Radiative mu and tau leptonic decays and the possibility to determine the tau dipole moments

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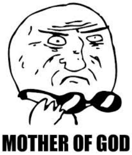
FCCP2015, Anacapri, September 11 2015

hep-ph:1301.5302, 1310.1081, 1506.03416

work in collaboration with:
D. Epifanov, S. Eidelman, L. Mercolli and M. Passera.



Electron

$$a_e = 1\,159\,652\,180.\,73\,(28) \cdot 10^{-12}$$

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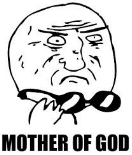
540 parts per billion! E821–Final Rep: PRD73 (2006) 072003



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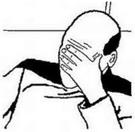


Tau

$$-0.052 \leq a_\tau \leq 0.013$$

DELPHI - EPJC 35 (2004) 159

Precise SM prediction: $a_\tau^{\mathrm{SM}} = 117\,721\,(5)\,10^{-8}$

S. Eidelman, M. Passera, Mod.Phys.Lett. A22 (2007) 159

The τ dipole moments: SM prediction

The Standard Model prediction for the tau g-2

$$
\begin{aligned}
a_\tau^{\rm SM} = \; & +117\,324\,(2) && \times 10^{-8} && \text{QED} \\
& +47.4\,(5) && \times 10^{-8} && \text{EW} \\
& +337.5\,(3.7) && \times 10^{-8} && \text{HLO} \\
& +7.6\,(2) && \times 10^{-8} && \text{HHO (vac)} \\
& +5\,(3) && \times 10^{-8} && \text{HHO (lbl)} \\
= \; & 117\,721\,(5) && \times 10^{-8} &&
\end{aligned}
$$

Eidelman & Passera, Mod.Phys.Lett. A22 (2007) 159

- $(m_\tau/m_\mu)^2 \sim 280$: good to look for NP!

	Electron	Muon	Tau
$a^{\rm EW}/a^{\rm H}$	1/56	1/45	1/7
$a^{\rm EW}/\delta a^{\rm H}$	1.6	3	10

- Lepton EDMs vanish up to 3 loop Pospelov & Khriplovich, SJNP 53 (1991) 638
- $d_l^{\rm SM} \sim 10^{-38} - 10^{-35}\ e{\cdot}$cm $\lll$ exp. sensitivity

The τ g-2: experimental bounds

- $\tau_\tau = 2.903\,(5) \times 10^{-13}$ s
- No beam of polarized τ
- DELPHI measurements at LEP 2 of $\sigma(e^+e^- \to e^+e^-\tau^+\tau^-)$

the PDG value

$$a_\tau = -0.018\,(17)$$

DELPHI - EPJC35 (2004) 159

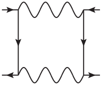
- Indirect bound from LEP1, SLC, and LEP2 data:

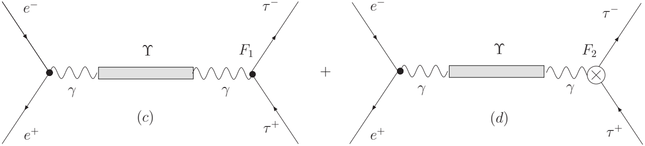
$-0.007 \leq a_\tau^{\mathrm{NP}} \leq 0.005$ González-Sprinberg et al., NPB 582 (2000) 2

- Belle bounds on τ EDM from $e^+e^- \to \tau^+\tau^-$:

$$2.2 \leq \mathrm{Re}(d_\tau) \leq 4.5\,(10^{-17} e \cdot \mathrm{cm})$$
$$2.5 \leq \mathrm{Im}(d_\tau) \leq 0.8\,(10^{-17} e \cdot \mathrm{cm})$$

The τ g-2: experimental bounds

- Idea: when  $\gg$  ?
- $F_{2V}(q^2 = M_\Upsilon^2)$ can be measured at B factories in $e^+e^- \to \Upsilon \to \tau^+\tau^-$ Bernabéu et al. NPB 790 (2008) 160.



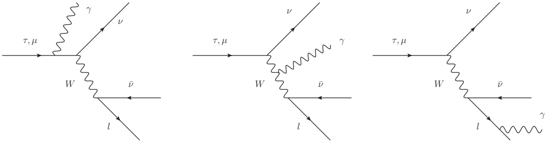
- Beam energy spread σ_W at Belle (Belle-II):

$$\Gamma_{\Upsilon(1S),\Upsilon(2S),\Upsilon(3S)} \sim \mathcal{O}(10\text{keV}) \quad \ll \quad \sigma_W = 5.24\,(5.45)\ \text{MeV}$$

Radiative μ and τ leptonic decays

Radiative μ and τ leptonic decays

- Proposal: probe a_τ and d_τ in radiative τ decays.



- Very clean, can be predicted with very high precision.
- Th. formulation in terms of Bouchiat-Michel-Kinoshita-Sirlin parameters.
- Additional BMKS-like parameters accessible in radiative decays.
- Irreducible SM background for CLFV decays:
  $\mu \to e\gamma$, $\tau \to l\gamma$, $\mu \to e^+ e^- e^+$.

Babar's branching ratios

RAPID COMMUNICATIONS

PHYSICAL REVIEW D **91**, 051103(R) (2015)

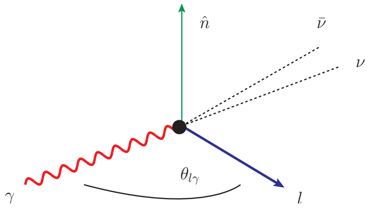
Measurement of the branching fractions of the radiative leptonic τ decays $\tau \to e\gamma\nu\bar{\nu}$ and $\tau \to \mu\gamma\nu\bar{\nu}$ at *BABAR*

B.R. of radiative τ leptonic decays ($\omega_0 = 10$ MeV)		
	$\tau \to e\bar{\nu}\nu\gamma$	$\tau \to \mu\bar{\nu}\nu\gamma$
$\mathcal{B}_{\mathrm{EXP}}$	$1.847\,(15)_{\mathrm{st}}(52)_{\mathrm{sy}} \times 10^{-2}$	$3.69\,(3)_{\mathrm{st}}(10)_{\mathrm{sy}} \times 10^{-3}$

- Babar experimental precision $\sim 3\%$.
- Previous CLEO results:
  $1.75\,(6)_{\mathrm{st}}(17)_{\mathrm{sy}} \times 10^{-2}$ $(\tau \to e\gamma\nu\bar{\nu})$,
  $3.61\,(16)_{\mathrm{st}}(35)_{\mathrm{sy}} \times 10^{-3}$ $(\tau \to \mu\gamma\nu\bar{\nu})$.
  T. Bergfeld et al., PRL 84 (2000) 830
- To be compared with $\mathcal{B}^{\mathrm{SM}}$ at NLO:
  $\sim (\alpha/\pi)\ln(m_l/m_\tau)\ln(\omega_0/m_\tau)$ 10% for $l = e$,
  3% for $l = \mu$.

Radiative μ and τ decays: definitions

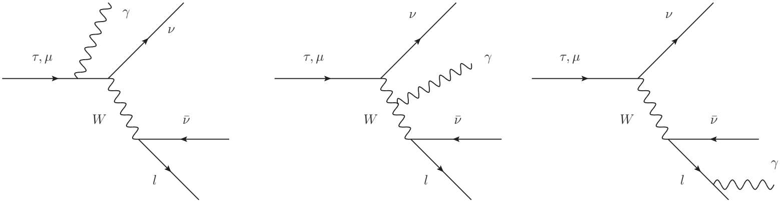
$$\mu \to e\gamma\nu\bar{\nu}$$
$$\tau \to l\gamma\nu\bar{\nu}\ (l = e, \mu)$$



- M: mass of the μ or τ.
- m_l: final charged lepton mass.
- $\hat{n}$: polarization vector.
- Mass ratio $r = m_l/M$.
- Minimum photon energy ω_0.

Differential decay rates

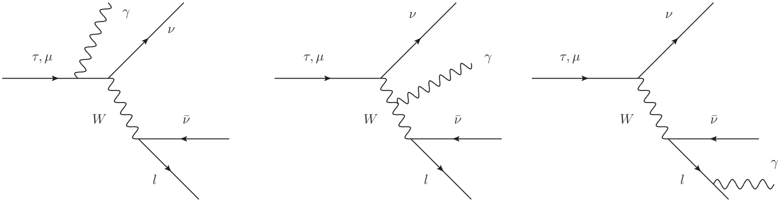
$$d\Gamma = d\Gamma_{\text{LO}} + \frac{m_{\mu,\tau}^2}{M_W^2}\, d\Gamma_W + \frac{\alpha}{\pi}\left[d\Gamma_{\text{virt}} + \int_0^{\omega_0'} d\omega'\, d\Gamma_{\gamma\gamma}\right]$$



Kinoshita, Sirlin, PRL 2 (1959) 177; Fronsdal, Uberall, PR 133 (1959) 654
Eckstein, Pratt, Ann. Phys. 8 (1959) 297; Kuno, Okada, RMP 73 (2001) 151

Differential decay rates

$$d\Gamma = d\Gamma_{\text{LO}} + \frac{m^2_{\mu,\tau}}{M^2_W}\, d\Gamma_W + \frac{\alpha}{\pi}\left[d\Gamma_{\text{virt}} + \int_0^{\omega_0'} d\omega'\, d\Gamma_{\gamma\gamma}\right]$$



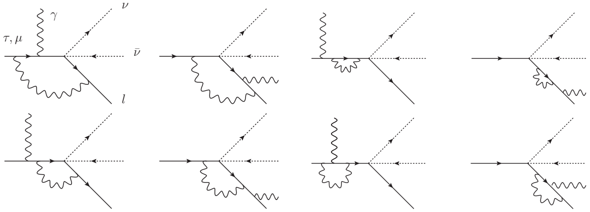
MF, L. Mercolli, M. Passera, PRD 88 (2013) 093011

- $m^2_\mu / M^2_W = 2 \times 10^{-6}$
- $m^2_\tau / M^2_W = 5 \times 10^{-4}$

Differential decay rates

$$d\Gamma = d\Gamma_{\text{LO}} + \frac{m_{\mu,\tau}^2}{M_W^2}\, d\Gamma_W + \frac{\alpha}{\pi}\left[d\Gamma_{\text{virt}} + \int_0^{\omega_0'} d\omega' \, d\Gamma_{\gamma\gamma} \right]$$

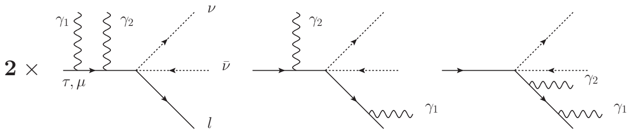
- NLO corrections computed in the effective Fermi Lagrangian.



Fischer et al., PRD 49 (1994) 3426; Arbuzov & Scherbakova, PLB 597 (2004) 285;
also as part of NNLO corrections to μ decay:
van Ritbergen & Stuart, PRL 82 (1999) 488;
Steinhauser & Seidensticke, PLB 467 (1999) 271;
Anastasiou, Melnikov, Petriello, JHEP 0709 (2007) 014;
Caola, Czarnecki, Liang, Melnikov, Szafron, PRD 90 (2014) 5.

Differential decay rates

$$d\Gamma = d\Gamma_{\text{LO}} + \frac{m^2_{\mu,\tau}}{M^2_W}\, d\Gamma_W + \frac{\alpha}{\pi}\left[d\Gamma_{\text{virt}} + \int_0^{\omega_0'} d\omega'\, d\Gamma_{\gamma\gamma}\right]$$



- Extra soft photon emission $\omega' < \omega_0$ integrated analytically:

$$d\Gamma^{\text{soft}}_{\gamma\gamma}(\omega_0') = \int_0^{\omega_0'} d\omega'\, d\Gamma_{\gamma\gamma} \quad (\omega_0 \ll M/2)$$

- IR divs of soft bremsstrahlung and virtual corrections cancel.
- Double "hard" bremsstrahlung integrated numerically $(\omega' > \omega_0)$.

Branching Ratios at LO

$$\Gamma_{\mathrm{LO}}(y_0) = \frac{G_F^2 M^5}{192\pi^3}\frac{\alpha}{3\pi}\Big[3\mathrm{Li}_2(y_0) - \frac{\pi^2}{2} - \frac{1}{2}\left(6+\bar{y}_0^3\right)\bar{y}_0 \ln \bar{y}_0$$

$$+\left(\ln r + \frac{17}{12}\right)\left(6\ln y_0 + 6\bar{y}_0 + \bar{y}_0^4\right) + \frac{1}{48}\left(125 + 45 y_0 - 33 y_0^2 + 7 y_0^3\right)\bar{y}_0\Big]$$

where $y_0 = 2\omega_0/M, \bar{y}_0 = 1 - y_0$

Kinoshita & Sirlin, PRL 2 (1959) 177; Eckstein & Pratt, Ann. Phys. 8 (1959) 297

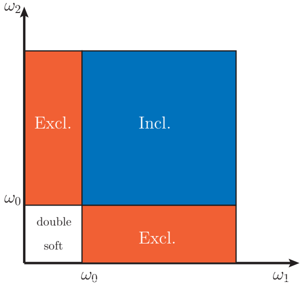
- Keep only $\ln r$ in $\Gamma_{\mathrm{LO}}(y_0)$, $\mathcal{O}(r)$ are neglected.
- Terms $\propto r^2$ cannot be neglected in the integrand $d\Gamma$ (also at NLO!).
  Lee & Nauenberg, PR 133 (1964) B1549; Sehgal, PLB 569 (2003) 25;

Approximate B.R. for $\omega_0 > 10$ MeV

	$\mathcal{B}_{\mathrm{LO}}$	$\mathcal{B}_{\mathrm{LO}}^{\mathrm{num}}$
$\mu \to e\gamma\nu\bar{\nu}$	1.31×10^{-2}	1.308×10^{-2}
$\tau \to e\gamma\nu\bar{\nu}$	1.83×10^{-2}	1.834×10^{-2}
$\tau \to \mu\gamma\nu\bar{\nu}$	3.58×10^{-3}	3.663×10^{-3}

Inclusive and exclusive branching ratios at NLO

$$\mathcal{B}^{\text{NLO}}(\omega_0) \propto \int d\Phi_4 \left(d\Gamma_{\text{LO}} + d\Gamma_{\text{virt}}\right) + \int d\Phi_5 \, d\Gamma_{\gamma\gamma}$$



- $\mathcal{B}^{\text{Exc}}(\omega_0)$: only one γ of energy $\omega > \omega_0$, additional second soft photon $\omega' < \omega_0$.

  $\mathcal{B}^{\text{Exc}}(\omega_0) =$ [red square]

- $\mathcal{B}^{\text{Inc}}(\omega_0)$: at least one γ of energy $\omega > \omega_0$.

  $\mathcal{B}^{\text{Inc}}(\omega_0) =$ [red square] $+$ [blue square]

Branching ratios: τ results

B.R. of radiative τ leptonic decays ($\omega_0 = 10$ MeV)	$\tau \to e\bar{\nu}\nu\gamma$	$\tau \to \mu\bar{\nu}\nu\gamma$
$\mathcal{B}_{\text{LO}}$	1.834×10^{-2}	3.663×10^{-3}
$\mathcal{B}^{\text{Inc}}_{\text{NLO}}$	$-1.06\,(1)_n(10)_N \times 10^{-3}$	$-5.8\,(1)_n(2)_N \times 10^{-5}$
$\mathcal{B}^{\text{Exc}}_{\text{NLO}}$	$-1.89\,(1)_n(19)_N \times 10^{-3}$	$-9.1\,(1)_n(3)_N \times 10^{-5}$
$\mathcal{B}^{\text{Inc}}$	$1.728\,(10)_{\text{th}}(3)_\tau \times 10^{-2}$	$3.605\,(2)_{\text{th}}(6)_\tau \times 10^{-3}$
$\mathcal{B}^{\text{Exc}}$	$1.645\,(19)_{\text{th}}(3)_\tau \times 10^{-2}$	$3.572\,(3)_{\text{th}}(6)_\tau \times 10^{-3}$
$\mathcal{B}^{\dagger}_{\text{EXP}}$	$1.847\,(15)_{\text{st}}(52)_{\text{sy}} \times 10^{-2}$	$3.69\,(3)_{\text{st}}(10)_{\text{sy}} \times 10^{-3}$

(n): numerical errors

(N): uncomputed NNLO cor. $\sim (\alpha/\pi) \ln r \ln(\omega/M) \times \mathcal{B}^{\text{Exc/Inc}}_{\text{NLO}}$

(th): combined $(n) \oplus (N)$

(τ): experimental error of τ lifetime: $\tau_\tau = 2.903(5) \times 10^{-13}$

	$\tau \to e\bar{\nu}\nu\gamma$	$\tau \to \mu\bar{\nu}\nu\gamma$
Δ^{Exc}	$2.02\,(57) \times 10^{-3} \to 3.5\sigma$	$1.2\,(1.0) \times 10^{-4} \to 1.1\sigma$

† BABAR - PRD 91 (2015) 051103

Probing a_τ and d_τ in $\tau \to l\gamma\nu\bar{\nu}$

Effective Lagrangian approach

$$\mathcal{L}_{\text{eff}} = C_{\tau W}(\bar{l}_L \sigma^{\mu\nu} \tau_R) \sigma^I \varphi W^I_{\mu\nu} + C_{\tau B}(\bar{l}_L \sigma^{\mu\nu} \tau_R) \varphi B_{\mu\nu} + \text{h.c.}$$

- New contribution to electromagnetic form factors

$$a_\tau = a_\tau^{\text{SM}} + \tilde{a}_\tau \text{ and } d_\tau = d_\tau^{\text{SM}} + \tilde{d}_\tau$$

- $\tilde{a}_\tau = \frac{2m_\tau}{e} \frac{\sqrt{2}v}{\Lambda^2} \left[\cos\theta_W \text{Re}\, C_{\tau B} - \sin\theta_W \text{Re}\, C_{\tau W}\right]$
- $\tilde{d}_\tau = \frac{\sqrt{2}v}{\Lambda^2} \left[\cos\theta_W \text{Im}\, C_{\tau B} - \sin\theta_W \text{Im}\, C_{\tau W}\right]$

The τ g-2 via radiative leptonic decays: a proposal

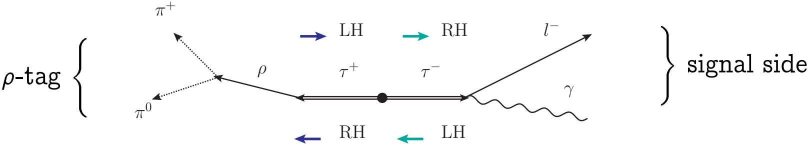
$$d\Gamma = d\Gamma_{\text{LO}} + \frac{m_\tau^2}{M_W^2} d\Gamma_W + \frac{\alpha}{\pi} d\Gamma_{\text{NLO}} + \tilde{a}_\tau \, d\Gamma_a + \tilde{d}_\tau \, d\Gamma_d$$

- Use radiation zero phenomena is too naïve: $U.L.(\tilde{a}_\tau) \simeq 2$.
  First suggest by Laursen et al., PRD 29 (1984) 2652
- Must employ the polarized differential decay rate.
- Study the full phase space:

$$\int d\Phi \left(d\Gamma^{\text{SM}} + \tilde{a} \, d\Gamma_a \right) \approx \begin{array}{ll} 1.6 \times 10^{-2} + \tilde{a} \, 8.6 \times 10^{-6}, & l = e \\ 3.6 \times 10^{-3} + \tilde{a} \, 8.2 \times 10^{-6}, & l = \mu \end{array}$$

A feasibility study for Belle and Belle-II

- Belle: $\int Ldt \simeq 1\text{ab}^{-1} \to N_{\tau\tau} \simeq 10^9$.
- Belle-II: higher statistics ($\times 50$) and better detector performance.
- Signal side: $\tau \to l\gamma\nu\bar{\nu}$
- Tag side: $\tau^\pm \to \rho^\pm\nu \to \pi^\pm\pi^0\nu$ $\longleftarrow$ spin analyzer



- Employ all information in the 12-dimensional $\Phi(l^\mp, \gamma, \pi^\pm, \pi^0)$.
- Extract $\tilde{a}$ and $\tilde{d}$ via unbinned maximum likelihood.

Feasibility study: results

Two scenarios:

- *ρ-tag* mode: $\tau \to \rho\nu_\tau$, BR= 25.5%.
- *full-tag* mode: six modes, BR= 90%.

Sensitivity to $\tilde{a}_\tau$ and $\tilde{d}_\tau$

	$\tilde{a}_\tau$	$\tilde{d}_\tau$
Belle (ρ-tag)	0.16	0.15
Belle-II (ρ-tag)	0.023	0.021
Belle (full tag)	0.085	0.080
Belle-II (full tag)	0.012	0.011
DELPHI†	0.017	—
Belle*	—	0.0015

†DELPHI - EPJC35 (2004) 159

*Belle coll. PLB 551 (2003) 16.

Conclusions

- We studied the SM prediction and $d\Gamma$ of $\tau \to l\gamma\nu\bar{\nu}$, $(l = \mu, e)$ and $\mu \to e\gamma\nu\bar{\nu}$.
- We calculated $\mathcal{B}^{\rm Exc/Inc}$ for a minimum photon energy $\omega_0 = 10$ MeV.
- Babar's recent measurement:

$$\mathcal{B}(\tau \to \mu\gamma\nu\bar{\nu}) ✓, \qquad \mathcal{B}(\tau \to e\gamma\nu\bar{\nu}) ✗ \to 3.5\sigma.$$

- Radiative τ leptonic decays can probe $\tilde{a}_\tau, \tilde{d}_\tau$, the $\mathcal{L}_{\rm eff}$ Wilson coeffs.
- Require full phase-space analysis → employ maximum informations.
- Feasibility study shows that Belle-II can ameliorate the current DELPHI result for $\tilde{a}_\tau$.

Backup slides

Branching ratios: μ results

B.R. of radiative μ leptonic decays ($\omega_0 = 10$ MeV)	
$\mathcal{B}_{\text{LO}}$	1.308×10^{-2}
$\mathcal{B}^{\text{Inc}}_{\text{NLO}}$	$-1.91\,(5)_n(6)_N \times 10^{-4}$
$\mathcal{B}^{\text{Exc}}_{\text{NLO}}$	$-2.25\,(5)_n(7)_N \times 10^{-4}$
$\mathcal{B}^{\text{Inc}}$	$1.289\,(1)_{\text{th}} \times 10^{-2}$
$\mathcal{B}^{\text{Exc}}$	$1.286\,(1)_{\text{th}} \times 10^{-2}$
$\mathcal{B}^{\dagger}_{\text{EXP}}$	$1.4\,(4) \times 10^{-2}$

†Crittenden *et al.*
PR 1961 (1961) 1823
*J. Adam *et al.*
arXiv:1312.3217 [hep-ex]
‡ D. Počanić *et al.*
1403.7416 [nucl-ex].

Preliminary new measurements		
	cuts	$\mu \to e\gamma\nu\bar{\nu}$
MEG*	$E_e > 45$ MeV, $\omega_0 > 40$ MeV	$6.03\,(14)_{\text{st}}\,(53)_{\text{sy}} \times 10^{-8}$
PIBETA‡	$\omega_0 > 10$ MeV, $\cos\theta_{e\gamma} > 30^\circ$	$4.365\,(9)_{\text{st}}\,(42)_{\text{sy}} \times 10^{-3}$

Inclusive and Exclusive Branching Ratios

$$\mathcal{B}(\omega_0) \propto \lim_{\omega_0' \to 0} \int d\Phi_4 \left(d\Gamma_{\rm LO} + d\Gamma_{\rm virt} + d\Gamma_{\gamma\gamma}^{\rm soft}(\omega_0') \right) + \int_{\omega_0'} d\Phi_5 \, d\Gamma_{\gamma\gamma}$$

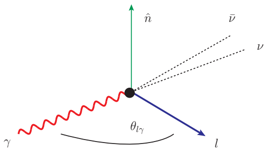
- Unphysical soft threshold $\omega_0' \ll \omega_0$ to cancel IR.
- Soft photon radiation $d\Gamma_{\gamma\gamma}^{\rm soft}(\omega_0')$ computed thanks to the factorization of the amplitude.
- Perform the limit $\omega_0' \to 0$.
- Integration complete real emission done numerically.
- Double "hard" emission also checked with MadGraph5.

Differential Decay Rates

$$\frac{d^6\Gamma}{dE_l\,d\omega\,d\Omega_l\Omega_\gamma} \propto G + \hat{n}\cdot\left[\hat{p}_l J + \hat{p}_\gamma K + (\hat{p}_l \times \hat{p}_\gamma) L\right]$$

$$G(\omega, E_l, \cos\theta_{le}) = G_{\rm LO} + \frac{m_l^2}{M^2} G_W + \frac{\alpha}{\pi} G_{\rm NLO}, \text{ same for } J \text{ and } K$$
$$L(\omega, E_l, \cos\theta_{le}) = \frac{\alpha}{\pi} L_{\rm NLO}$$

- Analytically integrated over neutrinos momenta and spin.
- Visible particle kinematics $\omega, E_l, \Omega_l, \Omega_\gamma$.
- G: isotropic function.
- J, K, L: spin dependent parts.



NLO corrections

- NLO corrections computed in the Femi V-A theory.
- One-loop integrals reduced via Passarino-Veltman reduction.
- Scalar integrals computed analytically.
- Extra soft photon emission $\omega' < \omega_0$ integrated analytically for (thanks to factorization of the amplitude):

$$d\Gamma^{\text{soft}}_{\gamma\gamma}(\omega'_0) = \int_0^{\omega'_0} d\omega' \, d\Gamma_{\gamma\gamma}$$

- IR divergence of soft bremsstrahlung is canceled by that arising from virtual corrections.
- Double “hard” bremsstrahlung integrated numerically (when ω and $\omega' > \omega_0$).

Comparison with previous results

$$\frac{d^6\Gamma}{dE_l\, d\omega\, d\Omega_l \Omega_\gamma} \propto G + \hat{n} \cdot \left[\hat{p}_l J + \hat{p}_\gamma K + (\hat{p}_l \times \hat{p}_\gamma) L \right]$$

- One-loop corrections to $\mu \to e\gamma\nu\bar{\nu}$ differential decay rate previously computed in:
  - I. Mohammad, A. Donnachie, (1976) CERN-TH-2127
    Unpublished, D_0 functions left unsolved for numerical integration.
  - A. Fischer et al., PRD 49 (1994) 3426
    Only the isotropic part G. Files with results unavailable.
  - A. Arbuzov, E. S. Scherbakova, PLB 597 (2004) 285
    Full spin dependence, but the calculation is performed setting $m_l = 0$.
    Additional collinear emission included to avoid $\ln r$.
    Perfect agreement for G, anisotropic sector (J, K) differs from ours.
    Also the function L has been overlooked.

The total differential decay for a polarized μ or τ lepton in the tau r.f. is

$$\frac{d^6\Gamma^{\rm NLO}}{dx\, dy\, d\Omega_l\, d\Omega_\gamma} = \frac{\alpha\, G_F^2 m_\tau^5}{(4\pi)^6} \frac{x\beta}{1+\delta_{\rm W}(m_\mu, m_e)} \Bigg[G(x,y,c)$$

$$+\; x\beta\, \hat{n}\cdot\hat{p}_l\, J(x,y,c) + y\, \hat{n}\cdot\hat{p}_\gamma\, K(x,y,c) + y\, x\beta\, \hat{n}\cdot(\hat{p}_l \times \hat{p}_\gamma)\; L(x,y,c) \Bigg]$$

where $x = 2E_l/m_\tau$, $y = 2E_\gamma/m_\tau$, $c = \cos\theta_{l\gamma}$. The polarization vector $n = (0, \vec{n})$ satisfies $n^2 = -1$ and $n \cdot p_\tau = 0$.

The function $G(x,y,c)$, and similarly for J and K, is given by

$$G(x,y,c) = \frac{4}{3yz^2} \left[g_{\rm LO}(x,y,z) + \frac{\alpha}{\pi}\, g_{\rm NLO}(x,y,z;y_{\rm min}) + \left(\frac{m_\tau}{M_W}\right)^2 g_{\rm W}(x,y,z) \right]$$

The total differential decay for a polarized μ or τ lepton in the tau r.f. is

$$\frac{d^6\Gamma^{\rm NLO}}{dx\, dy\, d\Omega_l\, d\Omega_\gamma} = \frac{\alpha\, G_F^2 m_\tau^5}{(4\pi)^6} \frac{x\beta}{1+\delta_{\rm W}(m_\mu, m_e)} \Bigg[G(x,y,c) + x\beta\, \hat{n}\cdot\hat{p}_l\, J(x,y,c) + y\, \hat{n}\cdot\hat{p}_\gamma\, K(x,y,c) + y\, x\beta\, \hat{n}\cdot(\hat{p}_l \times \hat{p}_\gamma)\; L(x,y,c) \Bigg]$$

where $x = 2E_l/m_\tau$, $y = 2E_\gamma/m_\tau$, $c = \cos\theta_{l\gamma}$. The polarization vector $n = (0, \vec{n})$ satisfies $n^2 = -1$ and $n \cdot p_\tau = 0$.
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Compared with previous work A. B. Arbuzov PLB 597 (2004) 285