

Incontro sulla fisica con ioni pesanti a
LHC

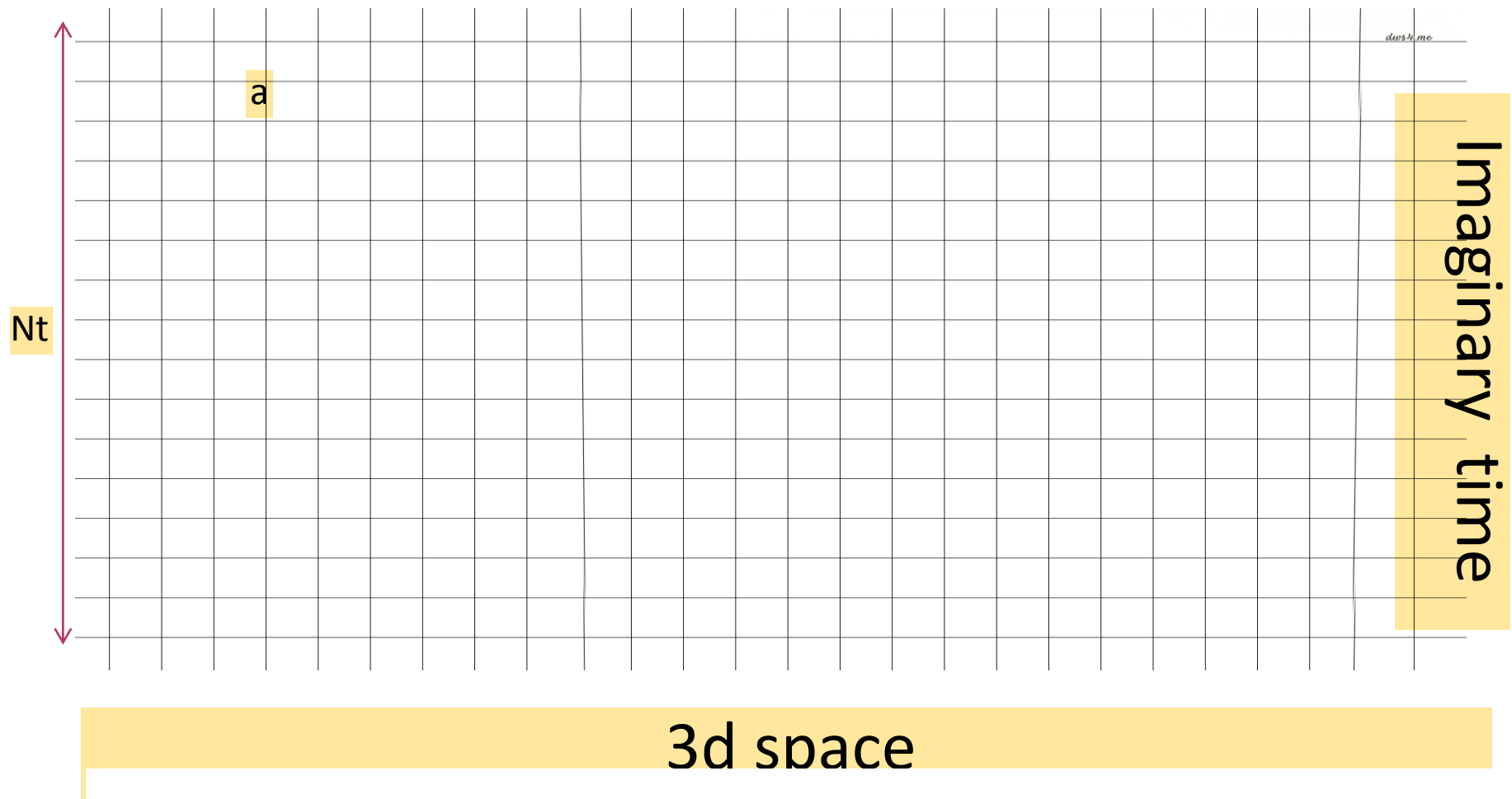
Bologna 26-27 Maggio 2015

Maria Paola Lombardo

Equation of state and transport from the lattice

The lattice:

With a rotation to imaginary time and
(anti)periodic boundary condition in time:
Statistical system in 3+1 dimension
Temperature = $1/ Nt a$



Spectrum and EoS

Lattice:
tool for
computing
Partition
Function

$$\mathcal{Z} = \text{Tr} \hat{\rho}$$
$$\hat{\rho} = e^{-(H - \mu \hat{N})/T}$$

$$\langle O \rangle = \text{Tr} O \hat{\rho} / \mathcal{Z}$$

Spectrum

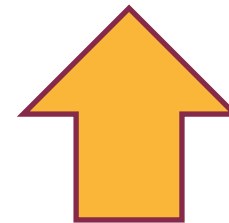
$$P = T \frac{\partial \ln \mathcal{Z}}{\partial V}$$
$$N = T \frac{\partial \ln \mathcal{Z}}{\partial \mu} \quad \text{EoS}$$
$$S = \frac{\partial T \ln \mathcal{Z}}{\partial T}$$
$$E = -PV + TS + \mu N$$

Designing a simulation

$$\mathbf{Z}(N_\sigma, N_\tau, \beta, m_q, \tilde{\mu}) = \int \prod_{n\nu} dU_{n,\nu} (\det Q^{KS}(m_q, \tilde{\mu}))^{n_f/4} e^{-\beta S_G}$$

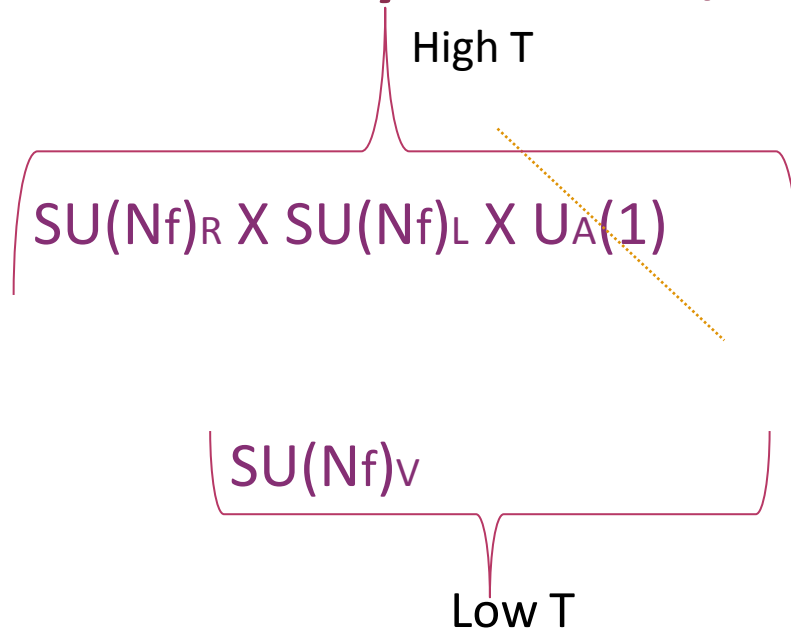


Physical choice:
Active Quarks

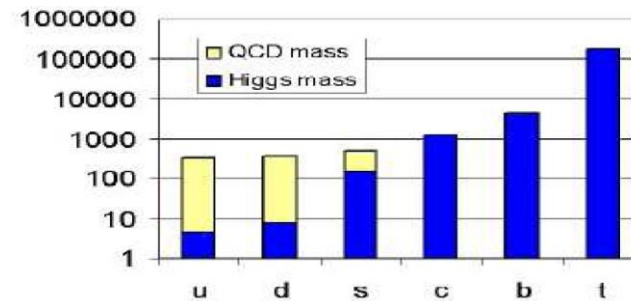


Technical choice:
Lattice Action

QCD Symmetries, lattice and the real world



c,b,t do not participate in the chiral dynamics around the critical temperature. Lattice simulations **around T_c** are then performed with **up,down, strange quarks – $N_f = 2+1$**



	$SU(N) \times SU(N)$	$U_A(1)$
Staggered	Remnant $U(1)$	Broken
Wilson	Broken	Broken
Domain Wall	Exact (for $L \rightarrow \infty$)	Exact (for $L \rightarrow \infty$)
Overlap	Exact	Exact
Wilson twisted	As good as staggered	Broken

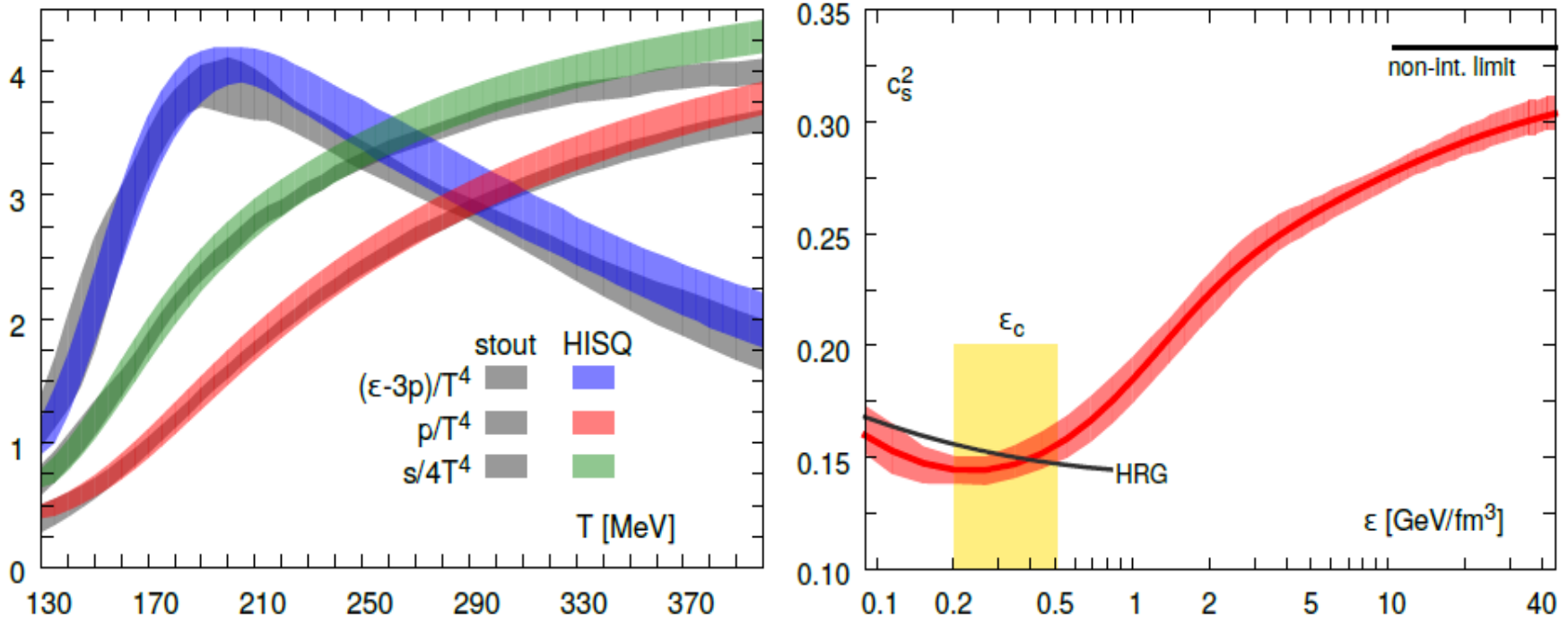
Input to Hydro:

EOS

$N_f = 2 + 1$

Eos Current Status

$$\langle \theta \rangle = \mathcal{E} - 3P$$

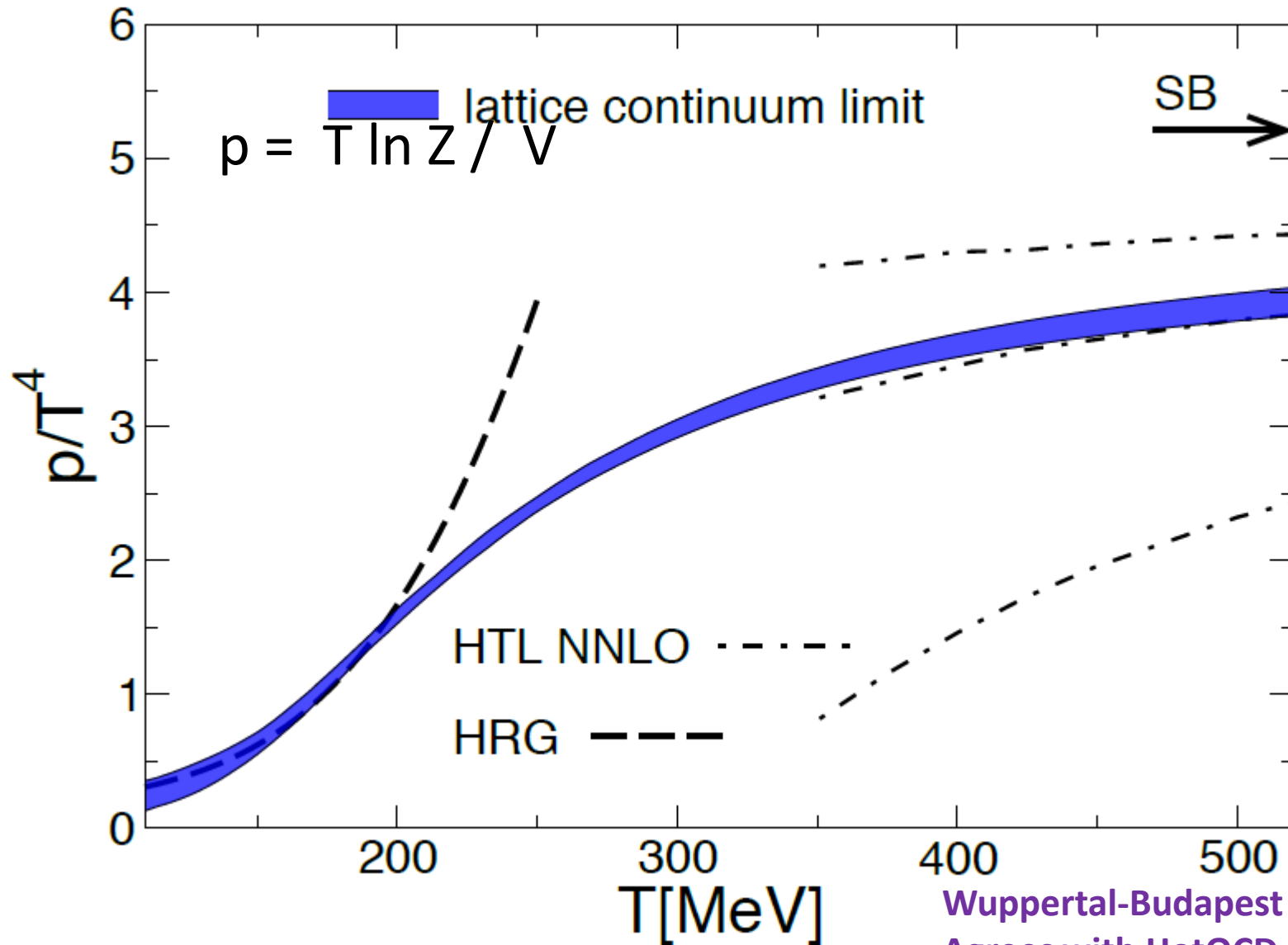


Wuppertal-Budapest 2014
Agrees with HotQCD
(Ding and Bazavov at QM2014)

$$\frac{\zeta}{\eta} \geq 2 \left(\frac{1}{3} - c_s^2 \right)$$

$N_f = 2+1$

EoS Current Status



Wuppertal-Budapest 2014
Agrees with HotQCD
(Ding and Bazavov at QM2014)

Tc

340 –380 MeV
RHIC AuAu
200 GeV

420-480 MeV
LHC
2.76 TeV

500- 600MeV
LHC hot spots
2.76 TeV

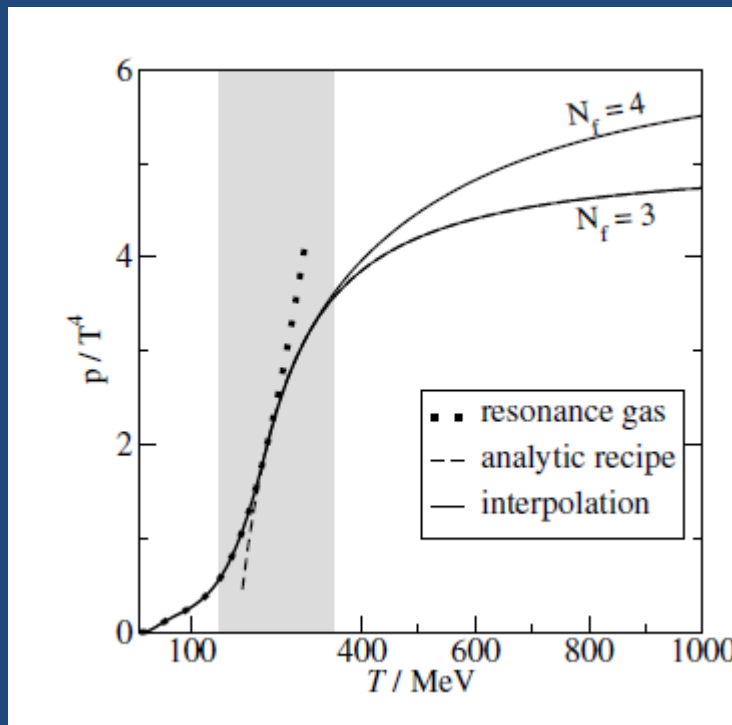


1 GeV
LHC
7 TeV

[Expected]

$\approx 200\text{MeV}$

Quark Gluon Plasma @ Colliders

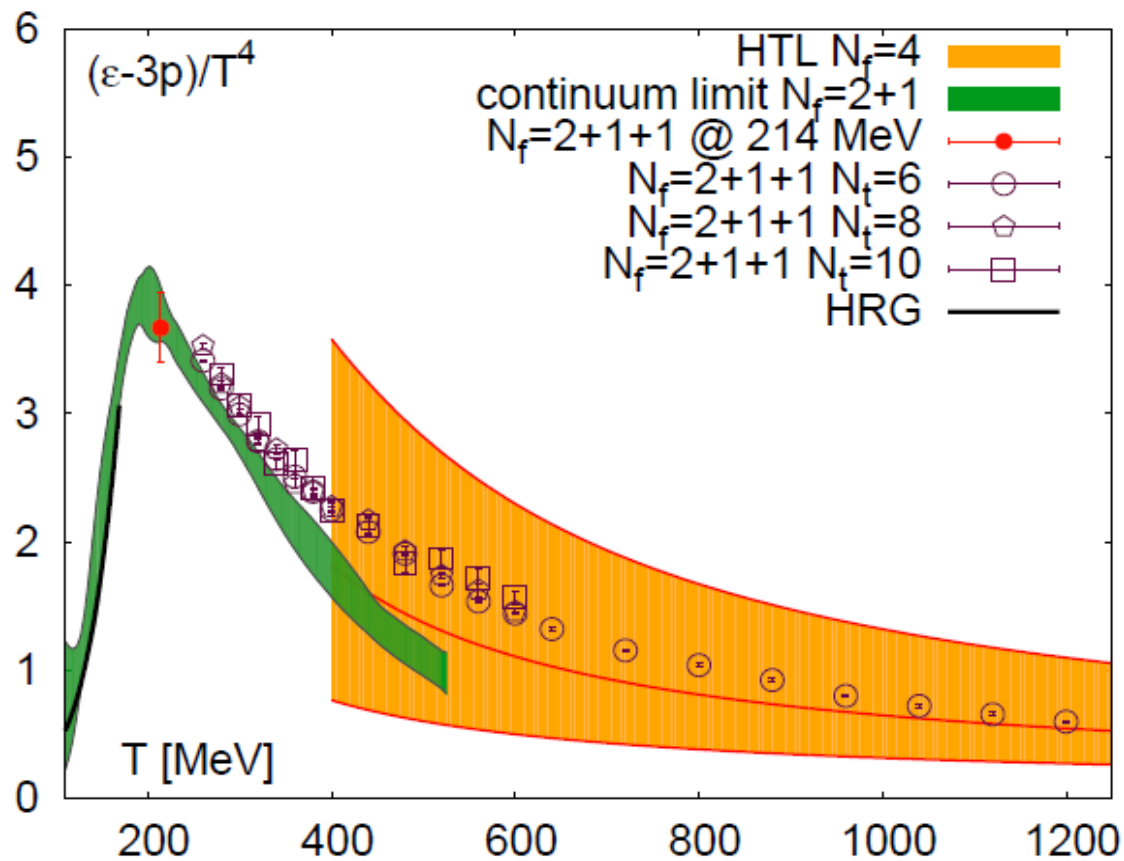


Analytic studies suggest that a dynamical charm becomes relevant above 400 MeV, well within the reach of LHC

Laine Schroeder 2006

Thermodynamics with a dynamical charm : $N_f = 2+1+1$:

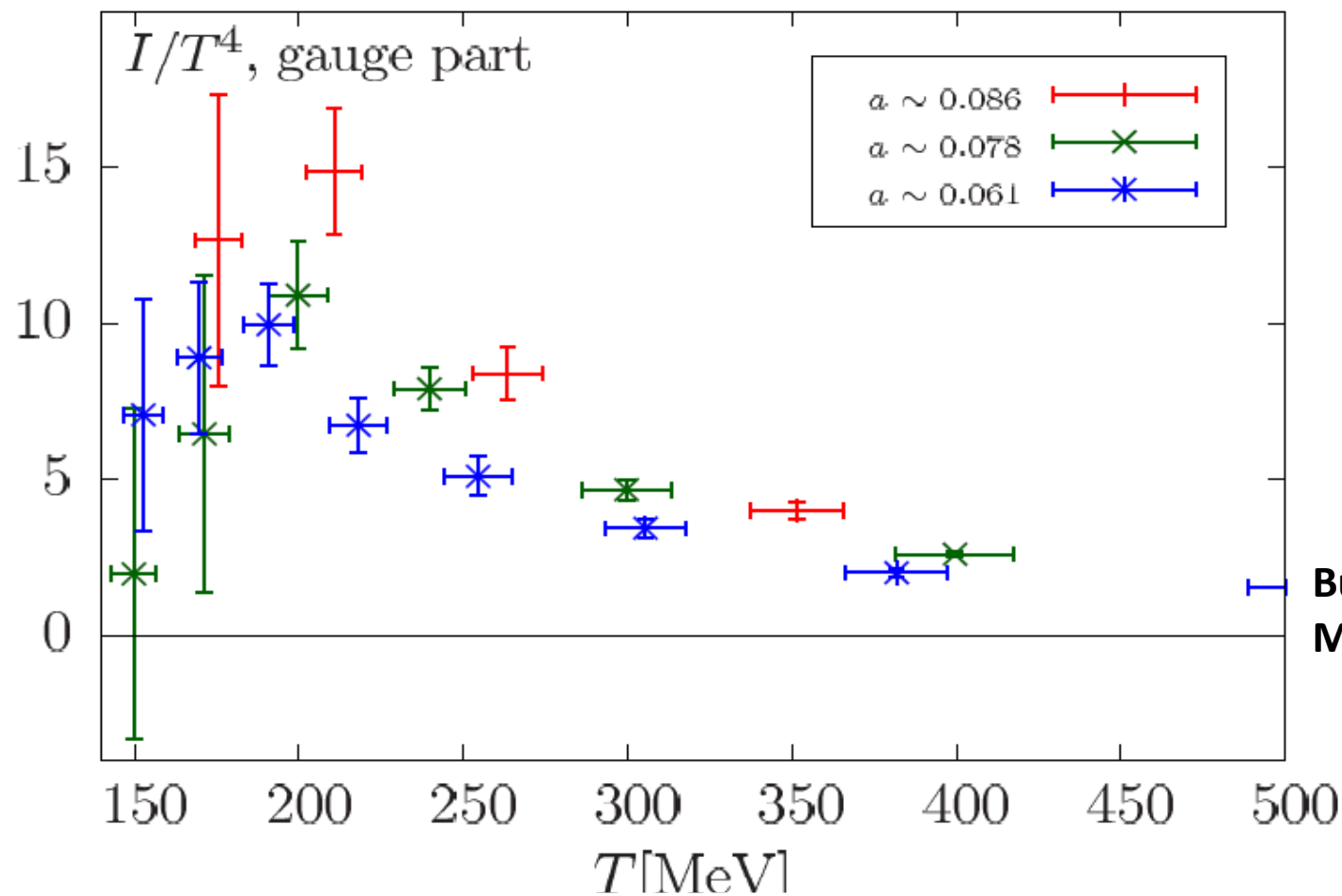
Staggered fermions (fixed N_t , varying spacing)



Budapest Wuppertal
2014

Thermodynamics with a dynamical charm : $N_f = 2+1+1$:

Wilson fermions (fixed a , varying N_t)



Burger, Ilgenfritz, MpL,
Mueller-Preussker 2014

In progress

EoS finished for $T < 400$ MeV!

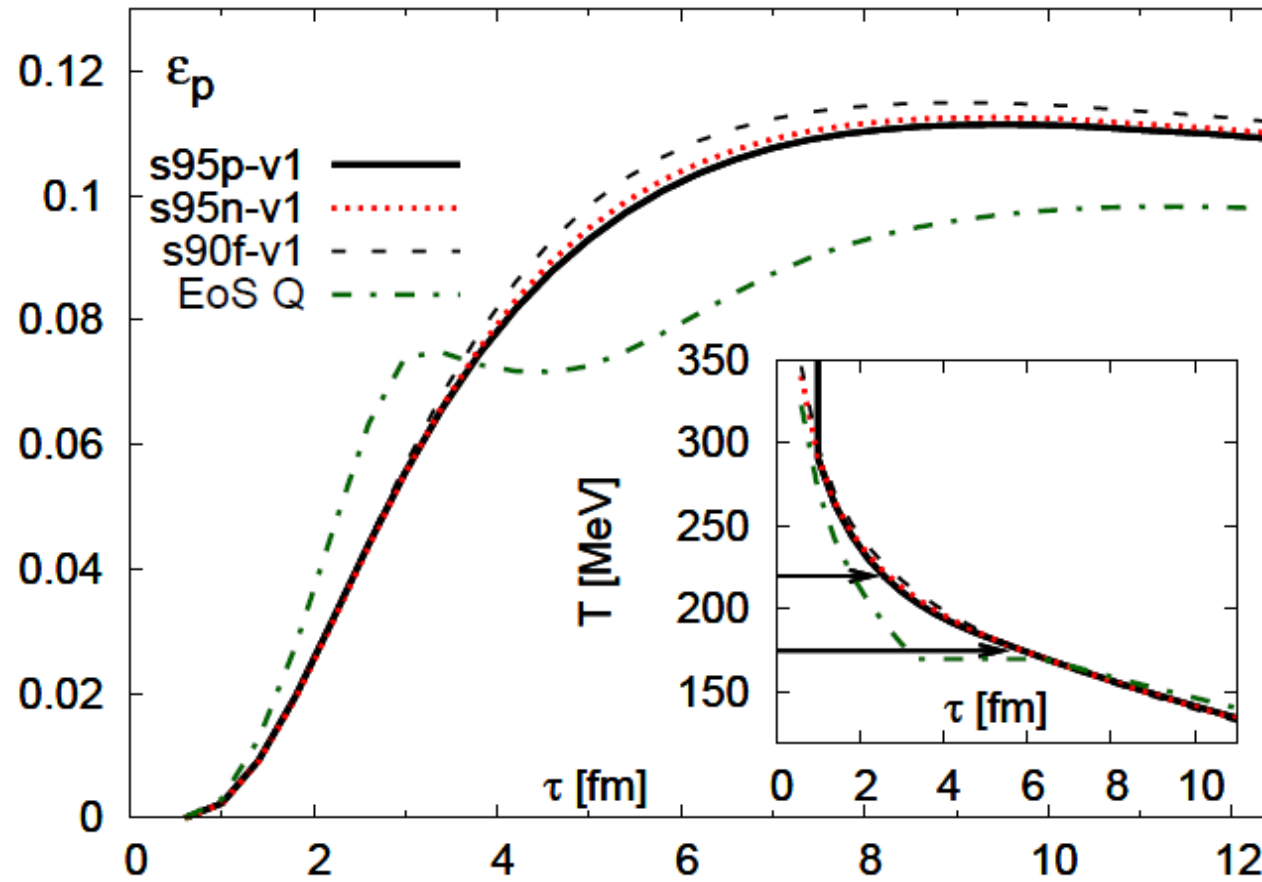
A controlled calculation of the Equation of State for $T > 400$ MeV requires a dynamical charm.

Studies with staggered and Wilson fermions are in progress

Is it possible to estimate the required accuracy from a phenomenological point of view?

For instance:

Impact of EoS on time evolution?



Huovinen Petreczky 2010

Beyond ideal hydro

TRANSPORT

Transport on the lattice



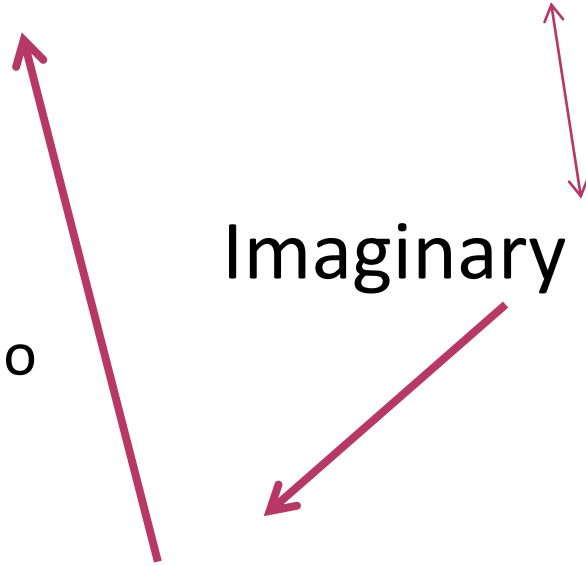
Analytic continuation in
principle possible

Transport on the lattice

Green-Kubo
relations

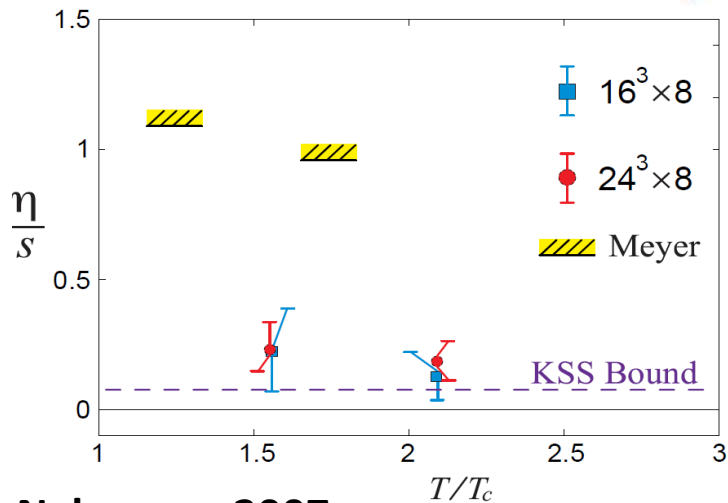
Imaginary time:

In practice, so far: **Spectral functions**
from Euclidean correlators computed in
imaginary time

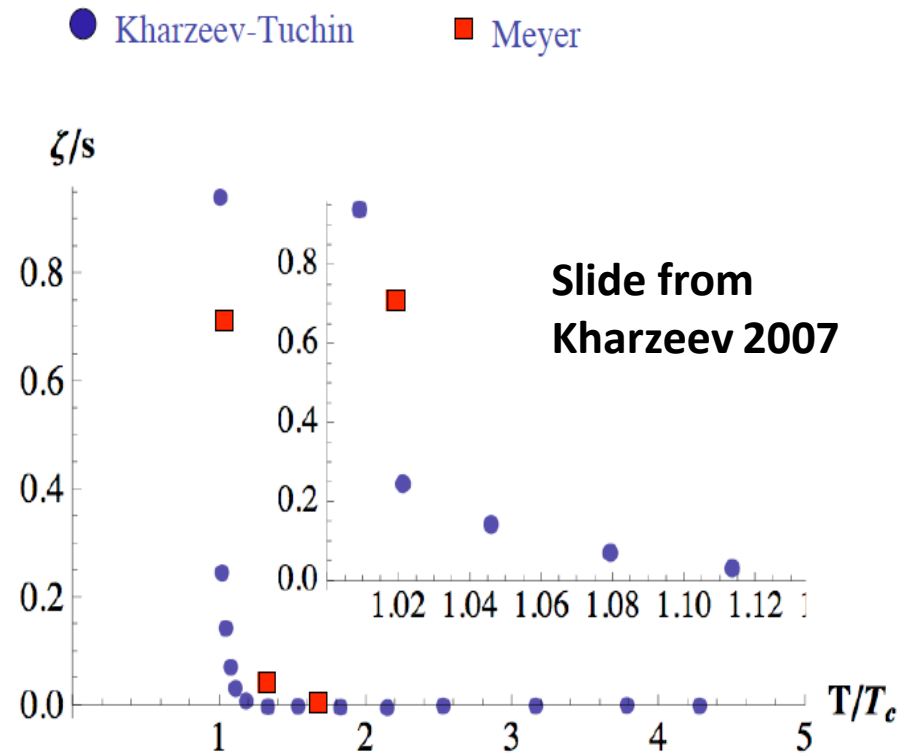
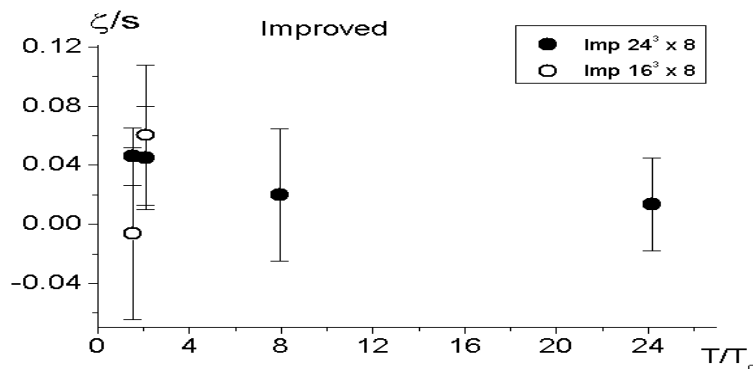


2007: Golden year for lattice transport...

$$\theta_{ij} = P_{eq}(\epsilon)\delta_{ij} - \eta \left(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\partial_k u_k \right) - \zeta \delta_{ij} \vec{\nabla} \cdot \vec{u}$$



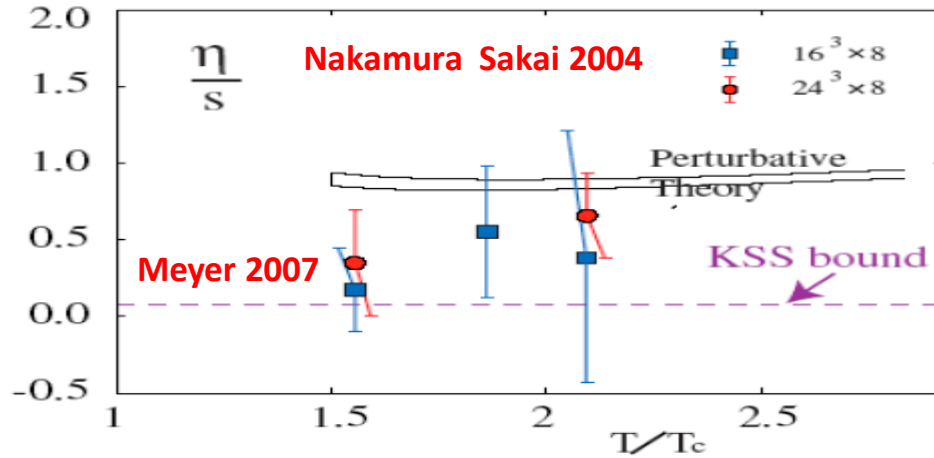
Nakamura 2007



Lattice and transport

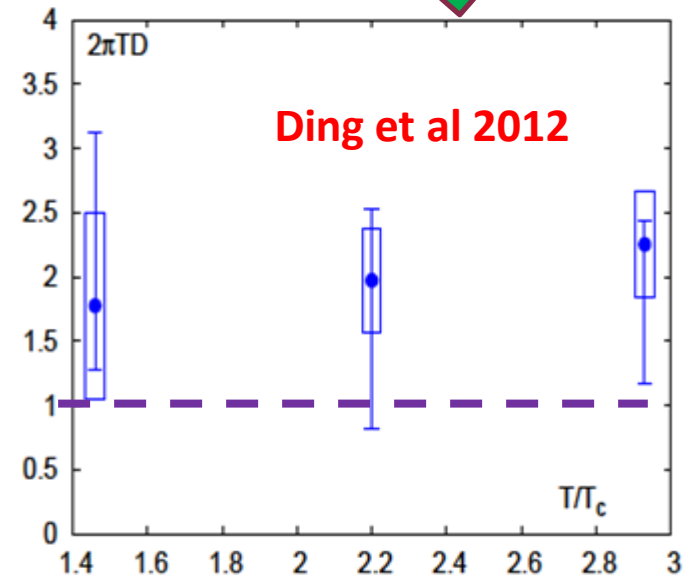
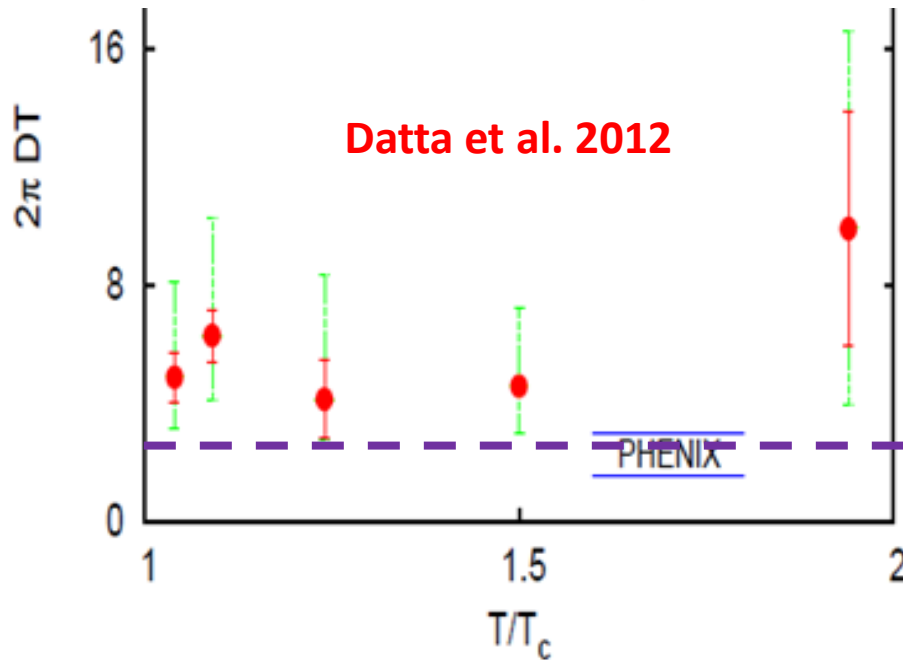
2012 status

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

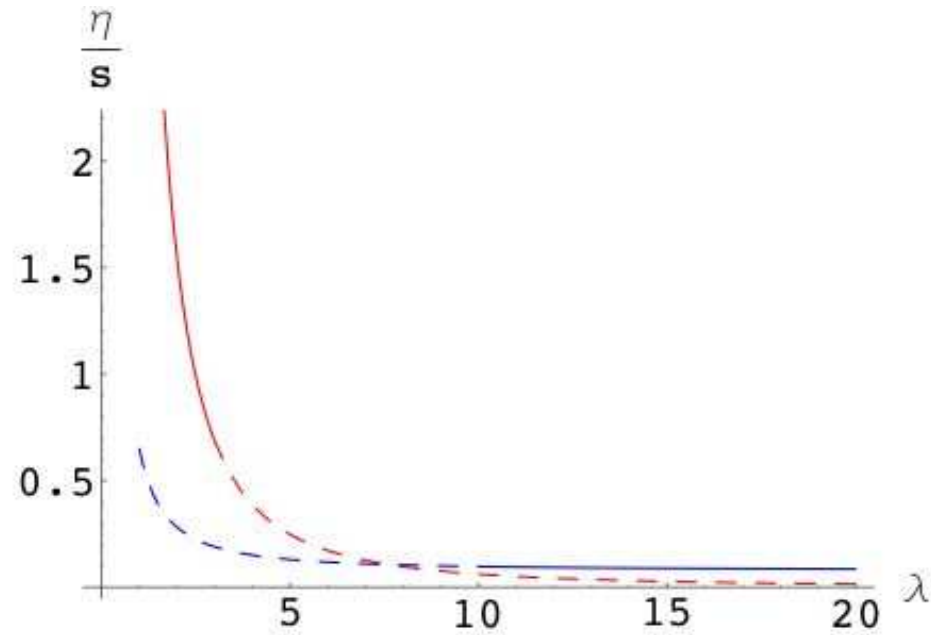


Early evidence for sQGP (slide from D. Kharzeev, 2007)

Recent results for the HQ diffusion coeff.



$$\eta/s$$



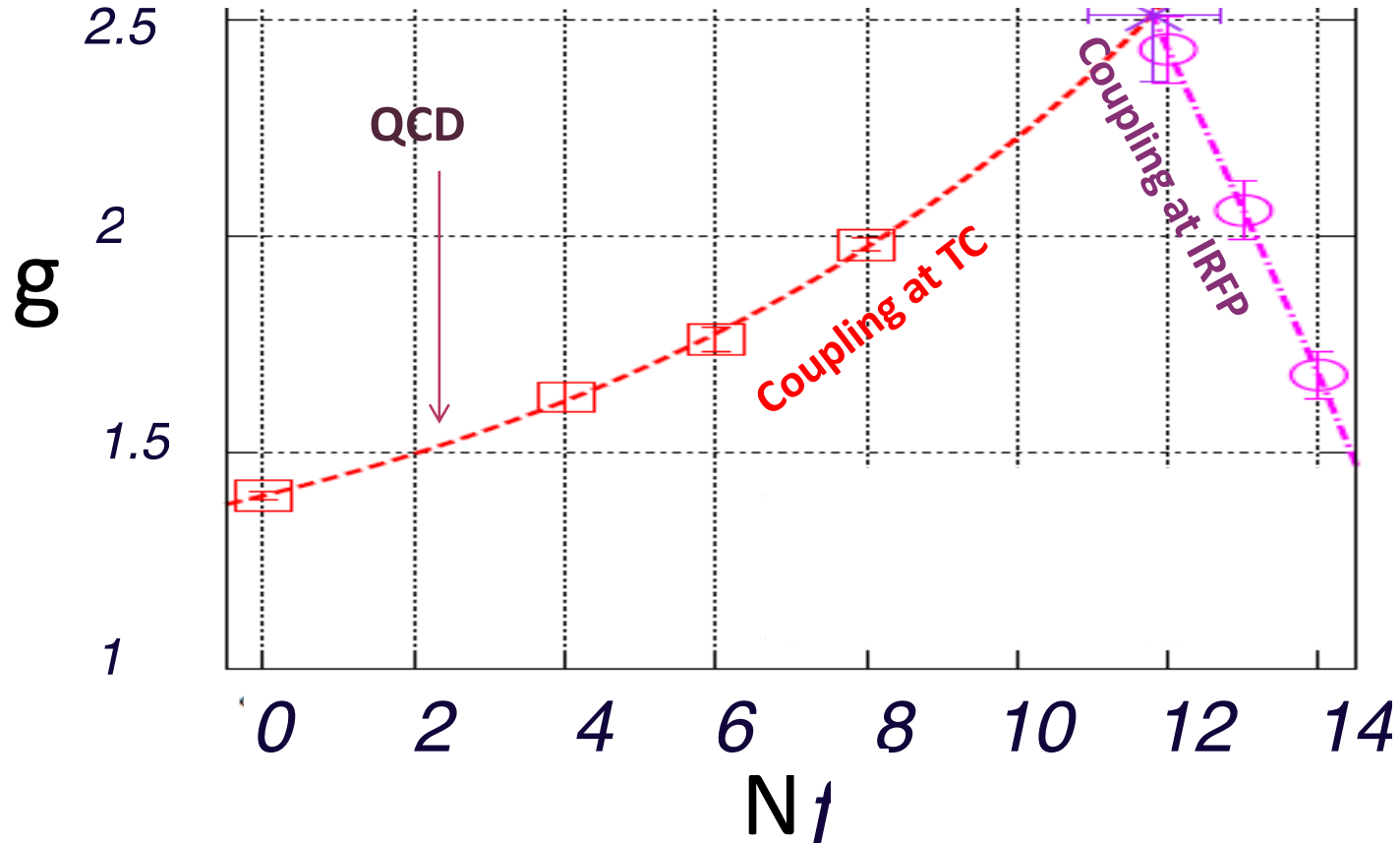
behaviour of η/s as a function of the 't Hooft coupling $\lambda = g_{YM}^2 N_c$

Kovtun-Son-Starinets (KSS) bound:

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

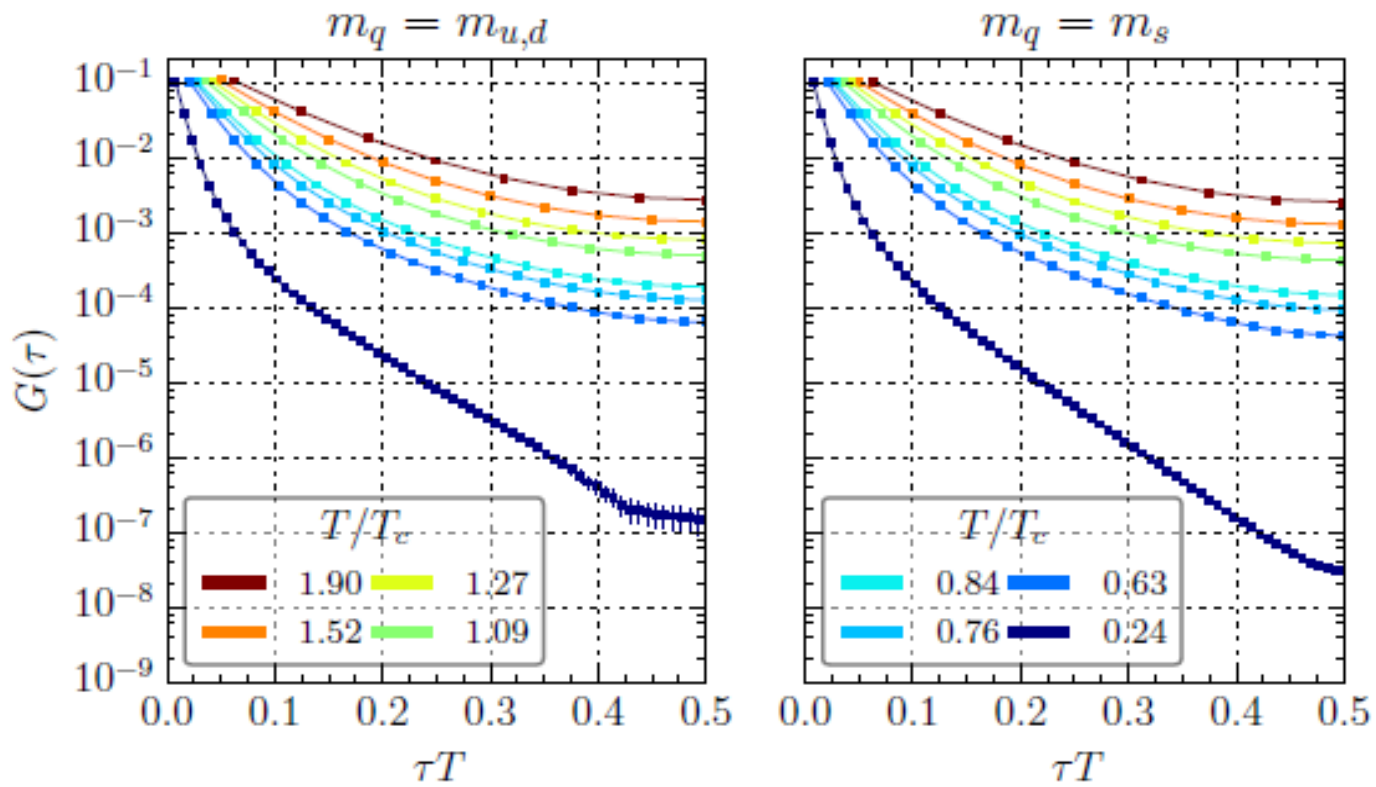
$\frac{\eta}{s} \approx 0.5$: border-value between sQGP and pQGP ?

Strength of the sQGP vs N_f



2015: transport on the lattice coming of age? Conductivity

$$G_{\mu\nu}^{\text{em}}(\tau) = \frac{1}{V} \int d^3x \langle j_{\mu}^{\text{em}}(\tau, \mathbf{x}) j_{\nu}^{\text{em}}(0, \mathbf{0})^{\dagger} \rangle$$

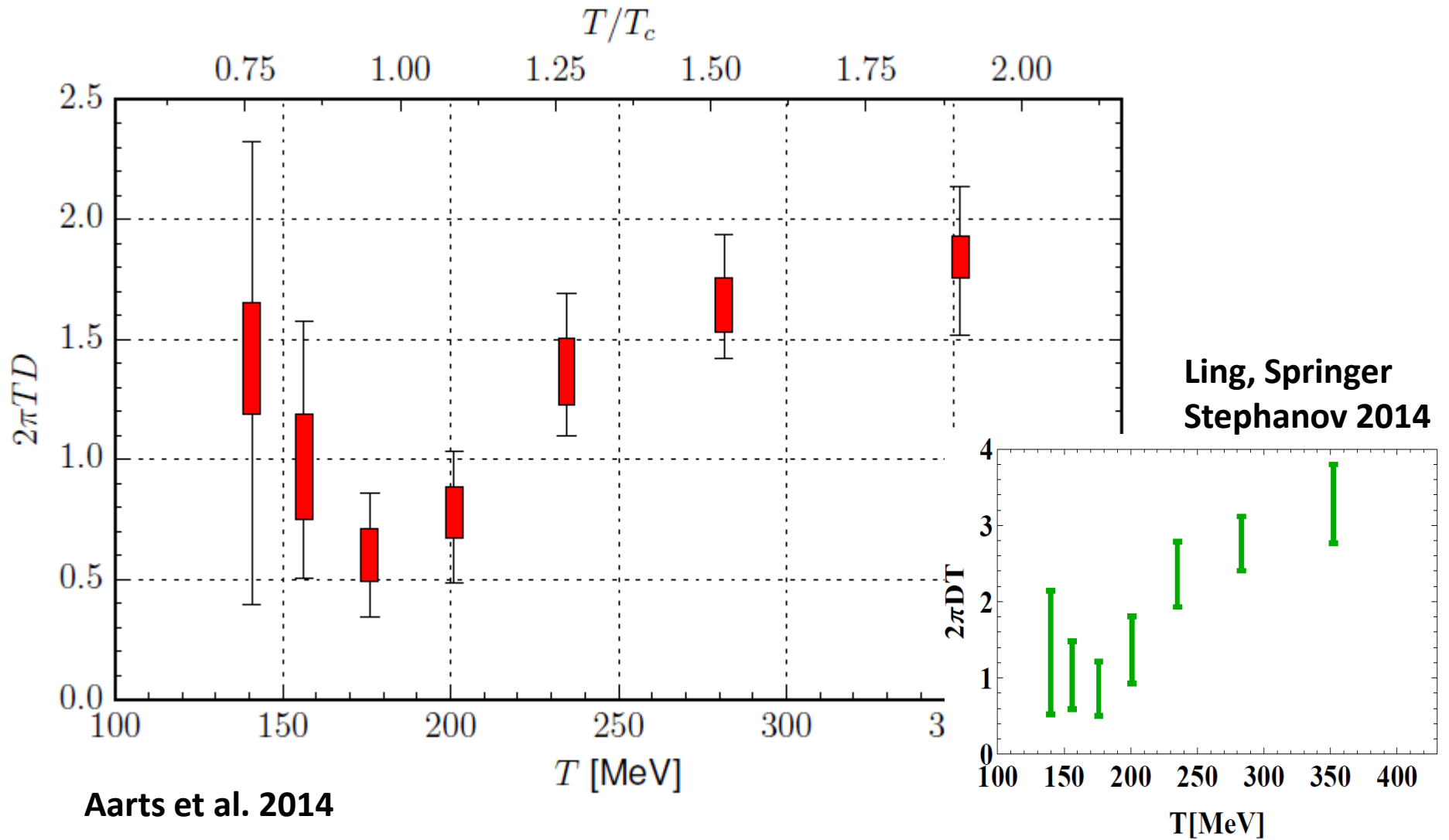


$$j_{\mu}^{\text{em}} = \sum_f (eq_f) j_{\mu}^f = \frac{2e}{3} j_{\mu}^u - \frac{e}{3} j_{\mu}^d - \frac{e}{3} j_{\mu}^s$$

Aarts et al. 2014
FASTSUM
Collaboration

Charge diffusion coefficient

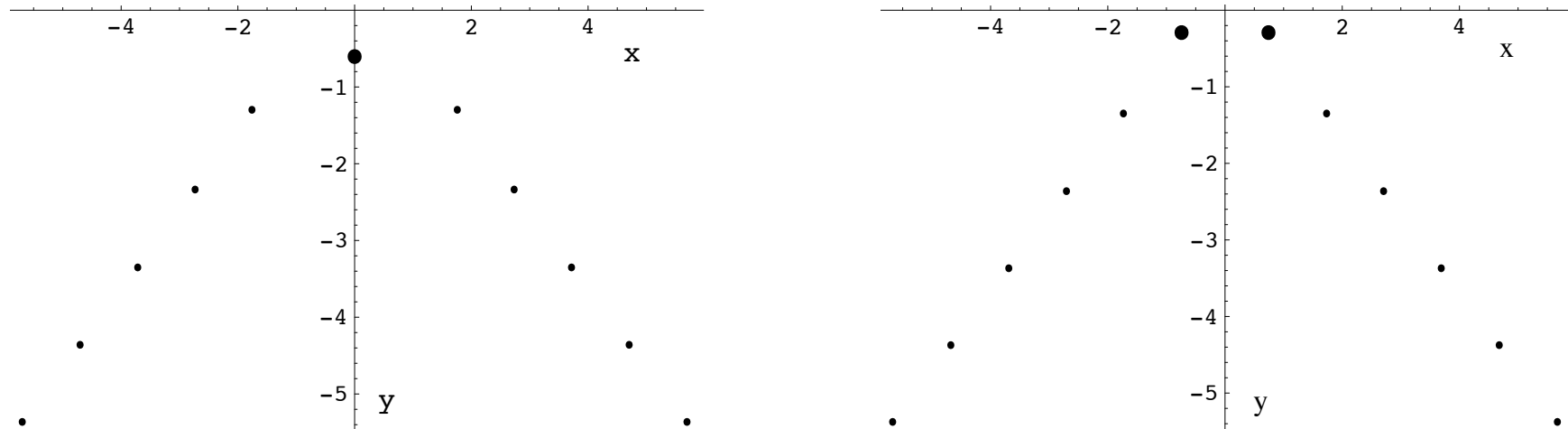
$$D = \sigma / \chi_Q$$



Beyond Hydro?

Quasinormal modes

hydrodynamic modes



quasinormal spectrum of black brane gravitational fluctuations in the shear and sound channels at fixed spatial momentum as a function of complex frequency

[Kovtun and Starinets (2005)]

hydrodynamic poles are marked by full dots, e.g. shear

$$\omega = -i \frac{\eta}{T_s} q^2 + O(q^3) = -i \frac{1}{4\pi T} q^2 + O(q^3)$$

approach the real frequency axis for $q \rightarrow 0$

[Kovtun, Son and Starinets (2005)]

Questions:

Equation of State: which is the needed accuracy

Transport coefficients: -which is the needed accuracy
-how many of them are needed

Approach to hydro: -quasinormal modes?

Physics mechanisms: -qualitative differences
between QGP and sQGP?
-Characterization of sQGP

*Emerging field: lattice + holography +
near-conformal model field theories*