What does Holographic QCD predict for Anomalous $(g\text{-}2)_\mu$?

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Holographic models of (Large-N_c) QCD

Pros

- A Lagrangian formulation, although in 5D space
- QCD partonic log at large Euclidean momentum Q^2 reproduced in HQCD models with (asymptotic) AdS metric
- Channels saturated by ∞ narrow resonances in models with IR cut-off (5D KK modes) or other suitable IR conditions
- Small number of parameters
- Many explicit calculations in a wide range of energy, with (surprisingly?) good results at low energy

Holographic models of (Large-N_c) QCD

Cons

Many features are model dependent

- Resonance spectra non-Regge for Hard Wall Model vs Regge in Soft Wall models
- Subleading (inverse powers of Q^2) at large Euclidean momentum not or badly reproduced
- Chiral Symmetry Breaking differently realized with different problematic issues
- (mainly for theoreticians) the only Top-Down model (Sakai-Sugimoto) not even reproducing the partonic log

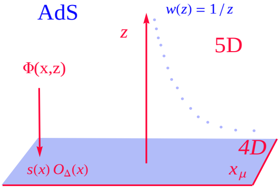
The Holographic Recipe

AdS/CFT duality (equivalence) between 4D (conformal) gauge theories (at strong coupling) and (classical) 5D field theories in a curved (AdS) gravitational background

Master Formula for 4D correlation functions

$$exp(iW[s(x)]) \equiv \langle exp\left(i \int d^4x\, s(x) O_\Delta(x)\right)\rangle_{QCD} = exp\left(i\, S_5(\Phi_0(z,x))\right)$$

with $\Phi_0(z,x)$ solution of the 5D EOM and $\Phi_0(z,x)|_{z\to 0} \to s(x)$



Holographic dictionary

4D	5D
source $s(x)$ coupled to $O_\Delta(x)$	on-shell $\Phi_0(x,z)$
conformal dimension Δ	mass m_Φ
global symmetry	gauge symmetry
conserved current $\bar{q}\gamma_\mu t^a q$	gauge field V_M^a

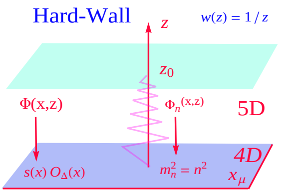
Lagrangians and fields of Holographic QCD

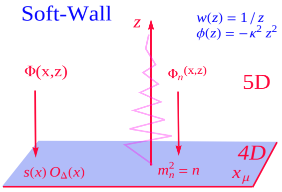
$$S_5 = S_{\rm YM} + S_X + S_{\rm CS} \ ,$$

$$\begin{aligned} S_{\rm YM} &= -{\rm tr} \int d^4x \int_0^{z_0} dz \ e^{-\Phi(z)} \frac{1}{8g_5^2} w(z) \left[\mathcal{F}_L^2 + \mathcal{F}_R^2 \right] \ , \\ S_X &= {\rm tr} \int d^4x \int_0^{z_0} dz \ e^{-\Phi(z)} w(z)^3 \left[D^M X D_M X^\dagger + V(X^\dagger X) \right] \ , \\ S_{\rm CS} &= \frac{N_c}{24\pi^2} \int {\rm tr} \left(\mathcal{A}_L \mathcal{F}_L^2 - \frac{i}{2} \mathcal{A}_L^3 \mathcal{F}_L - \frac{1}{10} \mathcal{A}_L^5 \right) - (L \to R) \end{aligned}$$

- $ds^2_{AdS} = w(z)^2 \left(dx_\mu^2 - dz^2 \right)$, with $w(z) = 1/z$
- $\mathcal{F}_{MN} = \partial_M \mathcal{A}_N - \partial_N \mathcal{A}_M - i[\mathcal{A}_M, \mathcal{A}_N]$ and $\mathcal{A}_{L,R} = V \mp A$,
- IR cut-off $z_0 < \infty$ (HW models) or $z_0 = \infty$ in SW models
- In SW models $\Phi(z) = -\kappa^2 z^2$ is a background dilaton field
- In some HW models, a 5D scalar field $X(x,z)$ (dual to $\bar{q}q$) triggers χSB

Example: Hard-Wall vs Soft-Wall models





- nice descriptions of χSB
  · HW1: X (dual to $\bar{\psi}\psi$) departure from chiral limit: $m_\pi \neq 0$, GOR
  · HW2: $U(x) = e^{2\pi(x)/f_\pi}$ from 5D Wilson line $\rightarrow$ Chiral Lagrangian terms
- Resonance not Regge: $m_n^2 = n^2$

- χSB has some problems
  · mostly modelled using X as in HW1
  · no chiral covariant formulation
- Resonance are Regge: $m_n^2 = n$

Large-Q^2 inverse power corrections (QCD condensates) to parton log absent or wrong

LHBL pion exchange diagram in HQCD

Neutral pion exchange contribution extracted from (models for the) electromagnetic pion form factor $F_{\gamma^*\gamma^*\pi^0}$

Questions we tried to address using HQCD:

- Which parameters of $F_{\pi^0\gamma^*\gamma^*}$ mostly affect the predicted value of $(g-2)_\mu$?
- What does HQCD predicts for linear and quadratic slopes (low-Q^2 expansion) of $F_{\pi^0\gamma^*\gamma^*}$
- Can we discriminate some models of HQCD wrt others?
- Is Vector Meson Dominance a valid approximation or more resonances give sizable contributions?
- Can we implement QCD low- and high-energy constraints in HQCD models ?

Holographic calculation of $F_{\pi^0\gamma^*\gamma^*}\left(Q_1^2, Q_2^2\right)$

In HQCD models one gets explicit (analytic) expressions for the relevant amplitudes.

$$F_{\pi^0\gamma^*\gamma^*}\left(Q_1^2, Q_2^2\right) = -\frac{N_C}{12\pi^2 f_\pi} K(Q_1^2, Q_2^2) \ ,$$

- Anomalous 3-point function $\langle AVV \rangle$ from 5D Chern-Simons action

$$S_{\mathrm{CS}} = \frac{N_c}{4\pi^2}\epsilon^{\mu\nu\rho\sigma}\int_0^{z_0} dz\,(\partial_z \alpha(z)) \int d^4x\ \pi^a\,(\partial_\rho V_\mu^a)\left(\partial_\sigma \hat{V}_\nu\right) \ .$$

$$K(Q_1^2, Q_2^2) = -\int_0^{z_0} \partial_z\alpha(z)\mathcal{J}(Q_1, z)\mathcal{J}(Q_2, z)\,dz$$

- vector bulk-to-boundary propagator $(Q^2 = -q^2)$

$$\mathcal{J}(Q, z) = \sum_{n=1}^{\infty} \frac{f_n}{Q^2 + m_n^2}\psi_n(z)$$

- "pion wave function" $\alpha(z)$ from the pion pole in the axial sector

A Hybrid Strategy L.C, D'Ambrosio, Cata '10

- Study the anomalous electromagnetic pion form factor $F_{\pi^0\gamma^*\gamma^*}$ within a set of holographic models.
- By comparing with the measured value of the linear slope, some models ruled out.
- From the remaining models obtain predictions for the low-energy quadratic slope parameters of $F_{\pi^0\gamma^*\gamma^*}$.
- Encode low-energy information in an interpolating form factor able to satisfy also QCD short-distance constraints (D'Ambrosio, Isidori, Portolez '97)
- Evaluate the (dominant) pion exchange diagram in the hadronic light by light scattering contribution to the muon anomalous magnetic moment.
- Estimate the theoretical uncertainty in $(g-2)_\mu$ coming from the different input: QCD short distances, experimental input and low-energy holographic predictions.

$F_{\pi^0\gamma^*\gamma^*}$ at low energies

- $K(Q_1^2, Q_2^2) \simeq 1 + \hat{\alpha}\ (Q_1^2 + Q_2^2) + \hat{\beta}\ Q_1^2 Q_2^2 = + \hat{\gamma}\ (Q_1^4 + Q_2^4)$

Model	$\hat{\alpha}$ (GeV^{-2})
HW1	-1.60
HW2 (AdS)	-1.81
SW	-1.66
SS	-2.04
HW2 (Flat)	-1.37

$\hat{\alpha} = -1.76 \pm 0.22\,\text{GeV}^{-2}$ (exp)

$$\Longrightarrow \begin{cases} \hat{\beta} = 3.33(32)\,\text{GeV}^{-4}, \\ \hat{\gamma} = 2.84(21)\,\text{GeV}^{-4}. \end{cases}$$

- (Lowest) Vector Meson Dominance:
  dominant contributions to $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ from first (second) terms in

$$\mathcal{J}(Q,z) = \sum_{n=1}^{\infty} \frac{f_n}{Q^2 + m_n^2} \psi_n(z) \quad \text{and} \quad K(Q_1^2, Q_2^2) = \sum_{k,l=1}^{\infty} \frac{B_{kl}}{(Q_1^2 + m_k^2)(Q_2^2 + m_l^2)}$$

I suggests that a good approximation to $K(Q_1^2, Q_2^2)$ already truncating the sum to double poles

High energy constraints on $F_{\pi^0\gamma^*\gamma^*}$

- Constraints satisfied in HQCD models with asymptotic AdS metric

$$\lim_{Q^2\to\infty} K(Q^2,Q^2) = \frac{8\pi^2 f_\pi^2}{N_c}\frac{1}{Q^2}$$

$$\lim_{Q^2\to\infty} K(0,Q^2) \sim \frac{1}{Q^2} \quad \text{Brodsky-Lepage}$$

- New off-shell constraint: $\pi_0\gamma^*\gamma^* \to \pi_0^*\gamma^*\gamma^*$ [Nyffeler '09]

$$\lim_{Q^2\to\infty} F_{\pi_0^*\gamma^*\gamma^*}(Q^2,Q^2,0) = \frac{f_\pi}{3}\chi_0 + \cdots$$

where χ_0 is a new order parameter: quark magnetic susceptibility

in literature $0 \leq \chi_0 \leq 9\ \text{GeV}^{-2}$

(more on possible independent determination of χ_0 in HQCD later !)

An interpolator for $F_{\gamma^*\gamma^*\pi^0}$

Promote the following ansatz (DIP) for the form factor:

$$K(q_1^2, q_2^2) = 1 + \lambda \left(\frac{q_1^2}{q_1^2 - m_V{}^2} + \frac{q_2^2}{q_2^2 - m_V{}^2} \right) + \eta \frac{q_1^2 q_2^2}{(q_1^2 - m_V{}^2)(q_2^2 - m_V{}^2)}$$

to an analytic interpolator valid to any value of the photon momenta

- first introduced (for Kaon decays) in [D'Ambrosio, Isidori, Portoles '97]
- low-energy parameters $\hat{\alpha}$, $\hat{\beta}$ and $\hat{\gamma}$ easily related to λ, η and m_V
- it allows to implement some high- and low-energy constraints and yeld predictions for the remaining parameters. For instance, imposing

  $$1+2\lambda+\eta = 0\ (OPE), \quad \lambda+\eta = -\frac{4\pi^2 f_\pi^2}{3m_V^2}, \quad (OPE), \quad \frac{\lambda}{m_V^2} = \hat{\alpha} = -1.76 \pm 0.22$$

  one gets

  $$m_V = (0.64^{+0.10}_{-0.13})\text{GeV}, \quad \chi_0 = \frac{N_c}{4\pi^2 f_\pi^2}(1+\lambda) = (2.42 \pm 0.17)\text{GeV}^{-2}$$

More importantly, DIP can be rewritten in a form which allows the use of the expressions of [Knecht, Nyffeler '02] for the evaluation of the 2-loop integral of the pion pole exchange in HLBL contribution to $(g-2)_\mu$

Numerical results

- In the numerical analysis a (mild) generalization of the interpolator has also been considered, showing, hovever, that numerical values are stable and very close to those otained with the original DIP.

$$a_\mu^{\pi^0} = 65.4(2.5) \cdot 10^{-11}$$

error mainly driven by the linear slope of $F_{\pi^0\gamma^*\gamma^*}$.

- Large values of the magnetic susceptibility χ_0 are disfavored, however, in the absence of stronger bounds on χ_0, an additional $(10-15)\%$ systematic uncertainty on the previous value for $a_\mu^{\pi^0}$ cannot be excluded.

Models for $F_{\pi^*\gamma\gamma}$	$a_\mu(\pi_0) \times 10^{-11}$	$a_\mu(\pi_0, \eta, \eta') \times 10^{-11}$
Modified ENJL [BPP '95, '96, '02]	59(9)	85(13)
VMD/HLS [HKS '95, '96; HK '98, '02]	57(4)	83(6)
VMD+V ($h_2 = 0$) [KN '02]	58(10)	83(12)
VMD+V ($h_2 = -10\,\text{GeV}^2$) [KN '02]	58(10)	83(12)
VMD+V (const.$F_{\pi_0\gamma^*\gamma}$) [MV '04, '08]	77(7)	114(10)
DSE [FGV '10, '11]	58(7)	84(13)
AdS/QCD [HoK '00]	69	107
AdS/QCD/DIP [CCD '10]	65.4(2.5)	-
RχT [KaNo '11]	65.8(1.2)	-

Beyond the DIP ?

A more thorough application of HQCD to the evaluation of $a_\mu^{\pi^0}$?

Pros

- Using, for the evaluation of $a_\mu^{\pi^0}$, the full analytic expressions for $K(Q_1^2, Q_2^2)$ provided by a given HQCD model
- Checking if HQCD models effectively comply with all the QCD constraints on the pseudoscalar exchange
- Understanding if one can have independent predictions on the magnetic susceptibility χ_0

Cons

- One has to consider a specific HQCD model. None is free from problems.
- Technical problem: $K(Q_1^2, Q_2^2)$ could not allow to use the simpler 2-loops integrations as it was the case with DIP
- The lagrangian and the set of fields has to be enlarged from vector to tensor fields, in order to get independent determinations of χ_0

I'll make a final comment on this last point

Some facts about χ_0

- In presence of a magnetic background, the tensor current

$$O^{T,a}_{\mu\nu}(x) = \bar{q}(x) t^a \sigma_{\mu\nu} q(x)$$

can acquire a non vanishing VEV

$$\langle \bar{q} t^a \sigma_{\mu\nu} q \rangle = e_q \chi_q \langle \bar{q} q \rangle F^{cl}_{\mu\nu} \quad \text{where} \quad F^{cl}_{\mu\nu} = \partial_\mu A^{cl}_\nu - \partial_\nu A^{cl}_\mu$$

- The operator O^T is odd under C but contains both parity even and odd parts and thus can create create both $J^{PC} = 1^{--}$ and 1^{+-} states $\rho^{(n)}$, $b_1^{(n)}$
- Estimated using QCD sum rules: it enters in the mixed VT 2-point function

$$\lim_{q^2 \to 0} \Pi_{VT}(q^2) = -\chi_0 \langle \bar{q} q \rangle$$

- Using OPE and pion dominance, [Vainshtein '03] studying the QCD $\langle AVV \rangle$ 3-point function, predicted

$$\chi_0 = -\frac{N_c}{4\pi^2 f_\pi}$$

HQCD predictions for χ_0

- $\langle AVV \rangle$ in HQCD: Vainhstein result almost (parametrically) recovered [Gorski, Krikun '12]
- Anomaly matching for resonances from HQCD models [Son, Yamamoto '10]: if true in QCD reproduces Vainshtein result (but soon criticized as being not true in QCD! [KPPdR '11])
- $\Pi_{VT}(q^2)$ in enlarged holographic models [GKKV '12] (with with 5D tensor field $B_{MN}(x,z)$ dual to $O^T_{\mu\nu}(x)$ [CCD '10, DHR '11,'12])

Defining $\chi_0 = -c_u \dfrac{N_c}{16\pi^2 f_\pi^2} = -c_u\,(2.22\,\mathrm{GeV}^{-2})$ one has

Non Holographic methods	c_u
Sum rules	4
Non Renorm. Th.	2
Lattice	0.96–1.04
Radiative Meson Decays	1.4

Holographic methods	c_u
$\langle AVV \rangle$ in HQCD [GK '10]	2.15
$\langle AVV \rangle$ in HQCD [SY '10]	2
HQCD + Tensors [GKKV '12]	0.06
HQCD +Tensors [DHR '13]	1–1.5

Conclusions

- HQCD is a set models which allow many analytic calculations of physical quantities which are relevant for non perturbative regime of QCD
- In particular, they capture important aspect of QCD at low and intermediate energies, reproducing features of chiral symmetry breaking, vector meson dominance, which are extended to whole families of resonances of increasing masses
- Hovever, there is not yet a model prevailing on the others: any of them has some problematic issue, eventually not shared by the others
- This state of affairs led us to consider in the evaluation of HLBL contribution to $(g.2)_\mu$ the idea of playing in the *s*pace of theories of a set of HQCD models, in order to extract generical predictions.
- Attempts to enlarge HQCD to a wider set of 5D field and a corresponding wider set of 4D operators have not led to an improvement of predictive power
- Do I have a favourite HQCD model? HW2 (Notice that SY result seem to favours it) What model would I like to have? SW2 !?!