Ab Initio Calculations of Light Hypernuclei

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Strangeness in nuclear many-body systems

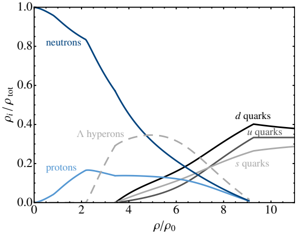
Interdisciplinary subject connecting particle physics, nuclear physics and astrophysics.

Related topical questions include:

- interaction of (anti)kaons with the nuclear medium
  - possible existence of deeply-bound K^-–nuclear states?
  - antikaons in dense matter?
- interaction of hyperons with the nuclear medium
  - S=-1 Λ hypernuclei, Σ-hypernuclei?
  - S=-2 $\Lambda\Lambda$-hypernuclei, Ξ hypernuclei
  - hyperons in dense nuclear matter and neutron stars?

Role of strangeness in dense nuclear matter?

- **admixtures of Λ and Σ hyperons in dense baryonic matter in neutron stars?**
- at baryon densities $\rho \gtrsim (2-3)\rho_0$,
  Λ hyperons can take role of neutrons if energetically favourable



(Hell, Weise, arXiv:1402:4098 [nucl-th])

- presence of hyperons results in considerable softening of EOS incompatible with recent astrophysical constraints – by reducing the maximum neutron star mass much below $2M_\odot$
- additional **repulsive** interactions are needed

Role of strangeness in dense nuclear matter?

- repulsive NNY forces?

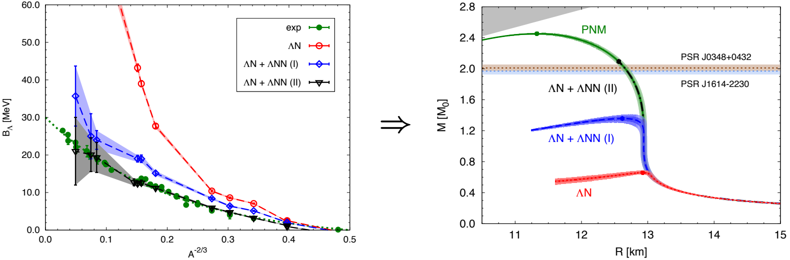


Figure: Hypernuclear binding energies (left) and neutron star mass–radius relations (right). (Taken from D. Lonardoni *et al.*)

Role of strangeness in dense nuclear matter?

- NLO chiral NY interactions
  $\rightarrow$ strong **repulsion** at higher energies

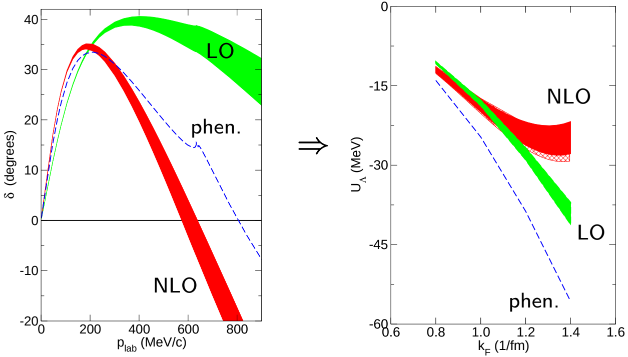


Figure: ${}^1S_0$ ΛN phase shifts (left) and Λ single-particle potential in matter (right).
(Haidenbauer, Meißner, NPA 936, 29 (2015))

Study of hypernuclei

- improve understanding of YN interaction
  - provide important constraints on YN interaction
  - precise experimental data on hypernuclear spectroscopy
  - supplement (very sparse) hyperon–nucleon scattering data base
- new precision experiments at J-PARC, J-Lab, FAIR, ...
- modern developments of YN interaction
  - based on SU(3) chiral EFT
  - require advanced many-body computational methods to confront with hypernuclear structure measurements

Ab initio calculations of light hypernuclei

- given microscopic NN (+NNN) and YN interactions, calculate the energy spectra of A-body hypernuclear system with controllable approximations
- calculations so far limited to A=3,4 hypernuclear systems (Faddeev, Faddeev–Yakubovsky equations)
- recent developments in computational many-body methods

Our aim:

- develop a method applicable to heavier A≥5 hypernuclei
- study available boson-exchange and chiral YN interaction models

No-core shell model for hypernuclei

Ab initio

- all particles are active (no rigid core)
- exact Pauli principle
- realistic 2- and 3-body interactions (accurate description of NN and YN data)
- controllable approximations

- Hamiltonian is diagonalized in a *finite* A-particle harmonic oscillator basis
- NCSM results converge to exact results

No-core shell model for hypernuclei

- two independent NCSM formulations developed:

Slater-determinant HO basis

+ starting with atisymmetrized basis
+ second quantization methods
− c.m. degree of freedom present $\Rightarrow$ huge basis

relative Jacobi-coordinate HO basis

+ c.m. d.o.f. removed
  $\Rightarrow$ smaller basis
  $\Rightarrow$ larger model space possible
− the basis has to be antisymmetrized

Input V_{NN}, V_{NNN}, and V_{NY} potentials

NN+NNN interaction

- chiral N3LO NN potential
  (Entem, Machleidt, PRC 68 (2003) 041001)
- chiral N2LO NNN potential
  (Navrátil, FBS 41 (2007) 14)

YN interaction

- phenomenological meson-exchange Jülich04 potential
  (Haidenbauer, Meißner, PRC 72 (2006) 044005)
- chiral LO potential
  NLO version recently developed (Haidenbauer *et al.*, NPA 915 (2013) 24)

$\Lambda N - \Sigma N$ mixing explicitly taken into account:

$$V_{NY} = \begin{pmatrix} V_{\Lambda N - \Lambda N} & V_{\Lambda N - \Sigma N} \\ V_{\Sigma N - \Lambda N} & V_{\Sigma N - \Sigma N} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & m_\Sigma - m_\Lambda \end{pmatrix}$$

Similarity renormalization group effective interaction

- "bare" interactions require large model spaces for reasonable convergence
- $\rightarrow$ series of unitary transformations of the original Hamiltonian H:

$$H_\lambda = U_\lambda H U_\lambda^\dagger, \; H = H_\infty$$

implemented as a flow equation in λ:

$$\frac{\mathrm{d}H_\lambda}{\mathrm{d}\lambda} = -4/\lambda^5 [[T, H_\lambda], H_\lambda]$$

- low- and high-momentum parts of H_λ being decoupled:

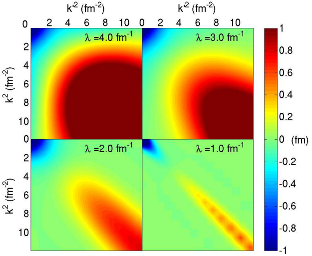


Figure: SRG evolved $^1S_0$ partial wave of chiral N3LO V_{NN}. (Bogner, Furnstahl, Perry, PRC 75, 061001 (2007))

Lee–Suzuki effective interaction

- preserves exactly part of the Hamiltonian spectrum

$$\sigma(H) = \{E_1, E_2, E_3, \ldots, E_P, \ldots, E_\infty\}$$

$$\sigma(H_{\text{eff}}) = \{E_1, E_2, E_3, \ldots, E_P\}$$

- decoupling condition

$$QXHX^{-1}P = 0 \rightarrow H_{\text{eff}} = PXHX^{-1}P$$

$$\begin{array}{c|c|c|}
 & \mathrm{P} & \mathrm{Q} \\
\hline
\mathrm{P} & H_{\text{eff}}^{(n)} & 0 \\
\hline
\mathrm{Q} & 0 & Q_n X_n H^{(n)} X_n^{-1} Q_n \\
\hline
\end{array}$$

- n-body cluster approximation ($2 \leq n \leq A$): $H_{\text{eff}}^{(n)}$ n-body operator
  $n = 2$: takes into account 2-body correlations from outside of the model space ($\leftarrow$ repulsive core)

s-shell hypernuclei: $^{3}_{\Lambda}$H

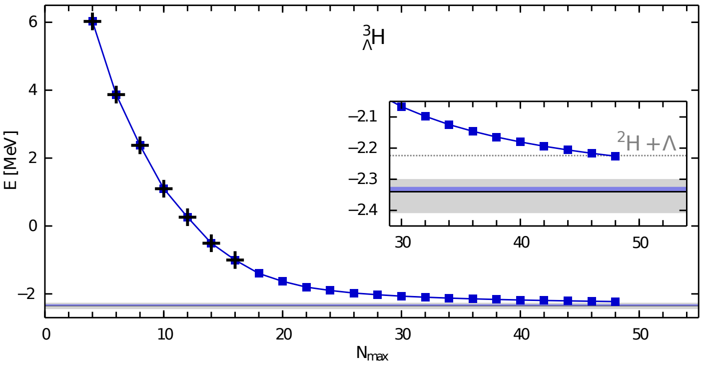


Figure: Ground state energy of $^{3}_{\Lambda}$H as a function of the size of the model space, with bare chiral LO @ 600 MeV interactions.

s-shell hypernuclei: ${}^{4}_{\Lambda}$H, ${}^{4}_{\Lambda}$He

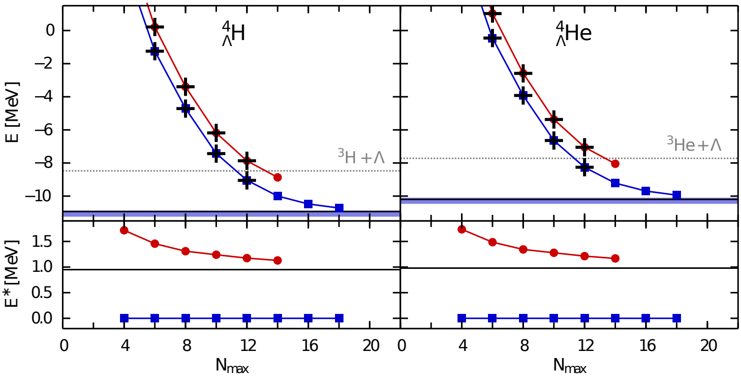


Figure: Ground state (blue) and excited state (red) energy of ${}^{4}_{\Lambda}$H and ${}^{4}_{\Lambda}$He as a function of the size of the model space, with chiral LO @ 600 MeV interactions.

s-shell hypernuclei: ${}^{4}_{\Lambda}$H with SRG interactions

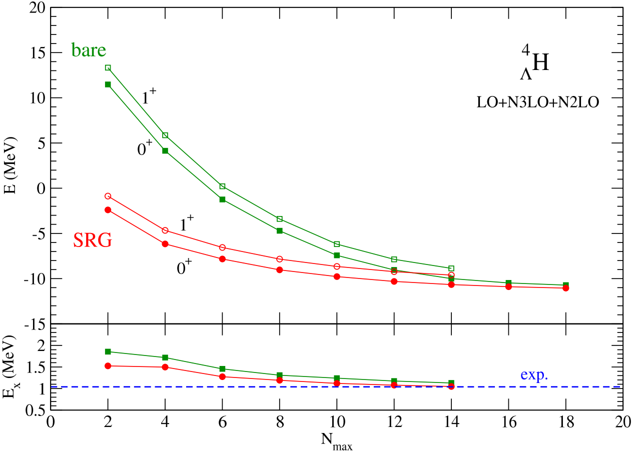


Figure: Ground state and excited state energies of ${}^{4}_{\Lambda}$H with bare and SRG effective interctions as a function of the size of the model space, with chiral LO @ 600 MeV interactions.

s-shell hypernuclei: ${}^{5}_{\Lambda}$He with Lee–Suzuki eff. interaction

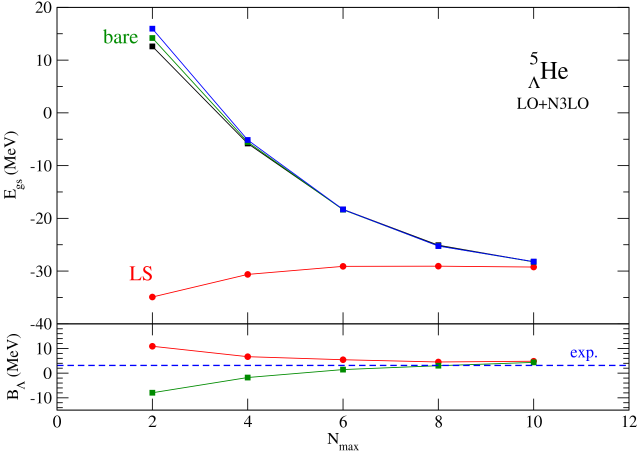


Figure: Ground state energy and Λ binding energy in ${}^{5}_{\Lambda}$He with bare and Lee–Suzuki effective interctions as a function of the size of the model space, with chiral LO @ 600 MeV interactions.

p-shell hypernuclei: ${}^{7}_{\Lambda}\mathrm{Li}$

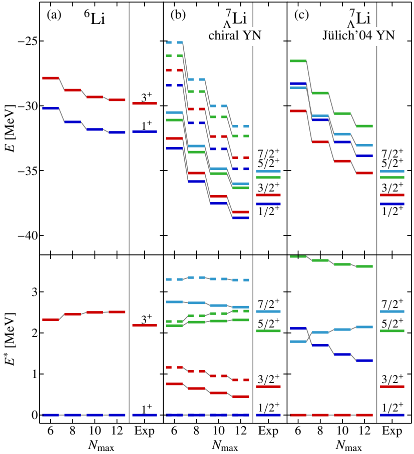


Figure: Calculations of ${}^{7}_{\Lambda}\mathrm{Li}$ with chiral LO @ 600@MeV (solid lines) and 700 MeV (dashed lines) and Jülich04 YN interactions.

p-shell hypernuclei: ${}^{9}_{\Lambda}$Be

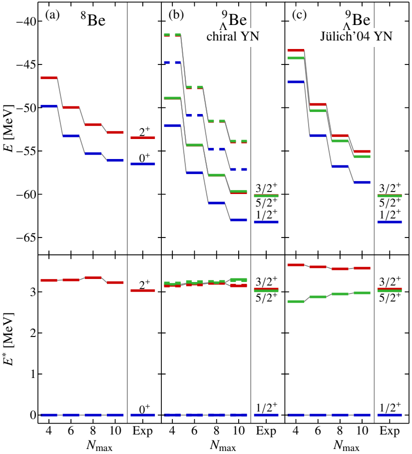


Figure: Calculations of ${}^{9}_{\Lambda}$Be with chiral LO @ 600@MeV (solid lines) and 700 MeV (dashed lines) and Jülich04 YN interactions.

p-shell hypernuclei: ${}^{13}_{\Lambda}$C

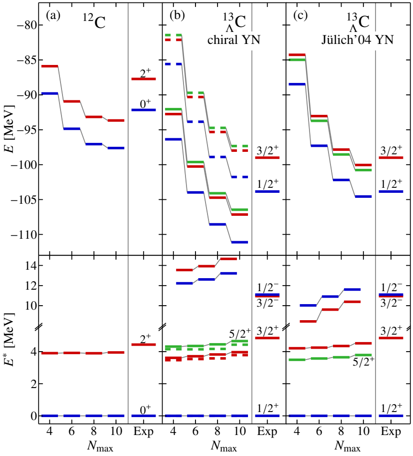


Figure: Calculations of ${}^{13}_{\Lambda}$C with chiral LO @ 600@MeV (solid lines) and 700 MeV (dashed lines) and Jülich04 YN interactions.

Summary

Calculations of light hypernuclei within NCSM

- reliable *ab initio* calculations of p-shell hypernuclei with microscopic interactions are now possible
- systematic study of p-shell hypernuclei improves understanding of YN interactions
- LO chiral YN interactions are consistent with measured low-lying energy levels of light hypernuclei
- indication of deficiencies for higher relative partial waves of LO chiral YN interactions

Gazda, Mareš, Navrátil, Roth, Wirth, Few-Body Syst. 55, 857 (2014).
Wirth, Gazda, Navrátil, Calci, Langhammer, Phys. Rev. Lett. 113, 192502 (2014).

Outlook

- benchmark calculations
- study chiral NLO NY interaction
- lattice NY interactions, $S=-2$ systems, ...