....=200.                 !sqrt(s_NN) (GeV)    ...
SIGMA_IN.=42.                  !total inelastic cros...
B........=7.                   !impact parameter (fm...
```

55 configuration options available

Additional fine-tuning is possible just changing only a few lines of code.

Please, look at the manual for a comprehensive overview:

http://theory.fi.infn.it/echoqgp/download/manual.pdf

About the output files

- The output is provided in binary format for primitive variables
- It is possible to choose individually which variables to print into the output files
- It is possible to choose whether to print or not the isothermal hypersurface data (as an ASCII tabulated file)
- It is possible to choose the frequency and the precision of the outputs
- It is possible to restart a simulation after a stop
- Postprocessing tools are provided to manage the output files (in Fortran and GDL)

About the output files

spectra routines

- All the particle-related outputs are in ASCII format
- For each particle specie the mean spectrum is given as a function of $(p_{\mathrm{T}}, \mathrm{y})$
- 2 separate lists of Monte Carlo generated particles are provided, with momentum and position (Bjorken and Minkowski coordinates)
- Histograms for such particles are automatically produced (and can also be produced at a subsequent stage)

MPI and OpenMP parallelization

ECHO-QGP can take advantage of multi-cores computers using the **Message Passing Interface**. Typical speedup on our 12 cores Intel Xeon E5645 server: $\sim$8X.

Particle spectra computation routines are partially parallelized using **OpenMP**, speedup on the same server: $\sim$3X.

Execution time strongly dependant on the configuration.

Just to have an idea: parallel (12 cpu) viscous run with a grid of 161x161x101 cells, max timestesp: 4E-03 fm/c, without f.o. computation, only 3 variables printed:

$\sim$7h 30' (of which $\sim$35' to print output files).

$\rightarrow$ Performance improvements planned for future releases.

Conclusions
ECHO-QGP

- ECHO-QGP is a **robust high-order shock-capturing** code, solving either **ideal** or **viscous** (Israel-Stewart) hydrodynamics
- Modules for 1D, 2D, and 3D Minkowsky and Bjorken available
- ECHO-QGP reproduces the standard analytic solutions
- ECHO-QGP is consistent with AZHYDRO, UVH2, MUSIC
- ECHO-QGP is highly customizable: EoS, IC, Decoupling
- ECHO-QGP is distributed under a free license: the **GPL v.2**
- ECHO-QGP is modular, parallel, user friendly: it can be downloaded from: `http://theory.fi.infn.it/echoqgp/`

The End!

Thank you!