Magnetic fields in HIC: inputs from Lattice QCD

Marco Mariti

Incontro sulla fisica con ioni pesanti a LHC

26/05/2015, Bologna

QCD IN MAGNETIC FIELD

- Quark and gluons $\rightarrow$ Dynamics ruled by strong interactions
- Quarks carry also electric charge $\rightarrow$ Can electromangetic interactions become relevant?
- External magnetic field B on the strong scale $\rightarrow eB \simeq m_\pi^2$

Why we are interested in QCD with magnetic fields

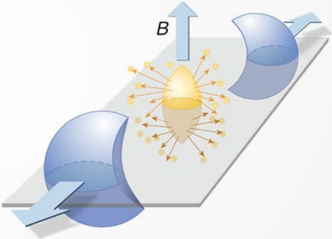
- Vacuum properties of the theory (modification of the confining potential, chiral condensate catalysis,...)
- Phase diagram of QCD ($T_c(B)$,...)
- New effects (CME,...)

MAGNETIC FIELD IN HIC

Off-central HIC $\rightarrow$ Largest magnetic field factory on Earth

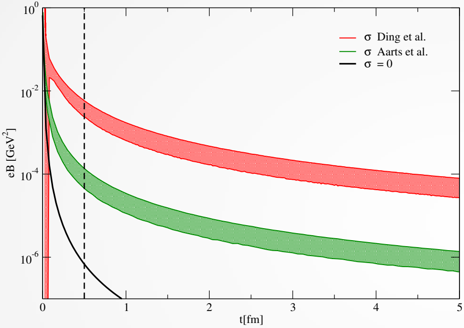
Pb+Pb collisions at LHC

- $R \sim 8$ fm $\longrightarrow$ Ion radius
- $Ze \sim 82\, e$ $\rightarrow$ Electric charge
- $b \sim 4$ fm $\longrightarrow$ Impact parameter
- $\sqrt{s} \sim 2.76$ TeV $\rightarrow$ Nuclei CM energy



$$eB \simeq \frac{\sqrt{s}}{2m_N} Ze^2 \frac{b}{R^3} \approx 15\, m_\pi^2 \approx 0.3 \text{ GeV}^2 \longrightarrow 10^{15} \text{ T}$$

MAGNETIC FIELD IN HIC



$$eB \simeq \begin{cases} \frac{Ze^2 b\gamma}{(b^2+\gamma^2 t^2)^{\frac{3}{2}}} & \text{if } \sigma = 0 \\ \frac{Ze^2 b\sigma}{2t^2} e^{-\frac{b^2\sigma}{4t}} & \text{if } \sigma \neq 0 \end{cases}$$

Tuchin, A. High Energy Phys. (2013) 490495

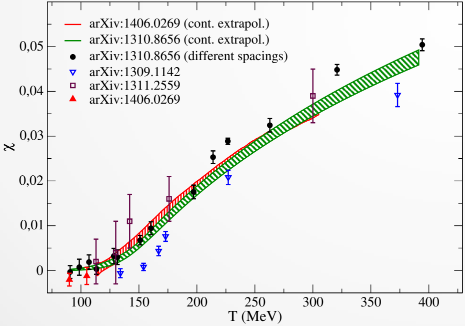
Medium properties

- $\sigma \to$ electrical conductivity
- thanks to σ, magnetic field freezes in the plasma after thermalization $\tau \sim 0.5\ \frac{\text{fm}}{c}$
- Lattice measurements:
  $\sigma = (5.8 \pm 2.9)\frac{T}{T_c}$ MeV
  [Ding et al., '11]
  $\sigma \approx (0.1 \pm 0.05)\frac{2}{3}e^2 \cdot 200$ MeV
  [Aarts et al., '14]
- Expansion of the medium extends magnetic field lifetime

QUARK GLUON PLASMA IN eB

Could eB modify the EoS?

$f \rightarrow$ free energy density $\qquad f(B,T) = f_0(T) - \frac{1}{2}\chi(T)B^2 + ...$



Assumptions

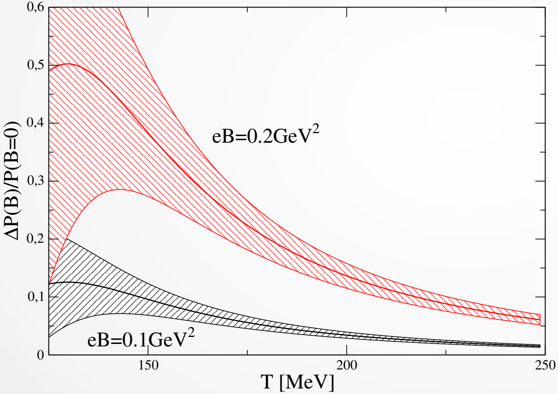
- Global th. equilibrium
- Uniform magnetic field
- $\mu_B = 0$

Results for $\chi(T)$

- Several determinations from the lattice. First study: Bonati et al., PRL 111 (2013) 182001
- QGP strongly paramagnetic

QUARK GLUON PLASMA IN eB

$\Delta P(B) = \frac{1}{2}\chi(eB)^2 \longrightarrow$ Magnetic contribution to the pressure



- (10-40)% magnetic contribution to the pressure for fields expected at LHC
- Mainly relevant near the transition point

Bonati et al., Phys. Rev. D89, 054506 (2014)

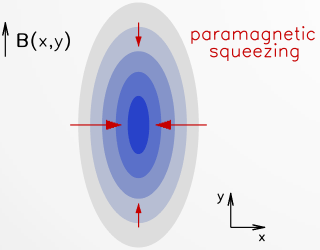
PARAMAGNETIC SQUEEZING

Non uniform magnetic field

$$B(x,y) = B_0\, e^{-\frac{x^2}{\sigma_x^2} - \frac{y^2}{\sigma_y^2}}; \qquad \sigma_x = \sigma_y/2 = 1.3\ \text{fm}; \qquad B_0 = 0.9\ \text{GeV}^2$$

Paramagnetism + Non uniform $B \rightarrow$ Force gradient

$$F = -\nabla f = -\frac{\partial f}{\partial(eB)} \nabla(eB) = M \nabla |eB|$$



Prevision

- Magnetic contribution to v_2
- $\Delta p_{ps} = 0.7 \frac{\text{GeV}}{\text{fm}^4}$ VS $\Delta p_g = 1 \frac{\text{GeV}}{\text{fm}^4}$

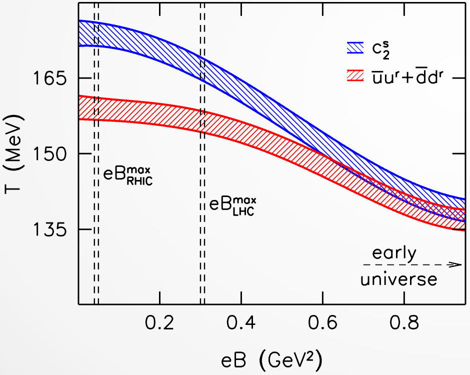
Strongly depends on

- Magnetic spatial and time evolution

Bali et al., Phys. Rev. Lett. 112, 042301 (2014)

T_c DEPENDENCE ON eB

- Light chiral condensate inflection point: $\langle\bar{\psi}_l\psi_l\rangle$, $\psi_l = u, d$
- Strange quark number susceptibility: $c_2^s = \frac{T}{V}\frac{1}{T^2}\frac{\partial^2 \log \mathcal{Z}}{\partial \mu_s^2}$



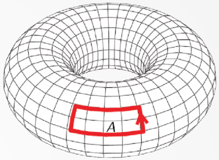
Bali et al., JHEP02(2012)044

- (3 - 6)% decrase of T_c with $eB = 0.3$ GeV^2
- No splitting of deconfinement-chiral symmetry restoration
- Relevant for initial conditions
- If eB freezed in the plasma, relevant for re-hadronization

THANK YOU

MAGNETIC FIELD ON THE LATTICE

- $U_\mu(n) \to U_\mu(n) \exp\left(iqa_\mu(n)\right)$

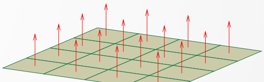


- $e^{iqBA} = e^{iqB(A - L_x L_y a^2)}$ da cui segue $qB = \frac{2\pi b}{L_x L_y a^2}$ con $b \in \mathbb{Z}$

- $\vec{B} = B\hat{z} \to$ gauge fixing $a_y = Bx$, then:

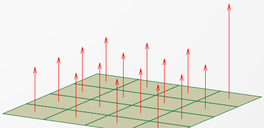
$$u_y^{(q)}(n) = e^{ia^2 qBn_x}$$

$$u_x^{(q)}(n)|_{n_x = L_x} = e^{-i\ a^2 qL_x Bn_y}$$



b=2.00000

- For $b \notin \mathbb{Z}$ string become visible. Non-uniform B



b=2.80000

HOW WE MEASURED $\chi(T)$

$$f(T,B) = f(T,0) - \frac{1}{2}\chi(T)B^2 + \mathcal{O}(B^3) \rightarrow \chi = - \left.\frac{\partial^2 f(T,B)}{\partial B^2}\right|_{B=0}$$

... **But** $\frac{\partial}{\partial B}$ not defined on the lattice!

- Compute **finite free energy differences**:
  $\Delta f(T,b) = f(T,b) - f(T,0)$
- Choose a path connecting two points in parameter space $A = (T;b) \rightarrow B = (T;0)$. Then $\Delta f(T,b) = -\frac{T}{V}\int_A^B \frac{\partial \log Z}{\partial \vec{p}} d\vec{p}$
- We choose to go straight in b introducing a real valued magnetic field $\rightarrow$ we can evaluate $\frac{\partial \log Z}{\partial b} \rightarrow \Delta f(T,b) = -\frac{T}{V}\int_0^b \frac{\partial \log Z}{\partial \tilde{b}} d\tilde{b}$
- For $eB = \frac{2\pi b}{L_x L_y a^2}$ with $b \notin \mathbb{Z}$ intermediate points does not corresponde to the uniform field case
- $\frac{\partial \log Z}{\partial b}$ is not the physical magnetization.