Hadronic Light-by-Light corrections to $(g\text{-}2)_\mu$

Marc Vanderhaeghen

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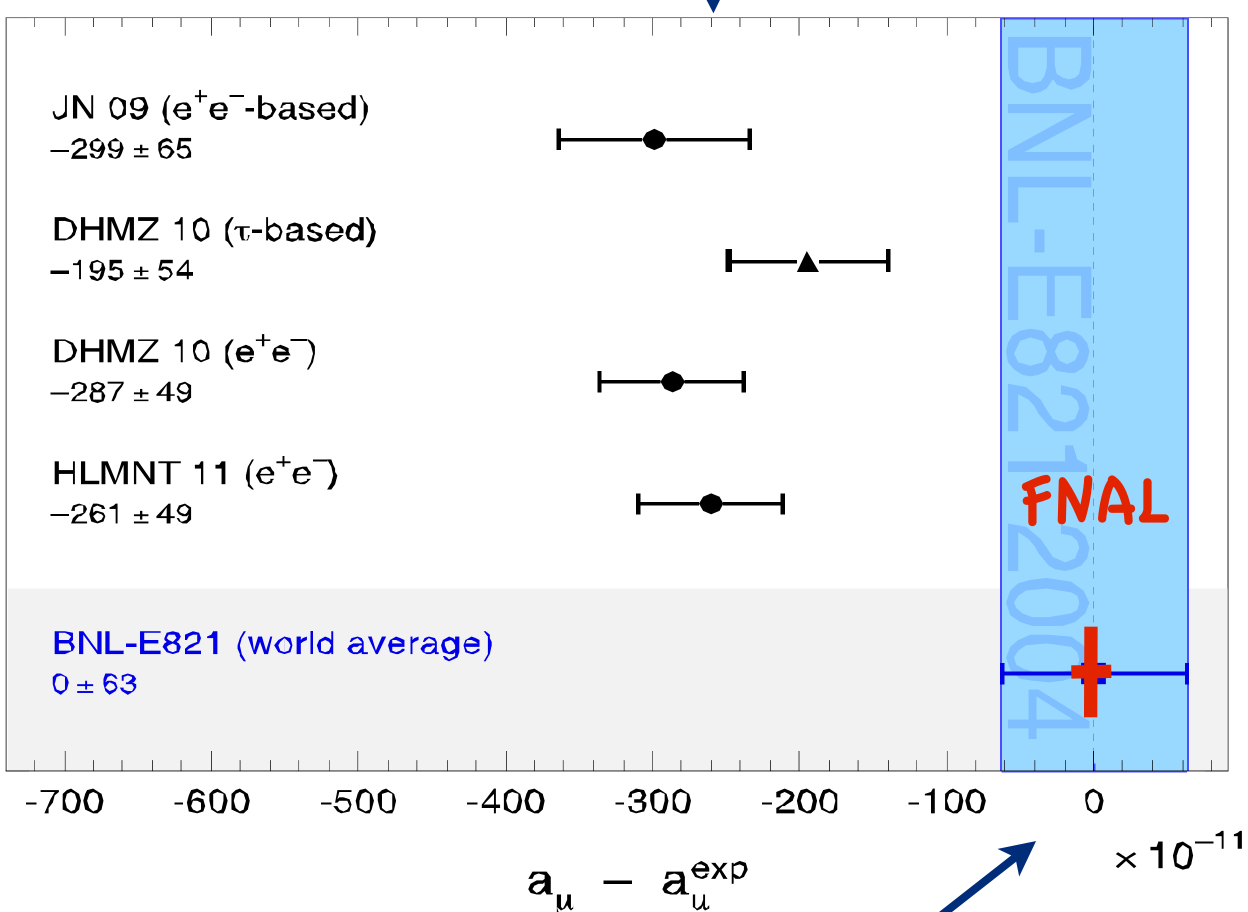
$(g-2)_\mu$: theory vs experiment

SM predictions for a_μ



BNL-E821 measurement of a_μ

$a_\mu^{exp} = (11\ 659\ 208.9 \pm 6.3) \times 10^{-10}$

$a_\mu^{exp} - a_\mu^{SM} =$
$(26.1 \pm 5.0_{th} \pm 6.3_{exp}) \times 10^{-10}$

Hagiwara et al. (2011)

3 - 4 σ deviation from SM value !

Errors or new physics ?

New FNAL experiment (2016)

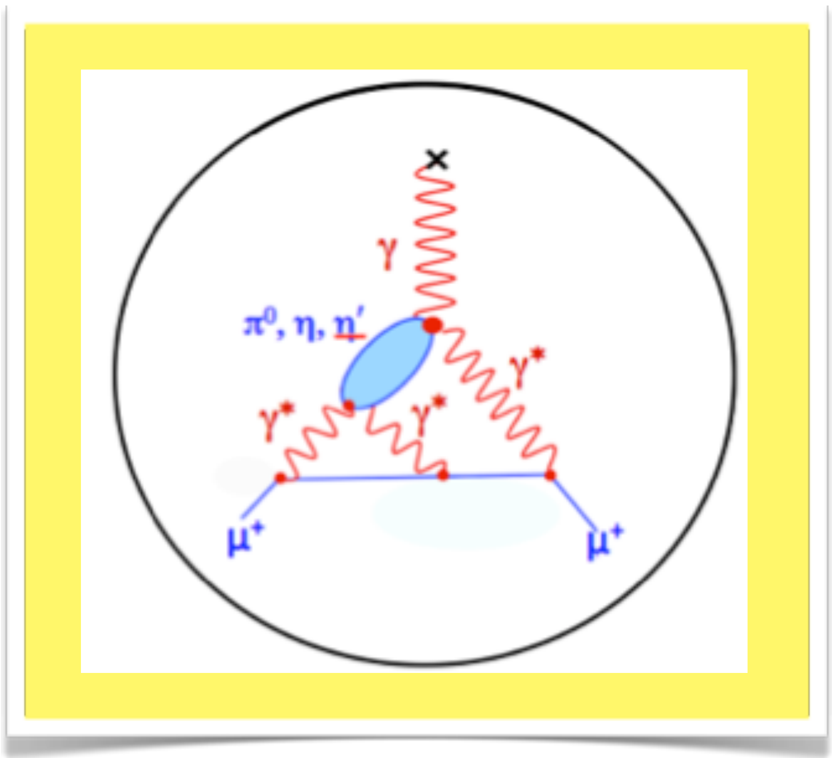
$\delta a_\mu^{FNAL} = 1.6 \times 10^{-10}$

factor 4 improvement in exp. error

-> Improve theory !

hadronic LbL corrections to $(g-2)_\mu$

New FNAL and J-Parc $(g-2)_\mu$ expt. : $\delta a_\mu^{exp} = 1.6 \times 10^{-10}$



$a_\mu^{\text{had, LbL}} = (11.6 \pm 4.0) \times 10^{-10}$

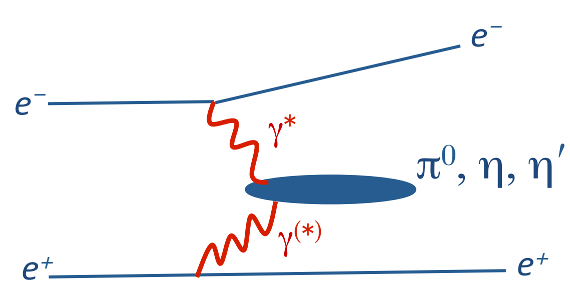
Jegerlehner, Nyffeler (2009)

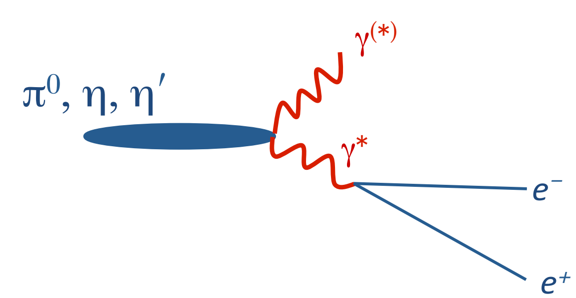
$a_\mu^{\text{had, LbL}} = (10.5 \pm 2.6) \times 10^{-10}$

Prades, de Rafael, Vainshtein (2009)

- **experimental input:** meson transition FFs, $\gamma^* \gamma^*$ -> multi-meson states, meson Dalitz decays

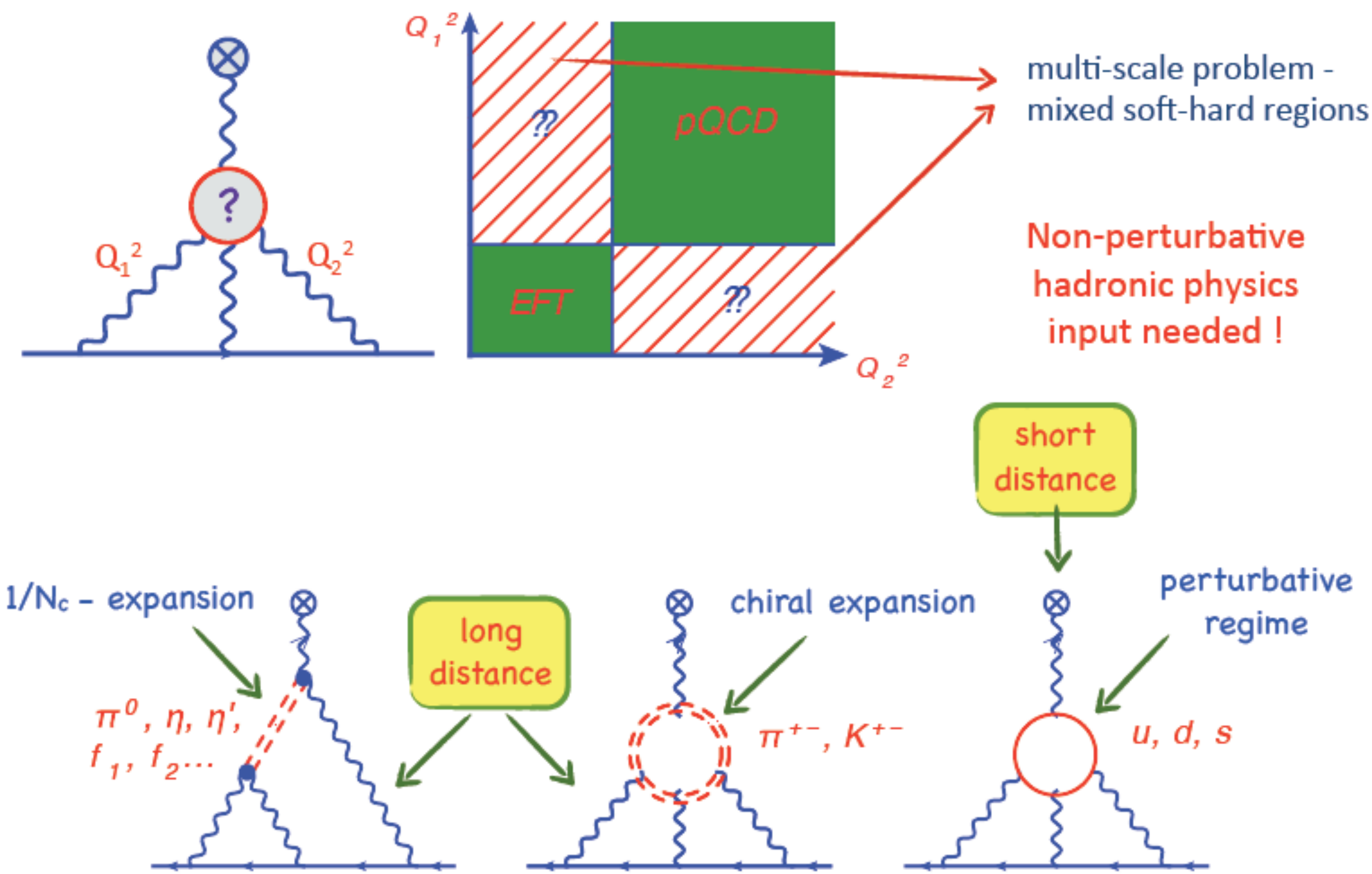
BES-III, MAMI, ...



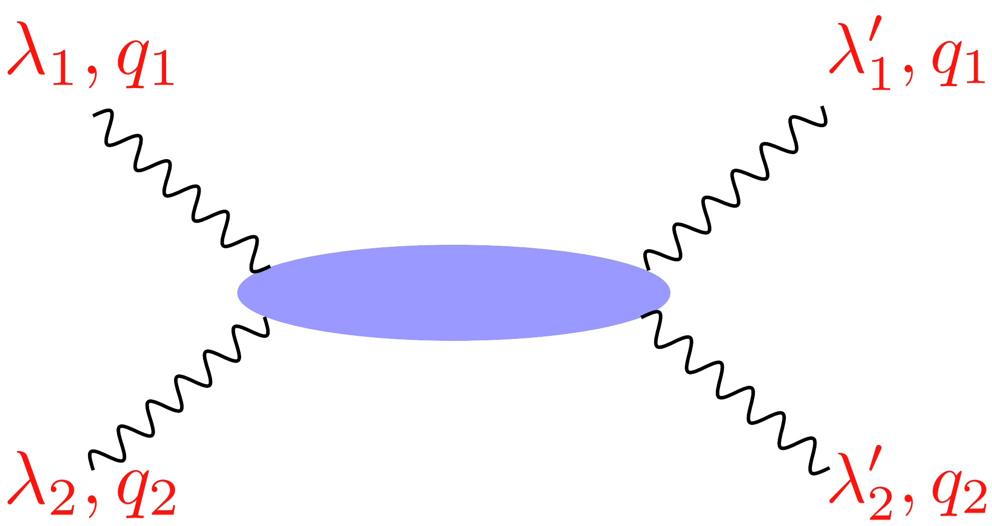


- **theory deveolpments:** models, sum rules, dispersion relations, lattice, ...

hadronic LbL corrections to $(g-2)_\mu$: relevant contributions



Theory: sum rules for LbL scattering (I)



$$\gamma^*(\lambda_1, q_1) + \gamma^*(\lambda_2, q_2) \to \gamma^*(\lambda_1', q_1) + \gamma^*(\lambda_2', q_2)$$

kinematical invariants:

$$s = (q_1 + q_2)^2, \qquad u = (q_1 - q_2)^2$$

$$\nu \equiv \frac{s-u}{4}, \quad Q_1^2 \equiv -q_1^2, \quad Q_2^2 \equiv -q_2^2$$

helicity amplitudes:

$$M_{\lambda_1'\lambda_2', \lambda_1\lambda_2}(\nu, Q_1^2, Q_2^2)$$

$$\lambda = 0, \pm 1$$

discrete symmetries:

$$P: \quad M_{\lambda_1'\lambda_2', \lambda_1\lambda_2} = M_{-\lambda_1' -\lambda_2', -\lambda_1 -\lambda_2}$$

$$T: \quad M_{\lambda_1'\lambda_2', \lambda_1\lambda_2} = M_{\lambda_1\lambda_2, \lambda_1'\lambda_2'}$$

8 independent amplitudes:

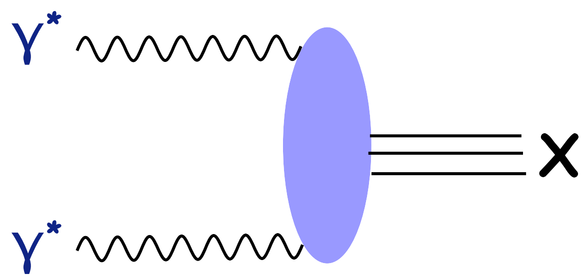
$M_{++,++},\ M_{+-,+-},\ M_{++,--},$ **T**

$M_{00,00},\ M_{+0,+0},\ M_{0+,0+},\ M_{++,00},\ M_{0+,-0}$ **T and L**

sum rules for LbL scattering (II)

Unitarity: link to $\gamma^* \gamma^* \to X$ cross sections

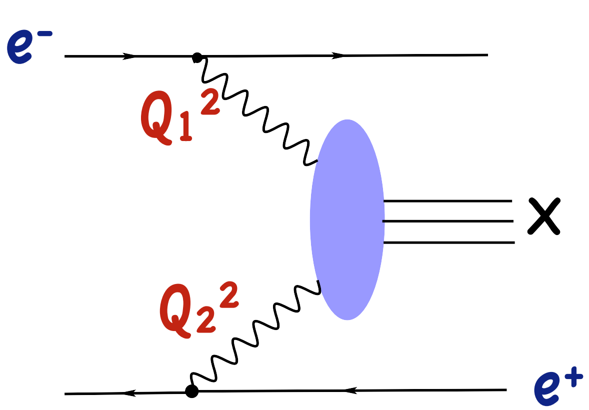
$$W_{\lambda_1'\lambda_2',\lambda_1\lambda_2} \equiv \mathrm{Im}\, M_{\lambda_1'\lambda_2',\lambda_1\lambda_2}$$



$$\begin{aligned}
W_{++,++} + W_{+-,+-} &\equiv 2\sqrt{X}\ (\sigma_0 + \sigma_2) = 2\sqrt{X}\ (\sigma_\parallel + \sigma_\perp) \equiv 4\sqrt{X}\ \sigma_{TT}, \\
W_{++,++} - W_{+-,+-} &\equiv 2\sqrt{X}\ (\sigma_0 - \sigma_2) \equiv 4\sqrt{X}\ \tau^a_{TT}, \\
W_{++,--} &\equiv 2\sqrt{X}\ (\sigma_\parallel - \sigma_\perp) \equiv 2\sqrt{X}\ \tau_{TT}, \\
W_{00,00} &\equiv 2\sqrt{X}\ \sigma_{LL}, \\
W_{+0,+0} &\equiv 2\sqrt{X}\ \sigma_{TL}, \\
W_{0+,0+} &\equiv 2\sqrt{X}\ \sigma_{LT}, \\
W_{++,00} + W_{0+,-0} &\equiv 4\sqrt{X}\ \tau_{TL}, \\
W_{++,00} - W_{0+,-0} &\equiv 4\sqrt{X}\ \tau^a_{TL}.
\end{aligned}$$

$$X \equiv \nu^2 - Q_1^2 Q_2^2$$

Experiment: $e^- e^+ \to e^- e^+ X$ cross sections

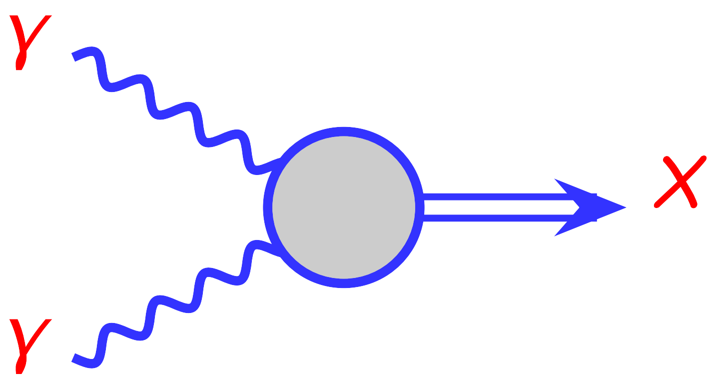


$$\begin{aligned}
d\sigma = &\frac{\alpha^2}{16\pi^4 Q_1^2 Q_2^2}\frac{2\sqrt{X}}{s(1-4m^2/s)} \cdot \frac{d^3\vec{p}_1{}'}{E_1'} \cdot \frac{d^3\vec{p}_2{}'}{E_2'} \\
&\times \{4\,\rho_1^{++}\,\rho_2^{++}\,\sigma_{TT} + \rho_1^{00}\,\rho_2^{00}\,\sigma_{LL} + 2\,\rho_1^{++}\,\rho_2^{00}\,\sigma_{TL} + 2\,\rho_1^{00}\,\rho_2^{++}\,\sigma_{LT} \\
&+2\ (\rho_1^{++}-1)\ (\rho_2^{++}-1)\ \left(\cos 2\tilde{\phi}\right)\ \tau_{TT} + 8\left[\frac{(\rho_1^{00}+1)\ (\rho_2^{00}+1)}{(\rho_1^{++}-1)\ (\rho_2^{++}-1)}\right]^{1/2} \left(\cos\tilde{\phi}\right)\ \tau_{TL} \\
&+ h_1 h_2\, 4\left[(\rho_1^{00}+1)\ (\rho_2^{00}+1)\right]^{1/2}\tau^a_{TT} + h_1 h_2\, 8\left[(\rho_1^{++}-1)\ (\rho_2^{++}-1)\right]^{1/2}\left(\cos\tilde{\phi}\right)\ \tau^a_{TL}\}
\end{aligned}$$

lepton beam polarization

ρ's, ϕ : kinematical quantities

sum rules for LbL scattering (III)

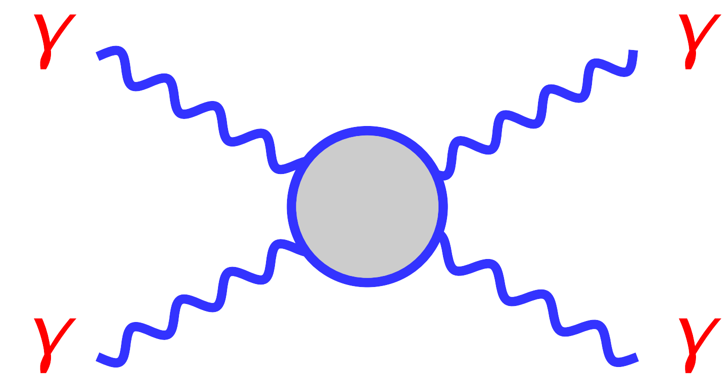


micro-causality

unitarity

dispersion theory

LETs



imaginary part of the amplitude - photon-photon fusion into leptons and hadrons:

$$\mathrm{Im} f^{(-)}(s) = -\frac{s}{8}[\sigma_2(s) - \sigma_0(s)]$$

$$\mathrm{Im} f^{(+)}(s) = -\frac{s}{8}[\sigma_{tot}(s)]$$

Sum rules

real part of the amplitude - low-energy structure of the elastic LbL scattering:

$$\mathcal{L}^{(8)} = c_1(F_{\mu\nu}F^{\mu\nu})^2 + c_2(F_{\mu\nu}\tilde{F}^{\mu\nu})^2$$

Euler, Heisenberg (1936)

$$\int_{s_0}^{\infty} \frac{ds}{s}\,[\sigma_2(s) - \sigma_0(s)] = 0$$

$$c_1 \pm c_2 = \frac{1}{8\pi}\int_{s_0}^{\infty} \frac{ds}{s^2}\,[\sigma_{\parallel}(s) \pm \sigma_{\perp}(s)]$$

sum rules for LbL scattering (IV)

3 superconvergent relations:

Pascalutsa, Pauk, Vdh (2012, 2014)

helicity difference sum rule →

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)} [\sigma_0 - \sigma_2]_{Q_2^2=0}$$

for $Q^2 = 0$: GDH sum rule
Gerasimov, Moulin (1975), Brodsky, Schmidt (1995)

sum rules involving longitudinal photons

$$0 = \int_{s_0}^{\infty} ds \frac{1}{(s+Q_1^2)^2} \left[\sigma_{\|} + \sigma_{LT} + \frac{(s+Q_1^2)}{Q_1 Q_2} \tau_{TL}^{a} \right]_{Q_2^2=0}$$

$$0 = \int_{s_0}^{\infty} ds \left[\frac{\tau_{TL}}{Q_1 Q_2} \right]_{Q_2^2=0}$$

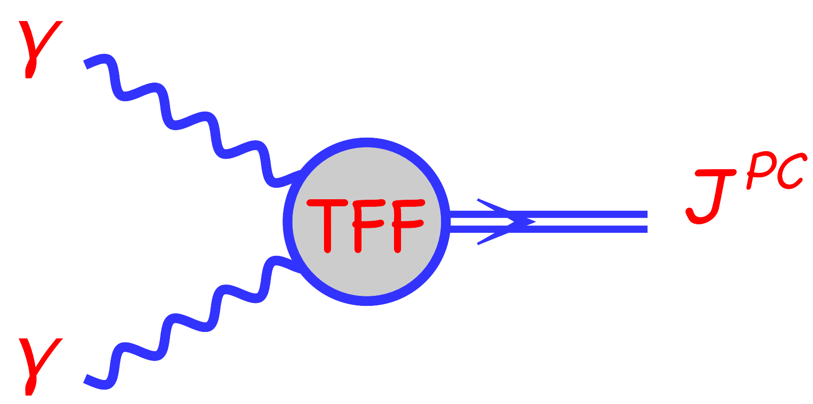
SRs involving LbL low-energy constants:

$$c_1 \pm c_2 = \frac{1}{8\pi} \int_{s_0}^{\infty} \frac{ds}{s^2} \left[\sigma_{\|}(s) \pm \sigma_{\perp}(s) \right]$$

+ 6 new LECs at next order

➡ sum rules have been tested in perturbative QFT both at tree-level and 1-loop level

single meson production in $\gamma\gamma$ collisions (I)



- two-photon state: produced meson has C=+1

- both photons are real: J=1 final state is forbidden (Landau-Yang theorem);

the main contribution comes from

J=0: 0^{-+} (pseudoscalar) and 0^{++} (scalar)

and J=2: 2^{++} (tensor)

– the SRs hold separately for channels of given intrinsic quantum numbers: isoscalar and isovector mesons, $c\bar{c}$ states

– input for the absorptive part of the SRs: γγ-hadrons response functions, can be expressed in terms of γγ→M transition form factors

$$\sigma_{\Lambda}^{\gamma\gamma\to M}(s) \approx (2J+1)16\pi^2 \frac{\Gamma_{\gamma\gamma}}{m_M}\delta(s-m_M^2)$$

meson contribution to the cross-section in the narrow-resonance approximation

$$\Gamma_{\gamma\gamma}(\mathcal{P}) = \frac{\pi\alpha^2}{4} m^3 \, |F_{\mathcal{M}\gamma^*\gamma^*}(0,0)|^2$$

two-photons decay rate for the meson

single meson production in $\gamma\gamma$ collisions (II)

the I=0 channel

	$\int \frac{ds}{s}(\sigma_2 - \sigma_0)$ [nb]	c_1 [10^{-4}GeV^{-4}]	c_2 [10^{-4}GeV^{-4}]
η	-191 ± 10	0	0.65 ± 0.03
η'	-300 ± 10	0	0.33 ± 0.01
$f_0(980)$	-19 ± 5	0.020 ± 0.005	0
$f_0'(1370)$	-91 ± 36	0.049 ± 0.019	0
$f_2(1270)$	449 ± 52	0.141 ± 0.016	0.141 ± 0.016
$f_2'(1525)$	7 ± 1	0.002 ± 0.000	0.002 ± 0.000
$f_2(1565)$	56 ± 11	0.012 ± 0.002	0.012 ± 0.002
Sum	-89 ± 66	0.22 ± 0.03	1.14 ± 0.04

dominant contribution to c_2 comes from η, η' and $f_2(1270)$

dominant contribution to c_1 comes from $f_2(1270)$

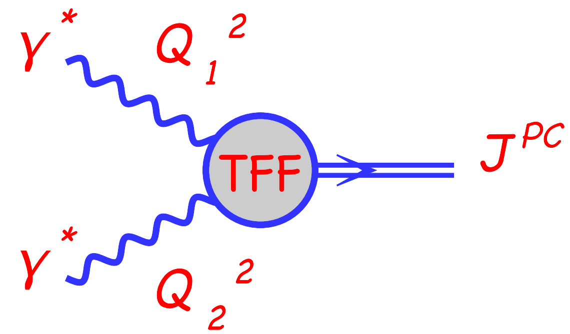
the I=1 channel

	$\int \frac{ds}{s}(\sigma_2 - \sigma_0)$ [nb]	c_1 [10^{-4} GeV^{-4}]	c_2 [10^{-4} GeV^{-4}]
π^0	-195 ± 13	0	10.94 ± 0.70
$a_0(980)$	-20 ± 8	0.021 ± 0.007	0
$a_2(1320)$	134 ± 8	0.039 ± 0.002	0.039 ± 0.002
$a_2(1700)$	18 ± 3	0.003 ± 0.001	0.003 ± 0.001
Sum	-63 ± 17	0.06 ± 0.01	10.98 ± 0.70

Pascalutsa, Pauk, Vdh (2012)

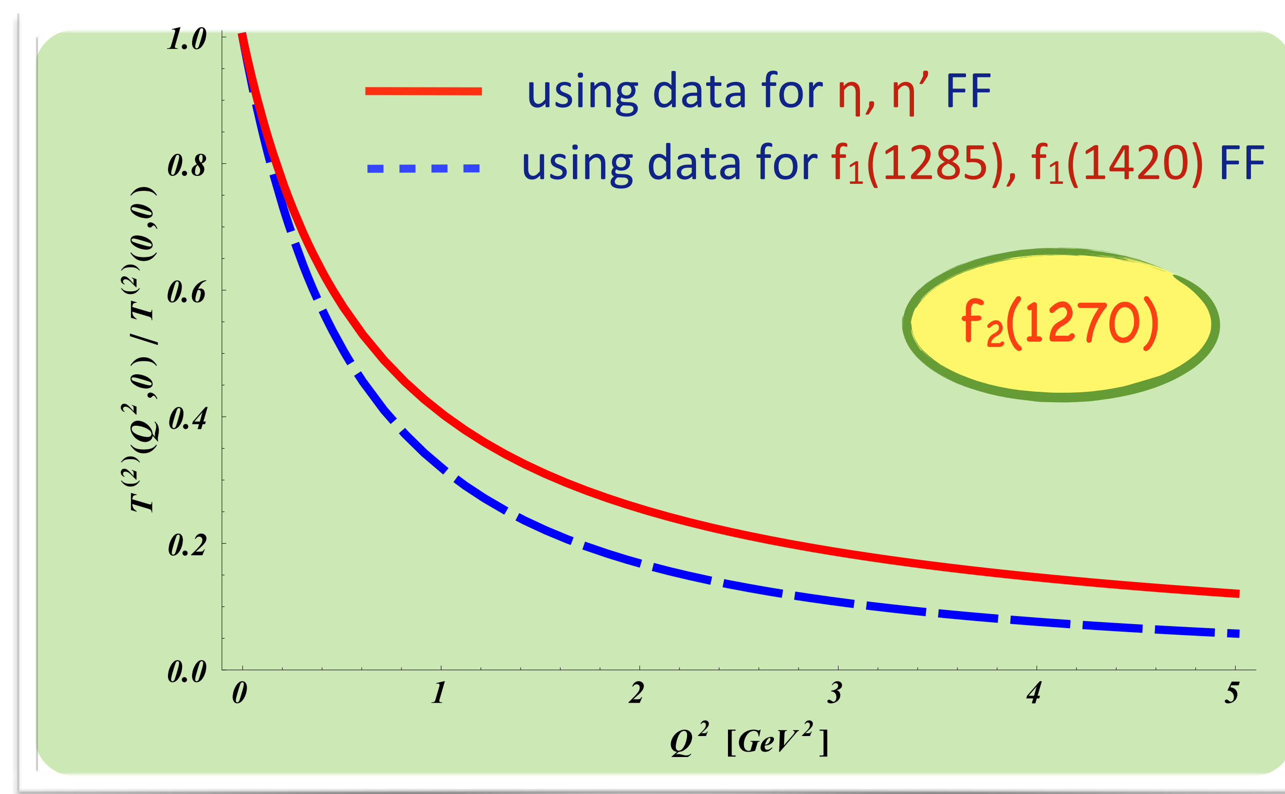
dominant contribution to c_2 comes from π^0

single meson production in $\gamma\gamma$ collisions (III)



- one photon is virtual Q_1^2, second is quasi-real $Q_2^2 \simeq 0$:
- axial-vector mesons 1^{++} are allowed
- $f_1(1285)$, $f_1(1420)$ transition FFs constrained from LEP (L3) data

	m_M [MeV]	$\Gamma_{\gamma\gamma}$ [keV]	$\int \frac{ds}{s^2} \sigma_\parallel(s)$ [nb / GeV^2]	$\int ds \left[\frac{1}{s}\frac{\tau^a_{TL}}{Q_1 Q_2}\right]_{Q_i^2=0}$ [nb / GeV^2]	$\int ds \left[\frac{1}{s^2}\sigma_\parallel + \frac{1}{s}\frac{\tau^a_{TL}}{Q_1 Q_2}\right]_{Q_i^2=0}$ [nb / GeV^2]
$f_1(1285)$	1281.8 ± 0.6	3.5 ± 0.8	0	-93 ± 21	-93 ± 21
$f_1(1420)$	1426.4 ± 0.9	3.2 ± 0.9	0	-50 ± 14	-50 ± 14
$f_0(980)$	980 ± 10	0.29 ± 0.07	20 ± 5	0	20 ± 5
$f_0'(1370)$	$1200 - 1500$	3.8 ± 1.5	48 ± 19	0	48 ± 19
$f_2(1270)$	1275.1 ± 1.2	3.03 ± 0.35	138 ± 16	$\gtrsim 0$	138 ± 16
$f_2'(1525)$	1525 ± 5	0.081 ± 0.009	1.5 ± 0.2	$\gtrsim 0$	1.5 ± 0.2
$f_2(1565)$	1562 ± 13	0.70 ± 0.14	12 ± 2	$\gtrsim 0$	12 ± 2
Sum					76 ± 36

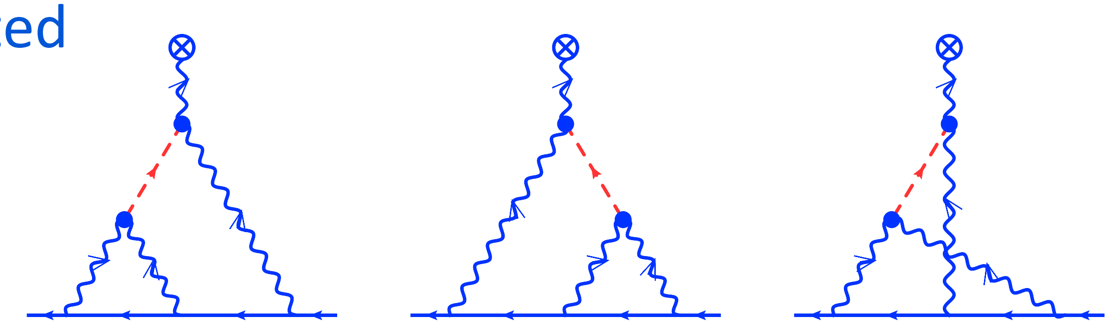


- sum rules allow to constrain so far unmeasured contributions, e.g. $\gamma^* \gamma^* \to$ tensor mesons

single meson contributions to a_μ (I)

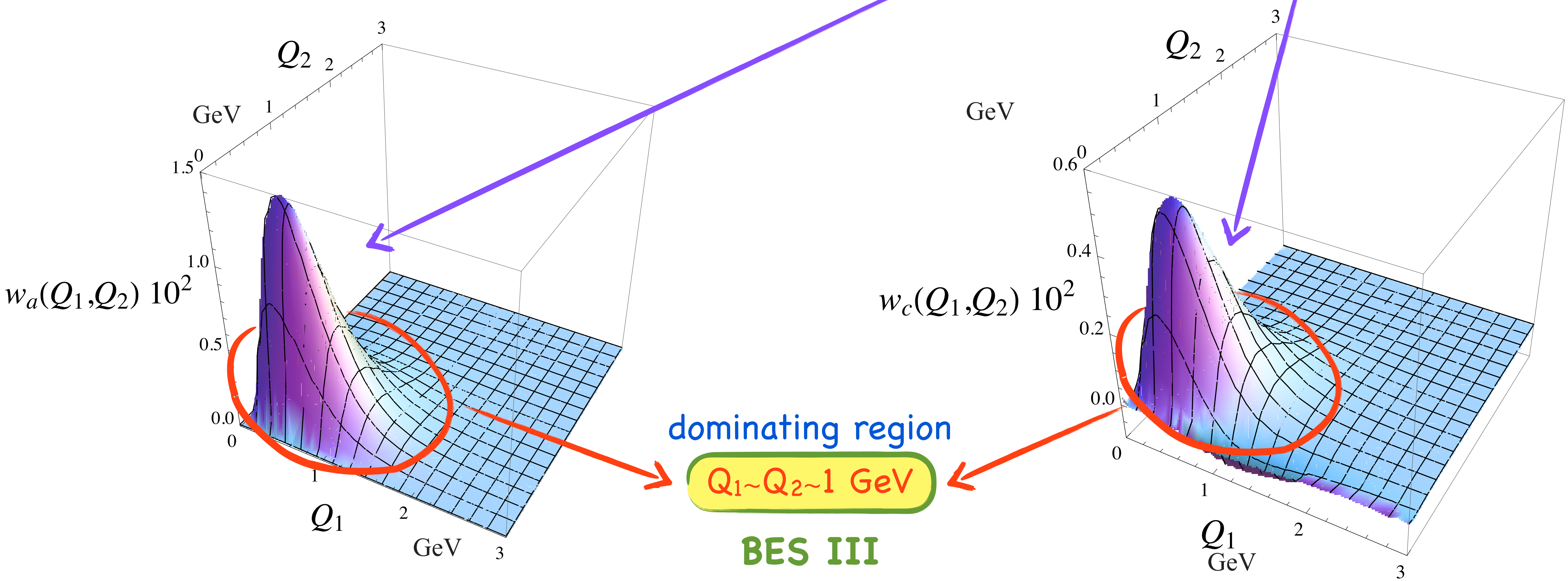
➡ **axial-vector meson** contribution to a_μ re-evaluated

- Landau-Yang theorem constraint implemented
- $f_1(1285)$, $f_1(1420)$ transition FFs from L3 data



Pauk, Vdh (2013)

$$a_\mu^{LbL} = \frac{\alpha}{(2\pi)^2} \frac{\Lambda_{A1}^6 \Lambda_{A2}^6 \tilde{\Gamma}_{\gamma\gamma}(A)}{m\, m_A^5} \int dQ_1 \int dQ_2 \left[2 w_a(Q_1, Q_2) + w_c(Q_1, Q_2)\right]$$

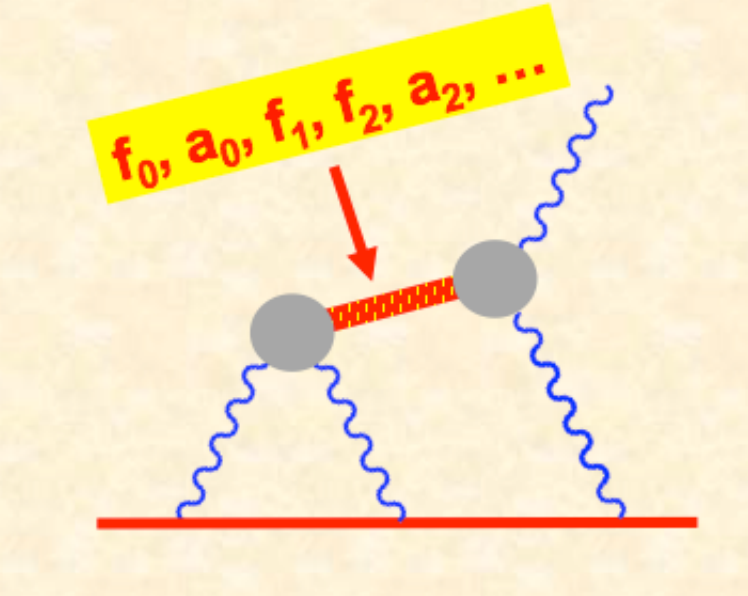


single meson contributions to a_μ (II)

➡ **axial-vector meson** re-evaluation was reported in 2 works

- implementation of Landau-Yang theorem constraint leads to difference with previous results

➡ **tensor mesons** evaluated for first time



	pseudo-scalars	axial-vectors	scalars	tensors
BPP	85 ± 13	2.5 ± 1.0	-7 ± 2	-
HKS	82.7 ± 6.4	1.7 ± 1.7	-	-
MV	114 ± 10	22 ± 5	-	-
KN	83 ± 12	-	-	-
J	93.9 ± 12.4	~ 7	-6.0 ± 1.2	-
this work	-	6.4 ± 2.0	$-(0.9 \sim 3.1) \pm 0.8$	1.1 ± 0.1

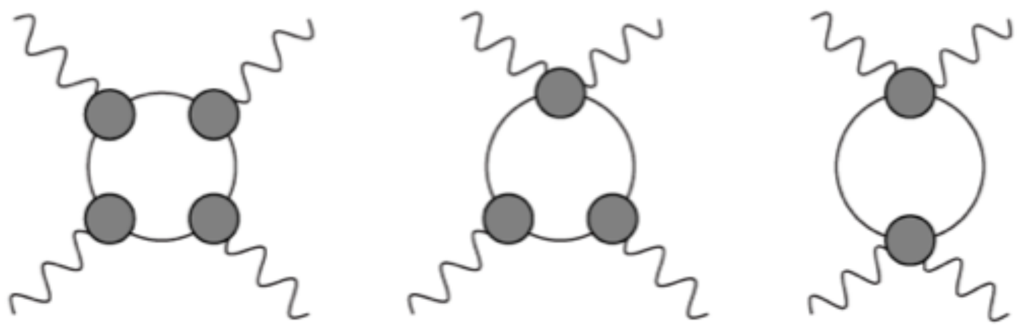
Jegerlehner

Pauk, Vdh

total $(6.6 \sim 4.4) \pm 2.9 \times 10^{-11}$

multi-meson production in $\gamma\gamma$ collisions (I)

- new estimate for **pion loop** contribution (with full VMD FF) **Bijnens (2014)**

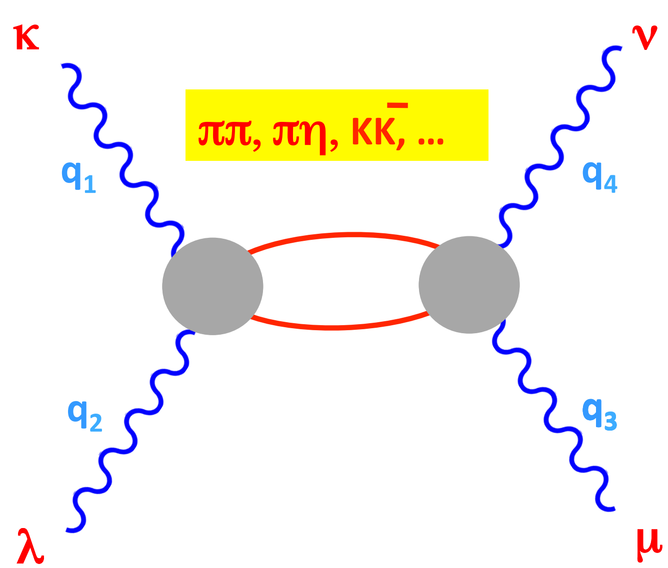


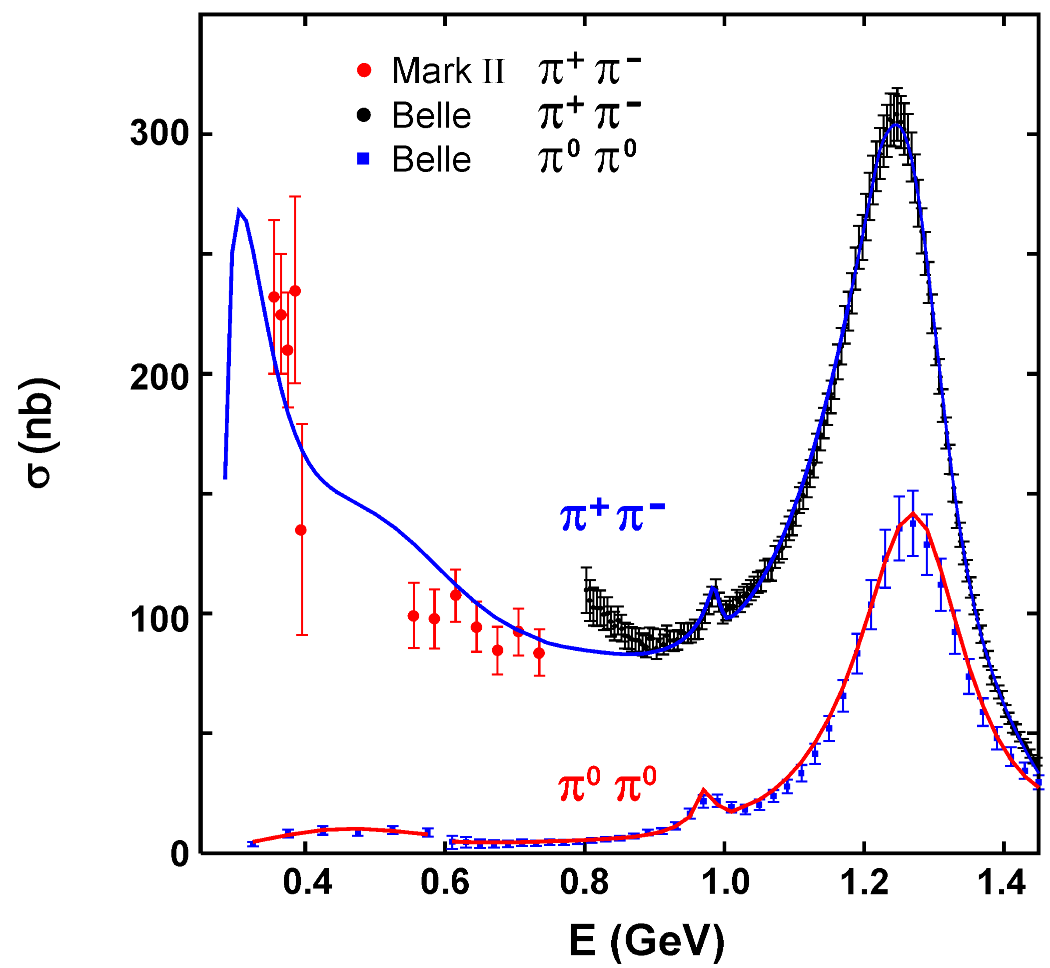
$a_\mu^{LbL\ \pi\text{-loop}} = (-\ 2.0 \pm 0.5) \times 10^{-10}$

integrating momenta in loop up to 1 GeV

- contribution of **multi-meson channels**

Dai, Pennington (2014)



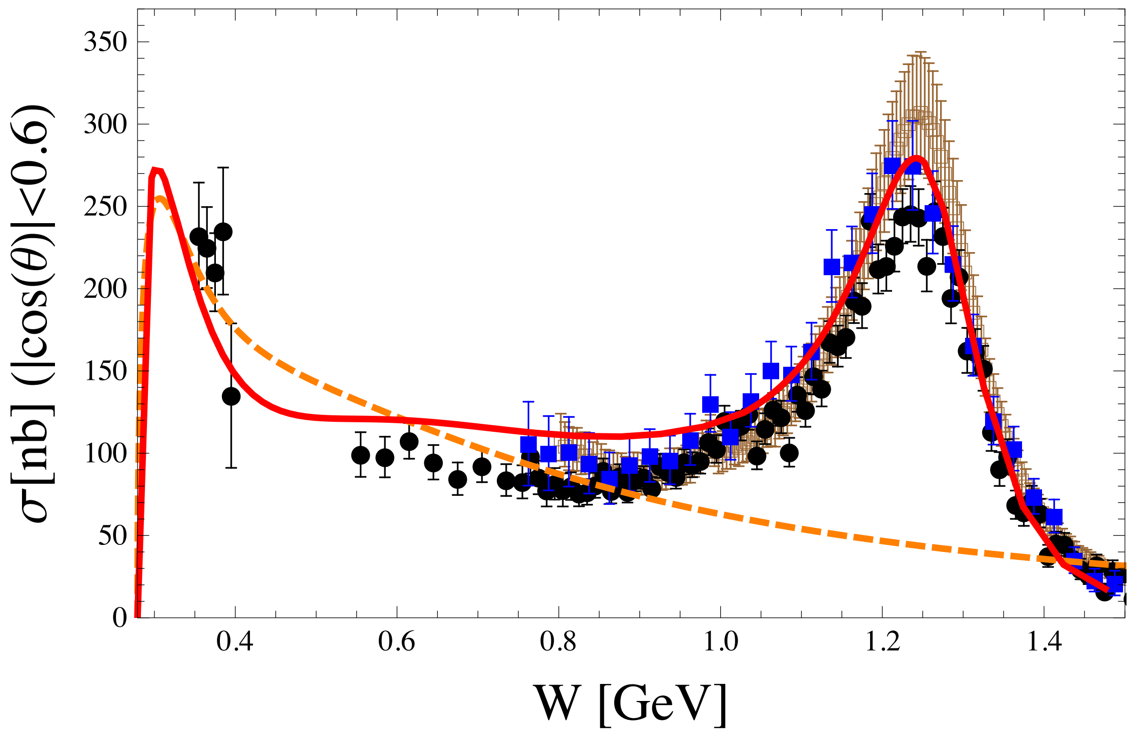


sum rules may be used as a consistency check of models

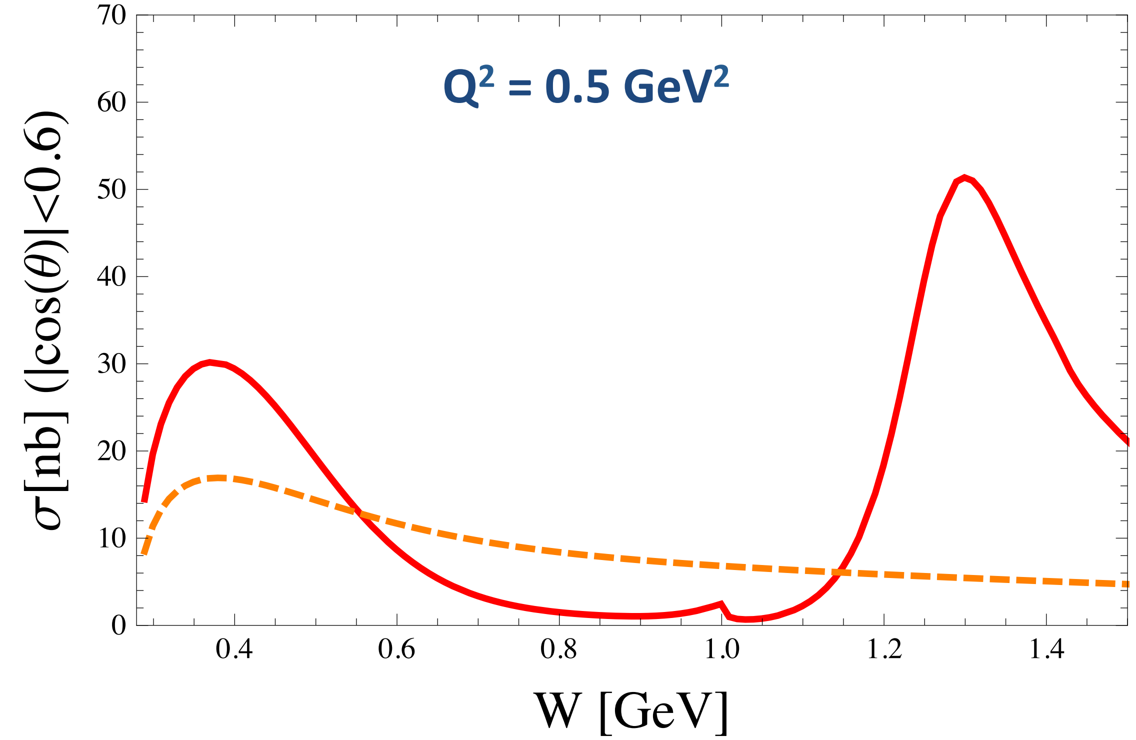
multi-meson production in $\gamma\gamma$ collisions (II)

➡ new **dispersion formalism** for $\gamma^* \gamma \to \pi \pi$

$\gamma\gamma \to \pi^+\pi^-$



$\gamma^*\gamma \to \pi^+\pi^-$



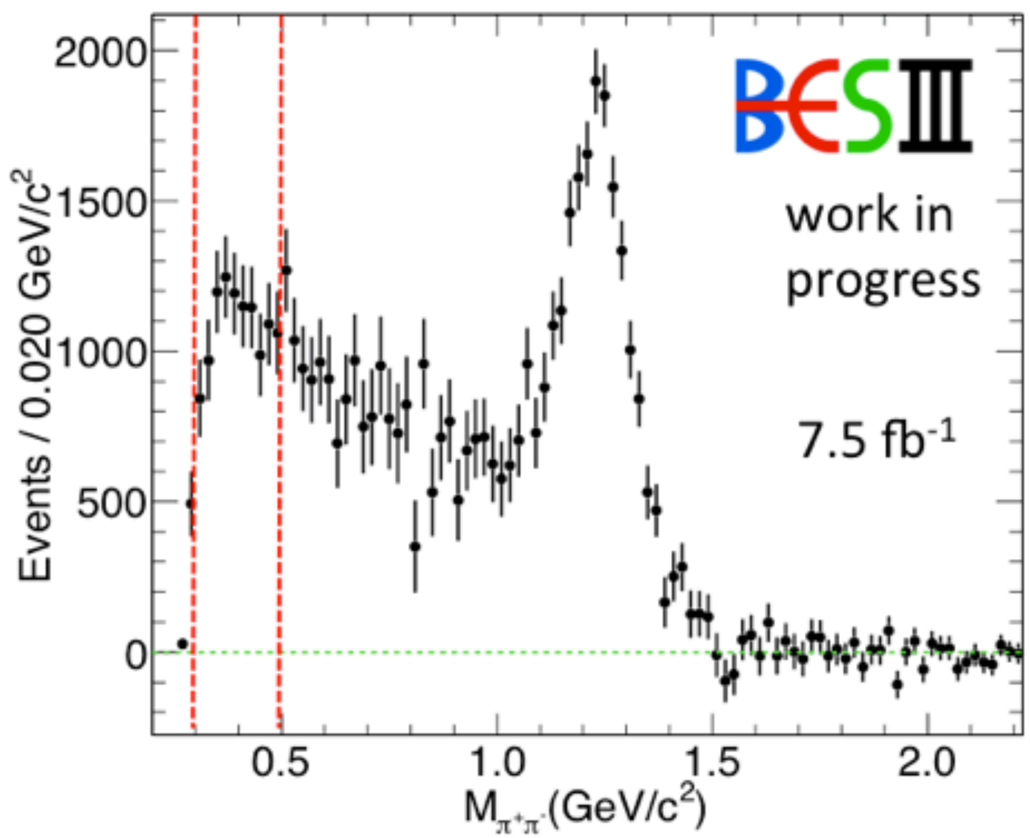
Masjuan, Sanchez Puertas et al. (2015)

- - - Born terms

— Unitarized s- and d-waves

multi-meson production in $\gamma\gamma$ collisions (III)

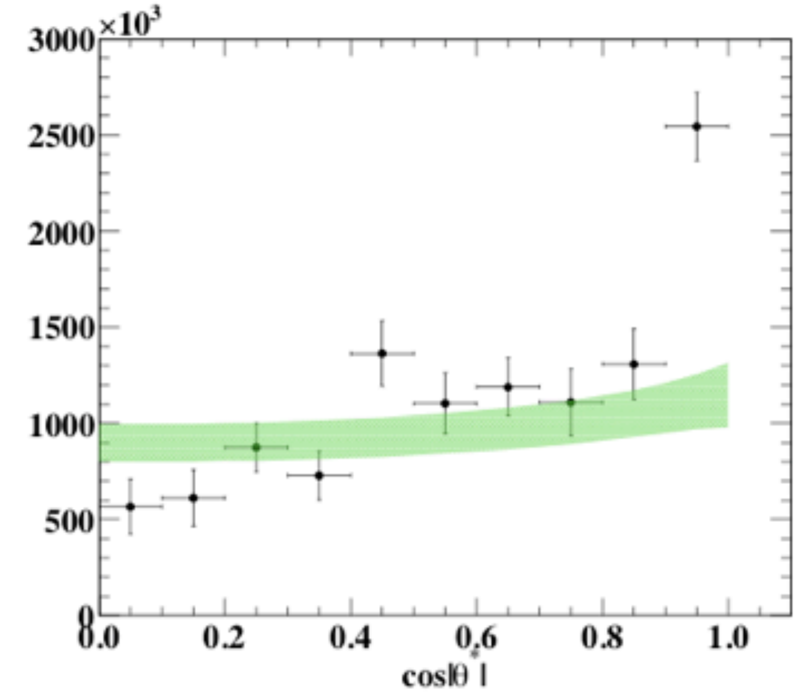
$\gamma^*\gamma \to \pi^+\pi^-$



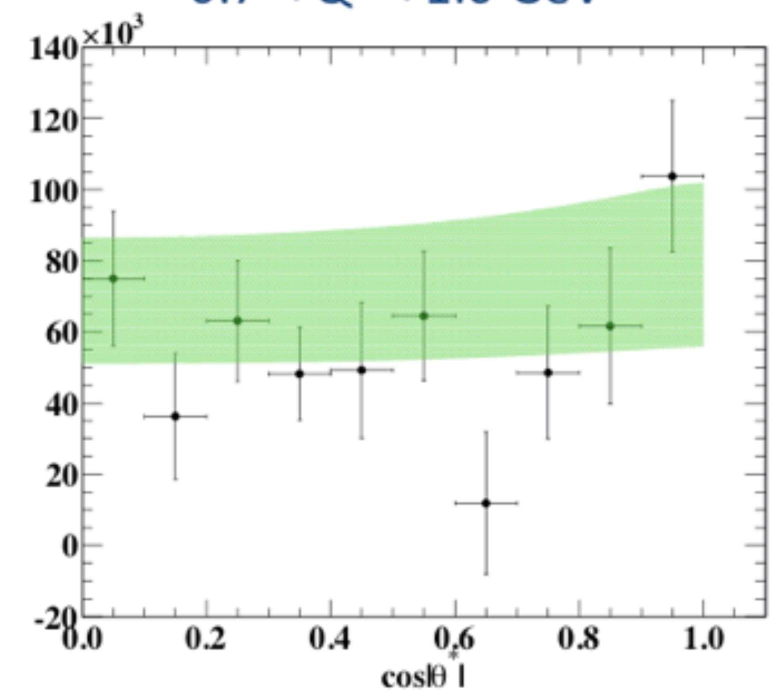
Guo et al.

- First single tag measurements of $\pi^+\pi^-$ channel
- Successful μ, π separation: neural network

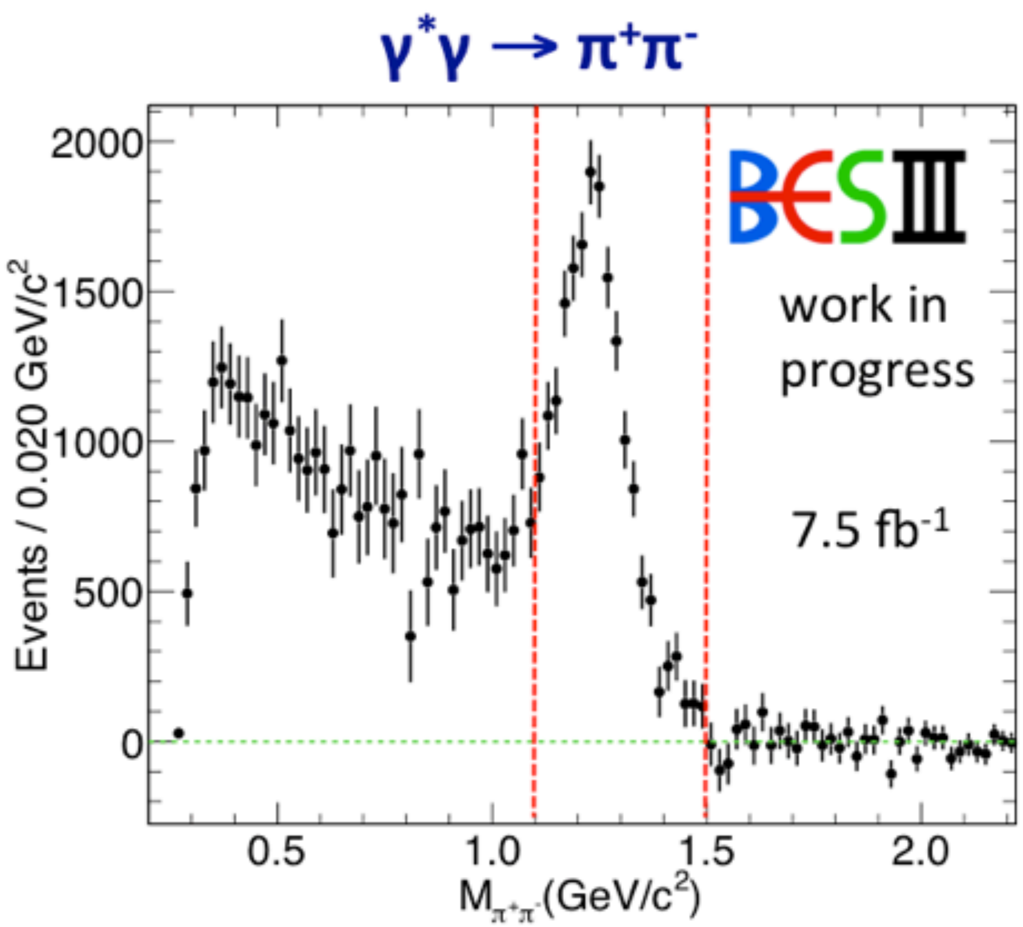
$0.3 < Q^2 < 0.5$ GeV^2



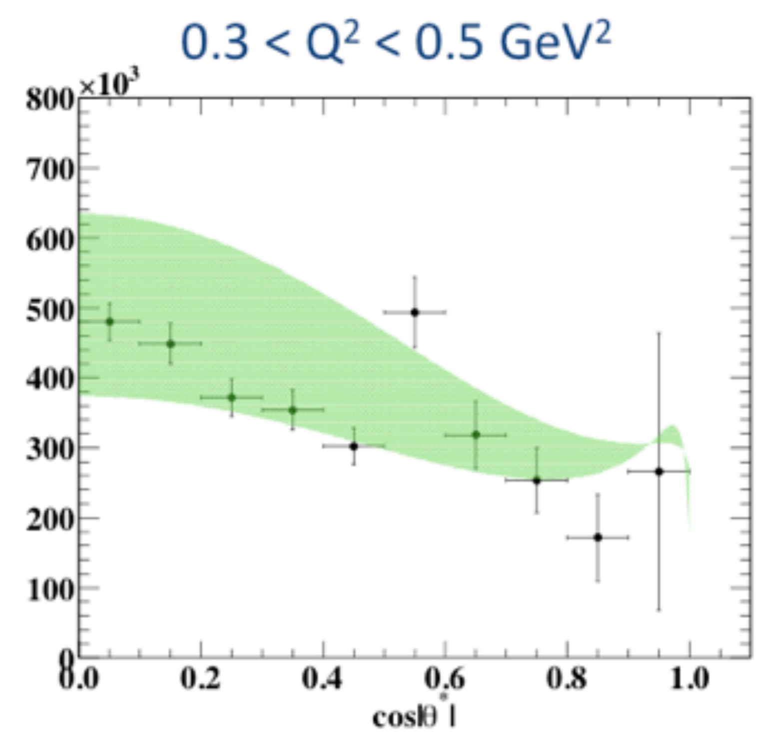
$0.7 < Q^2 < 1.0$ GeV^2

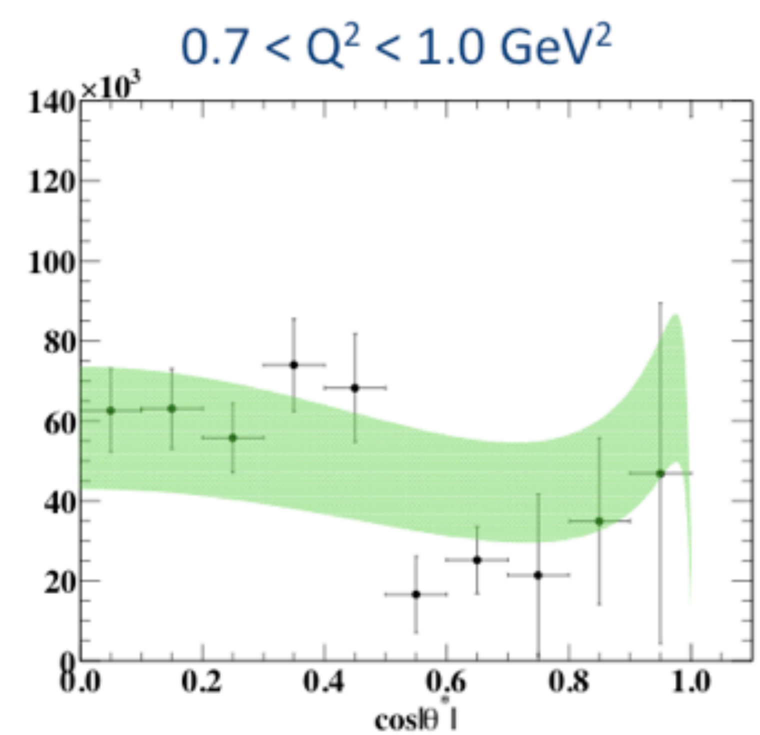


multi-meson production in $\gamma\gamma$ collisions (IV)



Guo et al.





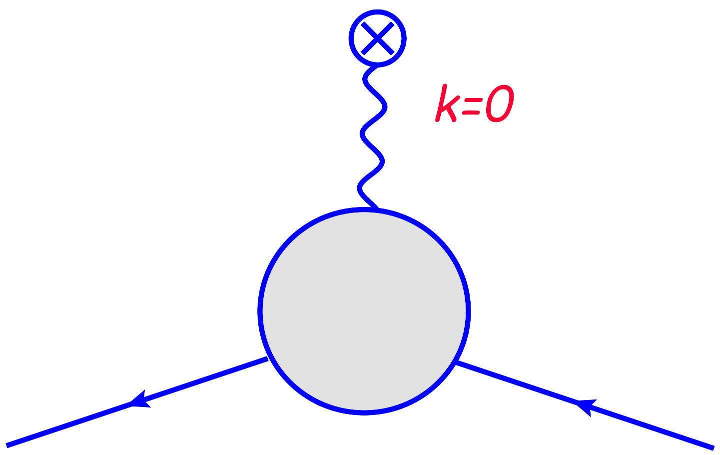
- First single tag measurements of $\pi^+\pi^-$ channel
- Successful μ, π separation: neural network

dispersion relation approaches for a_μ (I)

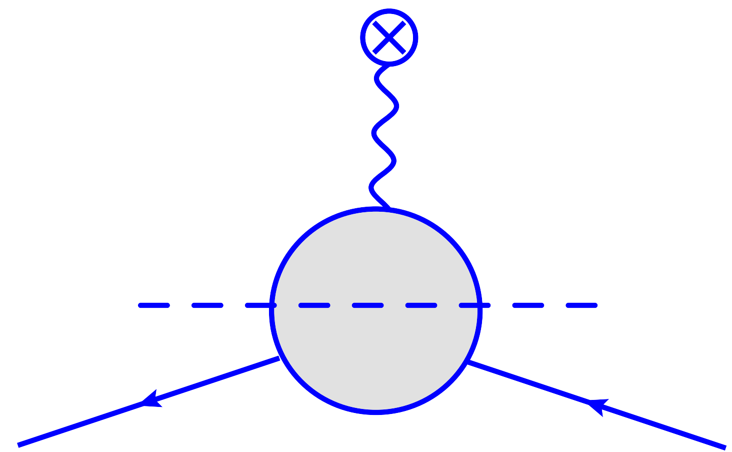
dispersion formalism directly for a_μ Pauk, Vdh (2014)

$$a_\mu = \lim_{k \to 0} F_2(k^2, (p+k)^2, p^2)$$

$$F_2(0) = \frac{1}{2\pi i} \int \frac{dk^2}{k^2} \mathrm{Abs}\, F_2(k^2)$$



$$a_\mu = F_2(0)$$



weighting functions (entire)

$$F_2(k^2) = e^6 \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda} (-1)^{\lambda+\lambda_1+\lambda_2+\lambda_3} \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} L_{\lambda_1\lambda_2\lambda_3\lambda}(p, p', q_1, k-q_1-q_2, q_2)$$

analytic structure →

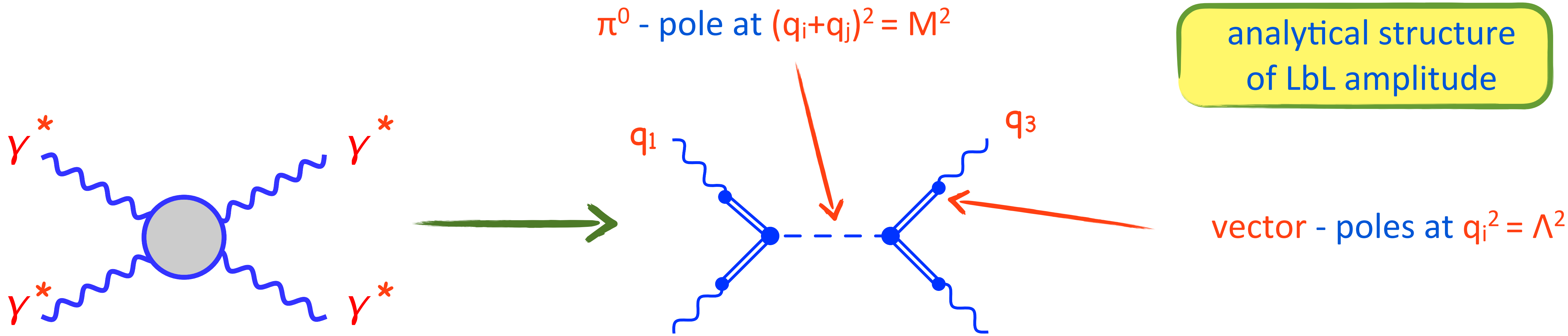
$$\times \frac{\Pi_{\lambda_1\lambda_2\lambda_3\lambda}(q_1, k-q_1-q_2, q_2, k)}{q_1^2 q_2^2 (k-q_1-q_2)^2 \left[(p+q_1)^2 - m^2\right] \left[(p+k-q_2)^2 - m^2\right]}$$

$$\Pi_{\lambda_1\lambda_2\lambda_3\lambda_4}(q_1, q_2, q_3) = \epsilon^\mu(q_1, \lambda_1)\epsilon^\nu(q_2, \lambda_2)\epsilon^\lambda(q_3, \lambda_3)\epsilon^\rho(q_4, \lambda_4)\Pi_{\mu\nu\lambda\rho}(q_1, q_2, q_3)$$

dispersion relation approaches for a_μ (II)

proof of principle: pole contributions

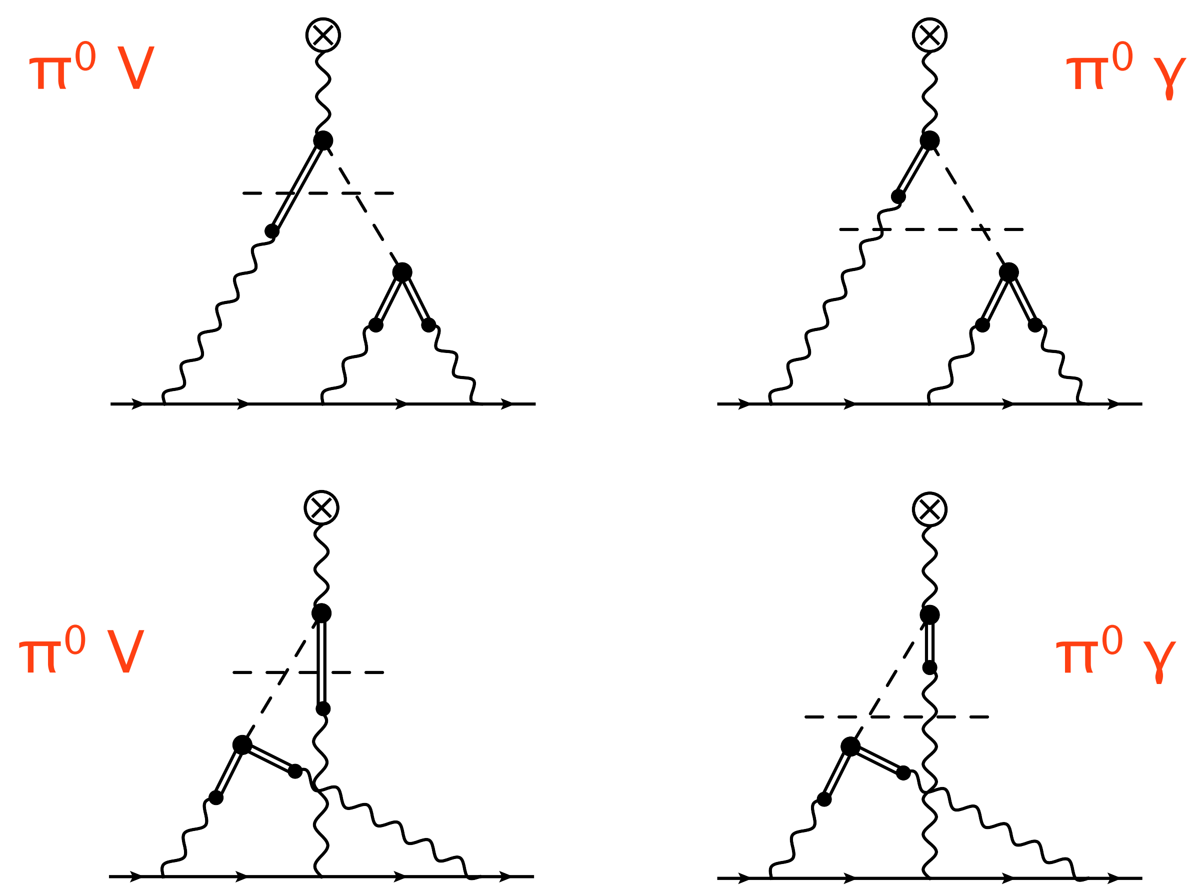
analytical structure of LbL amplitude



2-particle cuts

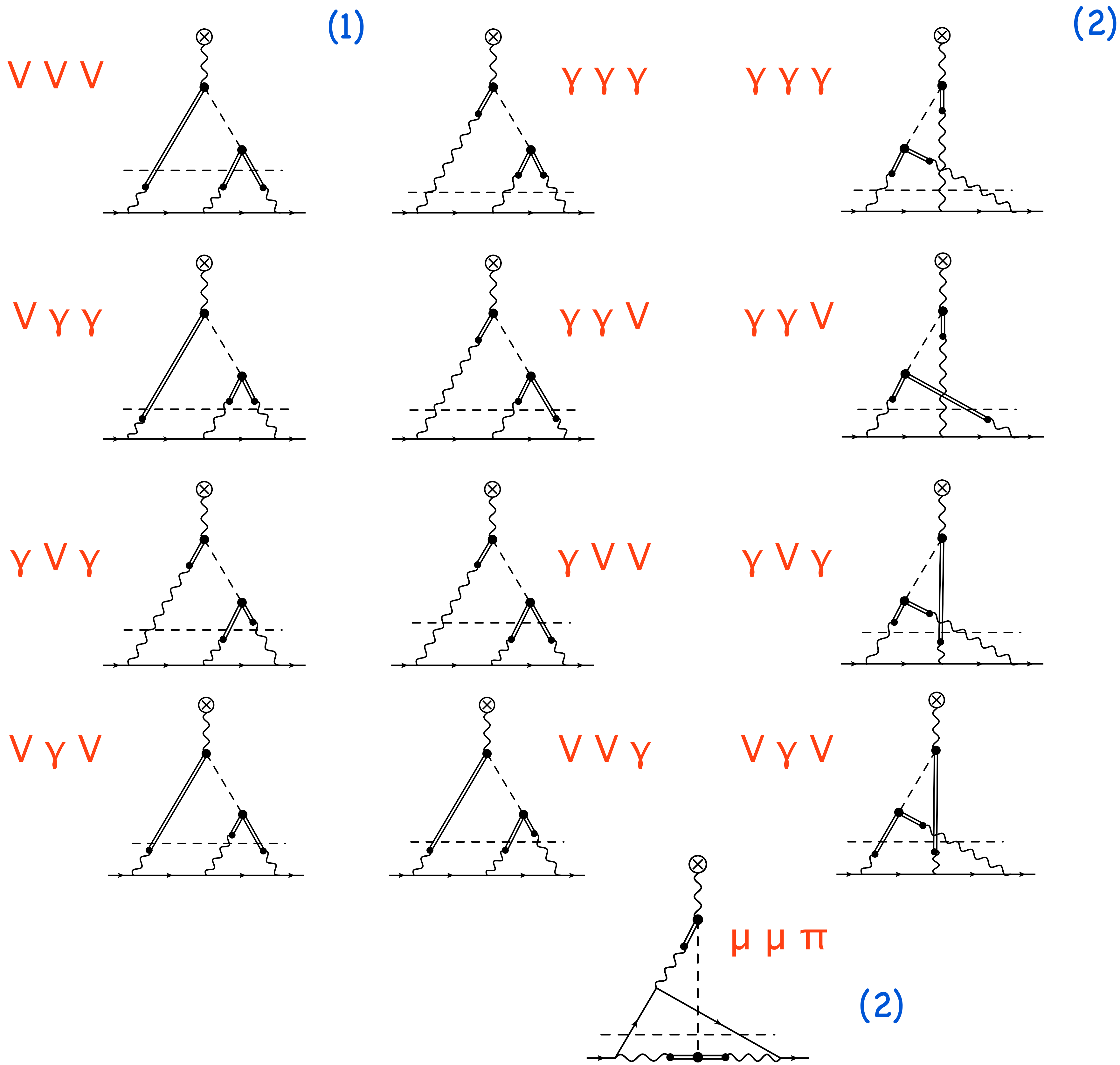
(1)

(2)



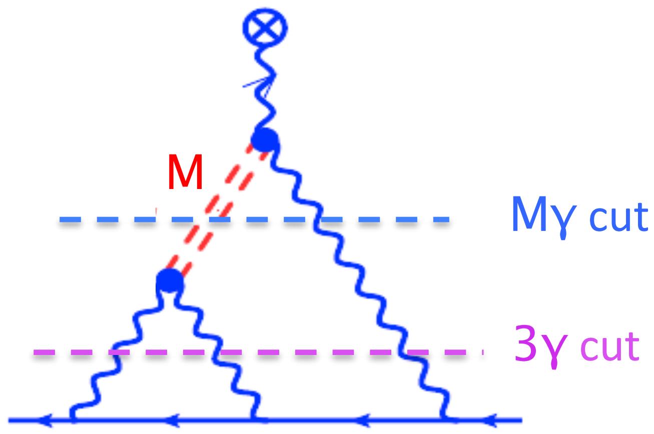
2-particle discontinuities

dispersion relation approaches for a_μ (III)



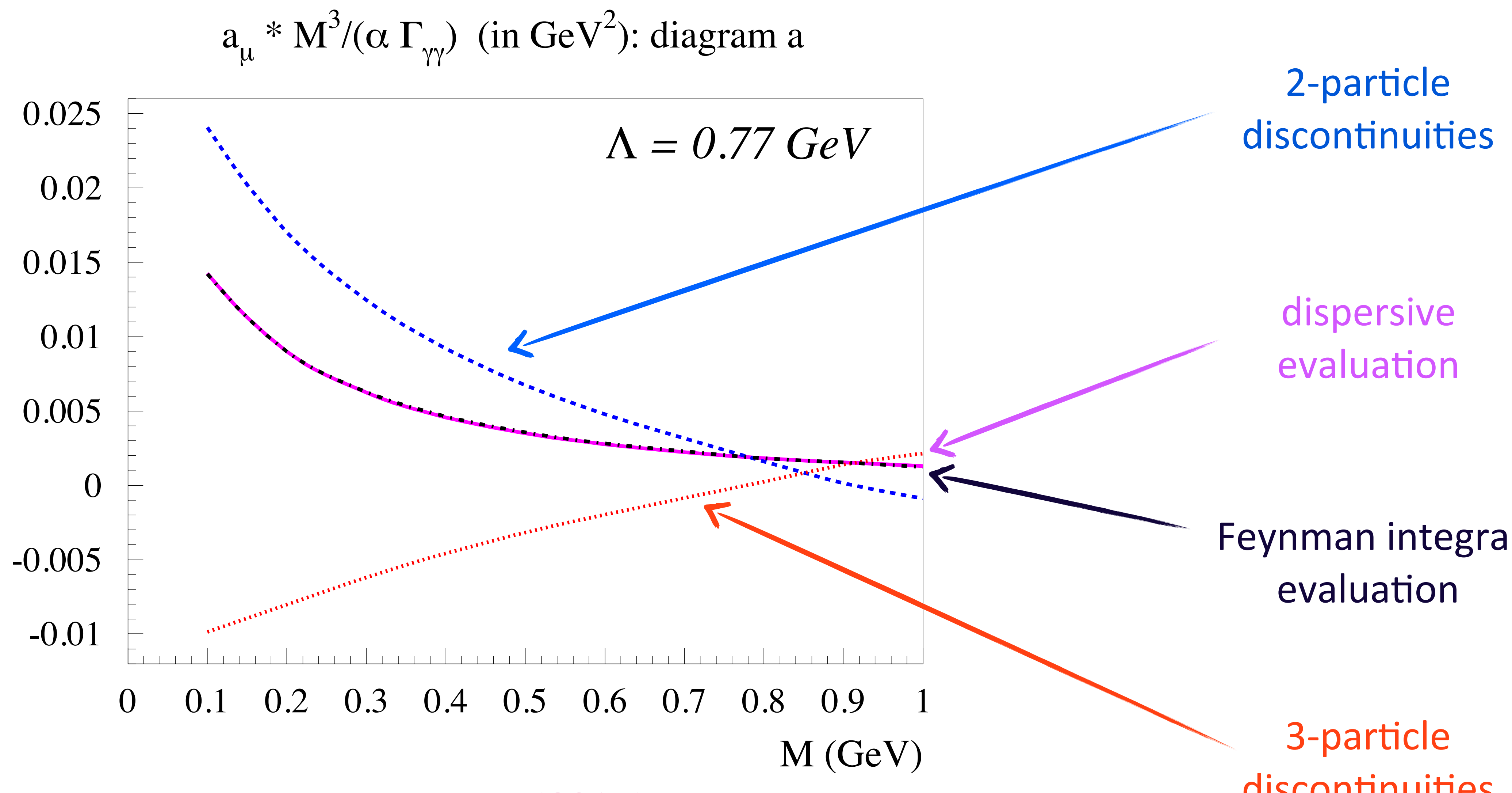
3-particle discontinuities

dispersion relation approaches for a_μ (IV)



reconstruction of a_μ from dispersion integral: **proof of principle**

$$F_2^{(i)}(0) = \frac{1}{2\pi i} \int_{M^2}^{\infty} \frac{dt}{t} \, \mathrm{Disc}_t^{(2)} F_2^{(i)}(t) + \frac{1}{2\pi i} \int_{0}^{\infty} \frac{dt}{t} \, \mathrm{Disc}_t^{(3)} F_2^{(i)}(t)$$



Pauk, Vdh (2014)

Summary and outlook

- **HVP:** new experimental program at BES-III
  - first results for $\mathbf{e^+ e^- \to \pi^+ \pi^-} \gamma$
  - aim: hadronic cross sections to 1% accuracy
  - strong lattice effort to rival experimental accuracy

- **HLbL: new theoretical tools** for $\gamma^* \gamma^* \to X$

  - sum rules, dispersive frameworks for transition FFs**:**
  allow to include experimental constraints

  - new evaluation of **heavier meson contributions:** $a_\mu = (6.6 \sim 4.4) \pm 2.9 \times 10^{-11}$

  - pioneering lattice efforts

- **new dispersion relation** frameworks for **HLbL** to $\mathbf{a_\mu}$:
  -> require close collaboration with experiment (spacelike, timelike, meson decays)

- Outcome of Mainz workshop:
  draft of roadmap for a data driven approach also in HLbL

- **goal: realistic error estimate on a_μ / reduce to 2 x 10^{-10} (20 % of HLbL)**