Equation of state of *A*-neutron matter from QMC calculations

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?

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Collaborators

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- Francesco Catalano (Trento)

 $\mathcal{L} = \mathcal{L} \mathcal{L}$ *Strangeness in Nuclei & Neutron Stars, Pisa, May 20-21, 2015*

Outline

- The "hyperon puzzle" in neutron stars.
- A non-relativistic model of the hyperon(Λ)-nucleon interaction (for "normal" baryonic matter).
- Connection to the existing experimental data: computation of BE in hypernuclei by QMC.
- Infinite Λ -neutron matter and prediction for the NS structure.
- Conclusions

Hyperons in dense matter

P. Haensel, A.Y. Potekhin, D.G. Yakovlev, Neutron Stars 1, Springer 2007

When hyperons appear, energy is reduced! $Q = +1:~ \mu_{Y^+} = \mu_n - \mu_e$ density nb. The baryon density is

A simple argument based on the properties of the Fermi gas tells us that at very large densities it is energetically favourable to change neutrons and protons into hyperons.

Hyperons might appear in the *inner core* of neutron stars.

Chemical equilibrium conditions:

$$
Q = -1 : \mu_{Y^-} = \mu_n + \mu_e
$$

$$
Q = 0 : \mu_{Y^0} = \mu_n
$$

$$
Q = +1 : \mu_{Y^+} = \mu_n - \mu_e
$$

Hyperon puzzle

The appearance of hyperons has an immediate consequence on the equation of state: it makes it **softer**, i.e. the pressure coming from the baryon-baryon interaction is reduced.

Softer EoS ➠ **lower star mass**

H. Ðapo, B.-J. Schaefer, and J. Wambach. Appearance of hyperons in neutron stars. Phys. Rev. C, 81(3): 035803 (2010) based on NN ("soft" and "stiff") EoS from M.Heiselberg, M.Hjort-Jensen, Phys. Rep. 328, 237 (2000) See also the work of the Catania group (H.-J. Schulze, MM Baldo, F. Burgio et al.)

Until 2010 observed masses of NS were distributed around the Chandrashekar mass $M_S=1.4 M_⊙$

Use of the equation of state of a p,n,e,μ leads to a maximum mass > 2 M⊙

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Many hyperon-nucleon model interactions, giving differen EoS and different predictions.

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Hyperon Puzzle

Recently a few NS with a large mass were observed. by using Shapiro delay measurements. The first (2010) was PSRJ1614-2230 pulsar with **M=1.97(4)M**⊙.

(P. B. Demorest, T. Pennucci, S. M. Ransom, M. S. E. Roberts and J. W. T. Hessels. A two-solar-mass neutron star measured using Shapiro delay measurements, Nature 467, 1081 (2010).

In a non relativistic framework (= pure baryonic stars) hyperons are problematic

Before 2010:

Maximum mass observed: 1.6M⊙ **Maximum mass predicted without hyperons: 2.3**⊙ **(still ok in principle) Maximum mass predicted with hyperons: 1.4-1.6M**⊙ **(good!)**

After 2010:

Observed mass: 2.0M⊙

Maximum mass predicted without hyperons: 2.3M⊙ **(good!)**

Maximum mass predicted with hyperons: 1.4-1.6M⊙ **(very bad…)**

Model Hyperon-nucleon interaction

 $\frac{1}{\frac{3}{2}}$ A fund**er in** ingredient to gain some understanding for this problem is to $r_{ij})$ $\mathcal{O}_{ij}^{\bm{\phi}}$ *NN* understand the structuræ of the baryon-baryon interaction.

Argonne \mathbf{v}_{18}

from Refs. [59] (open circles), and [70] (filled squares), $\frac{1}{2}$ (filled squares) $(Σ^-p → Σ^-p, Σ^+p → Σ^+p)$. The red/dark band shows the chiral EFT There are several ways of attacking the problem. This is our choice:

- **NON RELATIVISTIC APPROACH** (should be fine if the central density is not too large)
- YN INTERACTION CHOSEN TO FIT EXISTING **SCATTERING DATA** (with a hard-core)
- ⌦ **C** *Respectively* **o PHENOMENOLOGICAL YNN THREE-BODY**

 $\mathbf{F} \mathbf{F} \mathbf{F} \mathbf{F} \mathbf{F} \mathbf{F} \mathbf{F}$ \overline{X} **FORCES** with few parameters to be adjusted to R. H. Dalitz, R. C. Herndon, Y. C. Tang,
SI NGC ONN 61 *Pu*b 20 MG D. P. L reproduce light hypernuclei binding energies • **ALL THE OTHER RESULTS ARE PREDICTIONS**

ANTH NO OTHER ADJUSTABLE PARAMETERS o

obtained from an *accurate solution of the*

 \sim \sim \sim \sim *Schroedinger equation*.

Fig. 1. Total cross section for Ap scattering as a function of c.m. kinetic energy $E(MeV)$. The solid line is obtained with CSB potential of the form (2.9), while the dashed line is obtained

Model Hyperon-nucleon interaction

Model interaction (Bodmer, Usmani, Carlson):

A. Bodmer, Q. N. Usmani, and J. Carlson, Phys. Rev. C 29, 684 (1984).

from Kaon exchange terms (not considered explicitly in our calculations)

$$
V_{\Lambda i}(r) = v_0(r) + v_0(r)\varepsilon(P_x - 1) + \frac{1}{4}v_\sigma T_\pi^2(m_\pi r)\boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_i
$$

Two-body potential: accurately fitted on p-A scattering data

Q. N. Usmani and A. R. Bodmer, Phys. Rev. C 60, 055215 (1999).

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V_{\Lambda ij} = V_{\Lambda ij}^{2\pi} + V_{\Lambda ij}^D
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Two-body potential: accurately fitted on p- Λ scattering data

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Parameters to be determined from calculations

Input from experiment

We need to fit the three body interaction against some experimental data. There are available several measurements of the binding energy of Λ hypernuclei, i.e. nuclei containing a Λ hyperon. The idea is to compute such binding energies. We can then compute the **hyperon separation energy:**

where B_{hyp} is the total binding energy of a hypernucleus with A nucleons and one Λ , and B_{nuc} is the total inding $u\mu$ ergy of the $_{40}$ MeV corresponding nucleus with A nucleons. This number can be used to gauge the coefficients in the nucleon- Λ interaction. 1116 MeV u_g $\frac{u}{s}$ $\overline{\sum^{0}}$ 1200 MeV 3 -1

Hypernuclei data **Particle Physics** i uulu

J. Pochodzalla, Acta Phys. Polon. B 42, 833– 842 (2011)

ΠΥΡΟΠΙΟΝΟΝ ΟΡΟΔΙΟΔΟ

- The available data are very limited.
- There are several planned and ongoing systematic measurements.
- At present no proposals for gathering more Λ -nulceon scattering data
- Essentially no information on $\Lambda\Lambda$ interaction
- (Almost) nothing on Σ or \overline{E} hypernulcei

S. N. Nakamura, Hypernuclear workshop, JLab, May 2014 updated from: O. Hashimoto, H. Tamura, Prog. Part. Nucl. Phys. 57, 564 (2006)

Projection Monte Carlo

We compute ground state energies of nuclei by means of projection Monte Carlo methods. The ground state of a many-body system is computed by applying an "imaginary time propagator" to an arbitrary state that has to be non-orthogonal to the ground state (power method):

$$
\langle R|\Psi(\tau)\rangle=\langle R|e^{-(\hat{H}-E_0)\tau}|R'\rangle\langle R'|\Psi(0)\rangle
$$

In the limit of "short" τ (let us call it " $\Delta \tau$ "), the propagator can be broken up as follows (Trotter-Suzuki formula): $W(R,R',\Delta \tau)$

$$
\langle R|e^{-(\hat{H}-E_0)\Delta\tau}|R'\rangle \sim e^{-\frac{(R-R')^2}{2\frac{\hbar}{m}\Delta\tau}}e^{-\left(\frac{V(R)+V(R')}{2}-E_0\right)\Delta\tau}
$$

Notice term
Banple a new point from the Gaussian kernel
(2)
Caussian kernel
(R₁)
(R₂)
(P₂)
(P₂

Many-nucleon systems

PROBLEM

for realistic many-nucleon Hamiltonians, propagators must be evaluated on wave functions that have a number of components exponentially growing with A (spin/isospin singlet/triplet state for each pair of nucleons)

Very accurate results have been obtained in the years for the ground state and some excitation properties of nuclei with A≤12 by the Argonne based group (GFMC calculations by Pieper, Wiringa, Carlson, Schiavilla…). These calculations include twoand three-nucleon interactions. The set of the set of \sim 100

Courtesy of R. Wiringa, ANL

Stefano Fantoni & Kevin Schmidt, 1999

The computational cost of GFMC can be reduced by introducing a way of sampling over the space of states, rather than summing explicitly over the full set.

For simplicity let us consider only one of the terms in the interaction. We start by observing that: Linear combination

$$
\sum_{i
$$

Then, we can linearize the operatorial dependence in the propagator by means of an integral transform: auxiliary fields→Auxiliary Field Diffusion Monte Carlo

$$
e^{-\frac{1}{2}\lambda \hat{O}_n^2 \Delta \tau} = \frac{1}{\sqrt{2\pi}}
$$

 $\int\limits_{\pi}\,dx e^{-\frac{x^2}{2}}e^{-x\sqrt{\lambda\Delta\tau}}\hat{O}_n$

S. Gandolfi, F. Pederiva, S. Fantoni, and K. E. Schmidt, Phys. Rev. Lett. 99, 022507 (2007)

K. E. Schmidt and S. Fantoni, Phys. Lett. B 446, 99 (1999).
S. Gandolfi, F. Pederiva, S. Fantoni, and K. E. Schmidt, **Hubbard-Stratonovich transformation**

The operator dependence in the exponent has become linear.

In the Monte Carlo spirit, the integral can be performed by sampling values of x from the Gaussian $e^{-\frac{x^2}{2}}$ For a given x the action of the propagator will become:

$$
e^{-x\sqrt{\lambda \Delta \tau} \hat{O}_n} |S\rangle = \prod_{k=1}^{3A} e^{-x\sqrt{\lambda \Delta \tau} \phi_n^k \sigma_k} |S\rangle
$$

In a space of spinors, each factor corresponds to a rotation induced by the action of the Pauli matrices

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The crucial advantage of AFDMC is that the scaling of the required computer resources is no longer exponential: **the cost scales as** *A3* (the scaling required by the computation of the determinants in the antisymmetric wave functions) \rightarrow LARGER SYSTEMS **ACCESSIBLE!**

Problems

- The HS transformation can be used ONLY FOR THE PROPAGATOR Accurate wave functions require an operatorial dependence! "Cluster expansion" introduced and working!
- Extra variables \longrightarrow larger fluctuations and autocorrelations.
- Some problems in treating nuclear spin-orbit.
- Three-body forces (extremely important in nuclear physics) cannot be implemented in all cases

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Input from experiment

Assumption: use of a simplified NN interaction cancel in the difference and therefore the estimate of BΛ is accurate (verified!)

Only two parameters are relevant (one of them is essentially ineffective)

Hypernuclei

Results: hypperintered is that we resulted the process calculations is no The *nucleon-mucleor* interaction that we use in our hypernuclear calculations is not the full realistic one, but the simpler AV4'+ the central (repulsive) term of the Urbana IX

potential (*UIX_c*). Despite this simplification, the description of closed shell nuclei is at leas "not unrealistic". Here we report some results.

preliminary

preliminary

reasonable single particle densities and radii

Hypernuclei Results: hypernuclei

Diego Lonardoni, FP, Stefano Gandolfi ys. Rev. C 89, 014314 (2014)

Hypernuclei

Calculations can ber performed also for excited states: more information available on the structure of the interaction (e.g. spin orbit? charge symmetry breaking?)

Λ -neutron matter

Λ -neutron matter

neutrons $+$ neutrons

+

lambdas
 $\begin{cases}\n\rho_b = \rho_n + \rho_\Lambda \\
x_\Lambda = \frac{\rho_\Lambda}{\rho_b} \\
\rho_b\n\end{cases}\n\qquad\n\begin{cases}\n\rho_n = (1 - x_\Lambda)\rho_b \\
\rho_\Lambda = x_\Lambda \rho_b\n\end{cases}$ $\begin{cases} \rho_n = (1 - x_\Lambda)\rho_b \\ \rho_\Lambda = x_\Lambda \rho_b \end{cases}$ $\sqrt{ }$ \int \overline{a} $\rho_b = \rho_n + \rho_\Lambda$ $x_{\Lambda} =$ ρ_{Λ} ρ_b

$$
E_{\text{HNM}}(\rho_b, x_{\Lambda}) = \left[E_{\text{PNM}}((1 - x_{\Lambda})\rho_b) + m_n \right](1 - x_{\Lambda}) + \left[E_{\Lambda}^F(x_{\Lambda}\rho_b) + m_{\Lambda} \right] x_{\Lambda} + f(\rho_b, x_{\Lambda})
$$

Problem1: limitation in x_{Λ} due to simulation box Problem2: finite size effects Problem3: fitting procedure

$$
f(\rho_b, x_\Lambda)
$$
 cluster expansion $\frac{\rho_{\Lambda} \rho_n}{\rho_b}$, $\frac{\rho_{\Lambda} \rho_n \rho_n}{\rho_b}$, $\frac{\rho_{\Lambda} \rho_{\Lambda} \rho_n}{\rho_b}$, $\frac{\rho_{\Lambda} \rho_n \rho_n \rho_n}{\rho_b}$

Λ -neutron matter

$$
\begin{cases}\n\mu_n(\rho_b, x_\Lambda) = E_{\text{PNM}}(\rho_n) + \rho_n \frac{\partial E_{\text{PNM}}}{\partial \rho_n} + m_n + f(\rho_b, x_\Lambda) + \rho_b \frac{\partial f}{\partial \rho_n} \\
\mu_\Lambda(\rho_b, x_\Lambda) = E_\Lambda^F(\rho_\Lambda) + \rho_\Lambda \frac{\partial E_\Lambda^F}{\partial \rho_\Lambda} + m_\Lambda + f(\rho_b, x_\Lambda) + \rho_b \frac{\partial f}{\partial \rho_\Lambda}\n\end{cases}
$$

-neutron matter

-neutron matter

Within this model the repulsion \Box PSR J0348+0432 needed to correctly describe hypernuclear binding energy is so strong that **hyperons would not be present in 2M**[⊙] **stars!**

Diego Lonardoni, A. Lovato, S. Gandolfi, FP, arXiv:1407.4448 [nucl-th], submitted to Phys. Rev. Lett.

Diego Lonardoni, A. Lovato, S. Gandolfi, FP, arXiv:1407.4448 [nucl-th], submitted to Phys. Rev. Lett.

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In hypernuclei it is possible that the ANN interaction is not well constrained, especially in the isospin triplet channel:

We are doing the exercise of re-projecting the interaction in the isospin singlet and triplet we are doing the oxoroloo of the projecting the interaction in the locopin eniglet and triplet and try to explore the dependence of the hypernuclei binding energy on the relative strength. ر
F. strenath

$$
v^{2\pi, P} = -\frac{C_P}{6} \{X_{i\lambda}, X_{\lambda j}\} \vec{\tau}_i \cdot \vec{\tau}_j
$$

$$
v_{ij\lambda}^{\tau\tau} = -3v_{ij\lambda}^P \hat{P}_{ij}^T = 0 + C_T v_{ij\lambda}^P \hat{P}_{ij}^T = 1
$$

$$
v_{ij\lambda}^{\tau\tau} = \frac{3}{4} (C_T - 1) v_{ij\lambda}^P + \frac{1}{4} (3 + C_T) v_{ij\lambda}^P \vec{\tau}_i \cdot \vec{\tau}_j
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> C_T =1 gives the original potential, but we can choose an **arbitrary value**.

$$
v^{2\pi,P} = -\frac{C_P}{6} \left\{ X_{i\lambda}, X_{\lambda j} \right\} \vec{\tau}_i \cdot \vec{\tau}_j
$$
\n
$$
v_{ij\lambda}^{\tau\tau} = -3v_{ij\lambda}^P \hat{P}_{ij}^T = 0 + \underbrace{C_T v_{ij\lambda}^P \hat{P}_{ij}^T = 1}_{Q_T, S = C_S O_{ij\lambda}^{2\pi, S} \vec{\tau}_i \cdot \vec{\tau}_j
$$
\n
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Francesco Catalano, Diego Lonardoni, FP, unpublished

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Francesco Catalano, Diego Lonardoni, FP, unpublished

Conclusions

Status

- The three-body hyperon-nucleon force provides the necessary repulsion to reproduce the ground state physics of medium-light hypernuclei
- •The three-body hyperon-nucleon interaction plays a fundamental role in the softening of the EoS and for the consequent reduction of the predicted maximum mass.

Needs & Developments

- experimental inputs: scattering data, energy spectrum (gs+exc), CSB effects
- •benchmark calculations
- different NN(N) and YN(N) potentials: Nijmegen, chiral, isospin
- •projected realistic (hyper)nuclear matter in beta equilibrium
- medium-heavy mass hypernuclei [(e,e',K), proposal submitted at JLab]