Pseudoscalar transition form factors: $(g-2)_\mu$, $P\to e^+e^-$, η-η' mixing

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Work done in collaboration with
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SFB 1044
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

Cluster of Excellence Precision Physics, Fundamental Interactions and Structure of Matter
PRISMA

JG|U
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

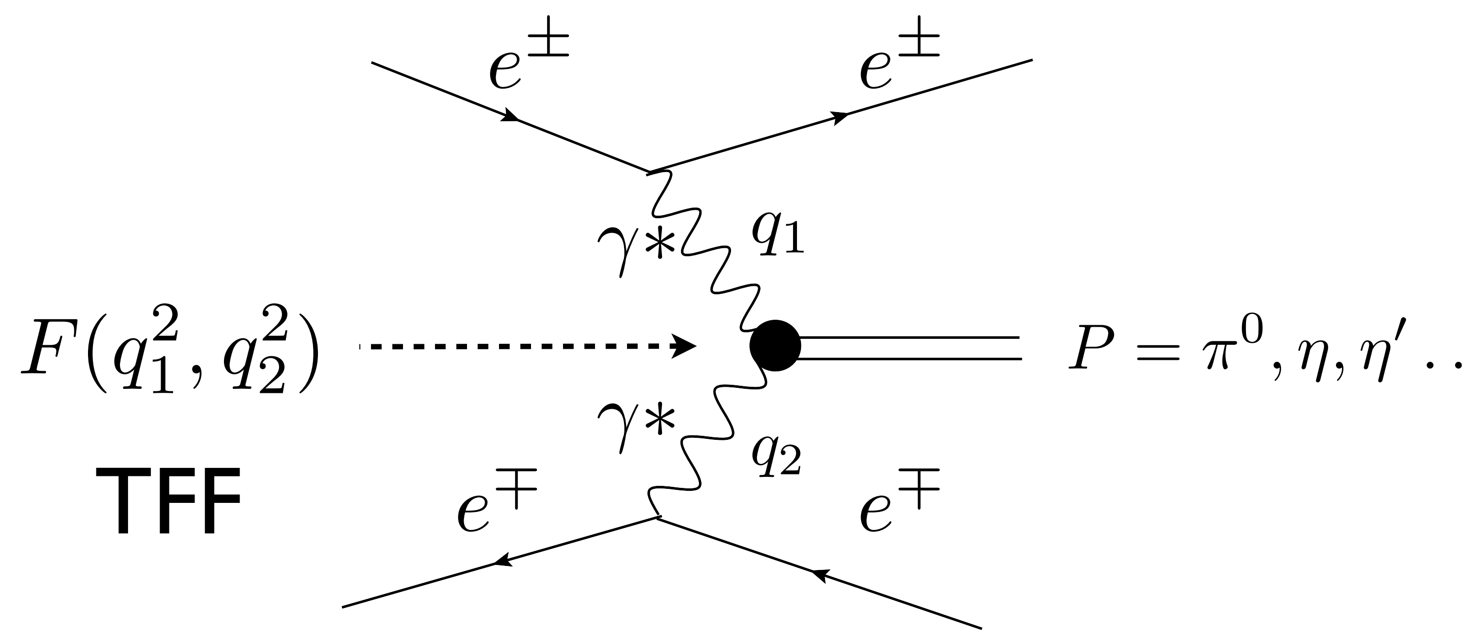
FCCP2015, Anacapri, 11th Sept.

Outline

- Pseudoscalar Transition Form Factors
- How to use data for dressing the TFFs
- Applications
  - $(g\text{-}2)_\mu$, $P\to e^+e^-$, η-η' mixing
- Conclusions and outlook

Pseudoscalar Transition Form Factors

- Study of ee→eeγ*γ* with γ*γ*→π,η,η' but also P→eeγ, 4e,2e



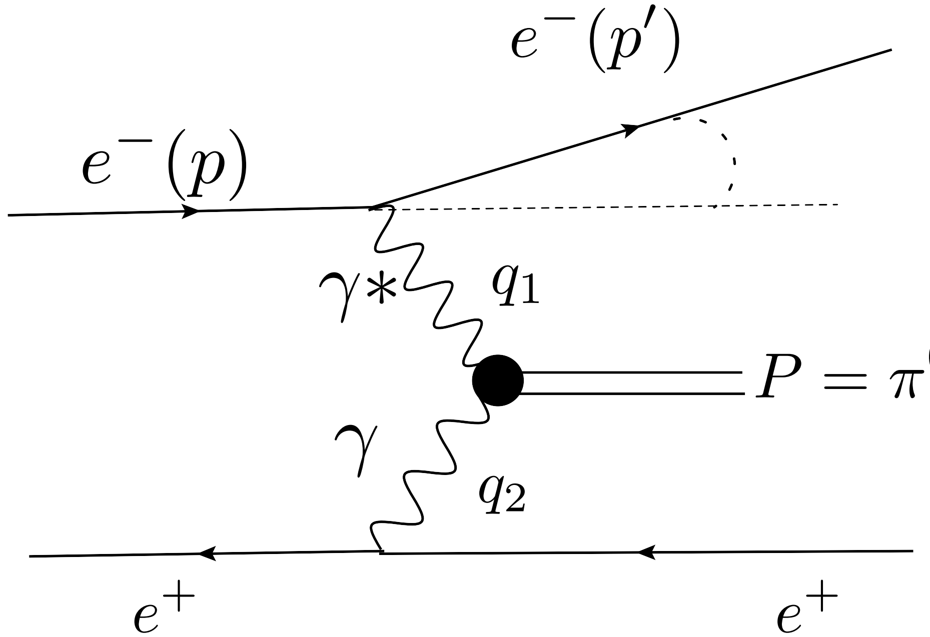
- Meson Structure
  - Transition Form Factors (TFF) give access to Meson Distribution Amplitudes

- Precision Tests of the Standard Model
  - Relation to mixing parameters, rare decays, and muon anomalous $(g-2)_\mu$

How do we do that?

- Single Tag Method can access the Meson Transition Form Factor

Selection criteria
- 1 e^- detected
- 1 e^+ along beam axis
- Meson full reconstructed

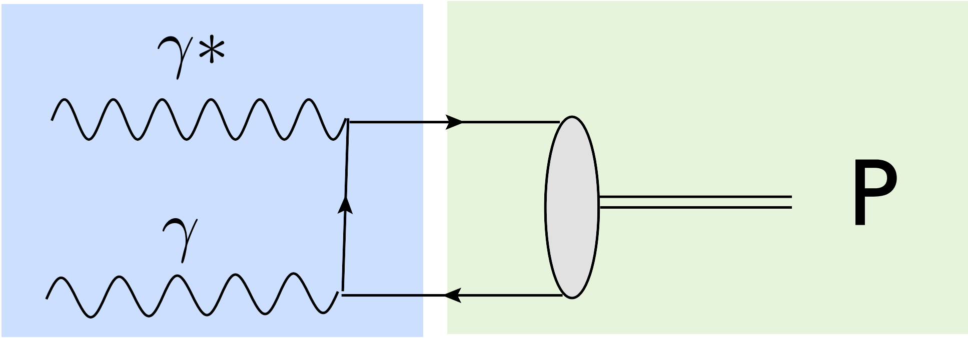


Momentum transfer
- tagged: $Q^2 = -q_1^2 = -(p-p')^2$
⇒highly virtual photon
- untagged: $q^2 = -q_2^2 \sim 0\,\mathrm{GeV}^2$
⇒quasi-real photon

How do we do that?

Cross section for P production depends only on $F(q_1^2, q_2^2)$

With the Single Tag Method: $F(q_1^2, q_2^2) \to F(Q^2)$



$$F(Q^2) = \int T_H(x, Q^2)\Phi_P(x, \mu_F)\mathrm{d}x$$

$T_H(\gamma^*\gamma \to q\bar{q})$ $\quad$ $\Phi_P(q\bar{q} \to P)$

- μ_F is scale between soft and hard
- x-dependence of $\Phi_P(x,Q^2)$ not known but models
- Experimental data on $F(Q^2)$ is needed

convolution of perturbative and nor-perturbative regimes

The role of experimental data

$$F_{P^*\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)$$

Use hadronic models constrained with chiral and large-Nc arguments

Use data from the Transition Form Factor for input calculations

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The role of experimental data

$$F_{P^*\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)$$

- We want a <u>method</u>, not a *model:* a TOOLKIT
- based on analyticity and unitarity
  - heading towards 10% errors
- Simple (not black box)
- Approaches yes (improvable), assumptions no
- Systematic:
  - easy to update with new data
  - error from approach
- Predictive (checkable)

The role of experimental data

Use data from
the Transition Form Factor
for numerical integral

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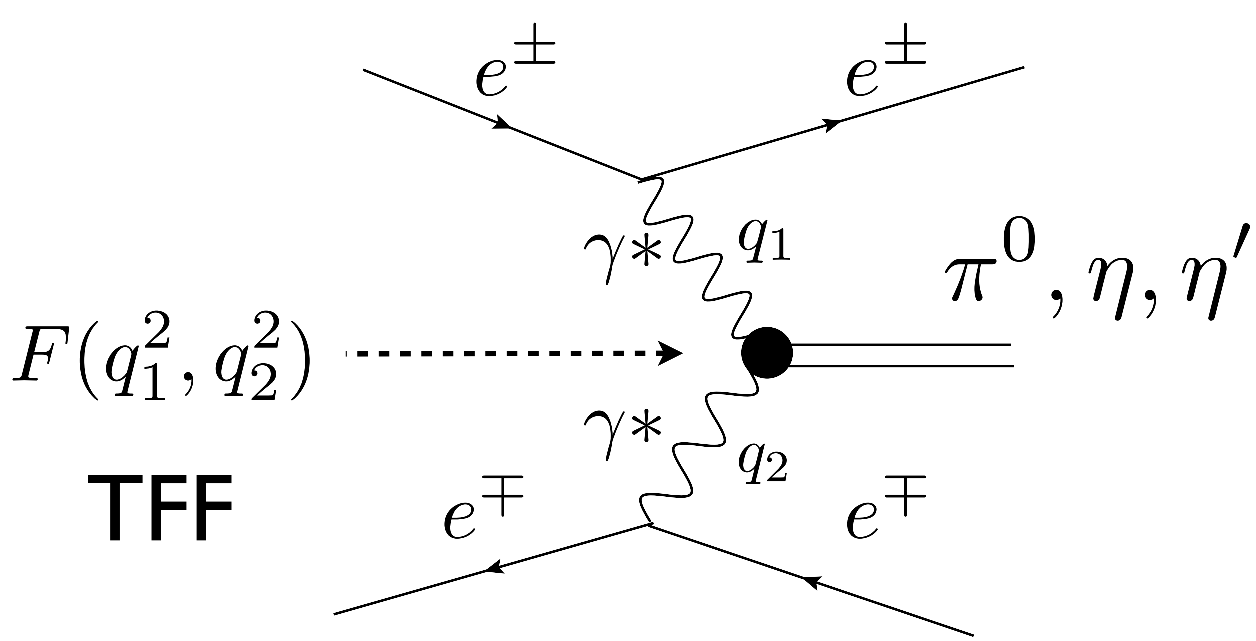
$$F_{P^*\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)$$

$$F_{P\gamma^*\gamma^*}(m_P^2, q_1^2, q_2^2)$$

double-tag method



The role of experimental data

~~Use data from the Transition Form Factor for numerical integral~~

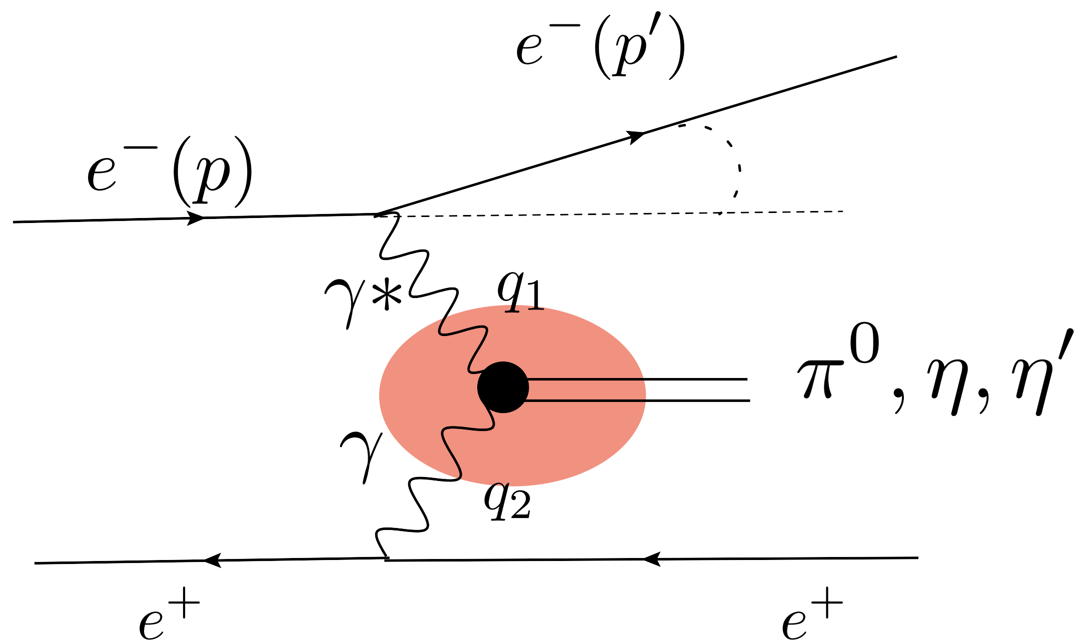
~~$$F_{P^*\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)$$~~

~~$$F_{P\gamma^*\gamma^*}(m_P^2, q_1^2, q_2^2)$$~~

Use data from the Transition Form Factor to constrain your hadronic model

$$\boxed{F_{P\gamma^*\gamma}(m_P^2, q_1^2, 0)}$$

single-tag method



The role of experimental data

Use data from
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$$F_{P^*\gamma^*\gamma^*}(q_3^2, q_1^2, q_2^2)$$

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Use data from
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$$F_{P\gamma^*\gamma}(m_P^2, q_1^2, 0)$$

How??

Nice synergy between experiment and theory

Simple, easy, systematic, user-friendly method

Our proposal: use Padé Approximants

[P.M.'12; P.M., M.Vanderhaeghen'12; R. Escribano, P.M., P. Sanchez-Puertas, '13, '15]

We need low-energy region (data driven) + high-energy tail
we don't want a model rather a method providing systematics

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$$F_{P\gamma*\gamma}(Q^2,0) = a_0^P \left(1 + b_P \frac{Q^2}{m_P^2} + c_P \frac{Q^4}{m_P^4} + \dots \right)$$

$\Gamma_{P\to\gamma\gamma}$ (→ a_0^P) slope (→ b_P) curvature (→ c_P)

We have published space-like data for $Q^2 F_{P\gamma*\gamma}(Q^2,0)$

$$Q^2 F_{P\gamma*\gamma}(Q^2,0) = a_0 Q^2 + a_1 Q^4 + a_2 Q^6 + \dots$$

$$P_M^N(Q^2) = \frac{T_N(Q^2)}{R_M(Q^2)} = a_0 Q^2 + a_1 Q^4 + a_2 Q^6 + \dots + \mathcal{O}((Q^2)^{N+M+1})$$

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$a_0^P \leftarrow \Gamma_{P\to\gamma\gamma}$; b_P: slope; c_P: curvature

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$$Q^2 F_{P\gamma*\gamma}(Q^2,0) = a_0 Q^2 + a_1 Q^4 + a_2 Q^6 + \dots$$

$$P_1^1(Q^2) = \frac{a_0 Q^2}{1 - a_1 Q^2} \longrightarrow \begin{array}{l} P_1^N(Q^2) = P_1^1(Q^2), P_1^2(Q^2), P_1^3(Q^2), \dots \\ P_N^N(Q^2) = P_1^1(Q^2), P_2^2(Q^2), P_3^3(Q^2), \dots \end{array}$$

sequence of approximations, i.e., <u>theoretical error</u>

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Convergence (making use of analytical properties):

$$\lim_{N\to\infty} P_1^N(Q^2) = F_{P\gamma^*\gamma}(Q^2,0)$$ Montessus Theorem

Conv. from pole at $-Q^2$ to Q^{*2}: good at LE, bad at HE. Fantastic for LEPs and cheap

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Convergence (making use of analytical properties):

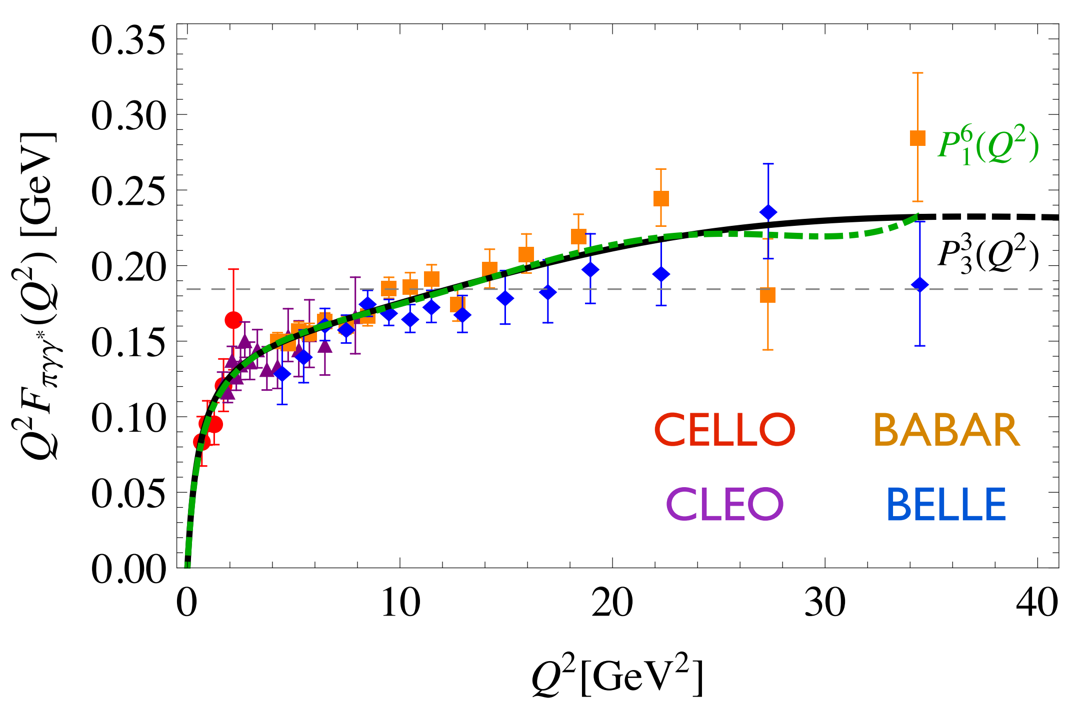
$$\lim_{N\to\infty} P_N^N(Q^2) = F_{P\gamma^*\gamma}(Q^2,0)$$ Pommerenke Theorem

Conv. from cut at $-Q^2$ to ∞: good at LE and HE. Good for LEPs and no cheap

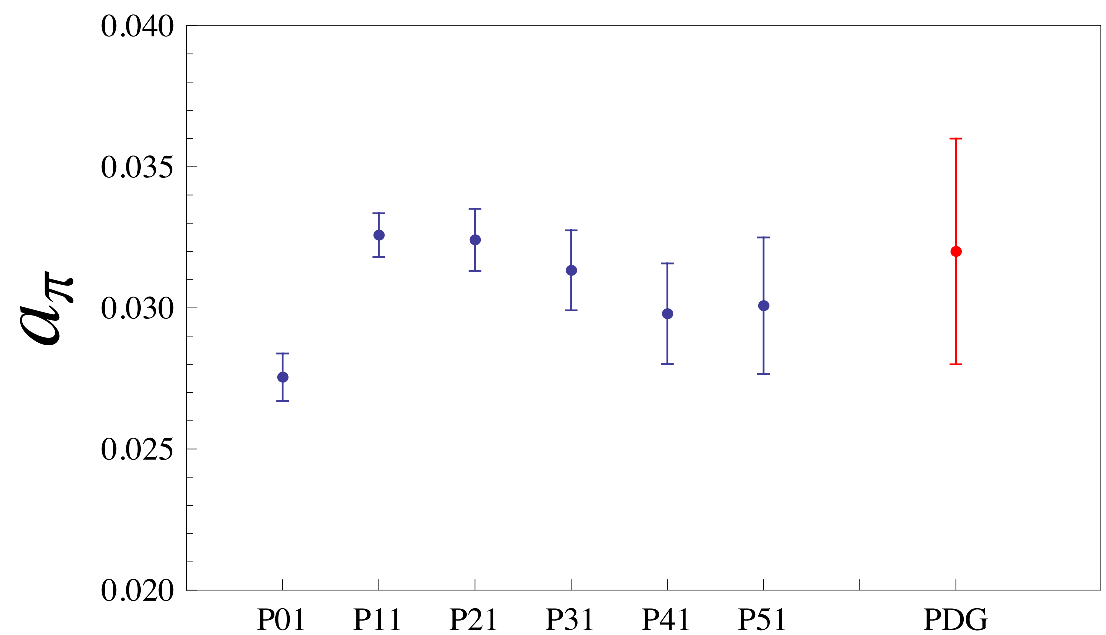
Our proposal: use Padé Approximants

[P.M.'12; P.M., M.Vanderhaeghen'12; R. Escribano, P.M., P. Sanchez-Puertas, '13, '15]

Fit to Space-like data: CELLO'91, CLEO'98, BABAR'09 and Belle'12



$P_1^N(Q^2)$ up to N=5 [P.M, '12]



$P_N^N(Q^2)$ up to N=3

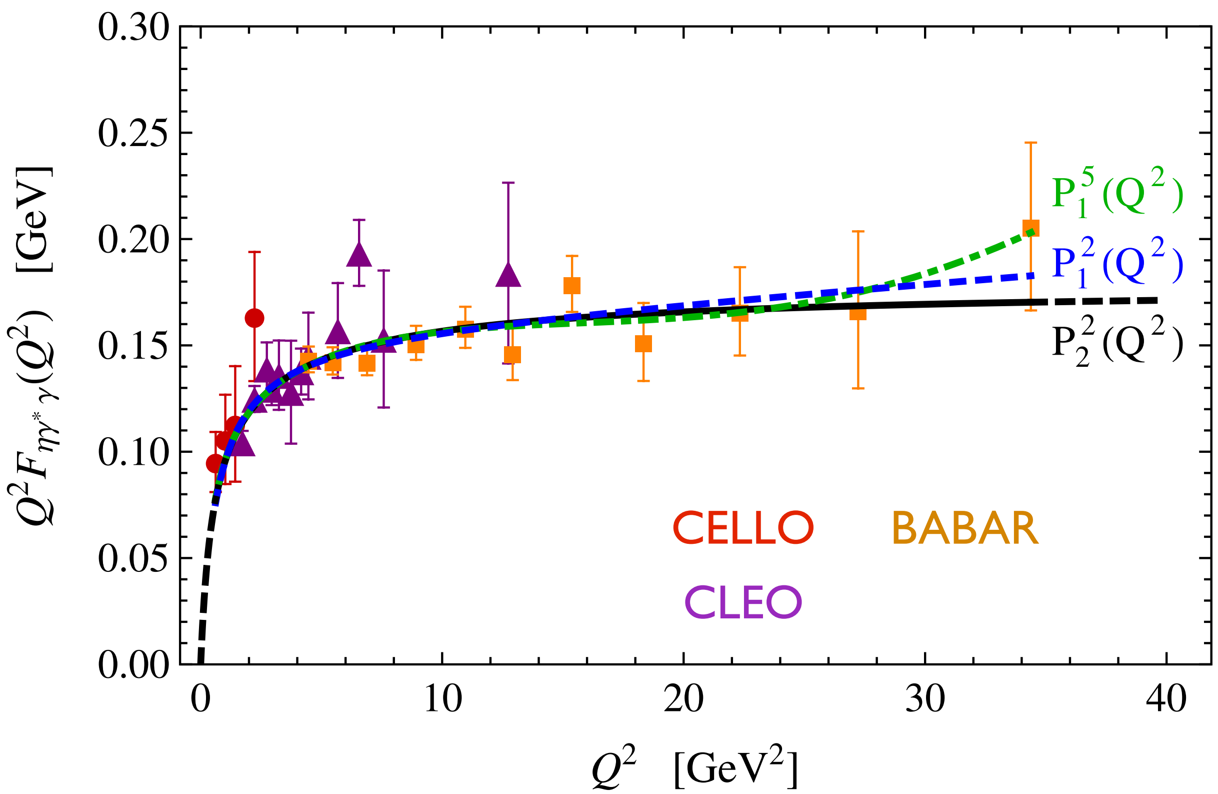
Accurate description of the low-energy region making full use of available experimental data

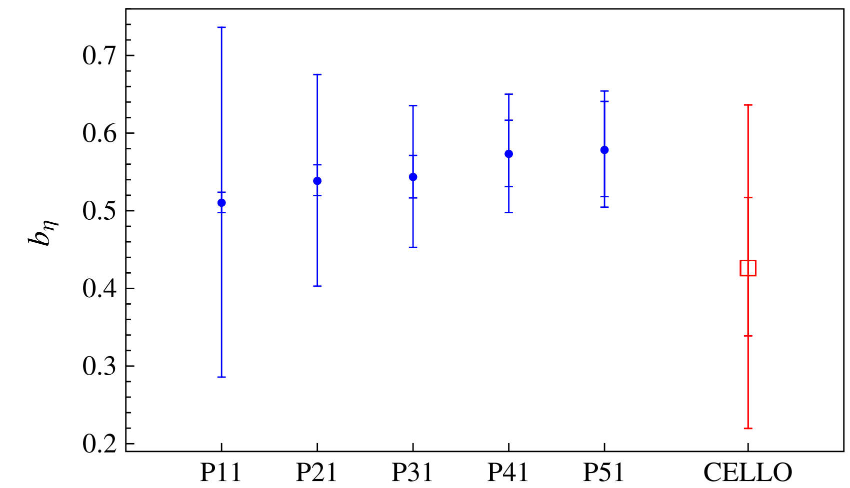
η-TFF

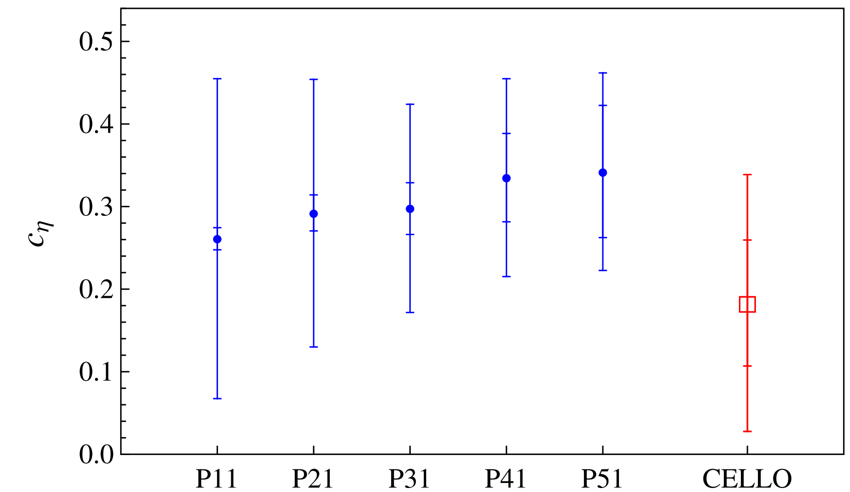
Fit to Space-like data: CELLO'91, CLEO'98, BABAR'11+ $\Gamma_{\eta\to\gamma\gamma}$

[R.Escribano, P.M., P. Sanchez-Puertas, '13]

$P_1^N(Q^2)$ up to N=4







$P_N^N(Q^2)$ up to N=2

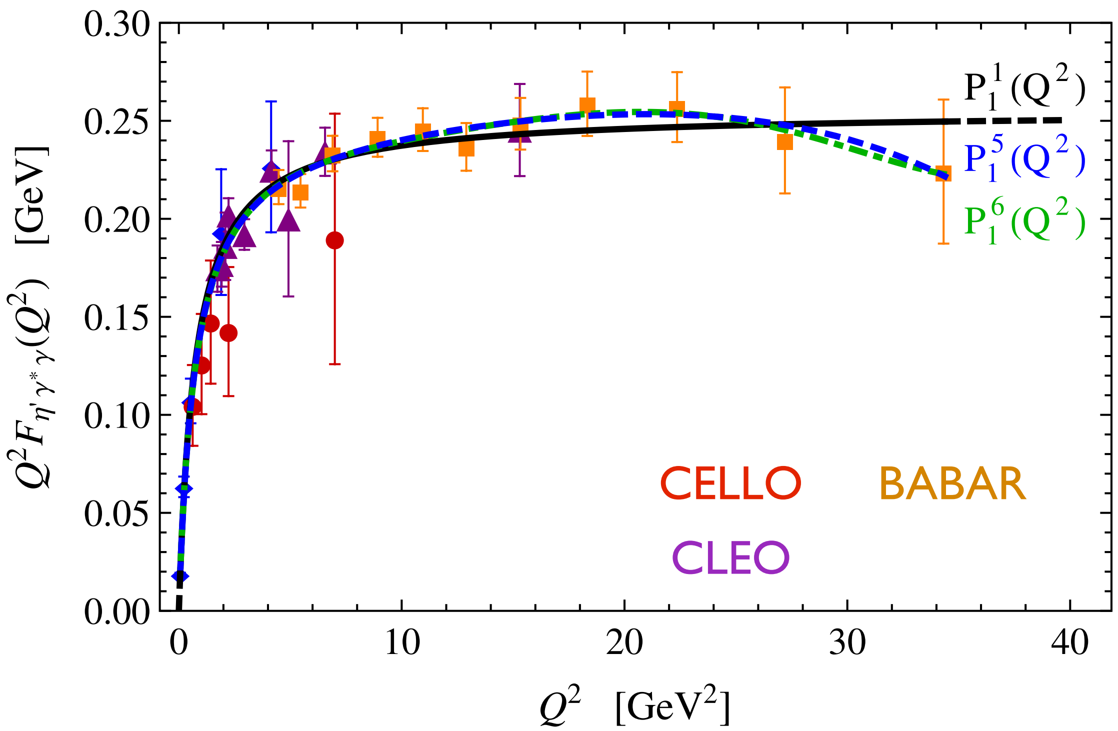
$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma*\gamma}(Q^2,0) = 0.164(2)GeV$$

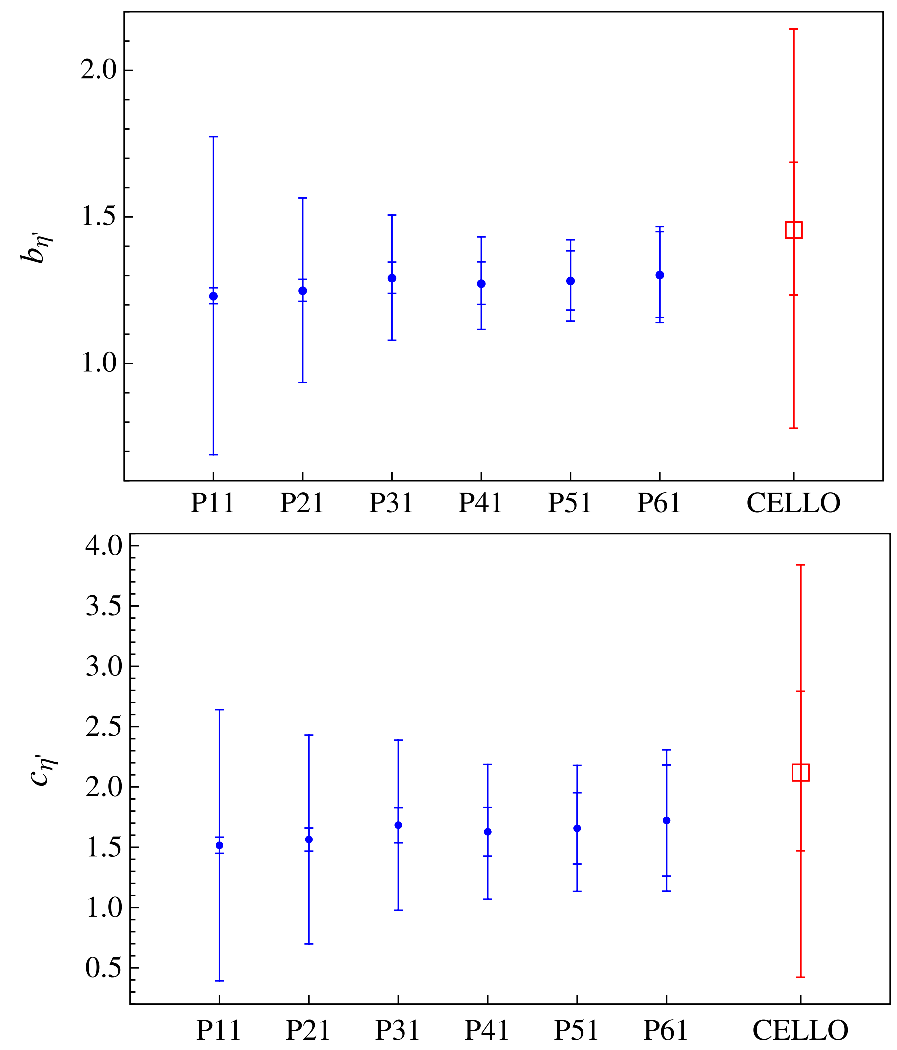
η'-TFF

Fit to Space-like data: CELLO'91, CLEO'98, L3'98, BABAR'11+ $\Gamma_{\eta' \to \gamma\gamma}$

[R.Escribano, P.M., P. Sanchez-Puertas, '13]

$P_1^N(Q^2)$ up to N=5





$P_N^N(Q^2)$ up to N=1

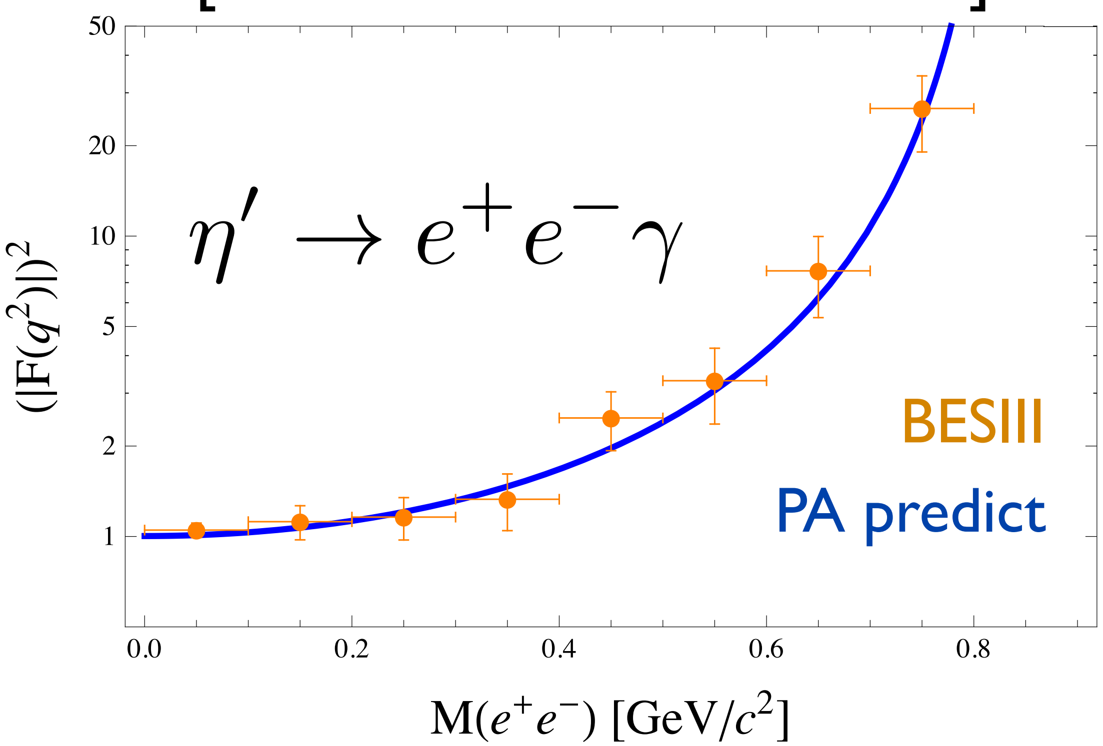
$$\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma*\gamma}(Q^2,0) = 0.254(4) GeV$$

time-like TFF

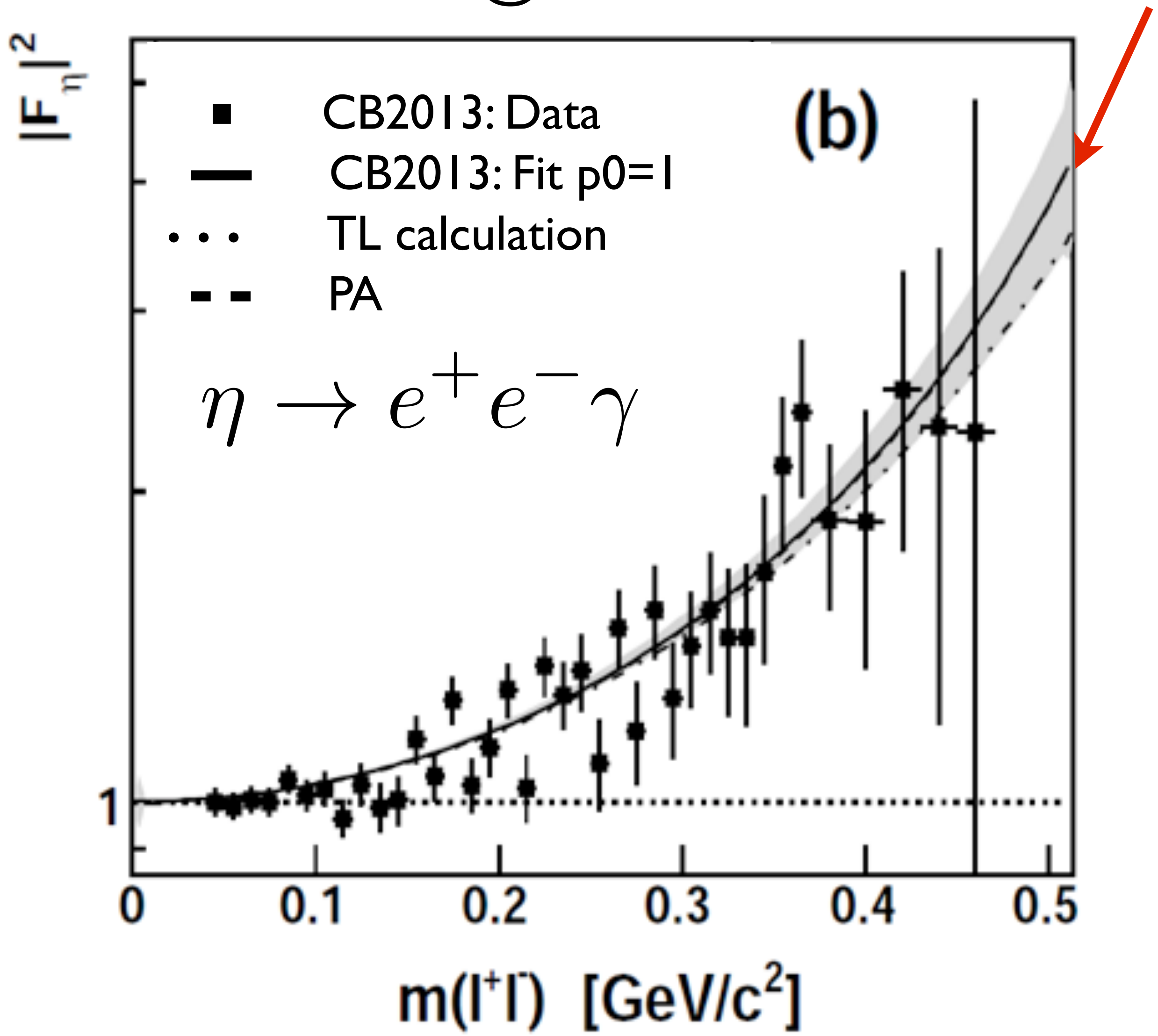
Predictive method!

- Study Dalitz decays $\eta(')\to\gamma^*\gamma\to e^+e^-\gamma$
- Prediction of the time-like from space-like data

[BESIII Coll. arXiv:1504.06016]



A2@MAMI

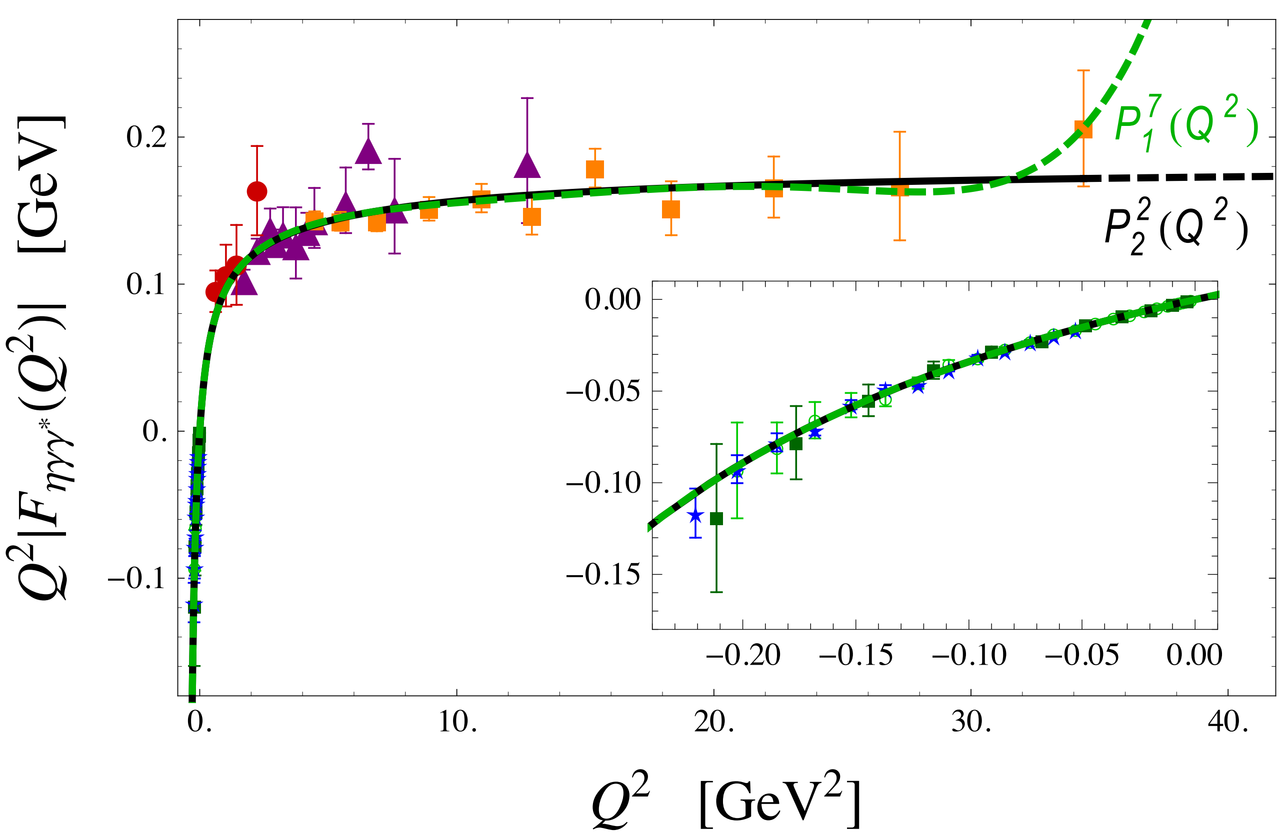


[A2 Coll. PRC89 2014]

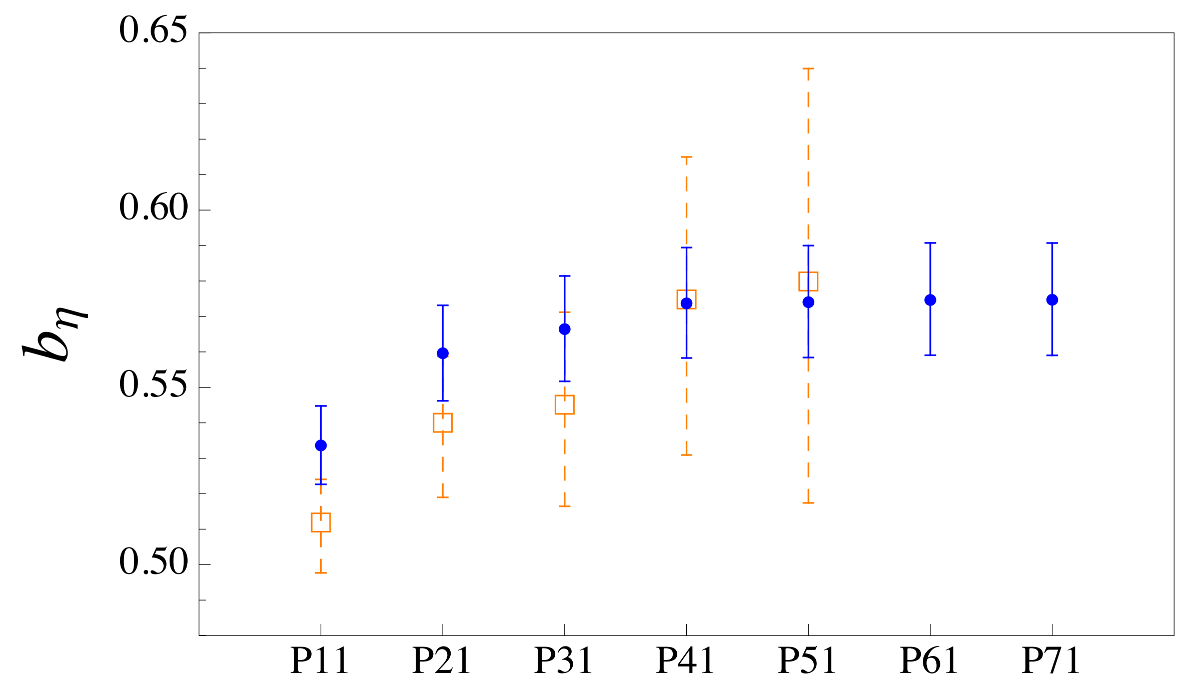
η-TFF

Fit to Space-like data [CELLO'91, CLEO'98, BABAR'11]+ $\Gamma_{\eta\to\gamma\gamma}$
+ Time-like data [NA60'09, A2'11, A2'13]

[R.Escribano, P.M., P. Sanchez-Puertas, '15]



$P_1^N(Q^2)$ up to N=7



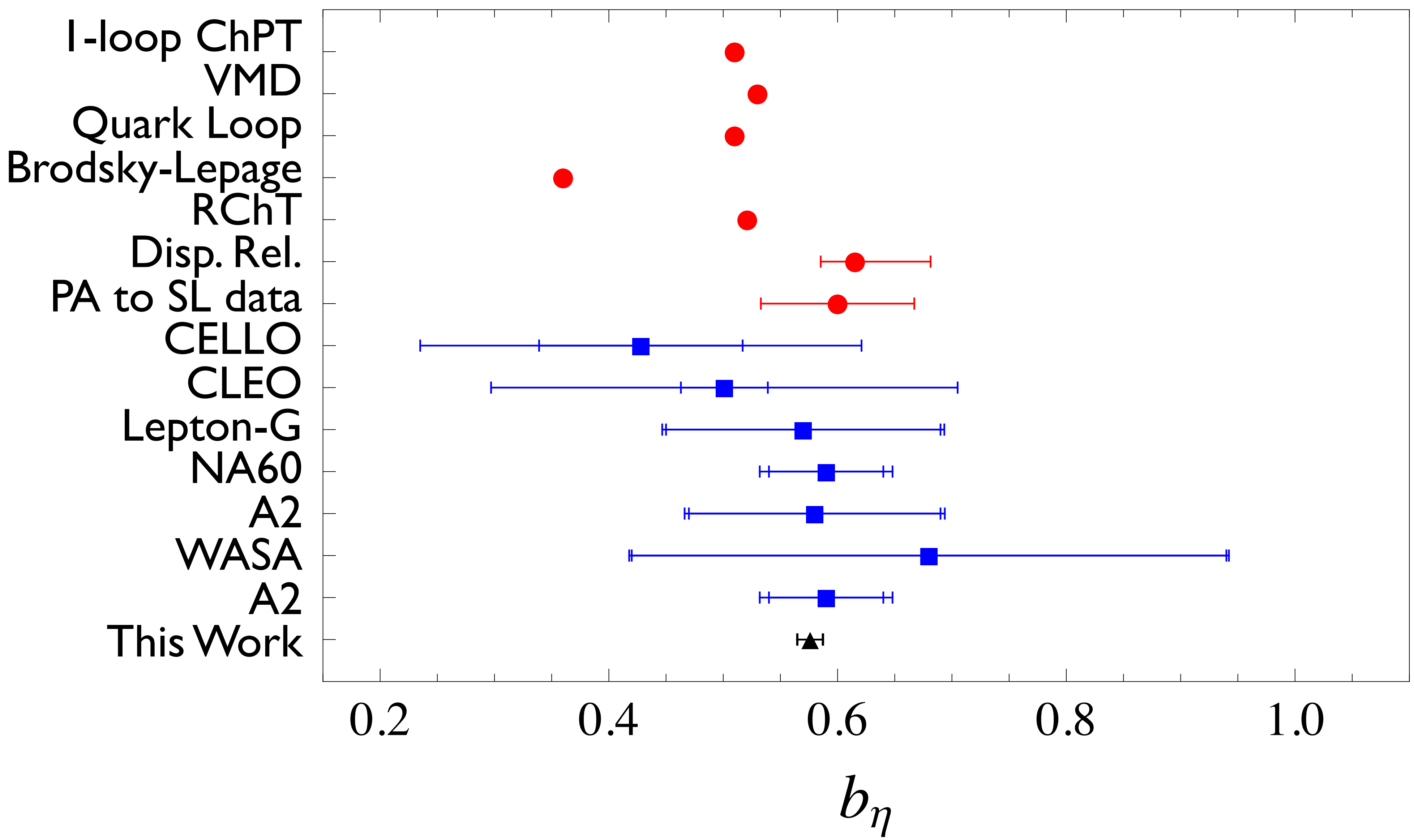
$P_N^N(Q^2)$ up to N=2

$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma*\gamma}(Q^2,0) = 0.177(15)GeV$$

η-TFF

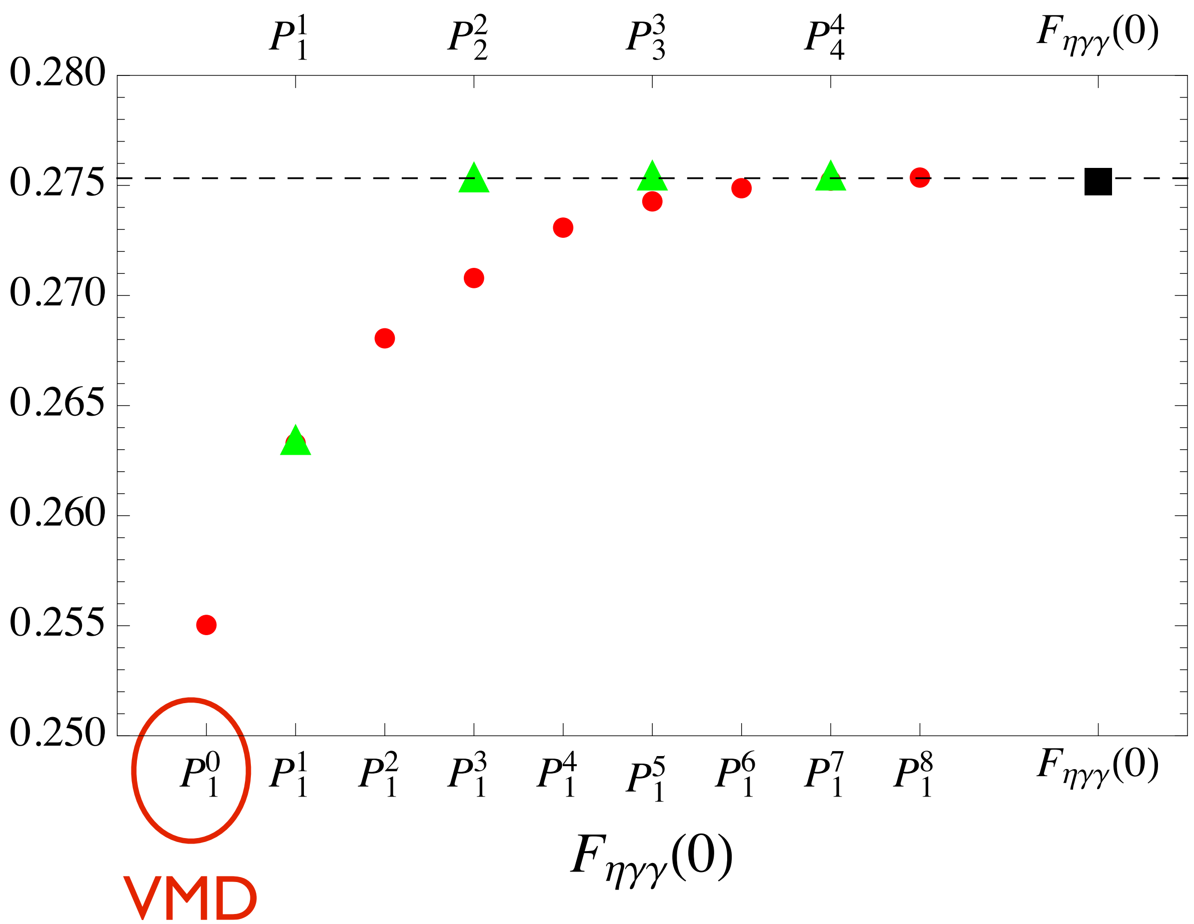
Fit to Space-like data [CELLO'91, CLEO'98, BABAR'11]+$\Gamma_{\eta\to\gamma\gamma}$
+ Time-like data [NA60'09, A2'11, A2'13]

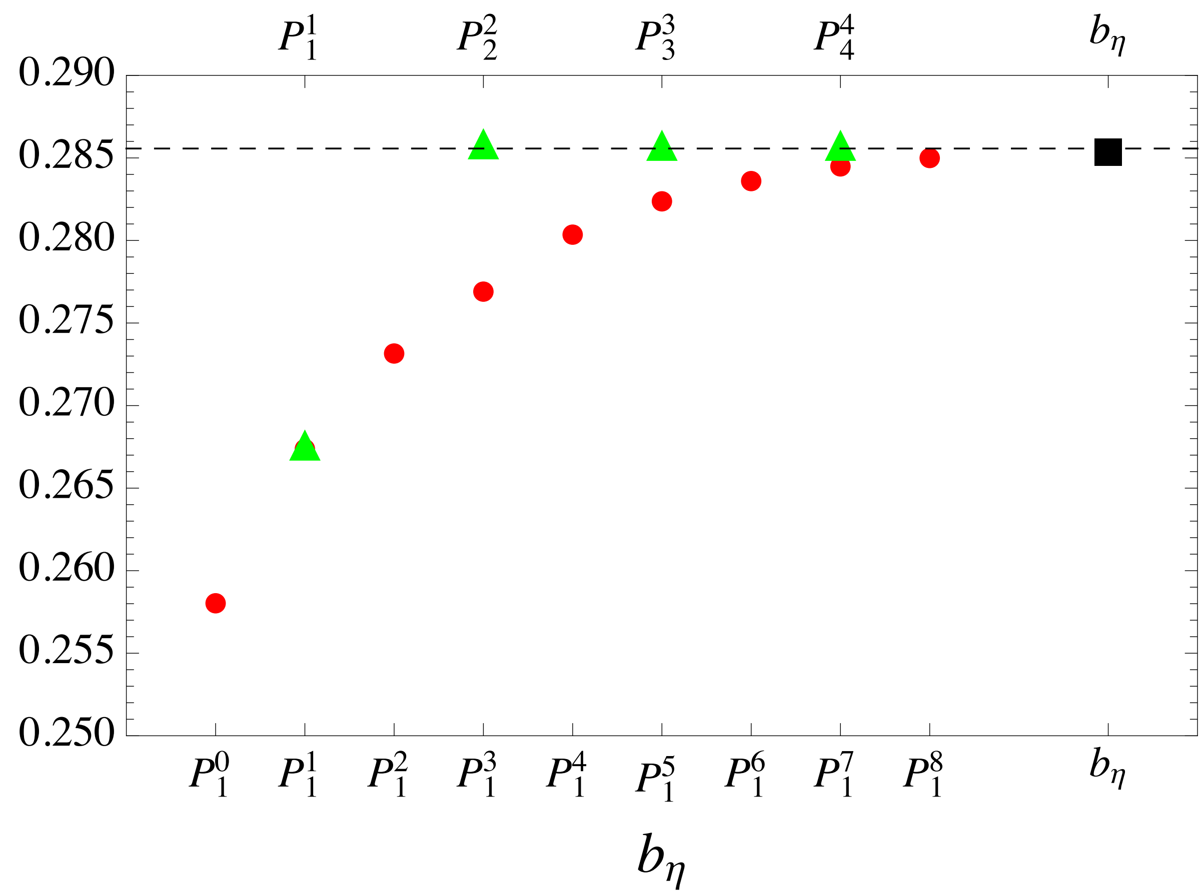
[R.Escribano, P.M., P. Sanchez-Puertas, '15]



A word on systematics

- Consider a model for η TFF
- Generate a pseudodata set emulating the physical situation (SL+TL)
- Build up your PA sequence
- Fit and compare





Why PAs?

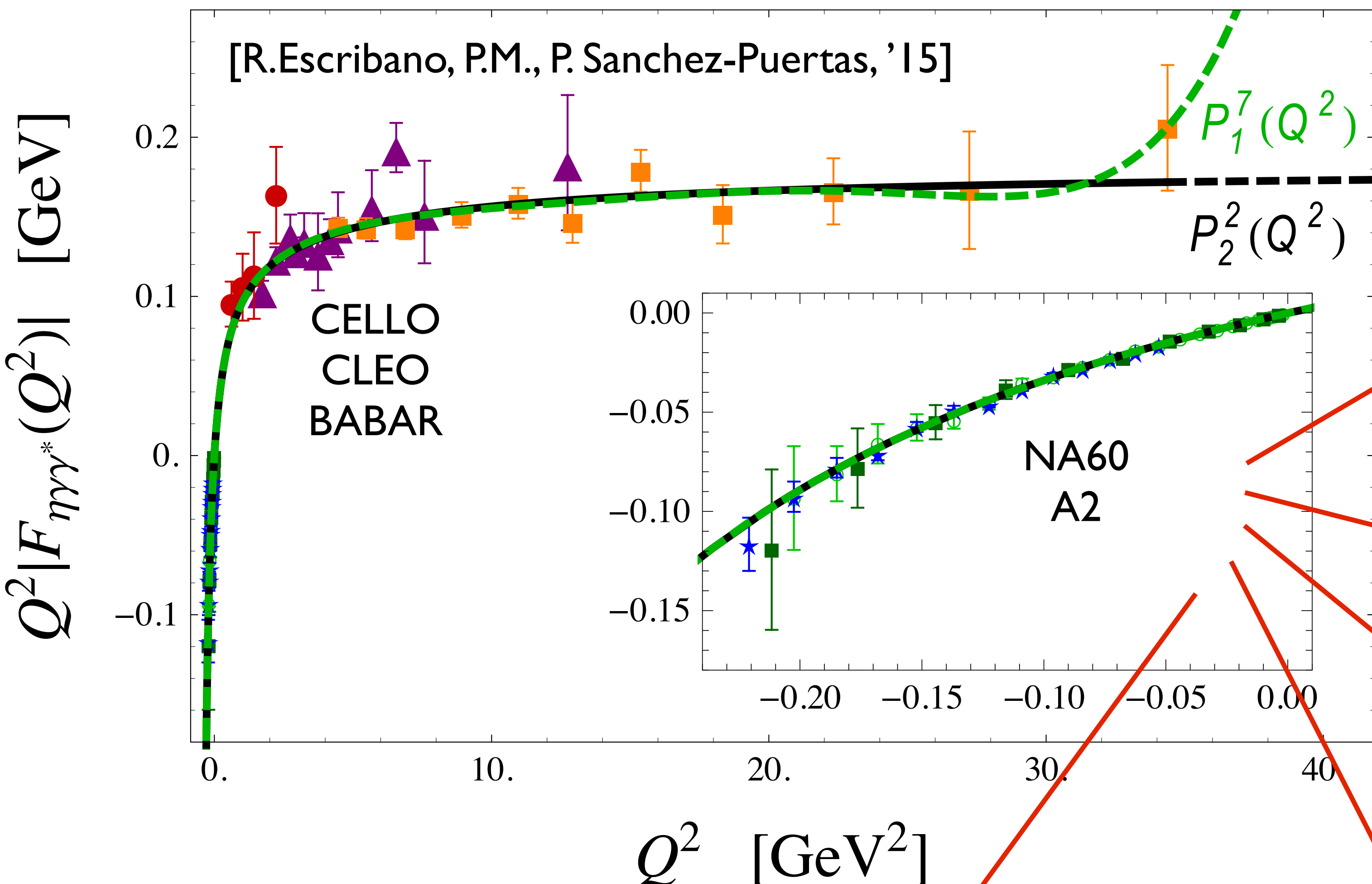
- To THEORY:
  - our problem is "the general rational Hermite interpolation problem": solution is PAs
  - in many cases we need $\int ds\, f(s)$ not $f(s)$
  - VMD, LMD, LMD+V, HQCD... are already a certain kind of PAs (PTs)
  - it is proven: PTs slower convergence than PAs
- To DISPERSION Theory:
  - excellent interpolation tool: bring DRs (TL!) to ∞(link with pQCD)
    - even more: PA can be the function to be used for the DR
    - or DR can provide LECs to feed in the PA
- To LATTICE:
  - used recently for lattice fitting of HVP: suggestion use it correctly!
- To EXPERIMENT:

  what energy and precision to measure: use PAs to identify!

  (high energies are as well important vs DRs)

PS-TFF

space-like and time-like data



Low-energy parameters up to the third derivative!

(g-2)$_\mu$

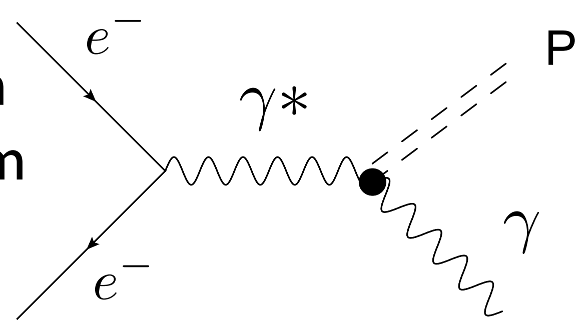
mixing parameters of the η-η' system

f_q, f_s, ϕ

Rare decays

$\Gamma_{P\to l^+l^-}$ $\Gamma_{P\to 4l}$

PS continuum on quarkonium region



beautiful synergy experiment - theory

Applications

1. Hadronic Light-by-Light contribution to muon (g-2)
2. PS decays into lepton pairs ($\pi^0 \to e^+e^-$)
3. η-η' mixing

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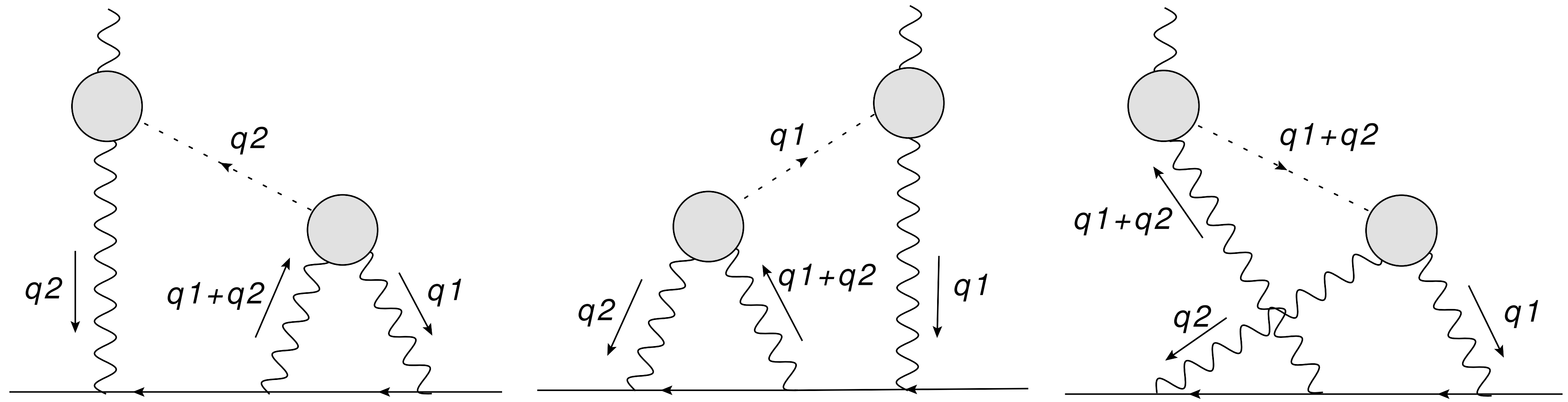
Dissection of the HLBL contribution



$$a_\mu^{LbL;P} = -e^6 \int \frac{d^4q_1}{(2\pi)^4} \int \frac{d^4q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (q_1+q_2)^2 [(p+q_1)^2 - m^2][(p-q_2)^2 - m^2]}$$

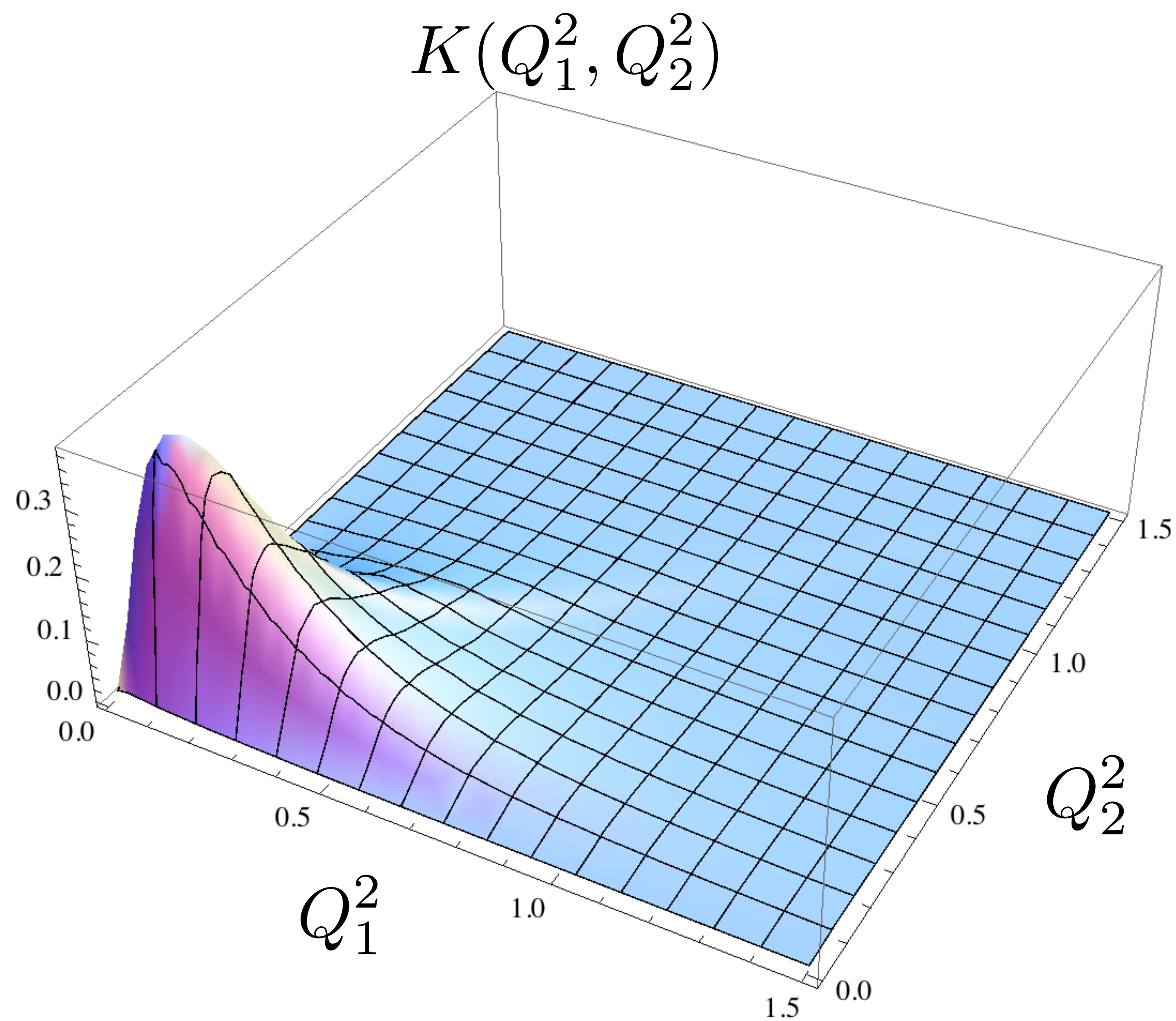
$$\times \left(\frac{F_{P^*\gamma^*\gamma^*}(q_2^2, q_1^2, (q_1+q_2)^2) F_{P^*\gamma^*\gamma^*}(q_2^2, q_2^2, 0)}{q_2^2 - M_P^2} T_1(q_1, q_2; p) \right.$$

$$\left. + \frac{F_{P^*\gamma^*\gamma^*}((q_1+q_2)^2, q_1^2, q_2^2) F_{P^*\gamma^*\gamma^*}((q_1+q_2)^2, (q_1+q_2)^2, 0)}{(q_1+q_2)^2 - M_P^2} T_2(q_1, q_2; p) \right)$$

Dissection of the HLBL contribution

$$a_\mu^{LbyL;\pi^0} = e^6 \int \frac{d^4Q_1}{(2\pi)^4} \int \frac{d^4Q_2}{(2\pi)^4} K(Q_1^2, Q_2^2)$$

Using $F_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2) \sim VMD(Q_1^2, Q_2^2)$



(main energy range from 0 to 1 GeV²)

Dissection of the HLBL contribution

a la Melnikov-Vainshtein

Central value: [Knecht-Nyffeler '01]

$$F^{LMD+V}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{f_\pi}{3}\frac{q_1^2q_2^2(q_1^2+q_2^2)+h_1(q_1^2+q_2^2)^2+h_2q_1^2q_2^2+h_5(q_1^2+q_2^2)+h_7}{(q_1^2-M_{V_1}^2)(q_1^2-M_{V_2}^2)(q_2^2-M_{V_1}^2)(q_2^2-M_{V_2}^2)}$$

Publication:

$$F_\pi = 92.4 \text{ MeV}$$
$$m_\rho = 769 \text{ MeV}$$
$$m_{\rho'} = 1465 \text{ MeV}$$
$$h_1 = 0 \,(\text{ BL limit})$$
$$h_5 = 6.93 \text{ GeV}^4$$
$$h_2 = -10 \text{ GeV}^2$$

$$\boxed{a_\mu^{\text{HLBL},\pi} = 7.7\times 10^{-10}}$$

Dissection of the HLBL contribution

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Publication:		Preliminary, using exp data
$F_\pi = 92.4$ MeV	→	$\Gamma_{\pi^0\to\gamma\gamma}$
$m_\rho = 769$ MeV		$m_\rho = 775$ MeV
$m_{\rho'} = 1465$ MeV	→	curvature
$h_1 = 0$ (BL limit)		$h_1 = 0$ (BL limit)
$h_5 = 6.93\ \text{GeV}^4$	→	slope
$h_2 = -10\ \text{GeV}^2$		$h_2 = -10\ \text{GeV}^2$
$a_\mu^{\text{HLBL},\pi} = 7.7\times10^{-10}$	→	$a_\mu^{\text{HLBL},\pi} = 9.8\times10^{-10}$

Dissection of the HLBL contribution

a la Knecht-Nyffeler '01

Central value:

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$$a_\mu^{\text{HLBL},\pi} = 6.3\times 10^{-10}$$

Dissection of the HLBL contribution

a la Knecht-Nyffeler '01

Central value:

$$F^{LMD+V}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{f_\pi}{3}\frac{q_1^2q_2^2(q_1^2+q_2^2)+h_1(q_1^2+q_2^2)^2+h_2q_1^2q_2^2+h_5(q_1^2+q_2^2)+h_7}{(q_1^2-M_{V_1}^2)(q_1^2-M_{V_2}^2)(q_2^2-M_{V_1}^2)(q_2^2-M_{V_2}^2)}$$

Publication:		Preliminary, using exp data:
$F_\pi = 92.4$ MeV	→	$\Gamma_{\pi^0\to\gamma\gamma}$
$m_\rho = 769$ MeV		$m_\rho = 775$ MeV
$m_{\rho'} = 1465$ MeV	→	curvature
$h_1 = 0\,(\text{ BL limit})$		$h_1 = 0\,(\text{ BL limit})$
$h_5 = 6.93\ \text{GeV}^4$	→	slope
$h_2 = -10\ \text{GeV}^2$		$h_2 = -10\ \text{GeV}^2$
$a_\mu^{\text{HLBL},\pi} = 6.3\times10^{-10}$	→	$a_\mu^{\text{HLBL},\pi} = 7.5\times10^{-10}$

Dissection of the HLBL contribution

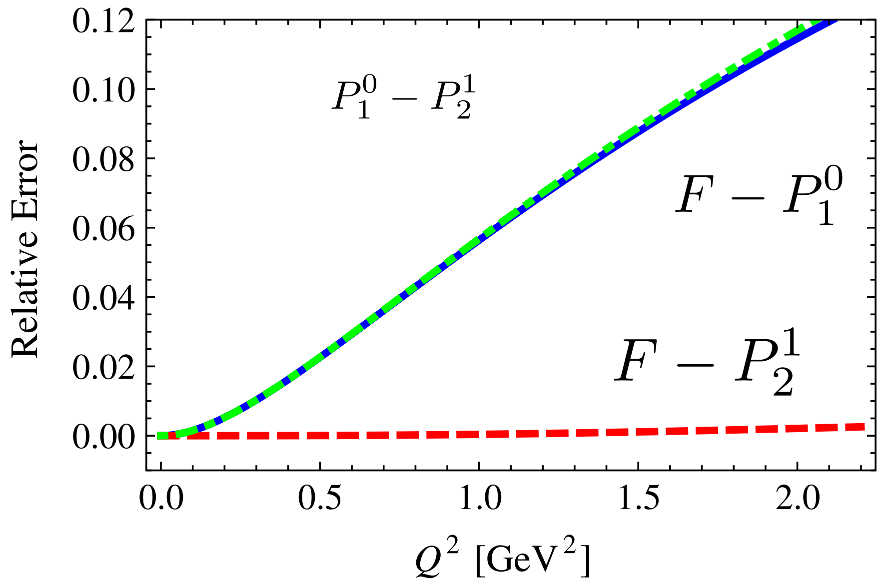
a la Padé

P.M., S. Peris, 07 P.M. '12
P.M., Vanderhaeghen'12
R. Escribano, P.M., P. Sanchez-Puertas, '13

$$F^{P01}_{P^*\gamma^*\gamma^*}(P_P^2,Q_1^2,Q_2^2) = a\frac{b}{Q_1^2+b}\frac{b}{Q_2^2+b}(1+cP_P^2)$$

$$F^{P12}_{P^*\gamma^*\gamma^*}(P_P^2,Q_1^2,Q_2^2) = \frac{a+bQ_1^2}{(Q_1^2+c)(Q_1^2+d)}\frac{a+bQ_2^2}{(Q_2^2+c)(Q_2^2+d)}(1+cP_P^2)$$

	b_P	c_P	$\lim_{Q^2\to\infty} Q^2 F_{P\gamma^*\gamma}(Q^2)$	$a_\mu^{\mathrm{HLBL;P}}$
π^0	0.0324(22)	$1.06(27)\cdot 10^{-3}$	$2f_\pi$	$6.49(56)\cdot 10^{-10}$
η	0.60(7)	0.37(12)	0.160(24)GeV	$1.25(15)\cdot 10^{-10}$
η'	1.30(17)	1.72(58)	0.255(4)GeV	$1.27(19)\cdot 10^{-10}$



Systematic error from approach:

$P_1^0(Q_1^2,Q_2^2)$ vs $P_2^1(Q_1^2,Q_2^2)$ → 5%

[P.M., Peris, '07]

Applications

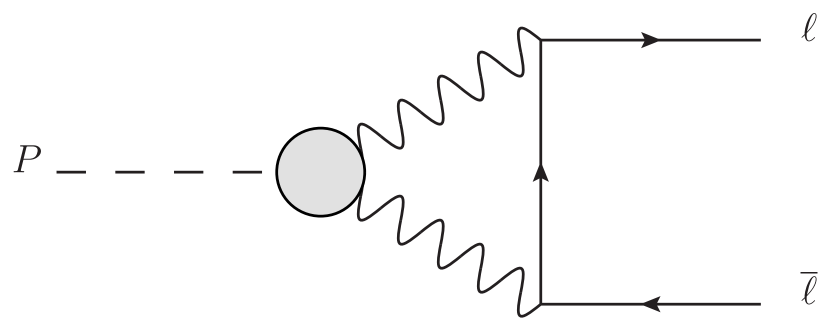
1. Hadronic Light-by-Light contribution to muon (g-2)

2. P decays into lepton pairs ($\pi^0 \rightarrow e^+e^-$)

3. η-η' mixing

Introduction and Motivation

Experiment



$$\frac{BR(P \to \bar{\ell}\ell)}{BR(P \to \gamma\gamma)} = 2\left(\frac{\alpha m_\ell}{\pi m_P}\right)^2 \beta_\ell(m_P^2)|\mathcal{A}(m_P^2)|^2$$

$$\sim 1.5 \cdot 10^{-10}$$

KTeV '07:

$$BR(\pi^0 \to e^+e^-(\gamma), x > 0.95) = (6.44 \pm 0.25 \pm 0.22) \times 10^{-8}$$

Extrapolation to x=1 + radiative correction + Dalitz decay background

$$BR_{\mathrm{KTeV}}^{w/o\,rad}(\pi^0 \to e^+e^-) = (7.48 \pm 0.29 \pm 0.25) \times 10^{-8}$$

(dominates de PDG)

Prague contribution: Radiative corrections

Vasko, Novotny '11 + Husek, Kampf, Novotny'14

$$\frac{\mathrm{BR}(\pi^0 \to e^+e^-(\gamma), x > 0.95)}{\mathrm{BR}(\pi^0 \to \gamma\gamma)} =$$

$$\frac{\Gamma(\pi^0 \to e^+e^-)}{\Gamma(\pi^0 \to \gamma\gamma)} [1 + \delta^{(2)}(0.95) + \Delta^{BS}(0.95) + \delta^{D}(0.95)]$$

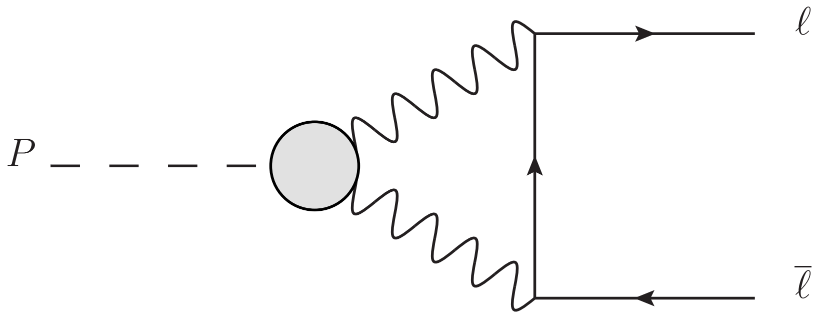
$$\delta^{(2)}(0.95) \equiv \delta^{\mathrm{virt.}} + \delta^{\mathrm{BS}}_{\mathrm{soft}}(0.95) = (-5.8 \pm 0.2)\,\% \qquad \mathrm{vs} \ \sim -13\%$$

$$\Delta^{\mathrm{BS}}(0.95) = (0.30 \pm 0.01)\,\% \qquad \delta^{D}(0.95) = \frac{1.75 \times 10^{-15}}{[\Gamma^{\mathrm{LO}}(\pi^0 \to e^+e^-)/\mathrm{MeV}]}$$

$$BR^{w/o\,rad}_{\text{"KTeV"}}(\pi^0 \to e^+e^-) = (6.87 \pm 0.36) \times 10^{-8}$$

Introduction and Motivation

Theory



$$\frac{BR(P \to \bar{\ell}\ell)}{BR(P \to \gamma\gamma)} = 2\left(\frac{\alpha m_\ell}{\pi m_P}\right)^2 \beta_\ell(m_P^2)|\mathcal{A}(m_P^2)|^2$$

The only unknown $\mathcal{A}(m_P^2)$ from loop calculation where the TFF enters.

$$\mathcal{A}(q^2) = \frac{2i}{\pi^2}\int d^4k \; \frac{q^2k^2 - (k\cdot q)^2}{k^2(k-q)^2((p-k) - m_\ell^2)} \frac{F_{P\gamma^*\gamma^*}(k^2,(q-k)^2)}{F_{P\gamma\gamma}(0,0)}$$

Dissection of $\pi^0 \to e^+e^-$

As model independent as possible:

Cutcosky rules provides the imaginary part

$$Im\mathcal{A}(q^2) = \frac{\pi}{2\beta_l(q^2)} ln\left(\frac{1-\beta_l(q^2)}{1+\beta_l(q^2)}\right); \quad \beta_l(q^2) = \sqrt{1-\frac{4m_l^2}{q^2}}$$

$$q^2 = m_P^2$$

Assuming $|\mathcal{A}|^2 \geq (\mathrm{Im}\mathcal{A})^2$

$$B(\pi^0 \to e^+e^-) \geq B^{\mathrm{unitary}}(\pi^0 \to e^+e^-) = 4.69 \cdot 10^{-8}$$

(doesn't depend on TFF)

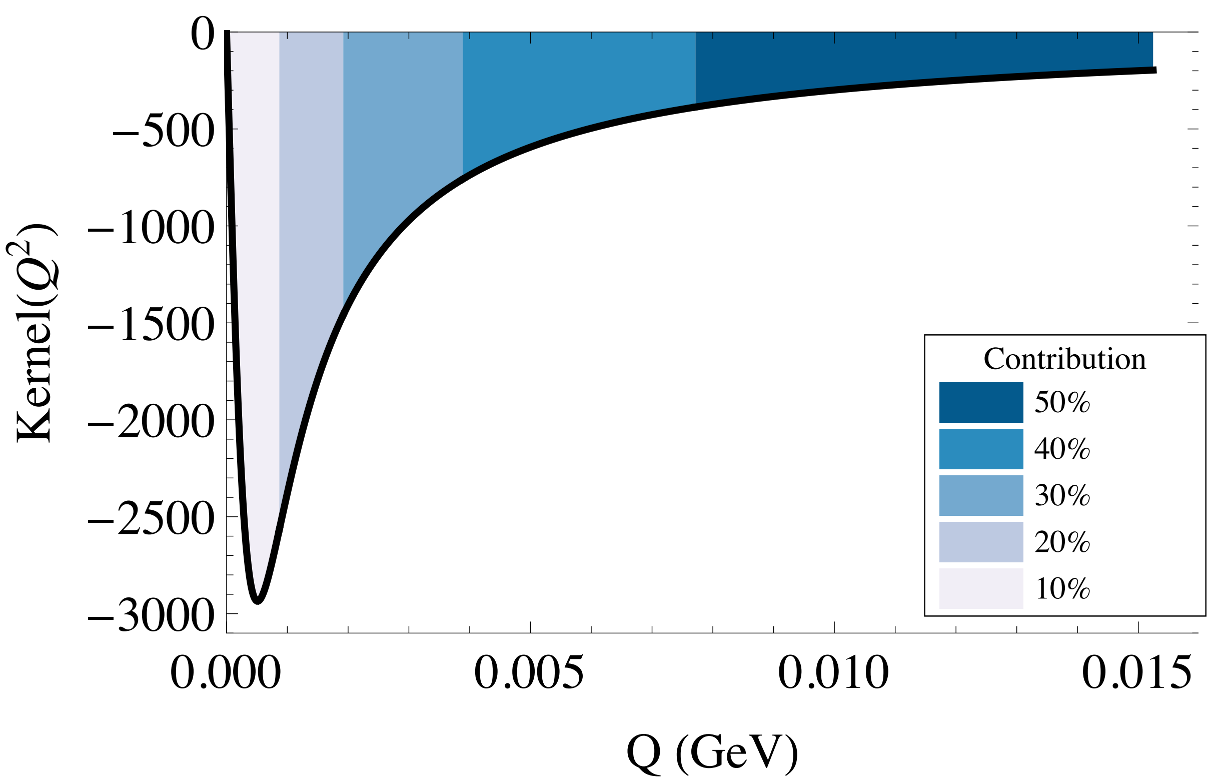
Dissection of $\pi^0 \to e^+e^-$

$$Re(\mathcal{A}(m_P^2)) = \left(-\frac{5}{4} + \int_0^\infty dQ^2\ Kernel(Q^2)\right) + \frac{\pi^2}{12} + ln^2\left(\frac{m_l}{m_P}\right)$$

$$Re(\mathcal{A}(m_P^2)) = \int_0^\infty dQ^2 Kernel(Q^2) + 30.7$$

Dissection of $\pi^0 \to e^+e^-$

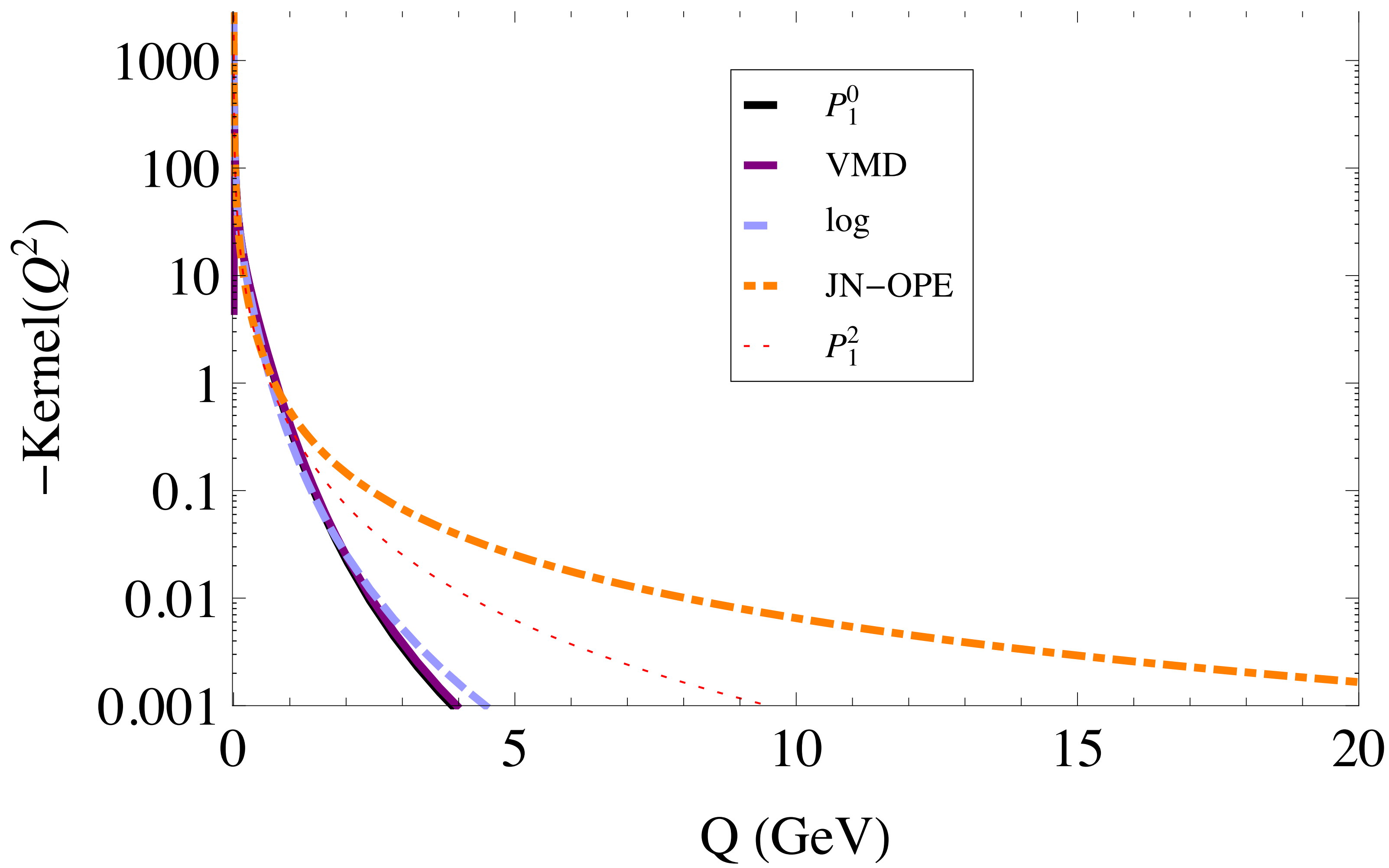
$$Re(\mathcal{A}(m_P^2)) = \int_0^\infty dQ^2 Kernel(Q^2) + 30.7$$



- Its contribution is negative: lowers the BR.
- Peaks at $\sim\ 2m_e$ and $\langle Q \rangle = 0.09$ GeV.
- Low energies relevant only: slope is enough.

Dissection of $\pi^0 \to e^+e^-$

$$Re(\mathcal{A}(m_P^2)) = \int_0^\infty dQ^2 Kernel(Q^2) + 30.7$$

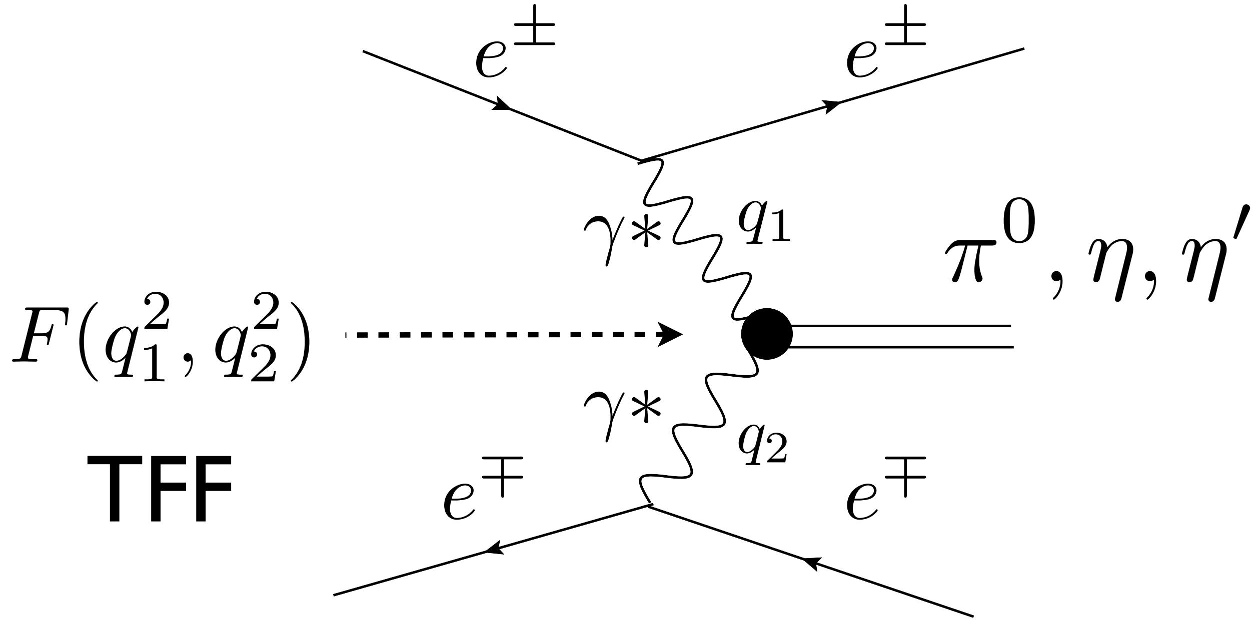


Mainz contribution: TFF parameterization

Use data from
the Transition Form Factor
for numerical integral

$$F_{P\gamma^*\gamma^*}(m_P^2, q_1^2, q_2^2)$$

double-tag method



Remember: only low-energy region is needed

Doubly virtual π⁰-TFF

[P.M., P. Sanchez-Puertas, '15]

For $BR_{SM}(\pi^0 \to e^+e^-)$ we need $F_{\pi^0\gamma^*\gamma^*}(Q^2, Q^2)$

Proposal: Canterbury PA Chisholm '73

$$P_M^N(Q_1^2, Q_2^2) = \frac{T_N(Q_1^2, Q_2^2)}{R_M(Q_1^2, Q_2^2)} = a_0 + a_1(Q_1^2 + Q_2^2) + a_{1,1}Q_1^2Q_2^2 + a_2(Q_1^4 + Q_2^4) + \cdots$$

$$P_1^0(Q_1^2, Q_2^2) = \frac{a_0}{1 + a_1(Q_1^2 + Q_2^2) + (2a_1^2 - a_{1,1})Q_1^2Q_2^2}$$

Doubly virtual π⁰-TFF

Chisholm '73

Proposal: Canterbury PA

$$P_1^0(Q_1^2, Q_2^2) = \frac{a_0}{1 + a_1(Q_1^2 + Q_2^2) + (2a_1^2 - a_{1,1})Q_1^2 Q_2^2}$$

a_1 from accurate study of space-like data

$a_{1,1}$ from a systematic fit to doubly virtual SL data

Doubly virtual π^0-TFF

Chisholm '73

Proposal: Canterbury PA

$$P_1^0(Q_1^2, Q_2^2) = \frac{a_0}{1 + a_1(Q_1^2 + Q_2^2) + (2a_1^2 - a_{1,1})Q_1^2 Q_2^2}$$

a_1 from accurate study of space-like data

$a_{1,1}$ from a systematic fit to doubly virtual SL data

OPE indicates: $\lim_{Q^2\to\infty} P_1^0(Q^2, Q^2) \sim Q^{-2}$ i.e., $a_{1,1} = 2a_1^2$

Doubly virtual π^0-TFF

Chisholm '73

Proposal: Canterbury PA

$$P_1^0(Q_1^2, Q_2^2) = \frac{a_0}{1 + a_1(Q_1^2 + Q_2^2) + (2a_1^2 - a_{1,1})Q_1^2 Q_2^2}$$

a_1 from accurate study of space-like data

$$0 \leq a_{1,1} \leq 2a_1^2$$

[P.M, P. Sanchez-Puertas,15]

$$BR_{SM}^{PA}(\pi^0 \to e^+e^-) = (6.20 - 6.41)(4) \times 10^{-8}$$

statistics+theoretical error

method checked for different models
+ to shrink the window: data (data-driven approach)

Doubly virtual π^0-TFF

[P.M, P. Sanchez-Puertas,15]

$$BR^{w/o\, rad}_{\mathrm{KTeV}}(\pi^0 \to e^+e^-) = (7.48 \pm 0.29 \pm 0.25) \times 10^{-8}$$

$$\Updownarrow \qquad \sim (3.3 - 2.8)\sigma$$

$$BR^{PA}_{SM}(\pi^0 \to e^+e^-) = (6.20 - 6.41)(4) \times 10^{-8}$$

[with new RC ~ 2 σ]

Impact of $\pi^0 \to e^+e^-$ on HLBL

[P.M, P. Sanchez-Puertas,15]

	Model	Published model		Modified model	
		$\pi^0 \to e^+e^-$ $(\times 10^8)$	$HLBL$ $(\times 10^{10})$	$\pi^0 \to e^+e^-$ $(\times 10^8)$	$HLBL$ $(\times 10^{10})$
Jegerlehner and Nyffeler '09	LMD+V	6.33	6.29	6.47	5.22
Dorokhov et al '09	VMD	6.34	5.64	6.87	2.44
Our proposal '14	PA	6.36	5.53	6.87	2.85

$$\Delta a_\mu^{SM} \sim 6 \times 10^{-10}$$

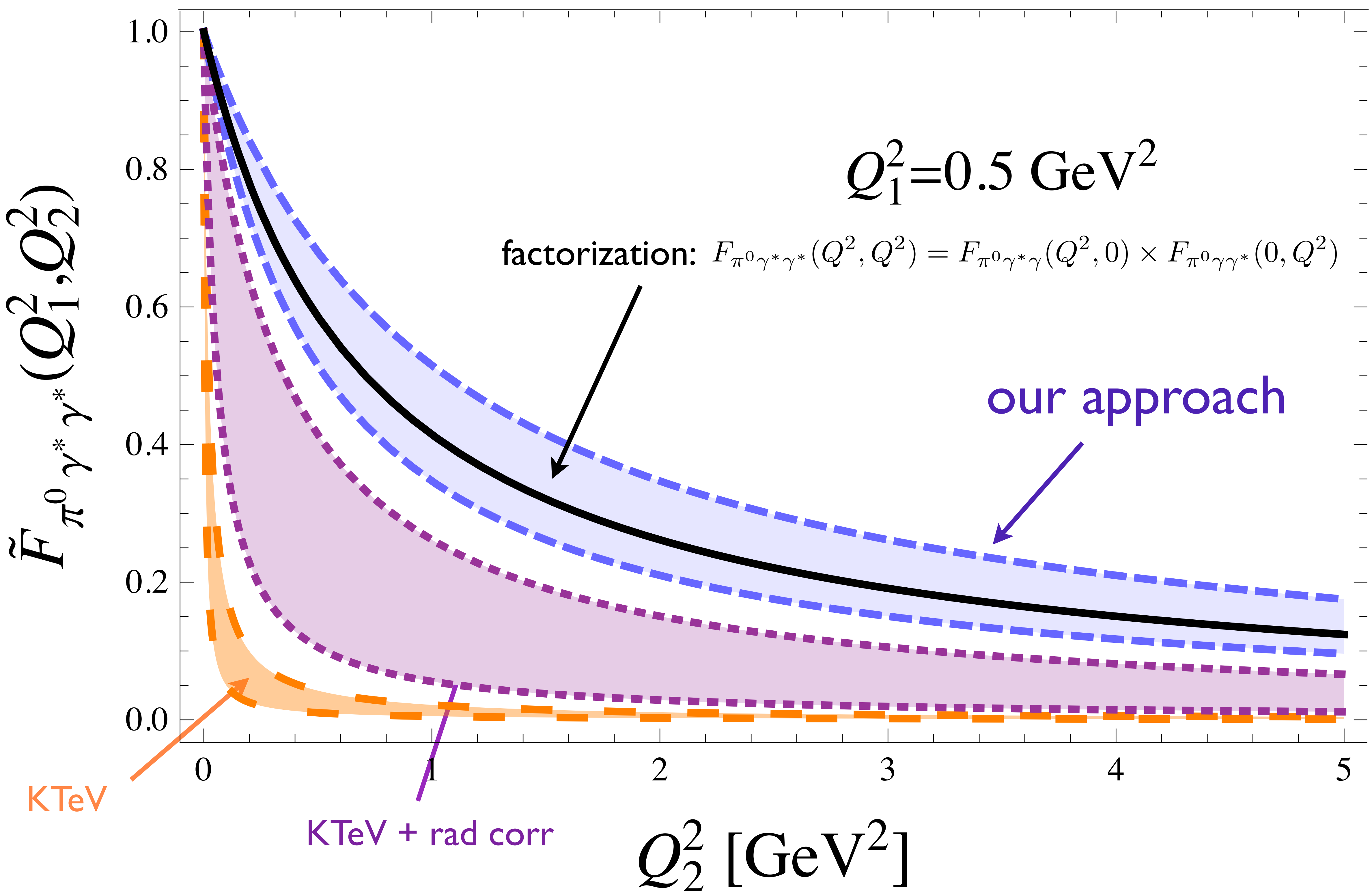
$$\Delta a_\mu^{HLBL} \sim 4 \times 10^{-10}$$

$$\Delta a_\mu^{HLBL;\pi^0 \to e^+e^-} \sim (2-3) \times 10^{-10}$$

+ similar effect for the η decay!

The role of doubly virtual TFF data

[P.M, P. Sanchez-Puertas,15]



Dissection of $\eta \to l^+ l^-$

[P.M, P. Sanchez-Puertas, in prep.]

PDG value dominated by the KTeV measurement

$$\frac{BR(P \to \bar{\ell}\ell)}{BR(P \to \gamma\gamma)} = 2\left(\frac{\alpha m_\ell}{\pi m_P}\right)^2 \beta_\ell(m_P^2)|\mathcal{A}(m_P^2)|^2 \quad \begin{aligned} &= 5.8(8)\cdot 10^{-6} \quad (\mu^+\mu^-) \\ &\leq 5.6\cdot 10^{-6} \quad (e^+e^-) \end{aligned}$$

Unitary Bound for the μμ case $= 4.37 \cdot 10^{-6}$

SM calculations with $m_\eta^2/\Lambda^2 \sim 0$ $= 4.99 \cdot 10^{-6}$

Our result from SL+TL (full result) $= 4.51(2) \cdot 10^{-6}$

Applications

1. Hadronic Light-by-Light contribution to muon (g-2)
2. PS decays into lepton pairs ($\pi^0 \rightarrow e^+e^-$)
3. η-η' mixing

η-η' mixing

η-η' mixing in the flavor basis

$$\begin{pmatrix} f_\eta^q & f_\eta^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = \begin{pmatrix} f_q \cos[\phi] & -f_s \sin[\phi] \\ f_q \sin[\phi] & f_s \cos[\phi] \end{pmatrix}$$

From the TFFs we can determine f_q, f_s, ϕ

$$\Gamma_{\eta\to\gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_\eta^3 \left(\frac{C_q \cos[\phi]}{f_q} - \frac{C_s \sin[\phi]}{f_s} \right)^2$$

$$\Gamma_{\eta'\to\gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_{\eta'}^3 \left(\frac{C_q \sin[\phi]}{f_q} + \frac{C_s \cos[\phi]}{f_s} \right)^2$$

$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma\gamma^*}(Q^2) = f_\eta^q \frac{10}{3} + f_\eta^s \frac{2\sqrt{2}}{3},$$

$$\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma\gamma^*}(Q^2) = f_{\eta'}^q \frac{10}{3} + f_{\eta'}^s \frac{2\sqrt{2}}{3}.$$

η-η' mixing

η-η' mixing in the flavor basis

$$\begin{pmatrix} f_\eta^q & f_\eta^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = \begin{pmatrix} f_q \cos[\phi] & -f_s \sin[\phi] \\ f_q \sin[\phi] & f_s \cos[\phi] \end{pmatrix}$$

From the TFFs we can determine f_q, f_s, ϕ [R.Escribano, P.M., P. Sanchez-Puertas, '15]

$$\Gamma_{\eta\to\gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_\eta^3 \left(\frac{C_q \cos[\phi]}{f_q} - \frac{C_s \sin[\phi]}{f_s} \right)^2$$

$$\Gamma_{\eta'\to\gamma\gamma} = \frac{9\alpha^2}{32\pi^3} M_{\eta'}^3 \left(\frac{C_q \sin[\phi]}{f_q} + \frac{C_s \cos[\phi]}{f_s} \right)^2$$

$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma\gamma^*}(Q^2) = f_\eta^q \frac{10}{3} + f_\eta^s \frac{2\sqrt{2}}{3},$$

$$\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma\gamma^*}(Q^2) = f_{\eta'}^q \frac{10}{3} + f_{\eta'}^s \frac{2\sqrt{2}}{3}.$$

[R.Escribano, P.M., P. Sanchez-Puertas, '14]

$$f_q = 1.07(1) f_\pi, \quad f_s = 1.39(14) f_\pi, \quad \phi = 39.3(1.3)^\circ$$

Update of Frere-Escribano '05 with PDG12 using 9 inputs

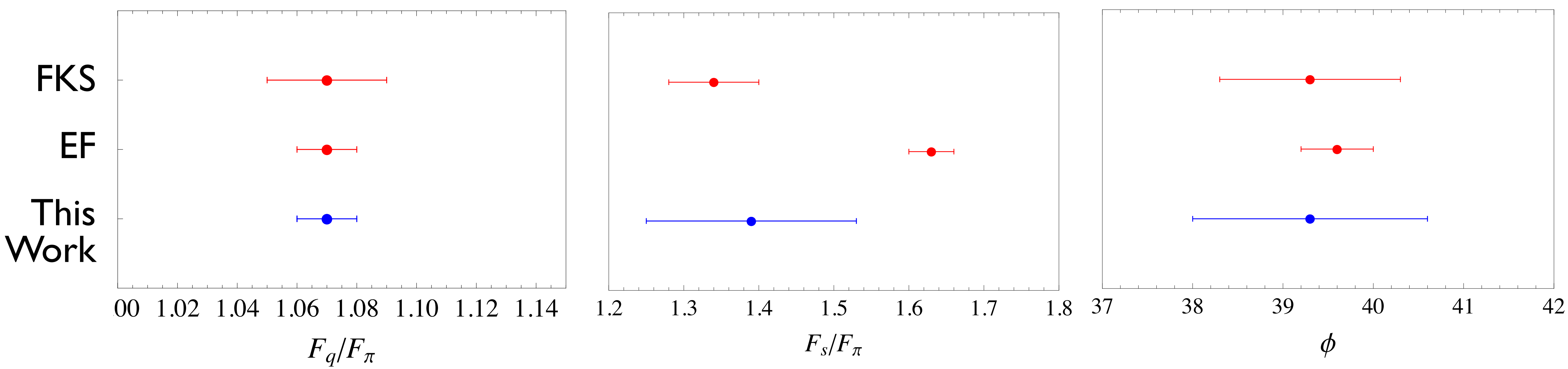
$$f_q = 1.07(1) f_\pi, \quad f_s = 1.63(2) f_\pi, \quad \phi = 40.4(0.3)^\circ$$

η-η' mixing

η-η' mixing in the flavor basis

From the TFFs we can determine F_q, F_s, ϕ

[R.Escribano, P.M., P. Sanchez-Puertas, '15]



FKS: Feldmann, Kroll, Stech, PLB 449, 339, (1999)
EF: Escribano, Frere, JHEP 0506, 029 (2005) updated in Escribano, P.M, Sanchez-Puertas, 2013.

Conclusions

- Transition Form Factors are a good laboratory to study meson properties (one and two virtualities)
- We propose Padé Approximants' method: is *easy*, *systematic* and can be *improved* upon by including new data.
- Considering space- and time-like data
  - provides very accurate LECs and asymptotic limits
  - provides insight in mixing scheme and meson structure
  - predicts $VP\gamma$, J/Ψ, rare decays, continuum...
  - <u>beautiful synergy experiment - theory</u>

Outlook

- To THEORY:
  - rethink VMD, LMD, LMD+V, HQCD in terms of PAs: data-driven and systematic
- To DISPERSION Theory:
  - excellent interpolation tool: bring DRs to ∞ (link with pQCD)
  - progress in PA theory is undergoing: better access TL, use unitary...
- To LATTICE:
  - used recently for lattice fitting of HVP: suggestion use it correctly!
- To EXPERIMENT:

  what energy and precision to measure: use PAs to identify!

Thanks!

Dissection of the HLBL contribution

a la Knecht-Nyffeler '01

Error budget:

$$F^{VMD}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = -\frac{N_c}{12\pi^2 f_\pi}\frac{M_V^2}{(q_1^2-M_V^2)}\frac{M_V^2}{(q_2^2-M_V^2)}$$

$$F^{LMD}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{f_\pi}{3}\frac{(q_1^2+q_2^2)-c_V}{(q_1^2-M_V^2)(q_2^2-M_V^2)}$$

$$F^{LMD+V}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{f_\pi}{3}\frac{q_1^2q_2^2(q_1^2+q_2^2)+h_1(q_1^2+q_2^2)^2+h_2q_1^2q_2^2+h_5(q_1^2+q_2^2)+h_7}{(q_1^2-M_{V_1}^2)(q_1^2-M_{V_2}^2)(q_2^2-M_{V_1}^2)(q_2^2-M_{V_2}^2)}$$

$$\Delta F_\pi \Rightarrow 2\Delta a_\mu^{\text{HLBL, P}}$$
$$\Delta\ \text{slope} \Rightarrow 0.75\Delta a_\mu^{\text{HLBL, P}}$$
$$\Delta\ \text{curv.} \Rightarrow 0.5\Delta a_\mu^{\text{HLBL, P}}$$
$$\Delta m_\rho = \Gamma/2 \Rightarrow 1.3\Delta a_\mu^{\text{HLBL, P}}$$

Current exp. precision:

$$\Delta F_\pi \sim 1.1\%$$
$$\Delta\ \text{slope} \sim 13\%$$
$$\Delta\ \text{curvature} \sim 25\%$$

Chiral limit $\quad F_0 \to F_\pi \sim 5\%$

1/Nc $\quad \Delta m_\rho \sim 10\%$

$$\Rightarrow \Delta a_\mu^{\text{HLBL},\pi} \sim 15\%$$

Dubna contribution: corrections m_e/m_π, m_e/Λ

Dorokhov and Ivanov, '08

$$\mathcal{O}\left(\frac{m_e}{\Lambda}\right)^2 \qquad \mathcal{O}\left(\frac{m_e}{\Lambda}\log\frac{m_e}{\Lambda}\right)^2$$

Dorokhov, Ivanov and Kovalenko '09

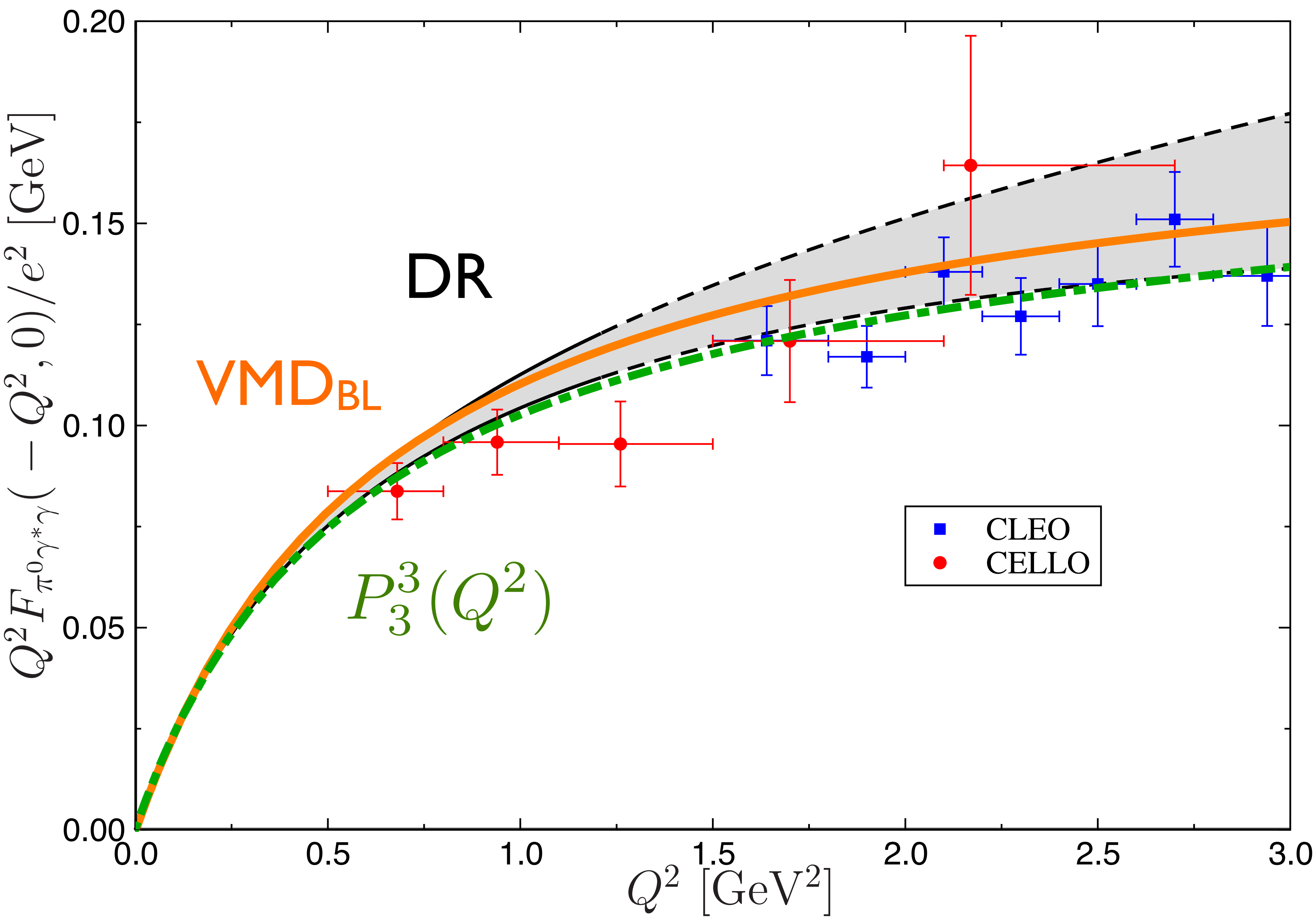
$$\mathcal{O}\left(\frac{m_\pi}{\Lambda}\right)^2 \qquad \mathcal{O}\left(\frac{m_e}{m_\pi}\right)^2$$

Λ
the cut-off
or
VMD "mass"

Resummation of power corrections using Mellin-Barnes techniques.
Conclusion: corrections negligible!

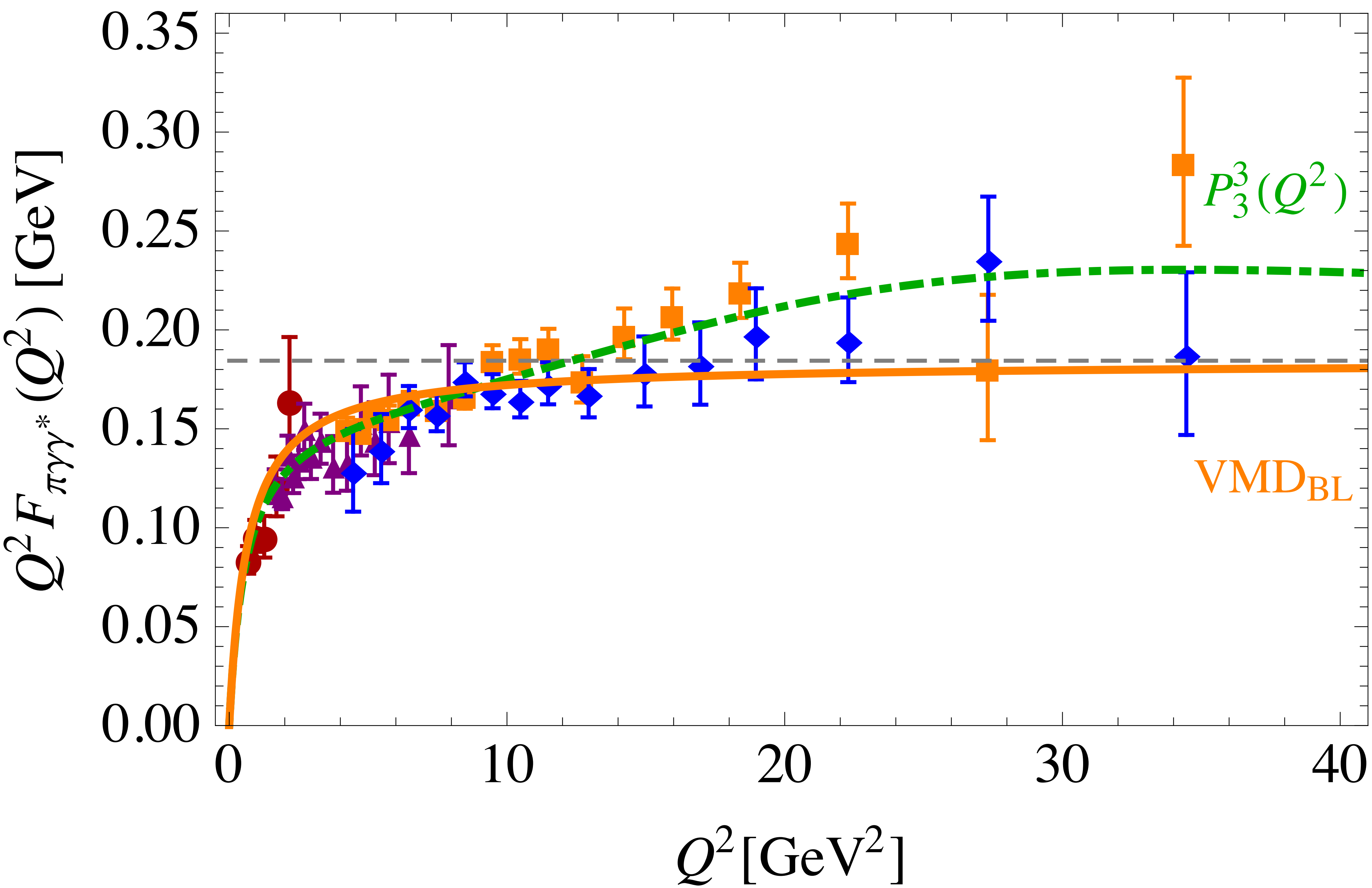
$$BR_{\rm SM}(\pi^0 \to e^+e^-) = (6.23 \pm 0.09) \times 10^{-8} \quad \sim 3\sigma$$

PA vs DR



0.20
0.15
0.10
0.05
0.00
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$Q^2 F_{\pi^0\gamma^*\gamma}(-Q^2,0)/e^2$ [GeV]
Q^2 [GeV^2]
DR
VMD$_{BL}$
$P_3^3(Q^2)$
CLEO
CELLO

PA vs DR



0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
10
20
30
40
$Q^2 F_{\pi\gamma\gamma^*}(Q^2)$ [GeV]
Q^2[GeV^2]
$P_3^3(Q^2)$
VMD$_{BL}$