Early Times Dynamics in Relativistic Heavy Ion Collisions

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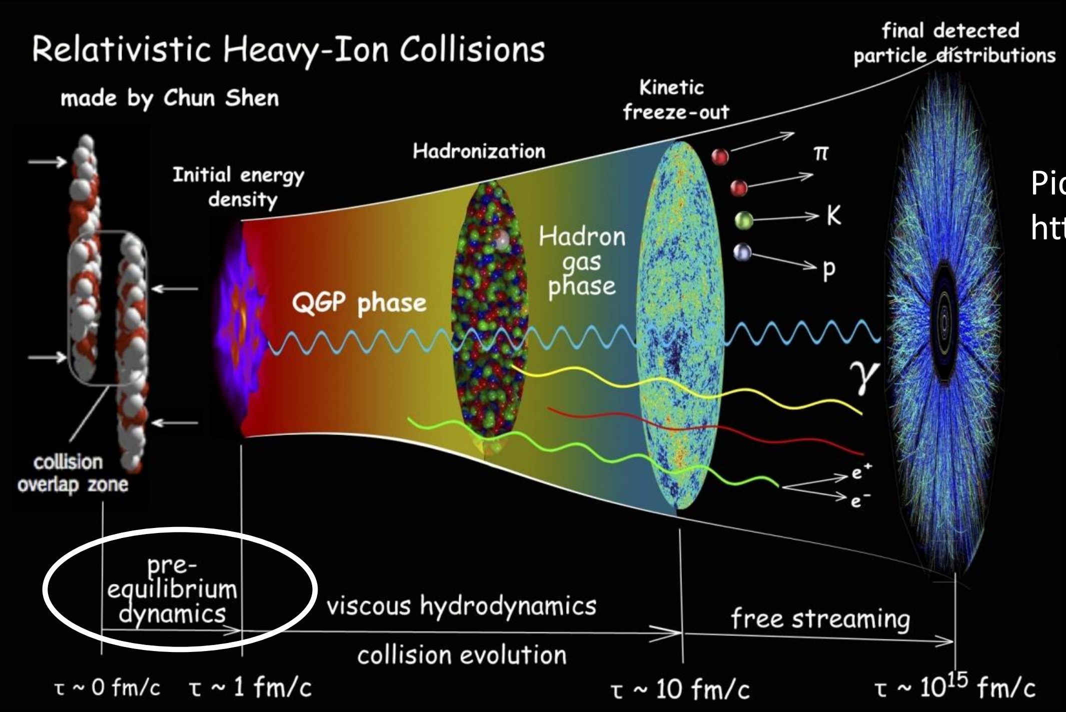
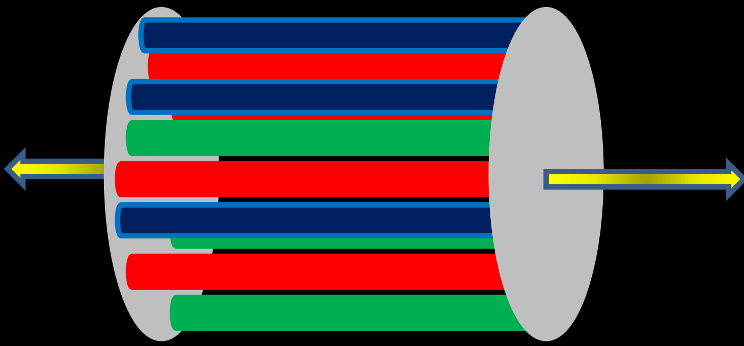
Bologna, 2015 May 26

Cartoon of a HIC

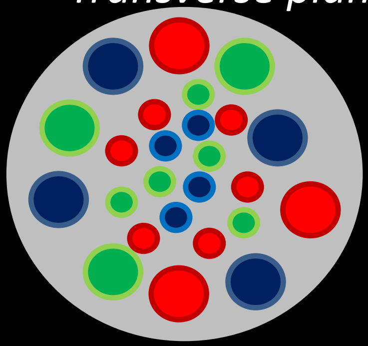


Picture taken from:
http://snelling.web.cern.ch/snelling/img/little_bang.jpg

Longitudinal view



Transverse plane



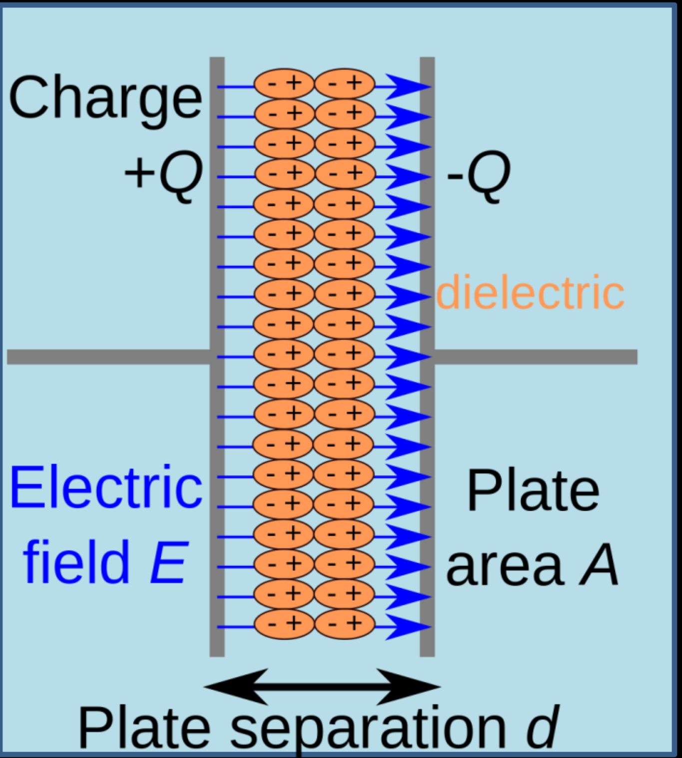
Problem:
how does the QCD dynamics leads to a thermalized and isotropic QGP, starting from a configuration of classical color fields?

Here we describe *one possible approach* to the problem,
based on the assumption that classical color fields decay to a QGP
via vacuum tunneling, namely via the ***Schwinger effect*** (Schwinger, 1951).

Euler-Heisenberg (1936)
J. Schwinger (1951)
Casher et al., PRD 20 (1979)

Schwinger effect in Electrodynamics

$$\mathcal{W}(x) = -\frac{|g\boldsymbol{E}|}{4\pi^3}\int d^2p_T \log\left(1 - e^{-\frac{\pi^2 E_T^2}{|g\boldsymbol{E}|}}\right) = \frac{g^2E^2}{4\pi^3}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-\frac{n\pi m^2}{|g\boldsymbol{E}|}\right)$$



WKB interpretation:

(.) Gives the p_0 and p_T spectrum of the produced pair

(.) Describes the Schwinger effect as a dipole formation in the vacuum; each dipole has moment

$$p = g \times 2 \times \frac{d}{2} = g\frac{2E_T}{|g\boldsymbol{E}|}$$

Casher et al., PRD 20 (1979)

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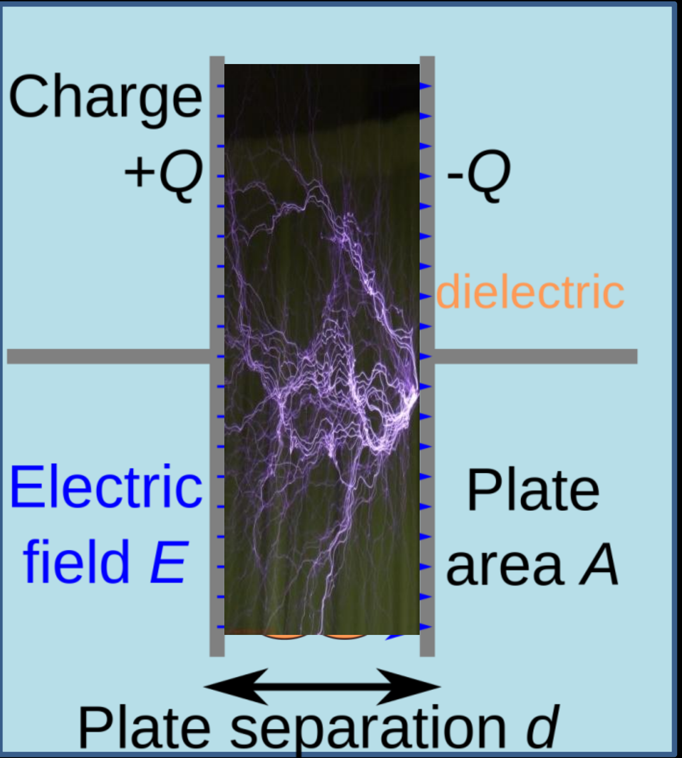


Once pairs pop up from the vacuum, charged particles propagate in real time producing electric currents:

$$\boldsymbol{J} = \sigma \boldsymbol{E}$$

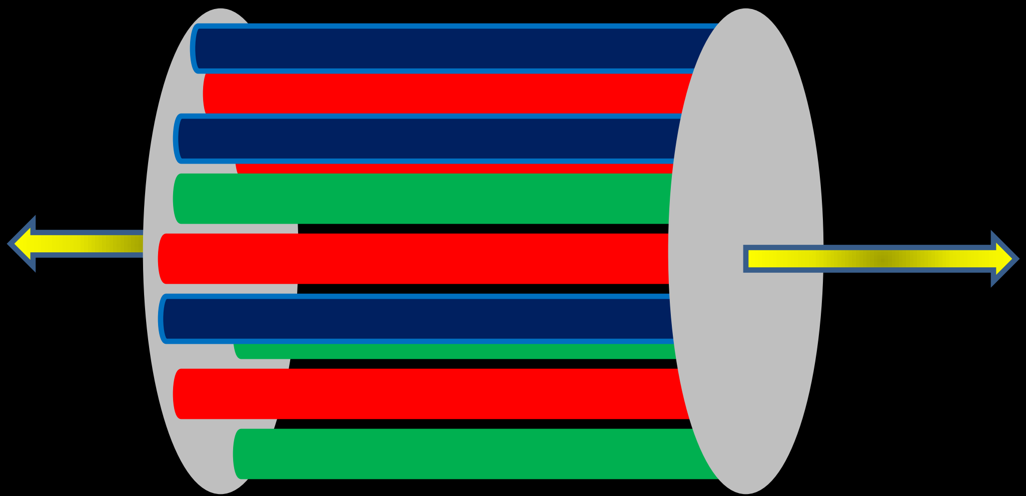
in linear response theory

Vacuum polarization
Electric current → ***Dielectric breakdown***

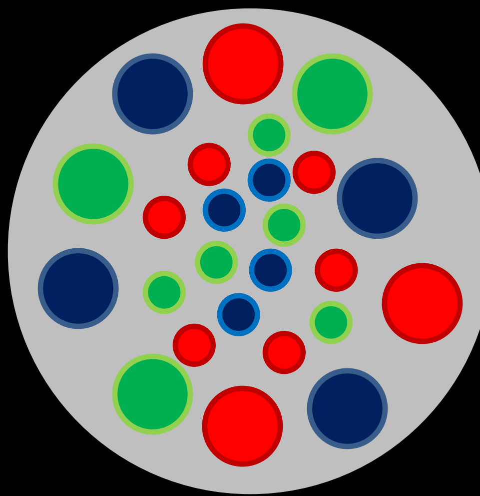
Schwinger effect in Chromodynamics

Abelian Flux Tube Model

Longitudinal view



Transverse plane view



Focus on a single flux tube:

(.) neglect color-magnetic fields;

(.) assume abelian dynamics for ***color-electric fields***;

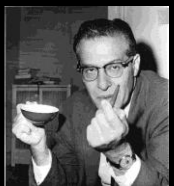
(.) assume ***Schwinger effect*** takes place:

Color-eletric color field decays into quark-antiquark as well as gluon pairs

Abelian Flux Tube Model

Boltzmann equation and QGP

In order to permit ***particle creation*** from the vacuum we need to add a ***source term*** to the rhs of the Boltzmann equation:



$$\left(p_\mu \partial^\mu + g Q_{jc} F^{\mu\nu} p_\mu \partial^p_\nu\right) f_{jc} = p_0 \frac{\partial}{\partial t} \frac{dN_{jc}}{d^3x d^3p} + \mathcal{C}[f]$$

Florkowski and Ryblewski, PRD 88 (2013)

Invariant source term

Invariant source term: change of ***f*** due to particle creation in the volume at ***(x,p).***

In our model, particles are created by means of the Schwinger effect, hence

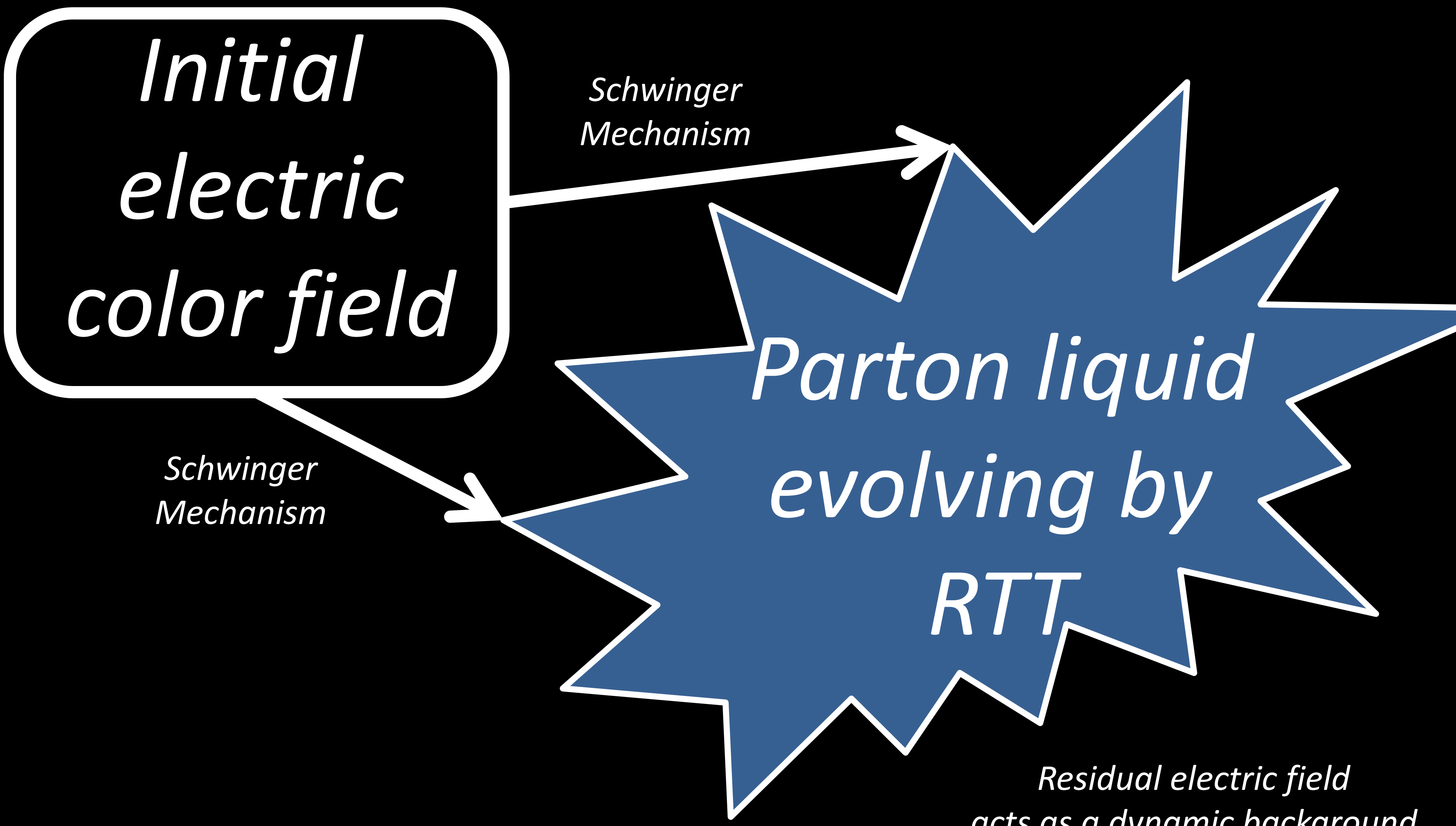
$$\frac{dN_{jc}}{d\Gamma} \equiv p_0 \frac{dN_{jc}}{d^4x d^2p_T dp_z} = \mathcal{R}_{jc}(p_T)\delta(p_z)p_0$$

$$\mathcal{R}_{jc}(p_T) = \frac{\mathcal{E}_{jc}}{4\pi^3}\left|\ln\left(1 \pm e^{-\pi p_T^2/\mathcal{E}_{jc}}\right)\right|$$

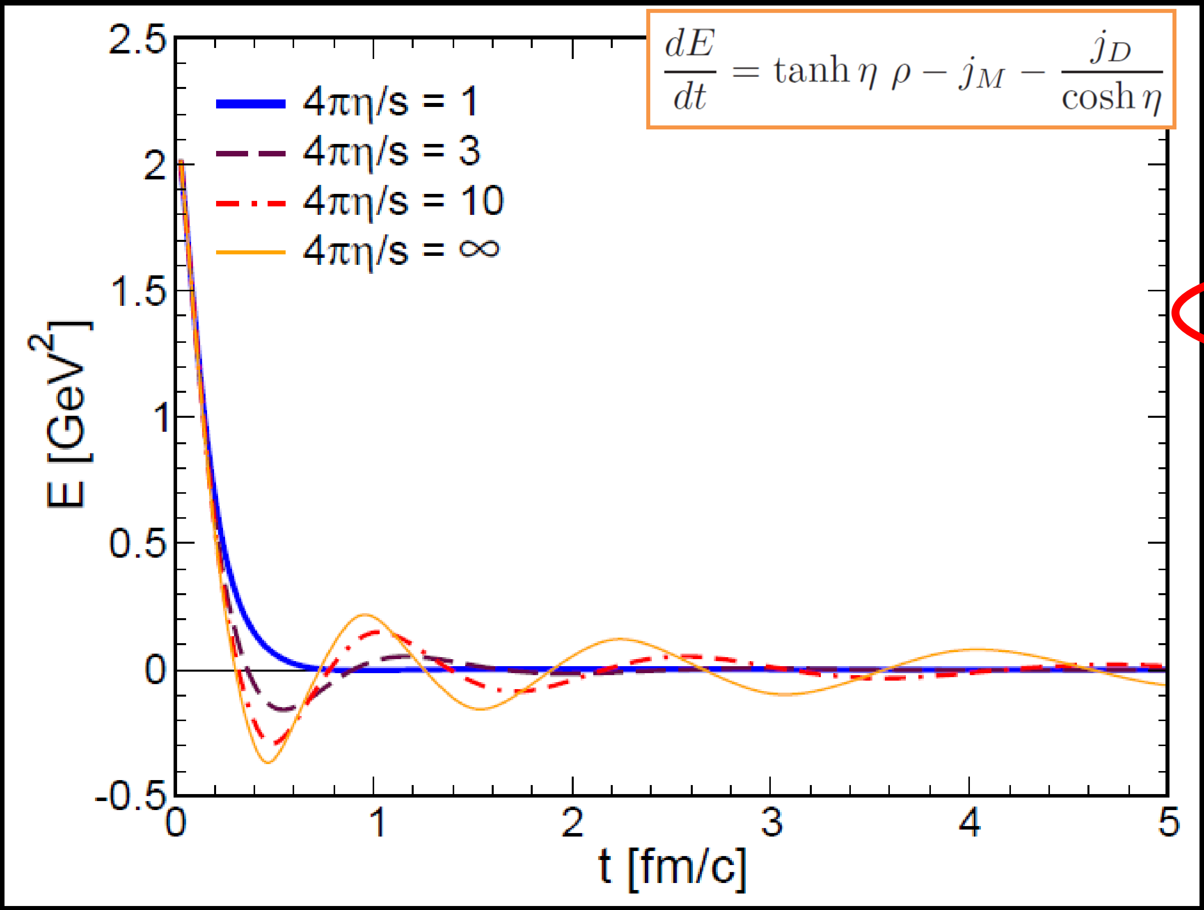
$$\mathcal{E}_{jc} = \left(g|Q_{jc}E| - \sigma_j\right)\theta\left(g|Q_{jc}E| - \sigma_j\right)$$

See also:
Gelis and Tanji, PRD 87 (2013)

Our early times dynamics



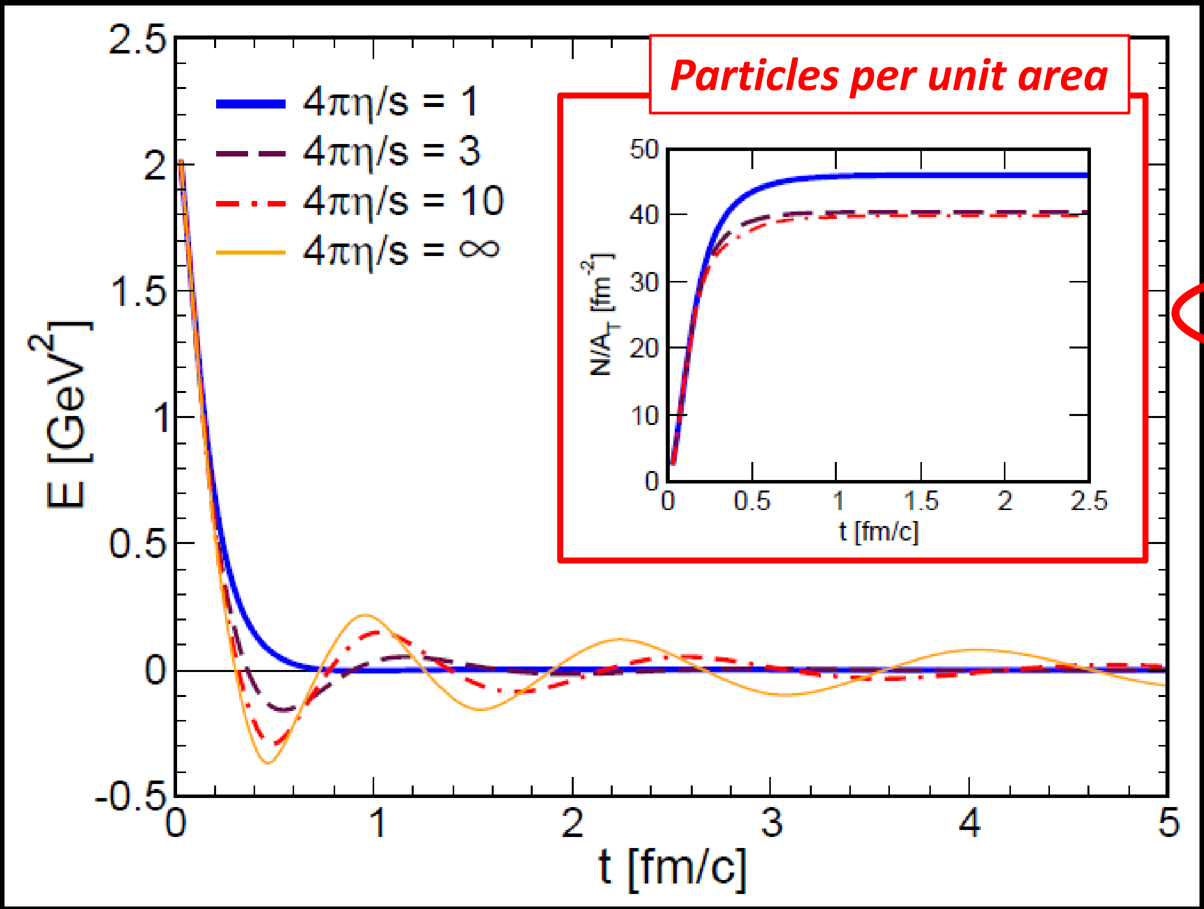
Flux tube evolution: ***field decay***



ρ	*charge density*
j_M	*electric current*
j_D	*polarization current*

Describes particles creation by Schwinger effect

Flux tube evolution: ***field decay***



ρ	*charge density*
j_M	*electric current*
j_D	*polarization current*

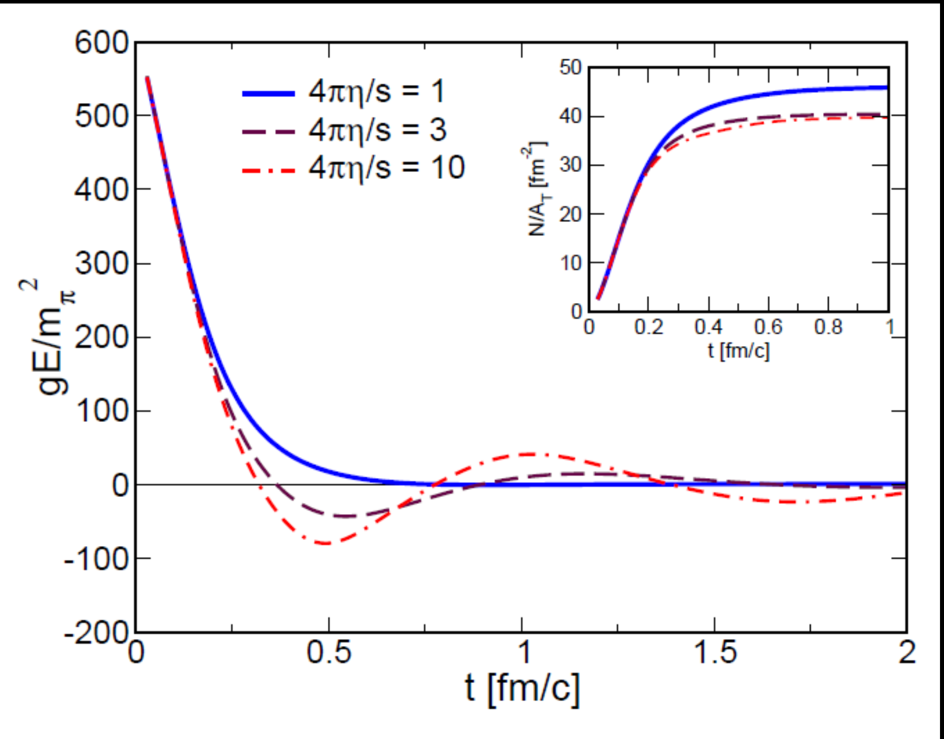
$$\tau_{form} \approx 0.6 \text{ fm/c}$$

Polarization current vanishes within a small fraction of fm/c: particles are dynamically produced in the very early stages.

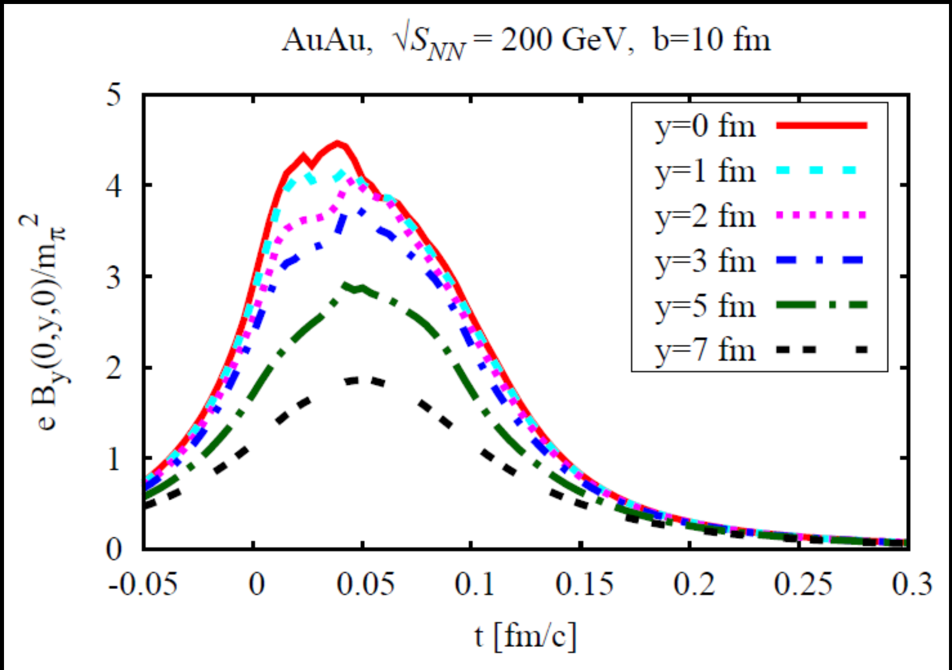
Flux tube evolution: ***field decay***

Comparison with electromagnetic fields

Color-electric field



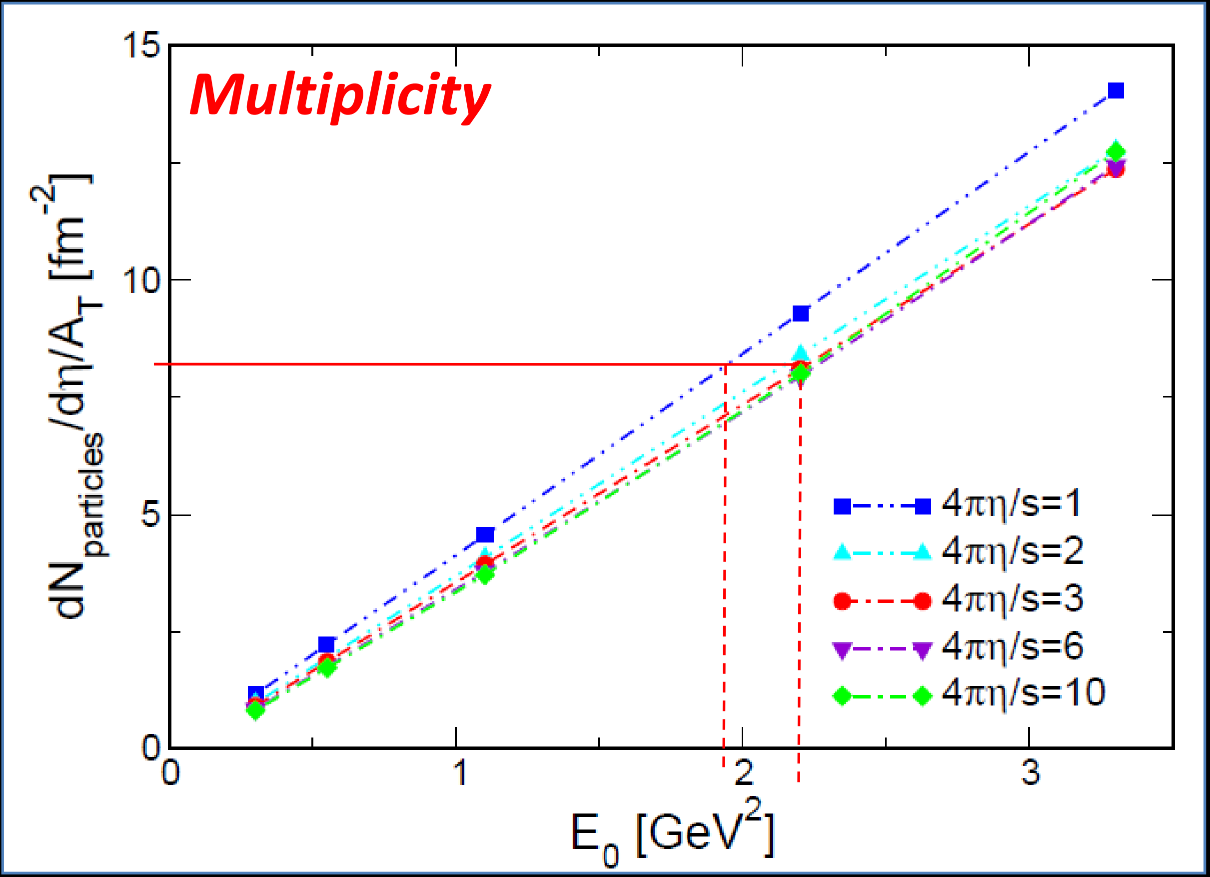
Magnetic field



Initial color-electric field in HICs:
gE: 1-10 GeV^2 (*i.e.* 50-500 m_π^2)

W. Cassing *et al.*, PRC 83 (2011)

Particles multiplicity



Multiplicity for a RHIC collision, b=2.5 fm:

$$\frac{dN}{dy} = \frac{dN}{d\eta} \approx 1040$$

Transverse area:

$$A_T \approx \pi R^2 \approx \pi (6.5)^2 \approx 137 \text{ fm}^2$$

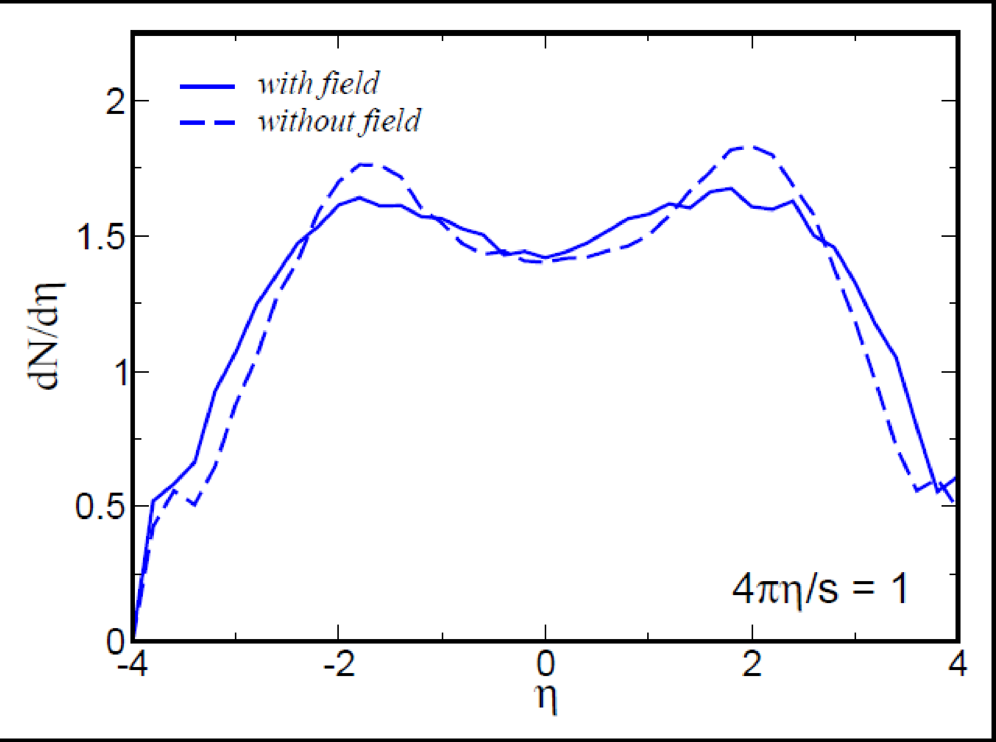
Multiplicity per transverse area:

$$\frac{dN}{A_T d\eta} \approx 8 \text{ fm}^{-2}$$

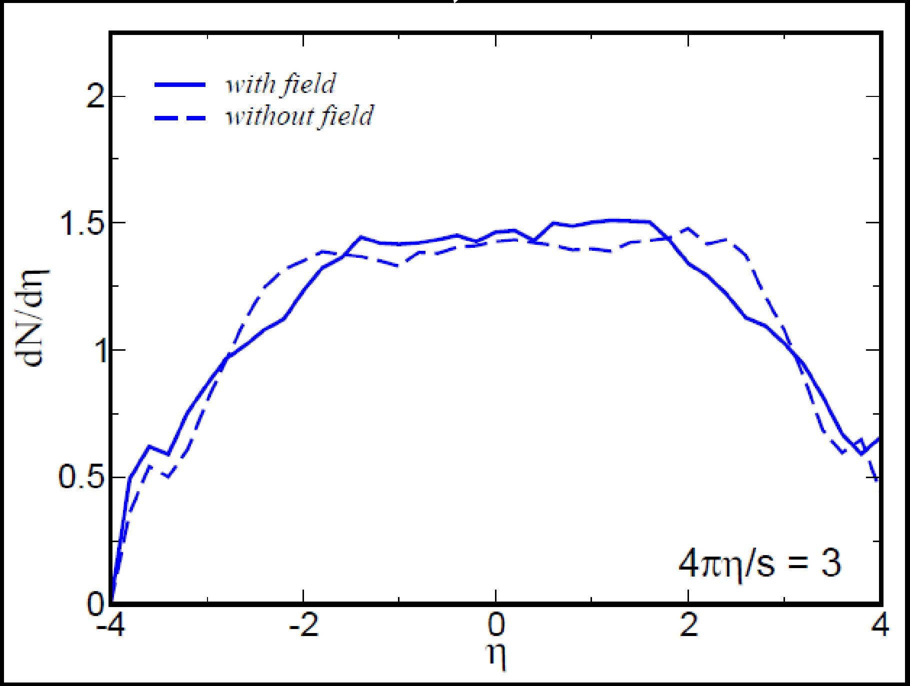
$$E_0 \approx 1.9 \div 2.2 \text{ GeV}^2$$

Very rough estimate:
it gives the proper order of magnitude,
leaving the exact number determination to a more realistic model of the initial tubes distribution

Rapidity distributions



No field calculations:
field is used to create particles, then it is switched off, causing no effect on QGP evolution



Mild effect of field dynamics onto final rapidity distribution

Pressure isotropization

$$T^{\mu\nu}_{field} = \mathrm{diag}\left(\varepsilon, P_T, P_T, P_L\right)$$
$$\propto \mathrm{diag}\left(\mathcal{E}^2, \mathcal{E}^2, \mathcal{E}^2, -\mathcal{E}^2\right)$$

$$T^{\mu\nu}_{particles} = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{p^\mu p^\nu}{E} f(\boldsymbol{x}, \boldsymbol{p})$$

$$T^{\mu\nu} = T^{\mu\nu}_{particles} + T^{\mu\nu}_{field}$$

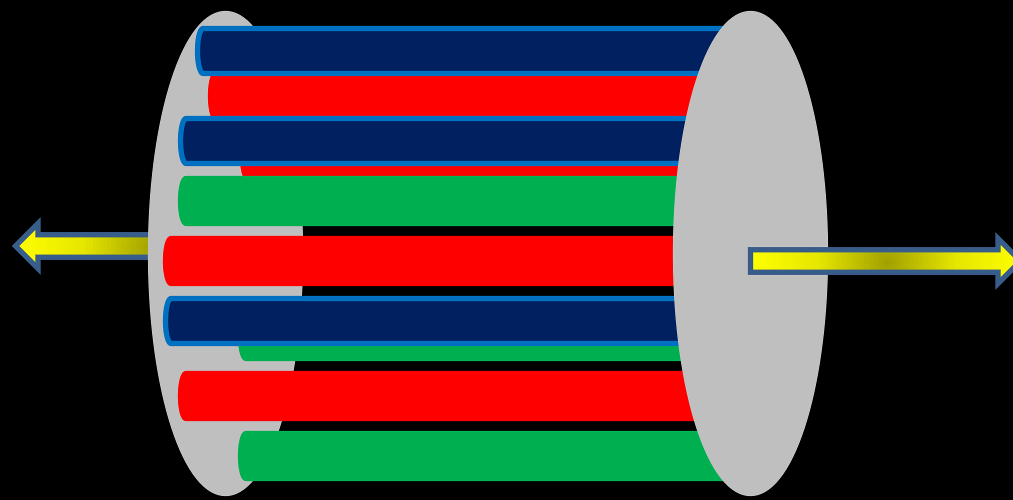
$$P_L = T_{zz}$$

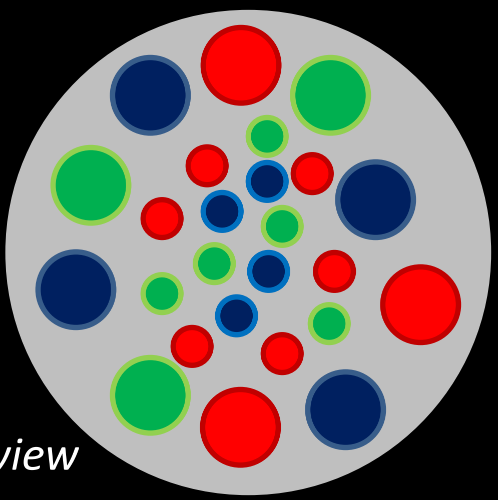
Along flight direction

$$P_T = \frac{T_{xx} + T_{yy}}{2}$$

On transverse plane

Longitudinal view

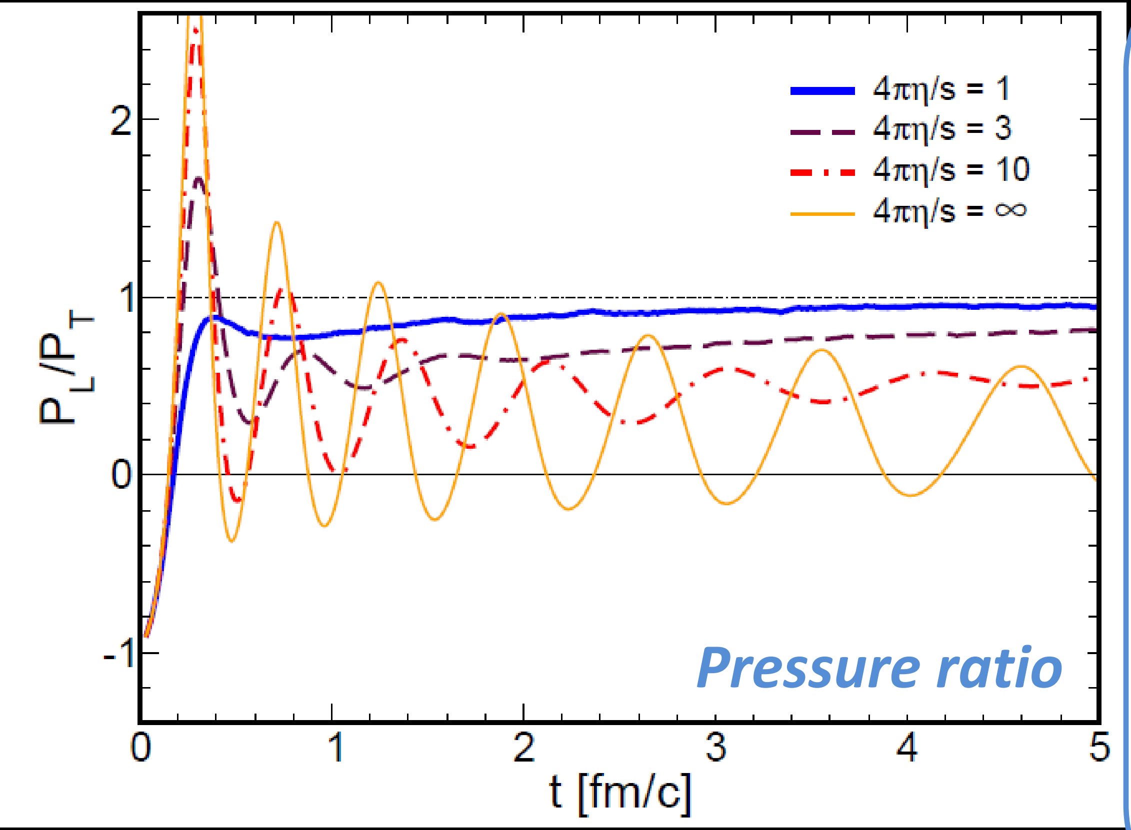




Transverse plane view

Isotropy is achieved if PL = PT

Pressure isotropization



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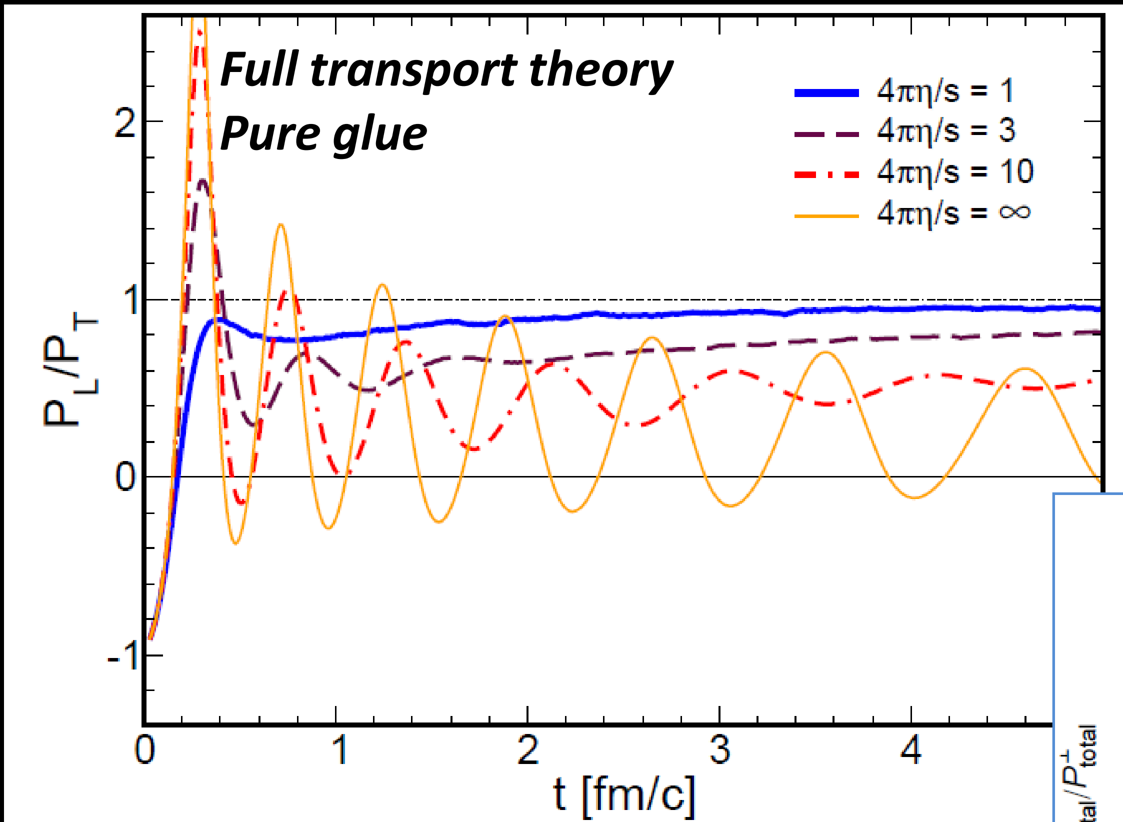
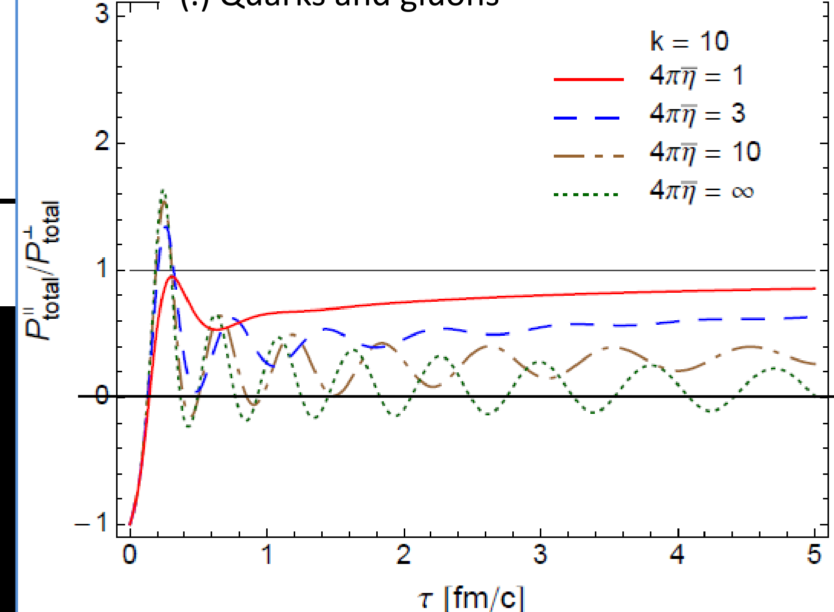
Pressure isotropization

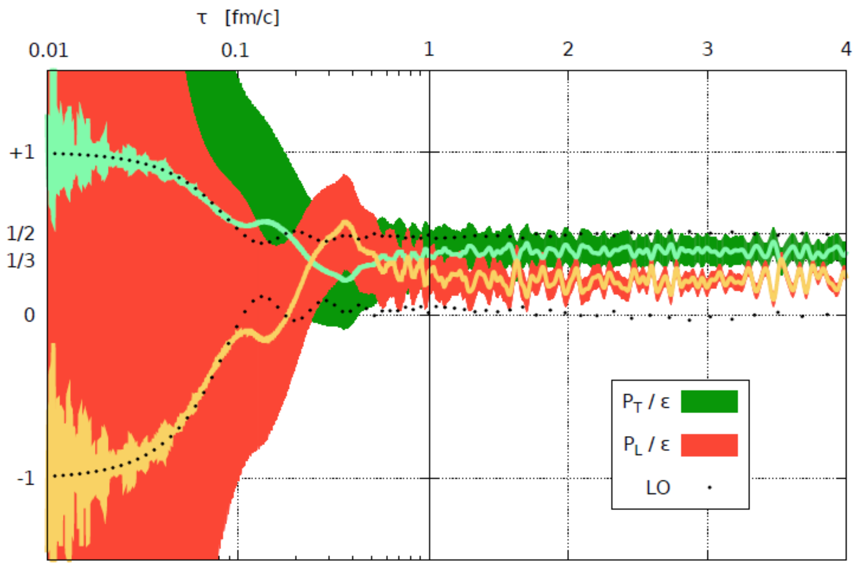


Florkowski and Ryblewski, PRD 88 (2013)
(.) RTA calculation
(.) Quarks and gluons

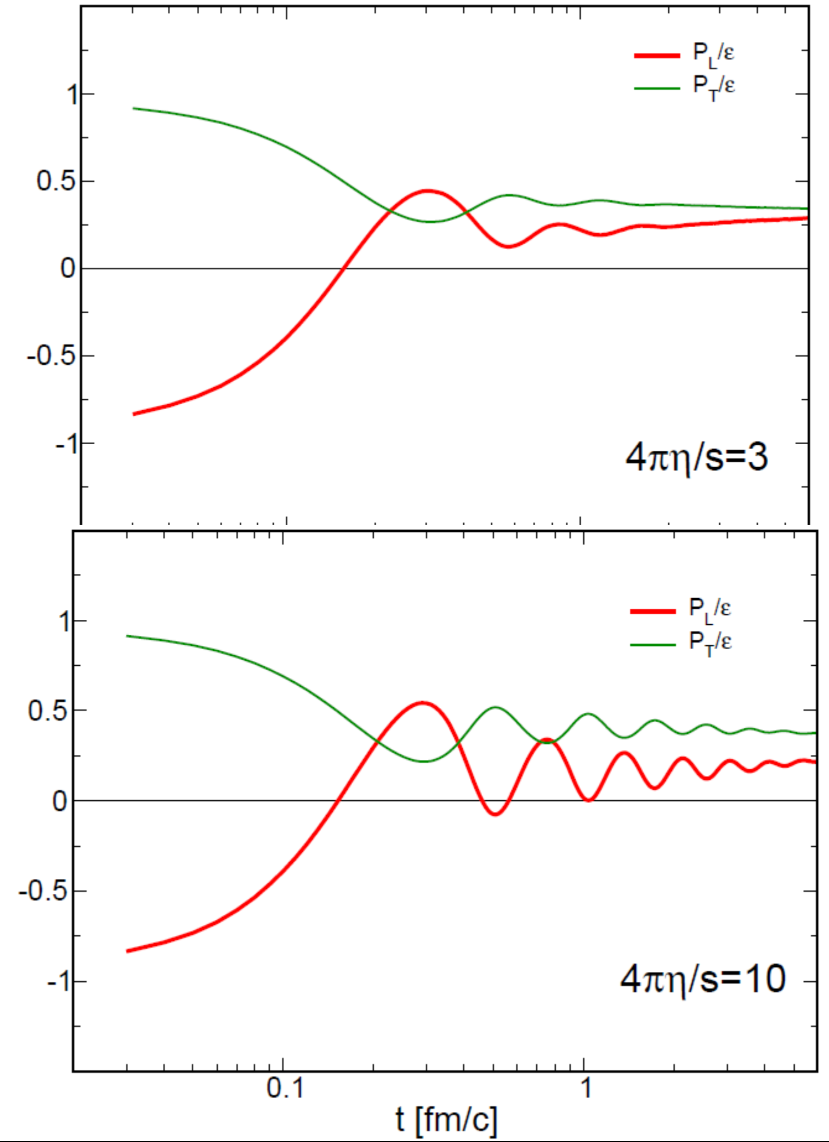


From the *qualitative* point of fiew the agreement is excellent.
Quantitatively, some difference arises because of different calculation scheme.

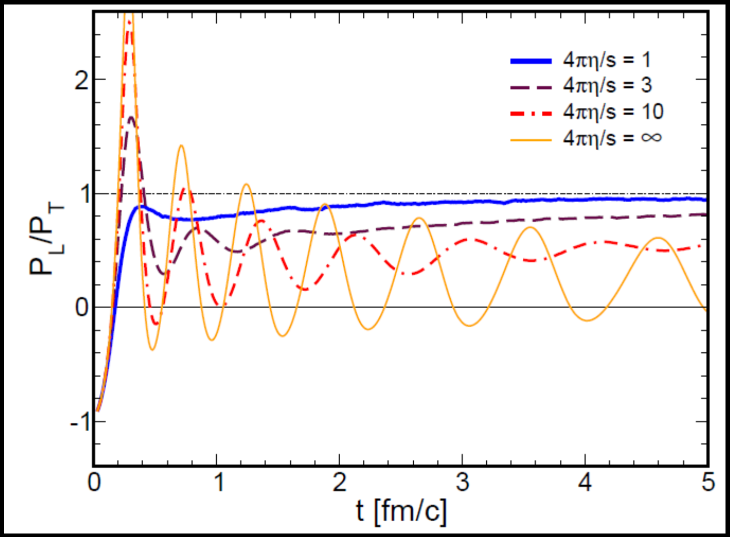
Pressure isotropization

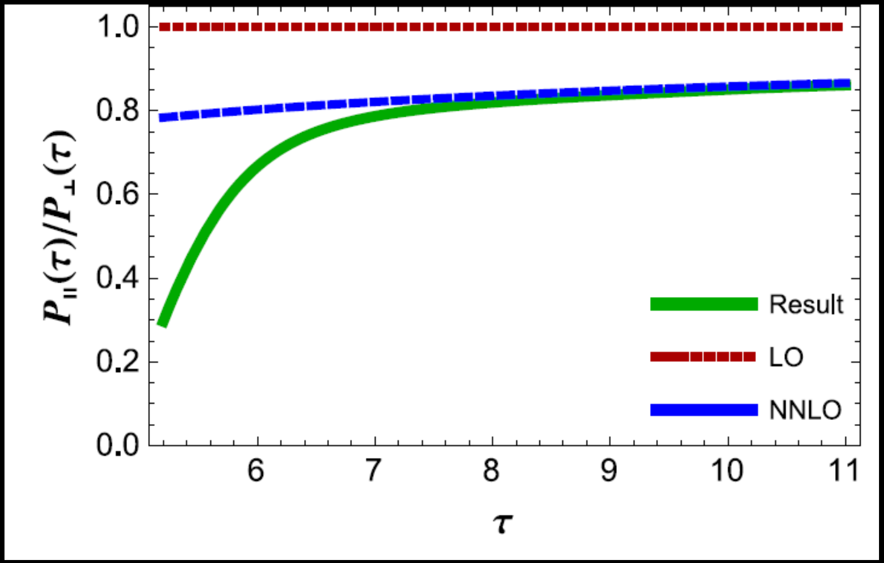


Epelbaum and Gelis, PRL 88 (2013)
(.) Classic Yang-Mills calculation, 3+1D
(.) Quantum fluctuations rather than Schwinger effect

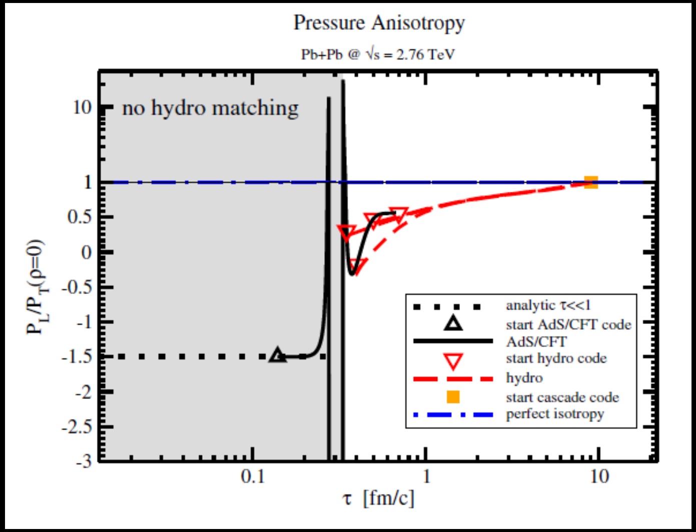


Pressure isotropization





P. Colangelo *et al.*, arXiv:1503.01977



P. Romatschke *et al.*, PRL 111 (2013)

AdS/CFT community is performing interesting calculations

Conclusions

- Relativistic Transport Theory, coupled to a decay mechanism for initial color fields, permits to study early times dynamics of heavy ion collisions.
- *Weakly coupled plasma* is characterized by plasma oscillations which are non negligible along the entire evolution of the system.
- *Strongly coupled plasma* does not experience important plasma oscillations, rather a hydro regime is reached in a very short time
- QGP production in less than 1 fm/c
- Isotropization time is less than 1 fm/c

Outlook

- Generalization to a 3+1D expansion *(work in progress)*
- Generalization to (Ex,Ey,Ez) and (Bx,By,Bz) *(work in progress)*
- Build up a *more realistic initial field configuration*, to describe more quantitatively the initial stage of AA as well as pA and pp collisions:
    - Several flux tubes;
    - One single tube, inhomogeneous on transverse plane;
- Initial rapidity fluctuations in the electric field.
- From abelian to non-abelian field evolution: implementing the simplest non-abelian generalization, namely the Euler-Heisenberg lagrangian.

What should we look for clear indications about the early times dynamics?

- *Photons?*
- *Signs of isotropization?*
- *Flows?*

Thanks for your attention

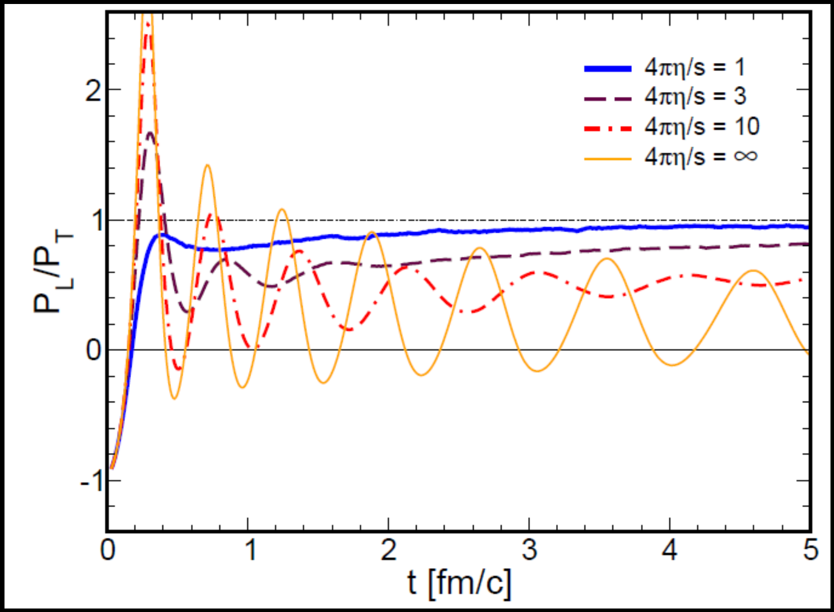
"BE YOURSELF...
EVERYONE ELSE IS ALREADY TAKEN"

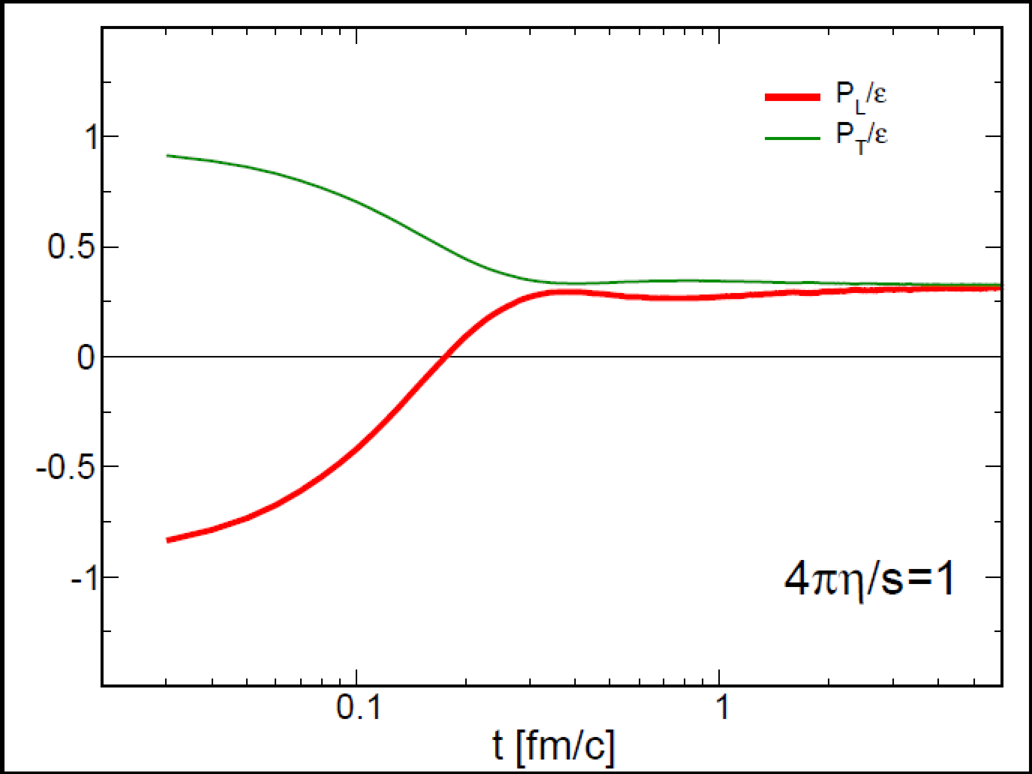
OSCAR WILDE

Appendix

APPENDIX

Pressure isotropization





Schwinger effect in Chromodynamics

Abelian Flux Tube Model

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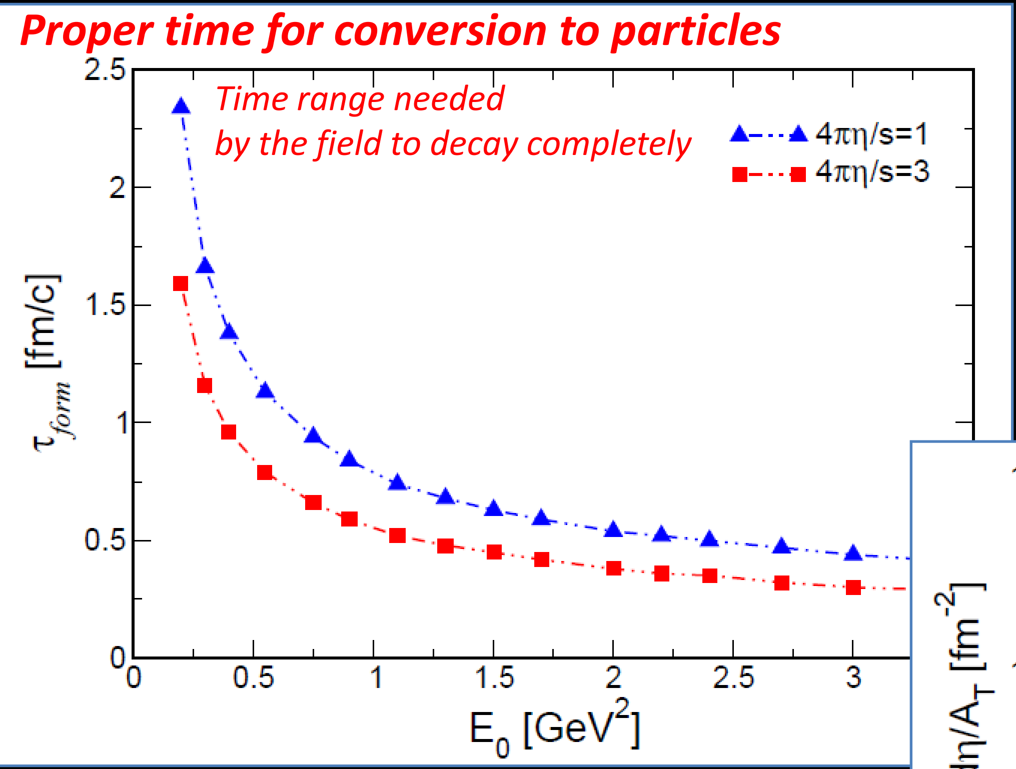
$$\mathcal{E}_{jc} = \left(g|Q_{jc}E| - \sigma_j\right)\theta\left(g|Q_{jc}E| - \sigma_j\right)$$

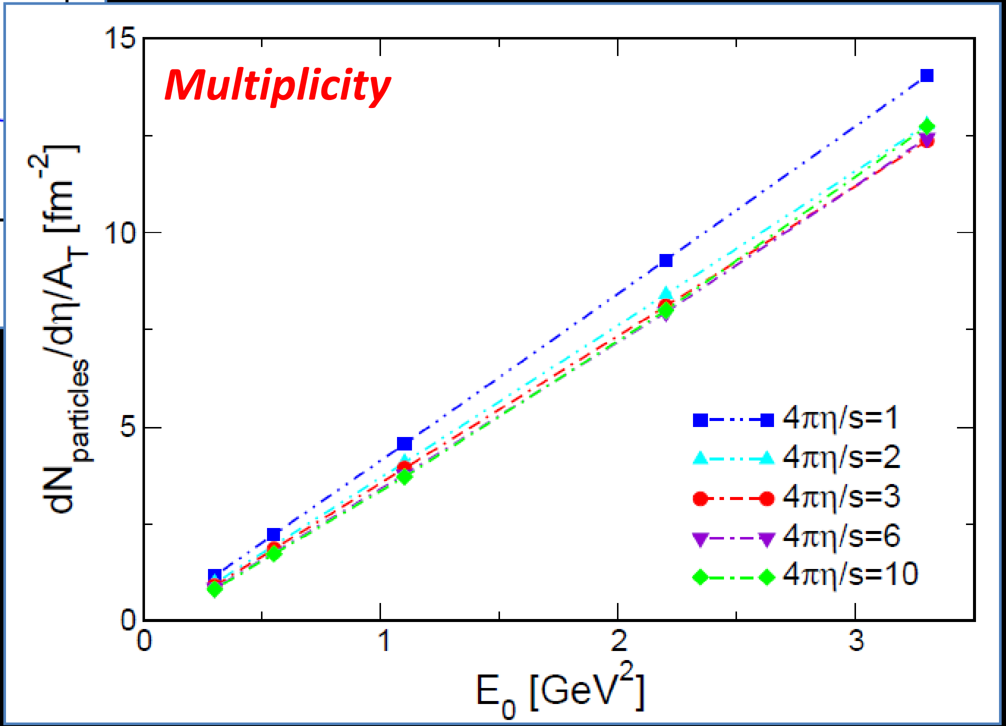
Particles formation



Field evolution satisfies:

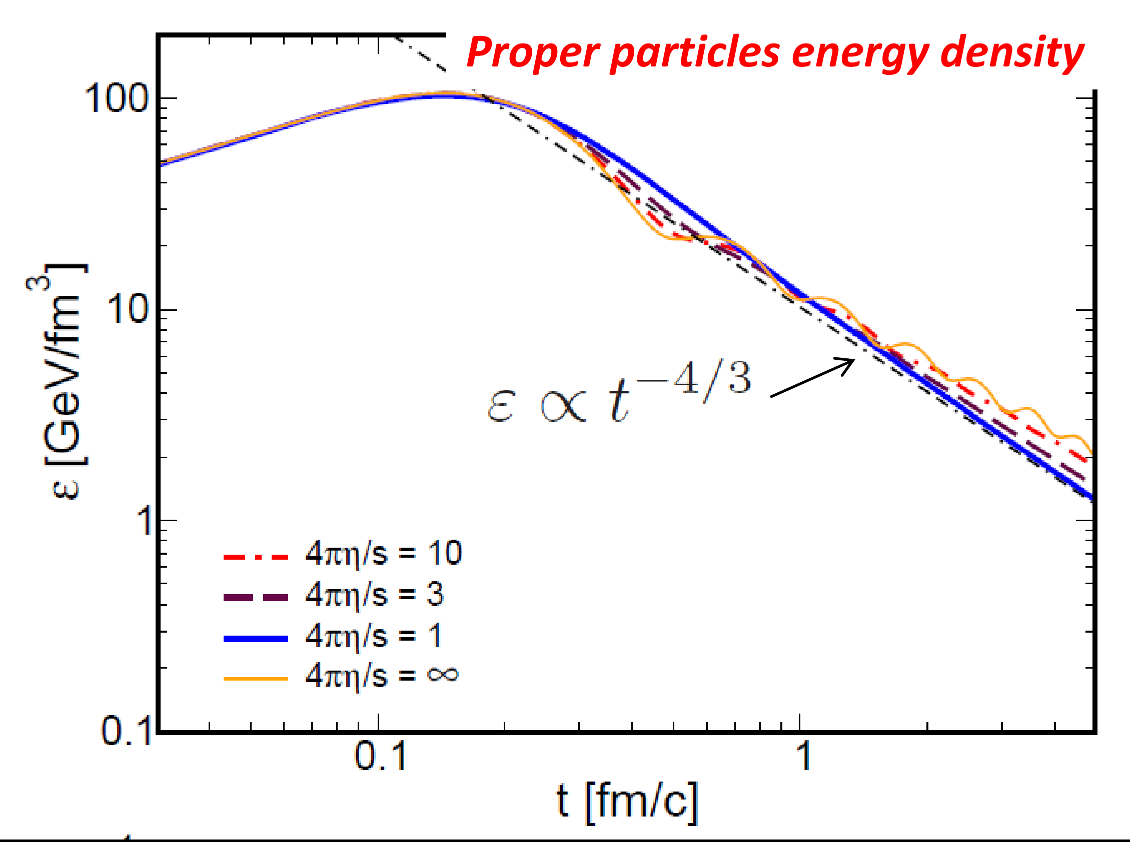
$$\frac{d}{dt}\left(\frac{\boldsymbol{E}^2}{2}\right) = -\boldsymbol{j}\cdot\boldsymbol{E}$$

hence, smaller field implies slower decay.



Unless initial field is very small, formation time is less than 1 fm/c

Flux tube evolution: energy density



Small η/s

After a short transient, the hydro regime begins:

$$\varepsilon \propto t^{-4/3}$$

Large η/s

After a short transient:

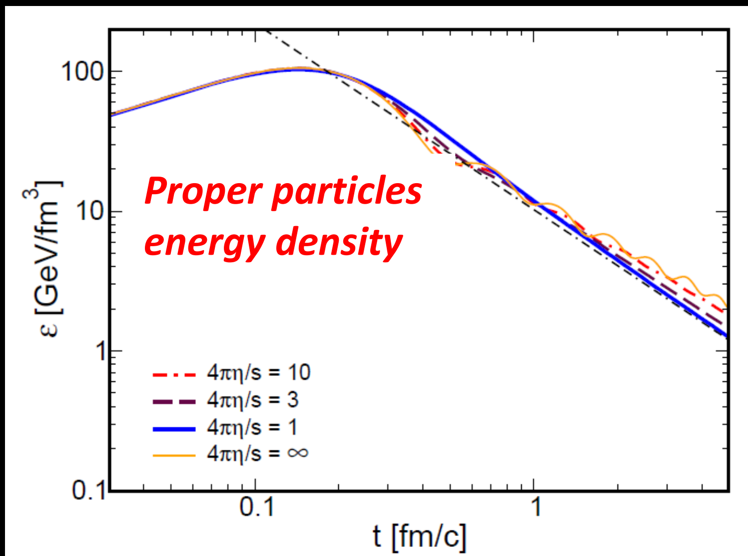
(.) dissipation keeps the system temperature higher;

(.) oscillations arising from the field superimpose to power law decay

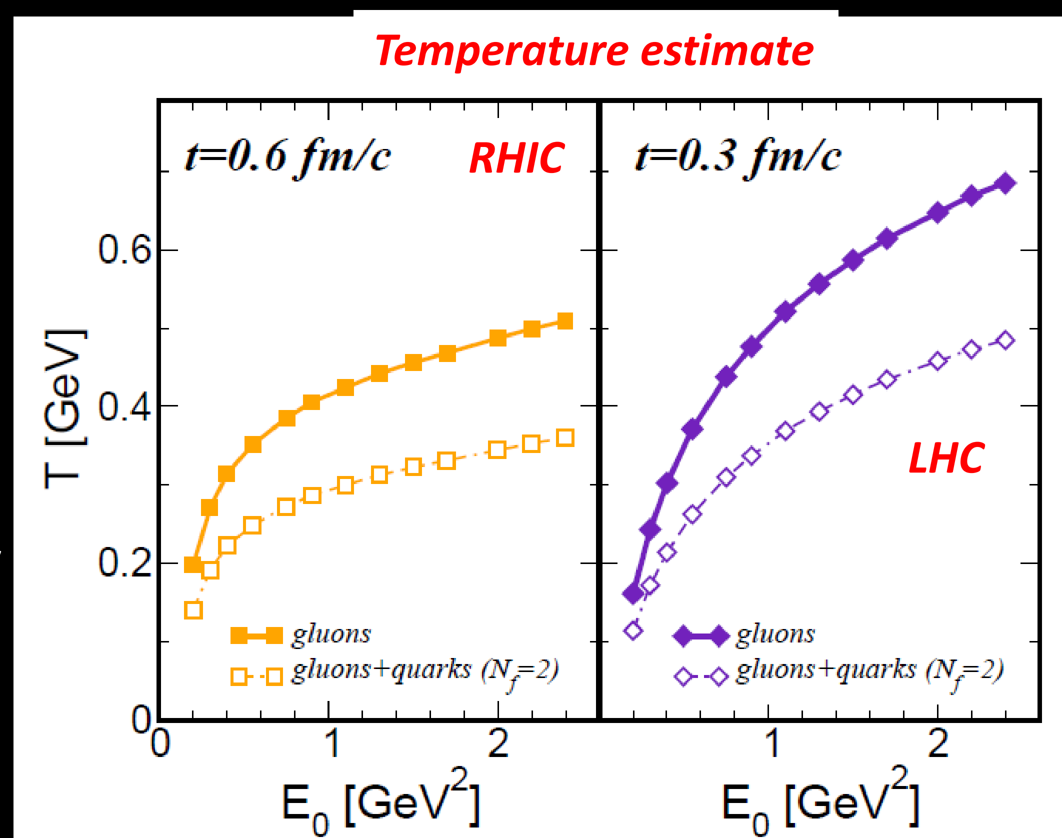
In agreement with ideal hydro calculations: Gatoff *et al.*, PRD 36 (1987)

This is quite interesting because it proves that transport theory is capable to describe, even in conditions of quite strong coupling (small η/s), the evolution of physical quantities in agreement with calculations based on hydrodynamics, once the microscopic cross section is put aside in favor of fixing η/s.

Flux tube evolution: local temperature



$$\varepsilon \propto T^4$$



For simulations at RHIC energy:
(.) free streaming up to t=0.6 fm/c
(.) assume a core temperature T=0.34 GeV

For simulations at LHC energy:
(.) free streaming up to t=0.3 fm/c
(.) assume a core temperature T=0.5 GeV

Schwinger effect in Electrodynamics

Maxwell equations: static box

We will be interested to very simple geometrical configurations, in which:
(.) Only one component of the electric field is non vanishing
(.) The electric field depends only on time and one space coordinate

$$\nabla \times \boldsymbol{B} = -j_M - \frac{\partial(\boldsymbol{E} + \boldsymbol{P})}{\partial t}$$

$\boldsymbol{P}(\boldsymbol{x}, t)$ *electric dipole moment* at point (***x***,*t*)

j_M *Conduction current* Due to charge movement

Given the symmetries of the problem:

$$\frac{dE}{dt} = -j_M - \frac{dP}{dt}$$

$$j_M = \sum_{species} g \int \frac{d^3\boldsymbol{p}}{|\boldsymbol{p}|} p_z f(|\boldsymbol{p}|, t)$$

The dipole moment is formed in the vacuum by the Schwinger effect:

Number of dipoles

$$\begin{aligned} \mathcal{W}(x) &= -\frac{|g\boldsymbol{E}|}{4\pi^3} \int d^2p_T \log\left(1 - e^{-\frac{\pi^2 E_T^2}{|g\boldsymbol{E}|}}\right) \\ &:= \frac{\partial}{\partial t}\left(\frac{dN}{d^3x d^3p}\right) = \frac{\partial \#_{dipoles}}{\partial t} \end{aligned}$$

Electric dipole moment

$$p = g \times 2 \times \frac{d}{2} = g\frac{2E_T}{|g\boldsymbol{E}|}$$

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$$j_D \equiv \frac{\partial P}{\partial t} = \int d^3p g \frac{2E_T}{gE} \times \frac{dN}{d^4x d^3p}$$

Schwinger effect in Electrodynamics

Maxwell equations: static box

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$\boldsymbol{P}(\boldsymbol{x},t)$ *electric dipole moment* at point (***x***,*t*)

j_M *Conduction current*
Due to charge movement

Given the symmetries of the problem:

$$\frac{dE}{dt} = -j_M - \frac{dP}{dt}$$

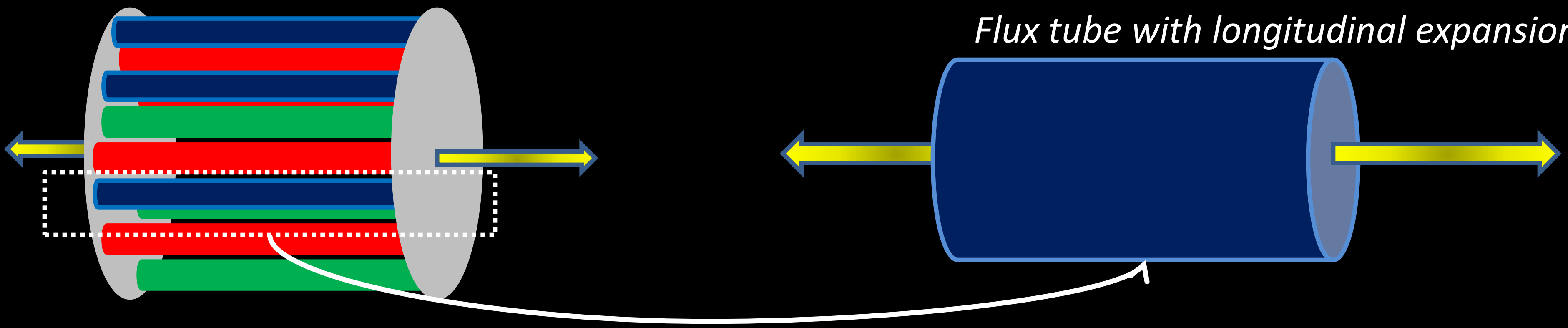
Hence Maxwell equation reads

$$\frac{dE}{dt} = -j_D - j_M$$

$$j_M = \sum_{species} g \int \frac{d^3\boldsymbol{p}}{|\boldsymbol{p}|} p_z f(|\boldsymbol{p}|, t)$$

$$j_D \equiv \frac{\partial P}{\partial t} = \int d^3p g \frac{2E_T}{gE} \times \frac{dN}{d^4x d^3p}$$

Boost invariant expansion



$$(p_\mu \partial^\mu + gQ_{jc}F^{\mu\nu}p_\mu \partial^p_\nu)\, f_{jc} = p_0 \frac{\partial}{\partial t}\frac{dN_{jc}}{d^3x d^3p} + \mathcal{C}[f]$$

We assume field dynamics is ***boost invariant.*** This means that, given

$$\tau = \sqrt{t^2 - z^2}$$ ***Proper time***

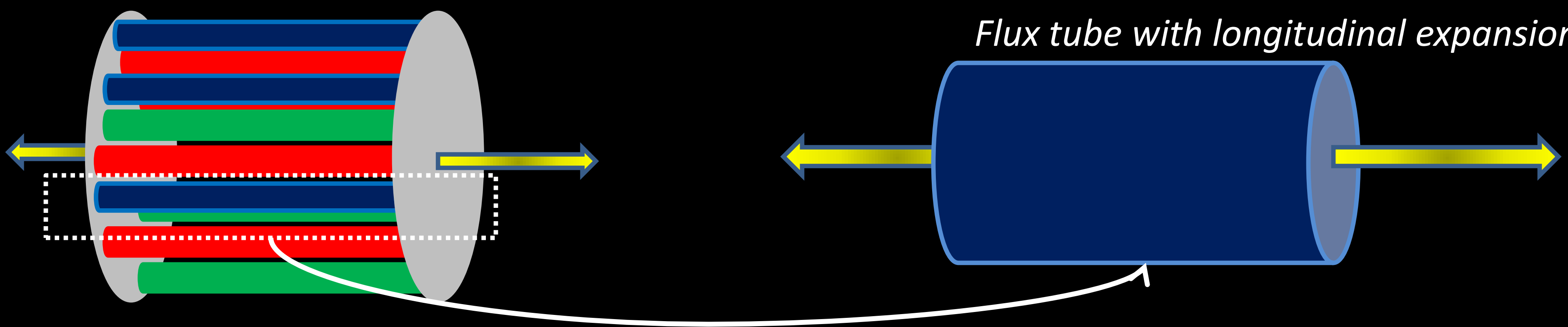
$$\eta = \frac{1}{2}\log\left(\frac{t+z}{t-z}\right)$$ ***Space-time rapidity***

$\Longleftrightarrow$

$$t = \tau\cosh\eta$$
$$z = \tau\sinh\eta$$

then E=E(t,z)=E(τ), hence independent on η.

Boost invariant expansion

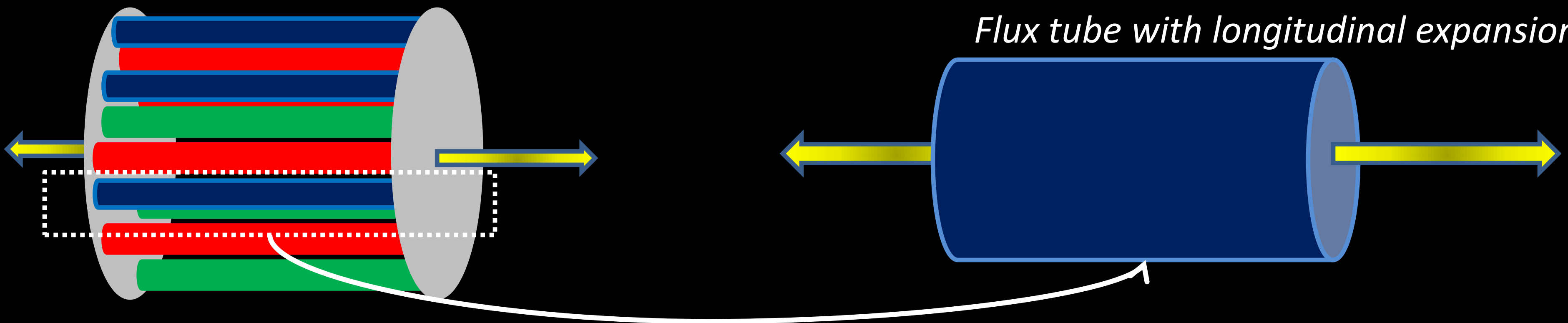


$$(p_\mu \partial^\mu + g Q_{jc} F^{\mu\nu} p_\mu \partial_\nu^p) f_{jc} = p_0 \frac{\partial}{\partial t} \frac{dN_{jc}}{d^3x d^3p} + \mathcal{C}[f]$$

We assume field dynamics is ***boost invariant.*** This means E=E(τ), hence independent on η:

$$\left.\begin{aligned} \frac{\partial E}{\partial z} &= \rho \\ \frac{\partial E}{\partial t} &= -j \end{aligned}\right\} \qquad \frac{dE}{dt} = \rho \tanh\eta - j_M - \frac{j_D}{\cosh\eta}$$

Boost invariant expansion



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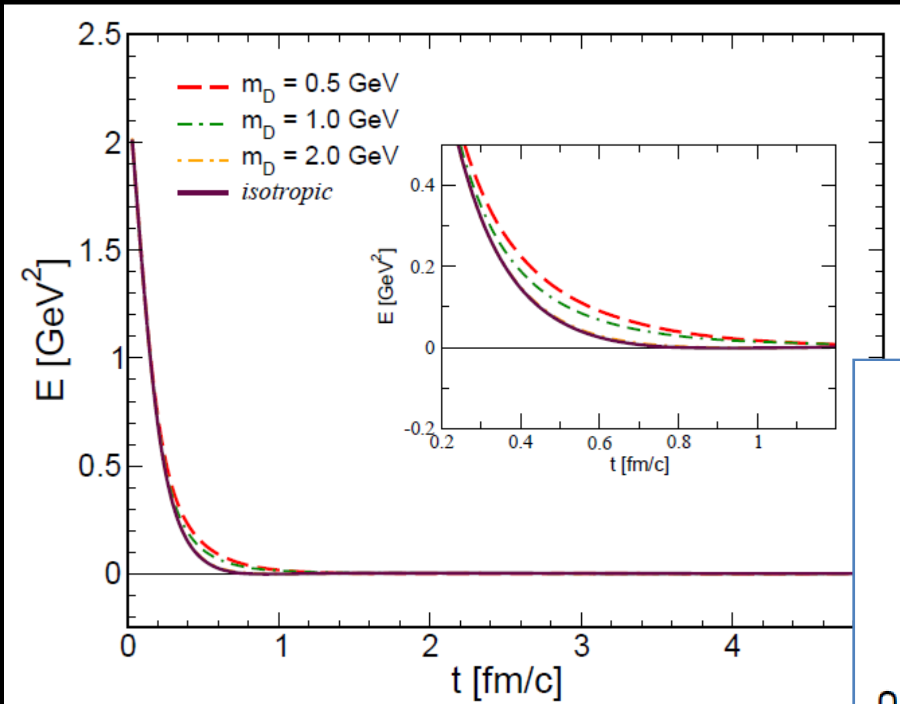
depend on distribution functions

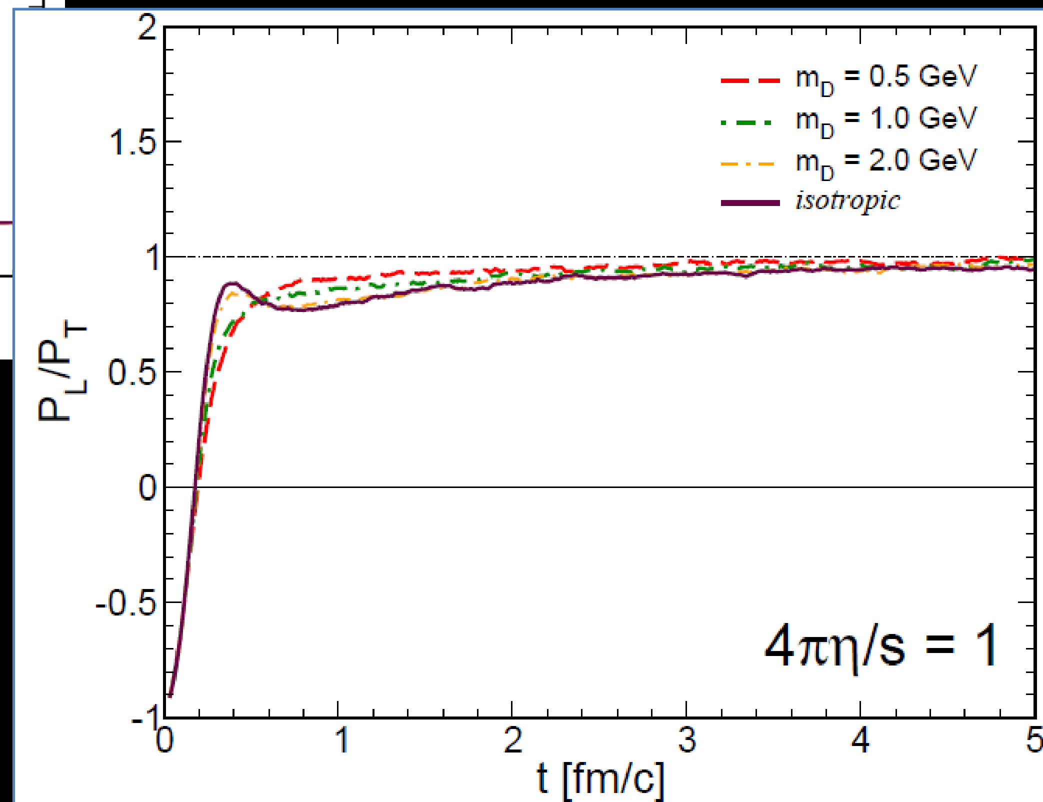
Link Maxwell equation to kinetic equation

Anisotropic cross section

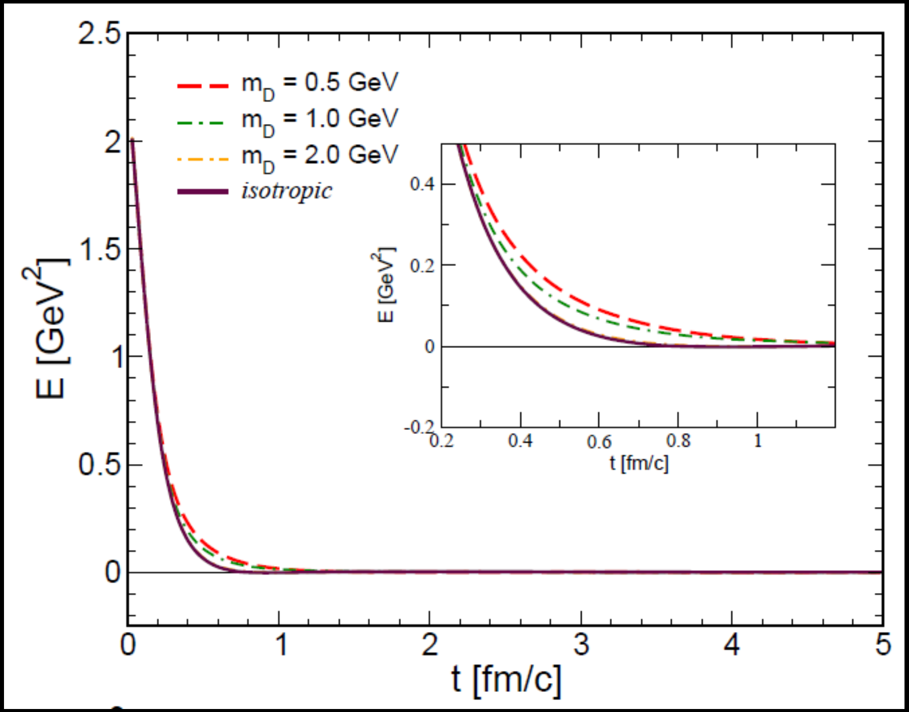


$$\frac{d\sigma}{dt} = \frac{9\pi\alpha_s^2}{2}\frac{1}{(t-m_D^2)^2}\left(1+\frac{m_D^2}{s}\right)$$

Increasing m_D results in more isotropic $d\sigma/dt$



Anisotropic cross section



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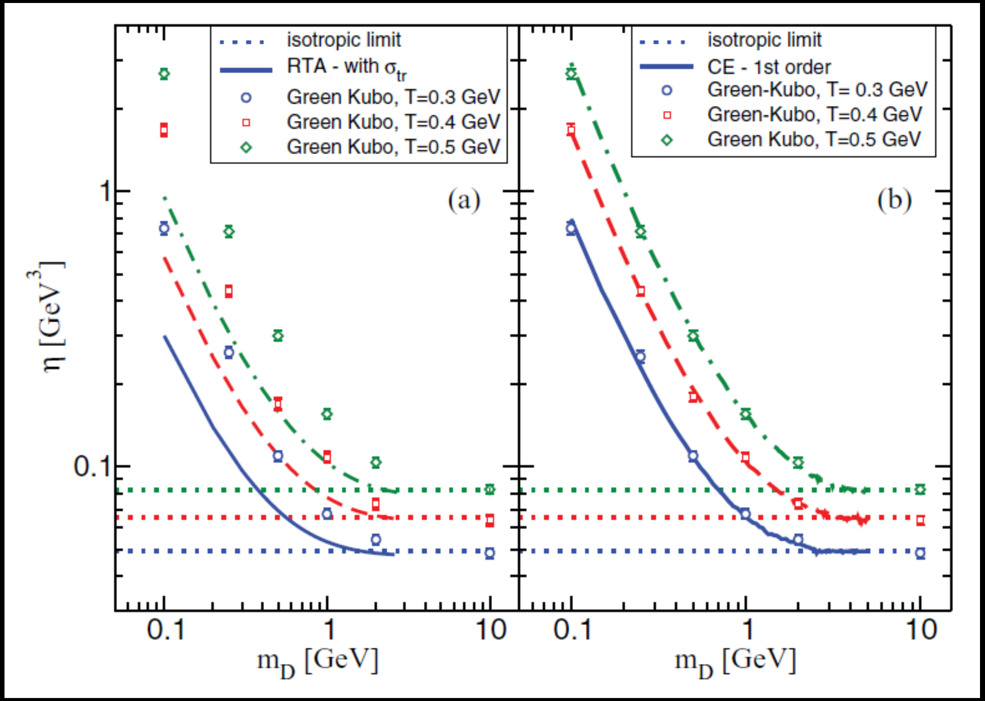
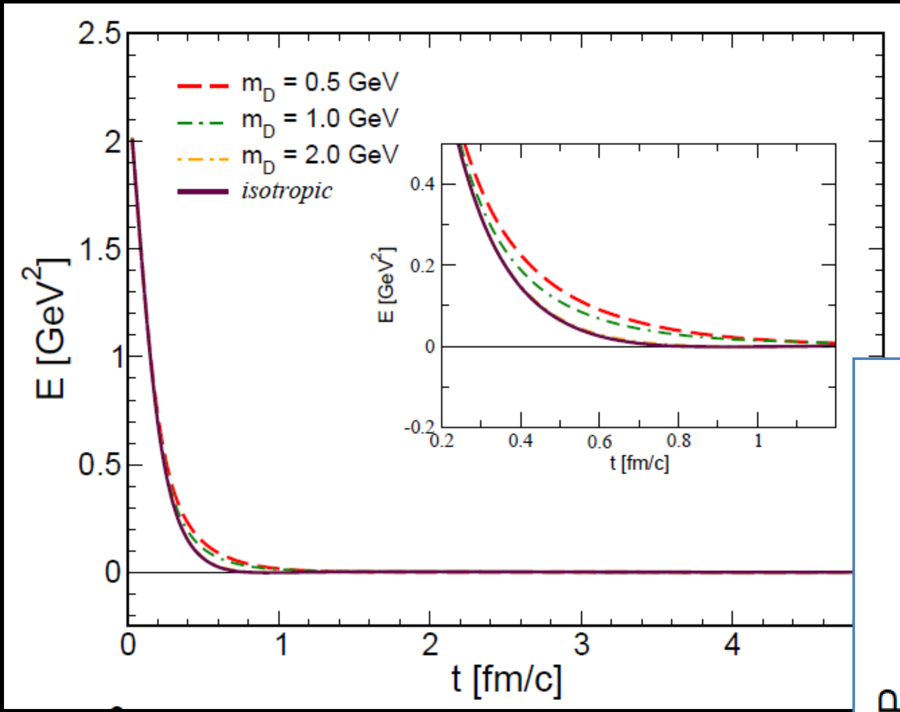


Lowering m_D at fixed η/s results in larger σ, hence in a smaller conductivity

Reminder:
conductivity enhances plasma oscillations.

Plumari et al., PRC86 (2012)

Anisotropic cross section



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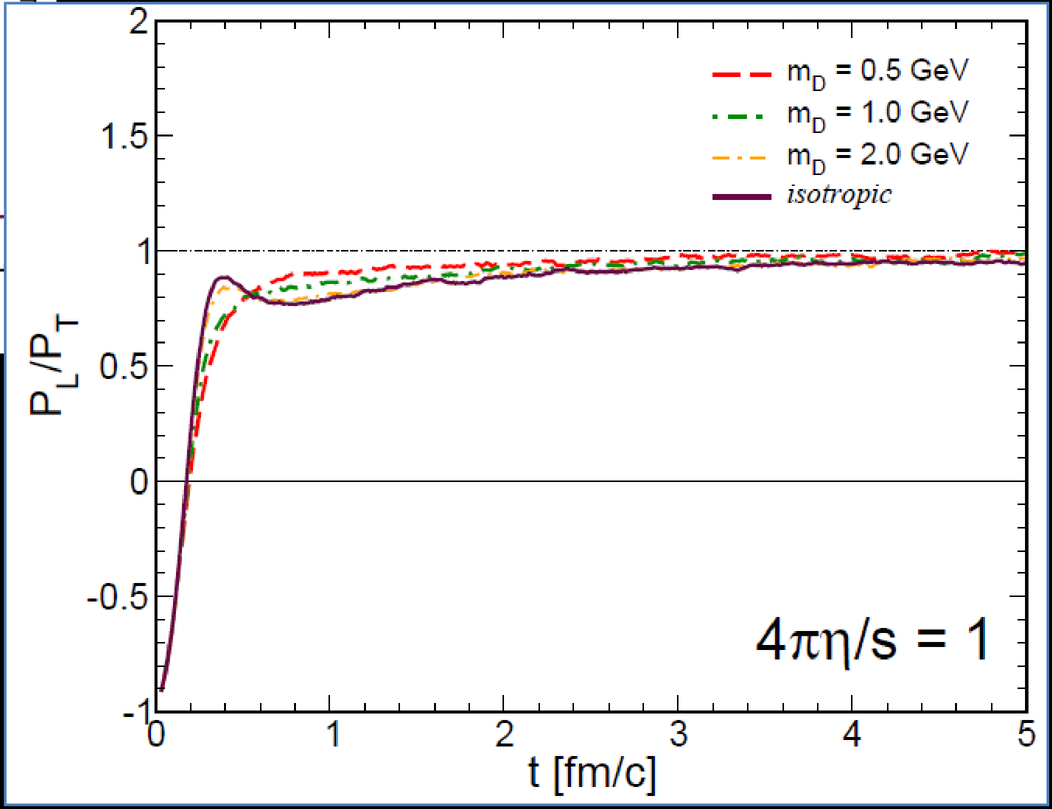
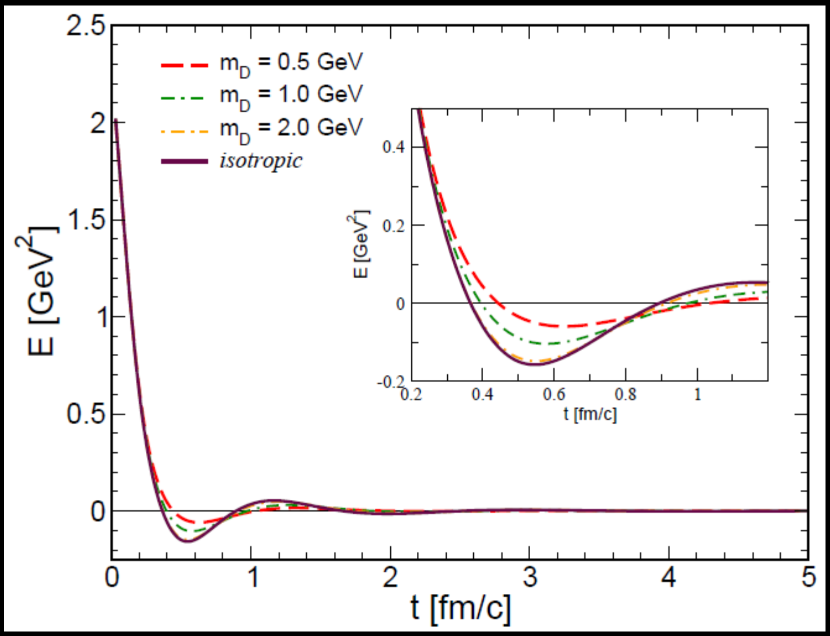


Changing m_D has some effect on the early times dynamics, however leaving practically unchanged the previous results:

(.) isotropization time

(.) isotropization efficiency

Anisotropic cross section



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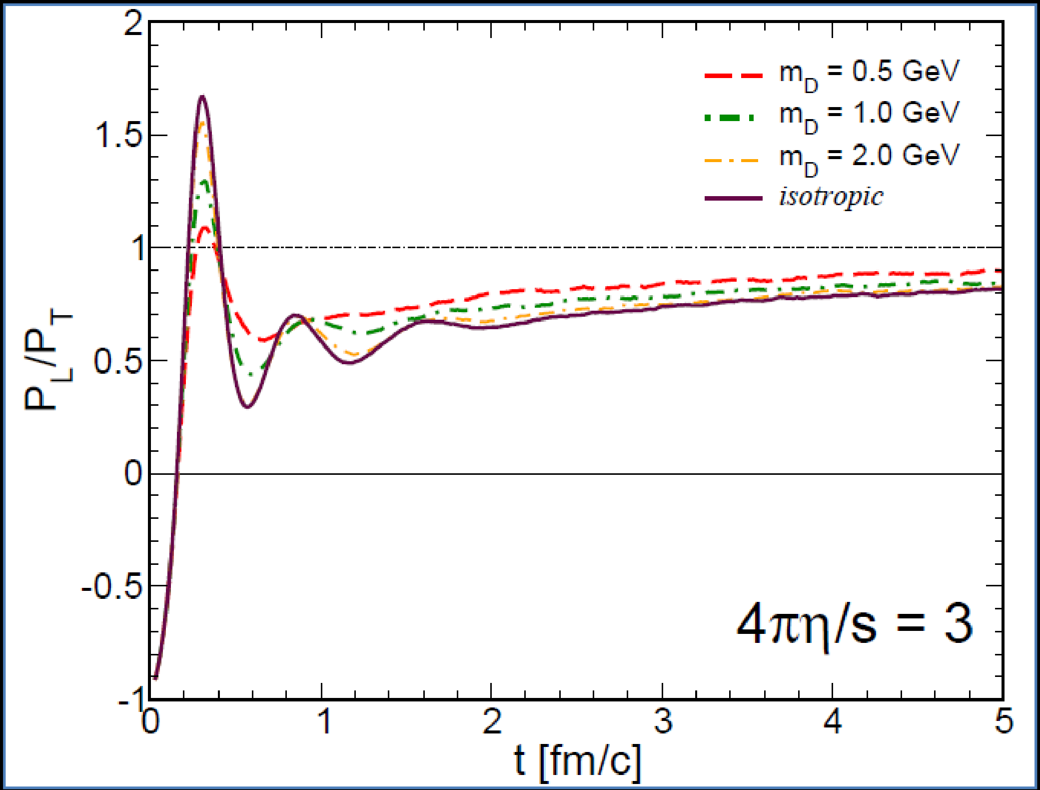

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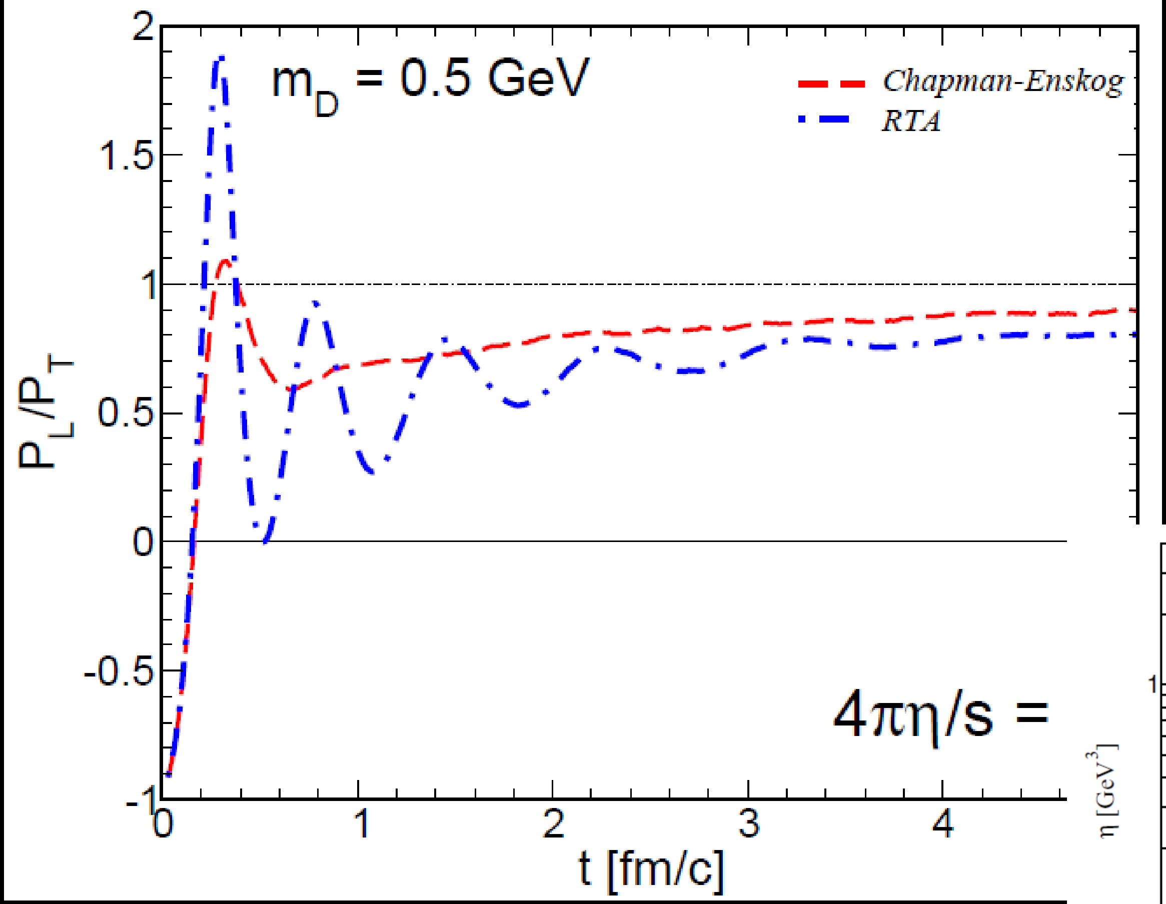
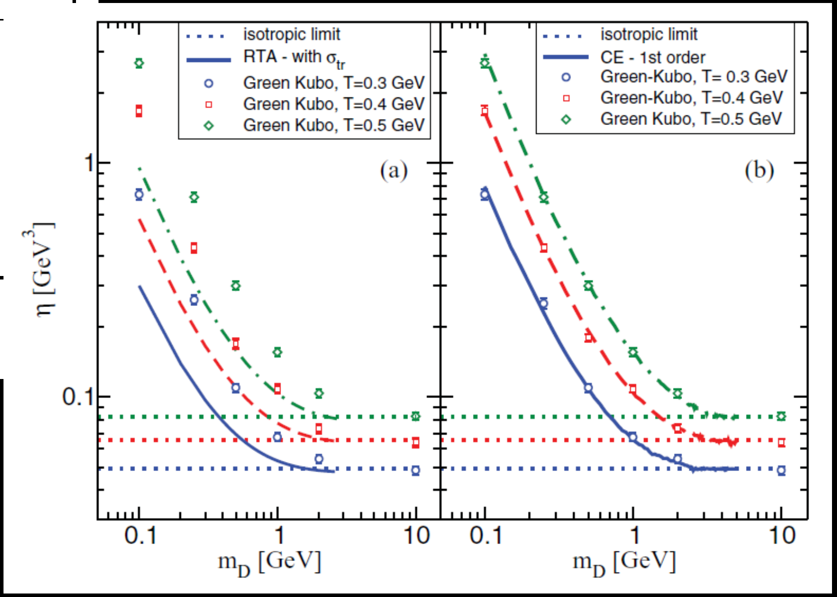
Relaxation time vs Chapman Enskog

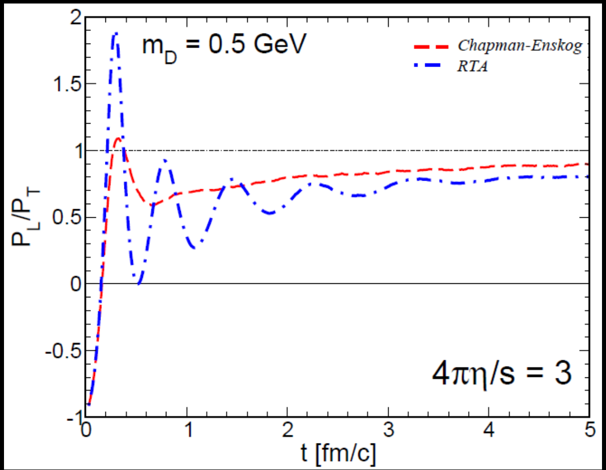


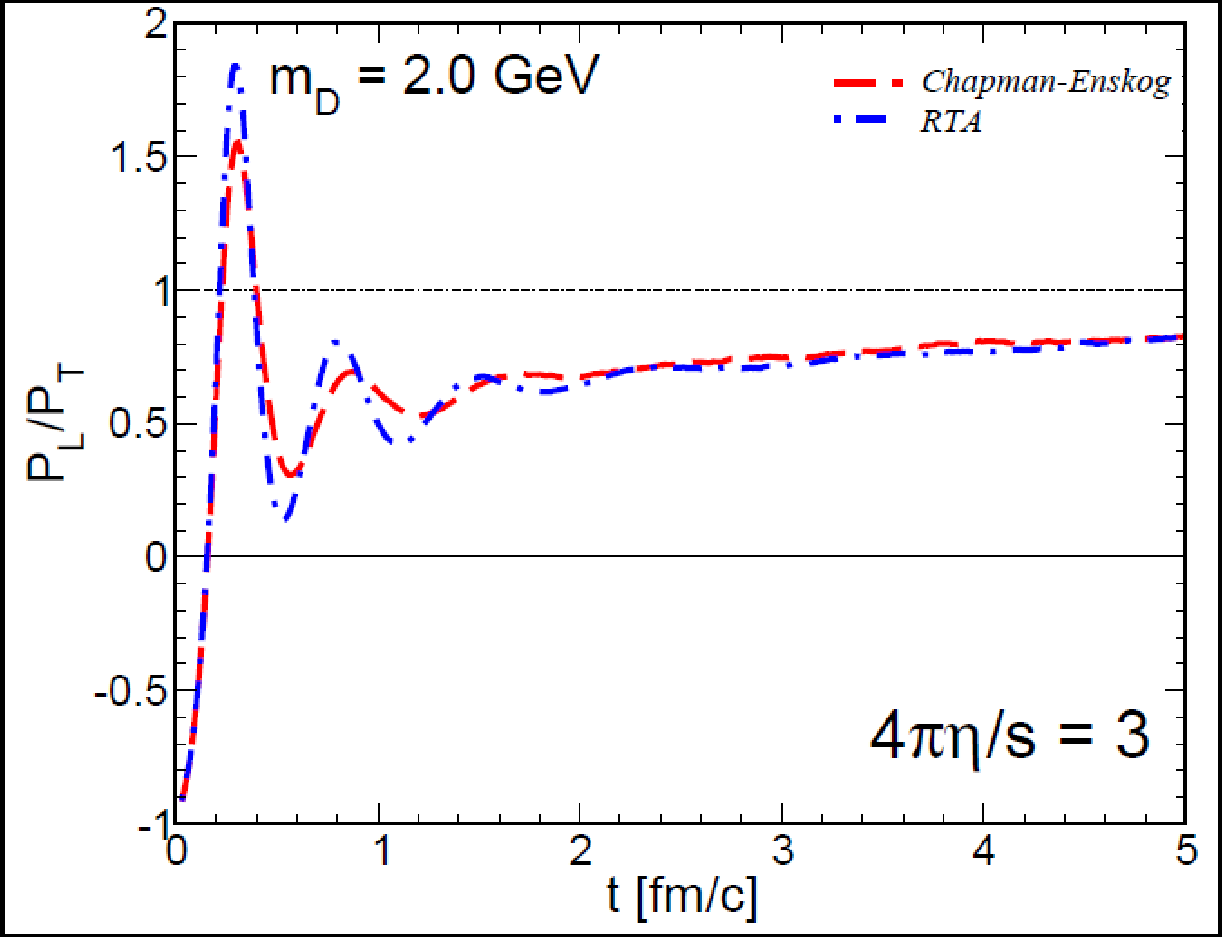


At a given m_D and fixed η/s,
RTA cross section is smaller than CE cross section, implying a larger conductivity in the former case.

Plumari et al., PRC86 (2012)

Relaxation time vs Chapman Enskog





Disagreement among RTA and CE less important for isotropic cross section.