

A New Approach To Evaluate The Leading Hadronic Corrections To Muon $g-2$

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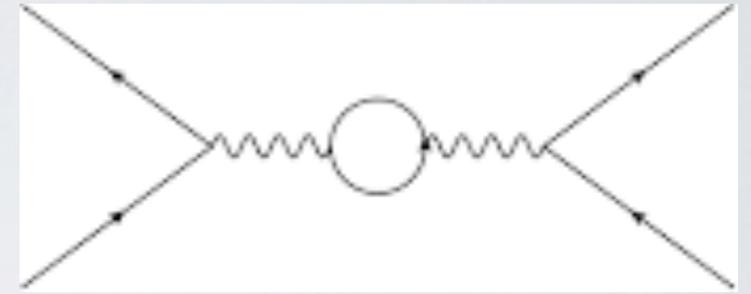
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Vacuum Polarization makes α_{em} running assuming a well defined “effective” value at any scale



vacuum polarization and the “effective charge” are defined by:

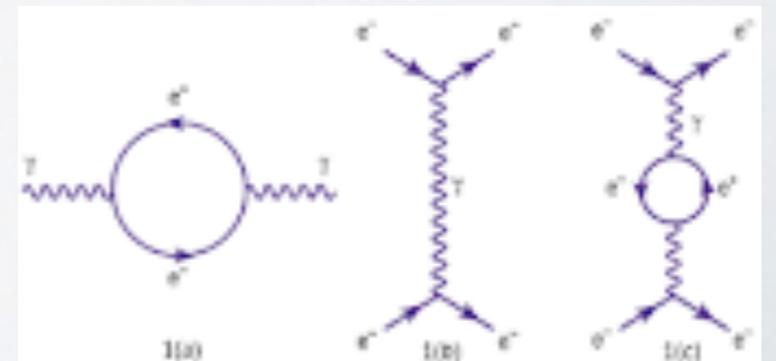
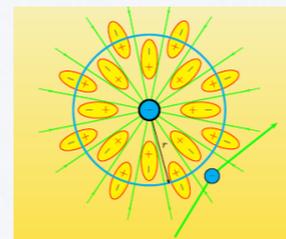
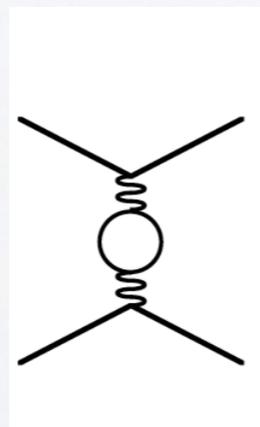
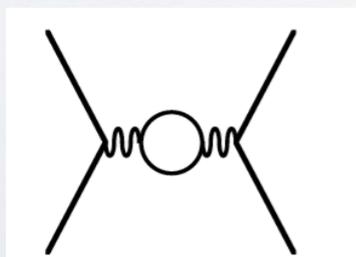
$$e^2 \rightarrow e^2(q^2) = \frac{e^2}{1 + (\Pi(q^2) - \Pi(0))} \quad \alpha(q^2) = \frac{\alpha(0)}{1 - \Delta\alpha}; \quad \Delta\alpha = -\Re e(\Pi(q^2) - \Pi(0))$$

$\Delta\alpha$ takes contributions from leptonic and hadronic and gauge bosons elementary states

Among these the non-perturbative $\Delta\alpha_{had}$

$$\Delta\alpha = \Delta\alpha_{leptonic} + \Delta\alpha_{gb} + \Delta\alpha_{had} + \Delta\alpha_{top}$$

α



The numerical prediction of electroweak observables involves the knowledge of $\alpha(q^2)$ usually for $q^2 \neq 0$

for example the knowledge of $\alpha(m_Z^2)$ is relevant to the evaluation of quantities measured by the LEP experiments.

This is achieved by evolving from $q^2 = 0$ to the Z mass.

The evolution is expressed by the quantity $\Delta\alpha(q^2)$ receives contributions from leptons, hadrons and the gauge bosons.

The hadronic contribution to the vacuum polarization, which cannot be calculated from first principles, is estimated with the help of a dispersion integral and evaluated by using total cross section measurements of $e^+e^- \rightarrow$ hadrons at low energies.

Therefore, any evolved value of $\Delta\alpha(q^2)$ and particularly for $q^2 \geq m_\pi^2$ is affected by uncertainties originating from hadronic contributions.

F. Jegerlehner: hep-ph/0308117

M. Davier and A. Hoecker: Phys. Lett. **B435** (1998) 427;

M. Davier, S. Eidelman, A. Hoecker and Z. Zhang: Eur. Phys. J. **C27** (2003) 497

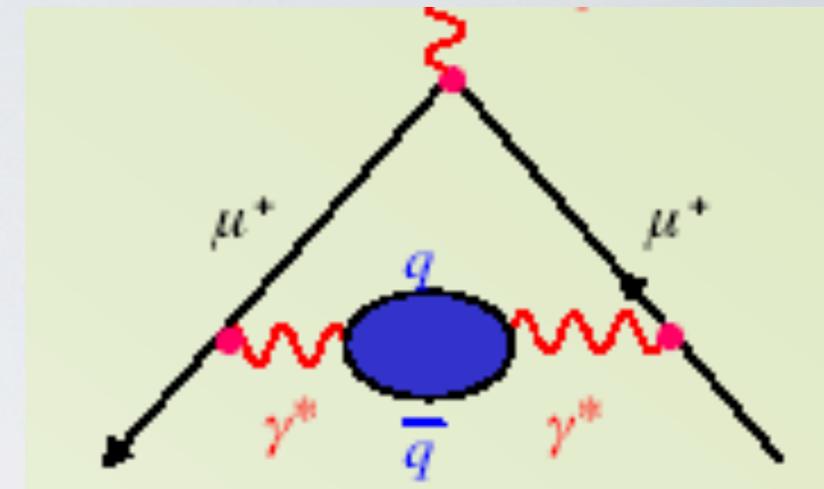
D. Karlen and H. Burkhardt: Eur. Phys. J. **C22** (2001) 39; hep-ex/0105065 (2001)

A.A. Pankov and N. Paver, Eur. Phys. J. **C29** (2003) 313

a_μ^{HLO} calculation, traditional way: time-like data

$$a_\mu^{\text{HLO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} \sigma_{e^+e^- \rightarrow \text{hadr}}(s) K(s) ds$$

$$a_\mu = \frac{g - 2}{2}$$



$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi^2} \int_0^1 dx \int_0^\infty \frac{ds}{s} K(s) \text{Im} \Pi_{\text{had}}(s)$$

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)} \sim \frac{1}{s}$$

$$\sigma_{e^+e^- \rightarrow \text{hadr}}(s) = \frac{4\pi}{s} \text{Im} \Pi_{\text{had}}(s)$$

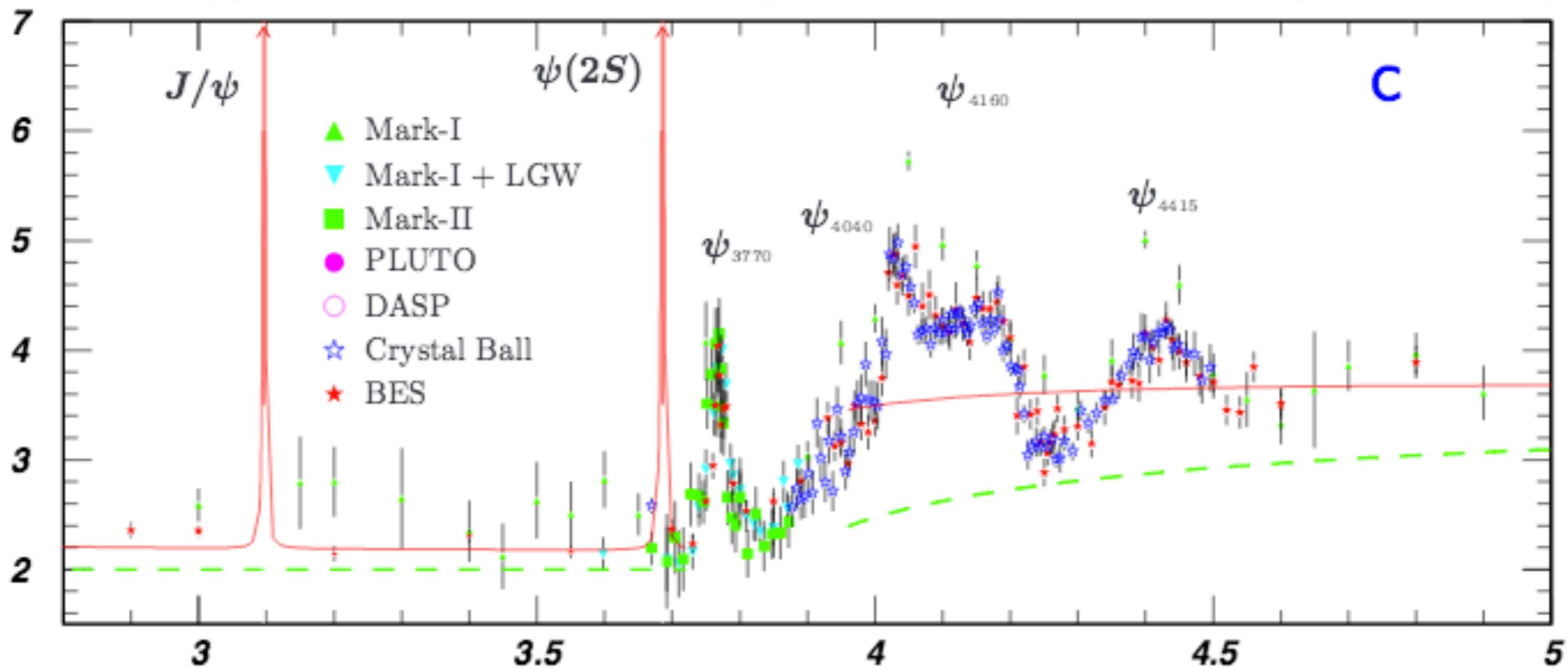
Old way : based on precise experimental (time-like) data:

$$a_\mu^{\text{had}} = (689.7 \pm 4.4) \cdot 10^{-10} \quad \text{The main contribution is in the low energy region}$$

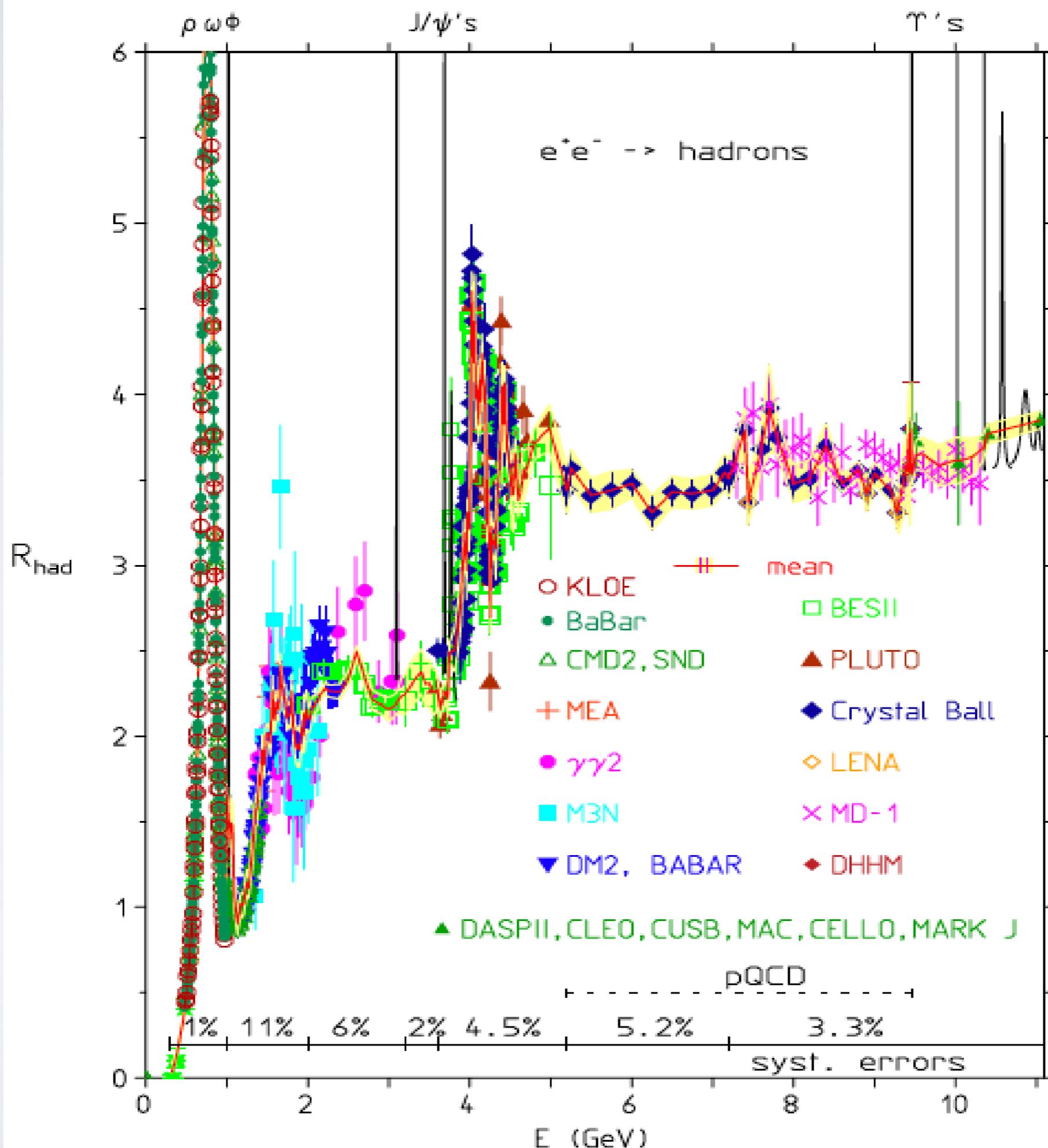
$\delta a_\mu^{\text{exp}} 1.5 \cdot 10^{-10} = 0.2 \% \text{ on } a_\mu^{\text{HLO}}$
 (from the 0.7 % of the present accuracy)

New measurements at
 FNAL and JPARC

R



with a more detailed plot....



Measurement of the running of α_{em}

A direct measurement of $\alpha_{em}(s/t)$ in space/ time-like regions can show the running of $\alpha_{em}(s/t)$

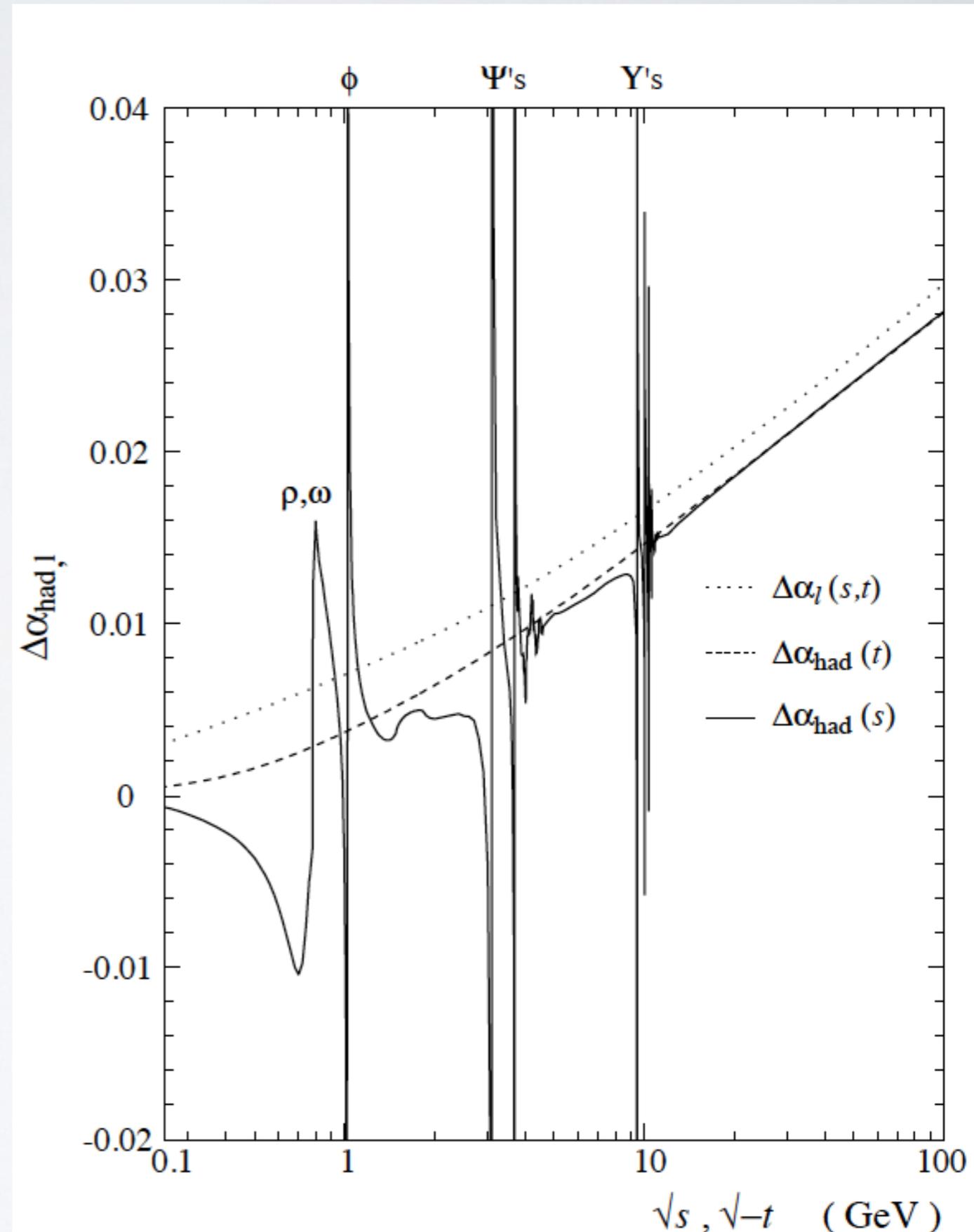
It can provide a test of “duality” (fare way from resonances)

It has been done in past by few experiments at e^+e^- colliders by comparing a “well-known” QED process with some reference (obtained from data or MC)

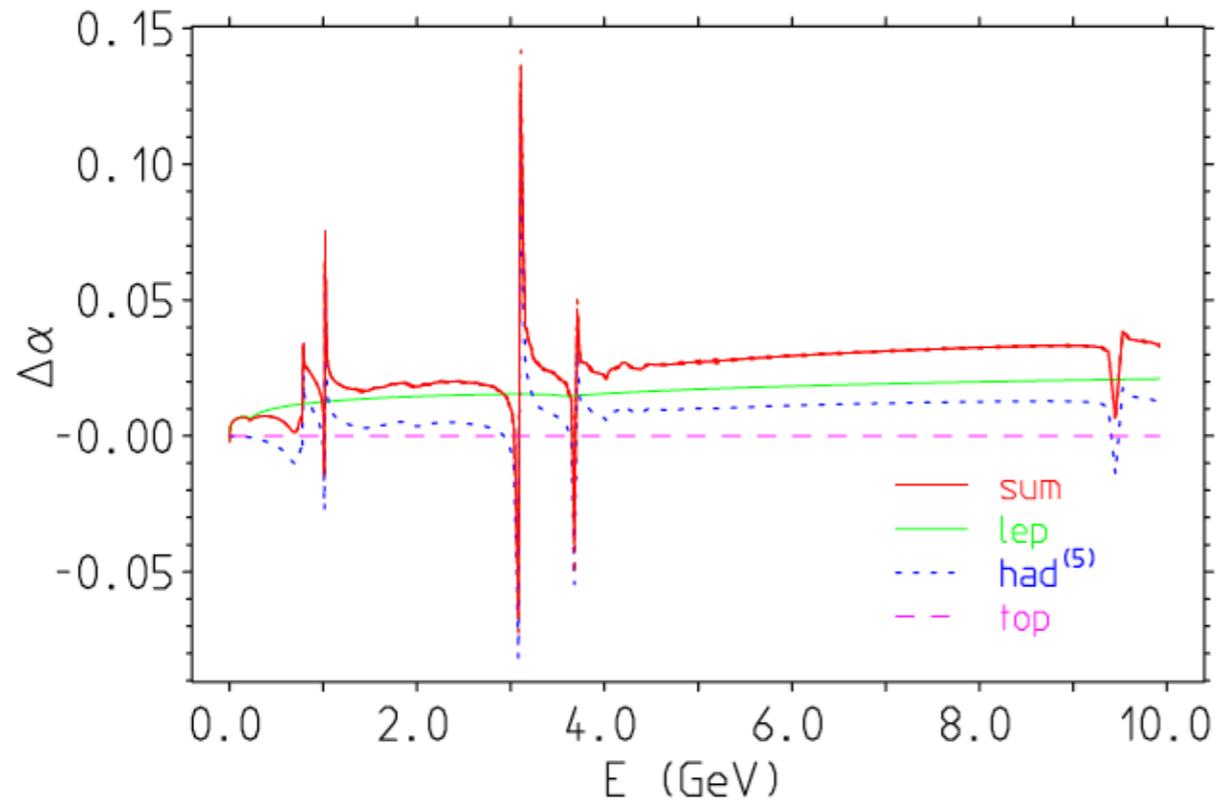
$$\left(\frac{\alpha(q^2)}{\alpha(q_0^2)} \right)^2 \sim \frac{N_{signal}(q^2)}{N_{norm}(q_0^2)}$$

N_{signal} can be any QED process, muon pairs, etc...

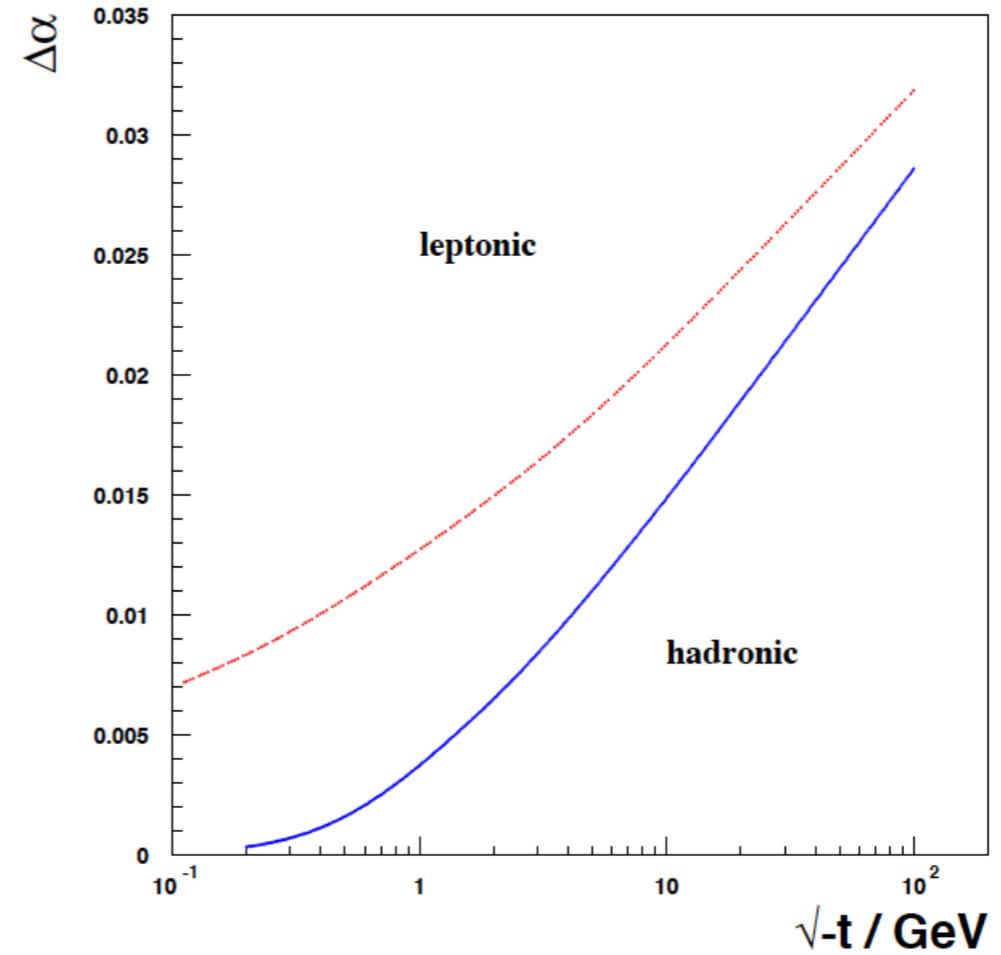
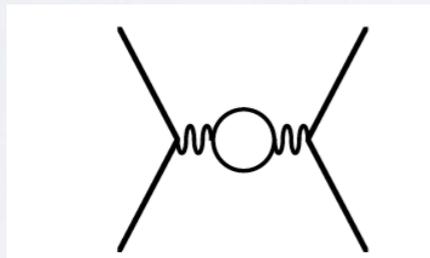
N_{norm} can be Bhabha process, pure QED as $\gamma\gamma$ pair production, a well as theory, or any other reference process.



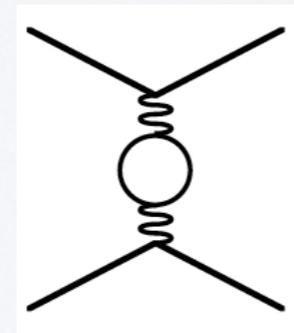
Running of α_{em}



time-like



space-like



$$\Delta\alpha_{had}^{(5)}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi} \text{Re} \int_{4m_\pi^2}^{\infty} ds \frac{R(s)}{s(s - M_Z^2 - i\epsilon)}$$



2. Theoretical framework

The leading-order hadronic contribution to a_μ is given by the well-known formula [4,15]

$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi^2} \int_0^\infty \frac{ds}{s} K(s) \text{Im}\Pi_{\text{had}}(s + i\epsilon),$$

where $\Pi_{\text{had}}(s)$ is the hadronic part of the photon vacuum polarization, $\epsilon > 0$,

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m_\mu^2)} \quad (2)$$

is a positive kernel function, and m_μ is the muon mass. As the total cross section for hadron production in low-energy e^+e^- annihilations is related to the imaginary part of $\Pi_{\text{had}}(s)$ via the optical theorem, the dispersion integral in Eq. (1) is computed integrating experimental time-like ($s > 0$) data up to a certain value of s [2,18,19]. The high-energy tail of the integral is calculated using perturbative QCD [20].

Alternatively, if we exchange the x and s integrations in Eq. (1) we obtain [21]

$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx (x-1) \bar{\Pi}_{\text{had}}[t(x)], \quad (3)$$

where $\bar{\Pi}_{\text{had}}(t) = \Pi_{\text{had}}(t) - \Pi_{\text{had}}(0)$ and

A new approach to evaluate the leading hadronic corrections to the muon $g-2$

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$$t(x) = \frac{x^2 m_\mu^2}{x - 1} < 0 \quad (4)$$

is a space-like squared four-momentum. If we invert Eq. (4), we get $x = (1 - \beta) (t/2m_\mu^2)$, with $\beta = (1 - 4m_\mu^2/t)^{1/2}$, and from Eq. (3) we obtain

$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi} \int_{-\infty}^0 \bar{\Pi}_{\text{had}}(t) \left(\frac{\beta - 1}{\beta + 1} \right)^2 \frac{dt}{t\beta}. \quad (5)$$


Eq. (5) has been used for lattice QCD calculations of a_μ^{HLO} [22]; while the results are not yet competitive with those obtained with the dispersive approach via time-like data, their errors are expected to decrease significantly in the next few years [23].

The effective fine-structure constant at squared momentum transfer q^2 can be defined by

$$\alpha(q^2) = \frac{\alpha}{1 - \Delta\alpha(q^2)}, \quad (6)$$

To summarize

$$a_{\mu}^{HLO} = -\frac{\alpha}{\pi} \int_0^1 (1-x) \Pi_{had} \left(-\frac{x^2}{1-x} m_{\mu}^2 \right) dx$$

$$t = \frac{x^2 m_{\mu}^2}{x-1} \quad 0 \leq -t < +\infty$$

$$a_{\mu} = (g-2)/2$$

$$x = \frac{t}{2m_{\mu}^2} \left(1 - \sqrt{1 - \frac{4m_{\mu}^2}{t}} \right); \quad 0 \leq x < 1;$$

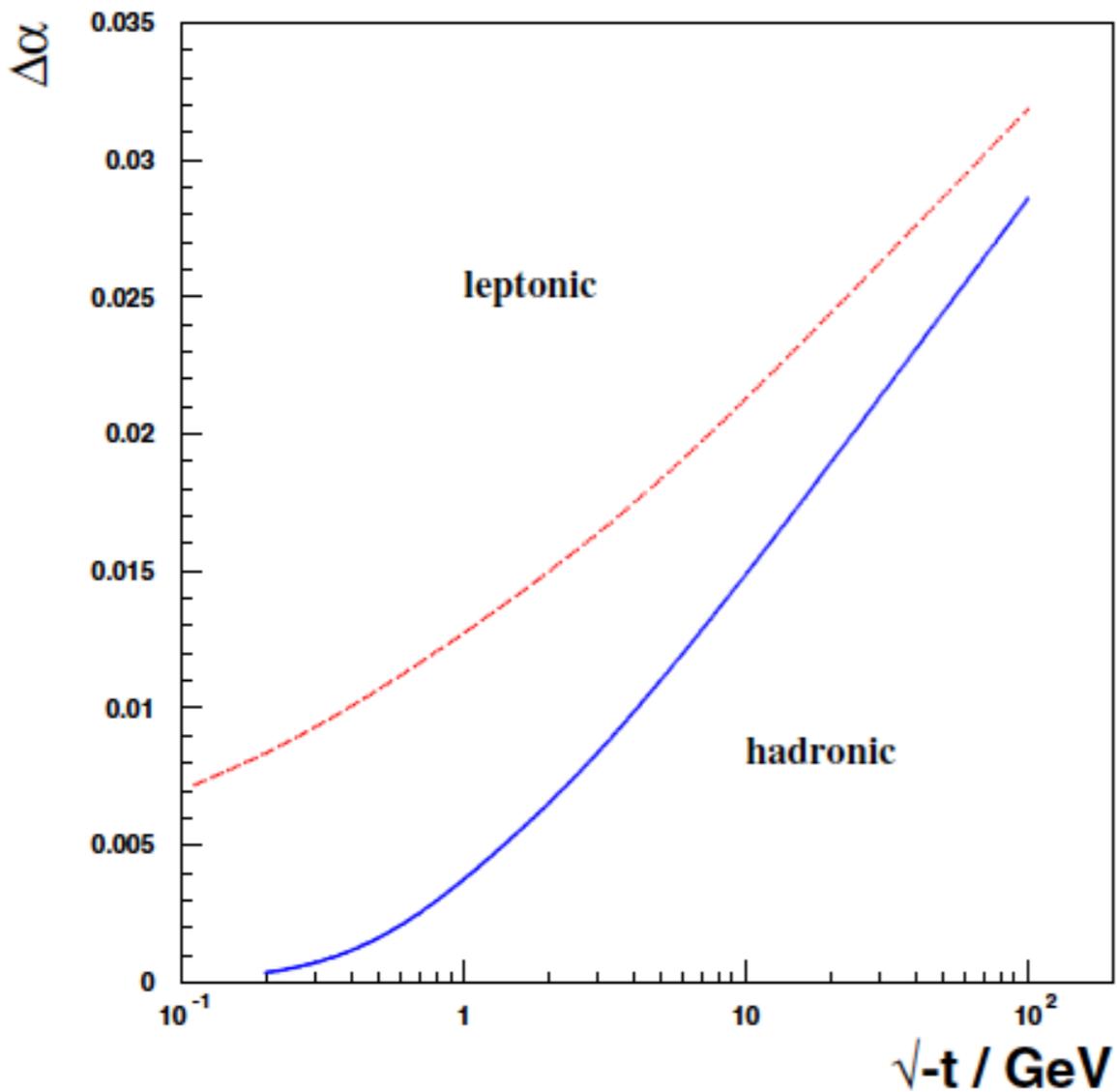
$$t = -s \sin^2 \left(\frac{\vartheta}{2} \right)$$

$$\Delta \alpha_{had}(t) = -\Pi_{had}(t) \quad \text{for } t < 0$$

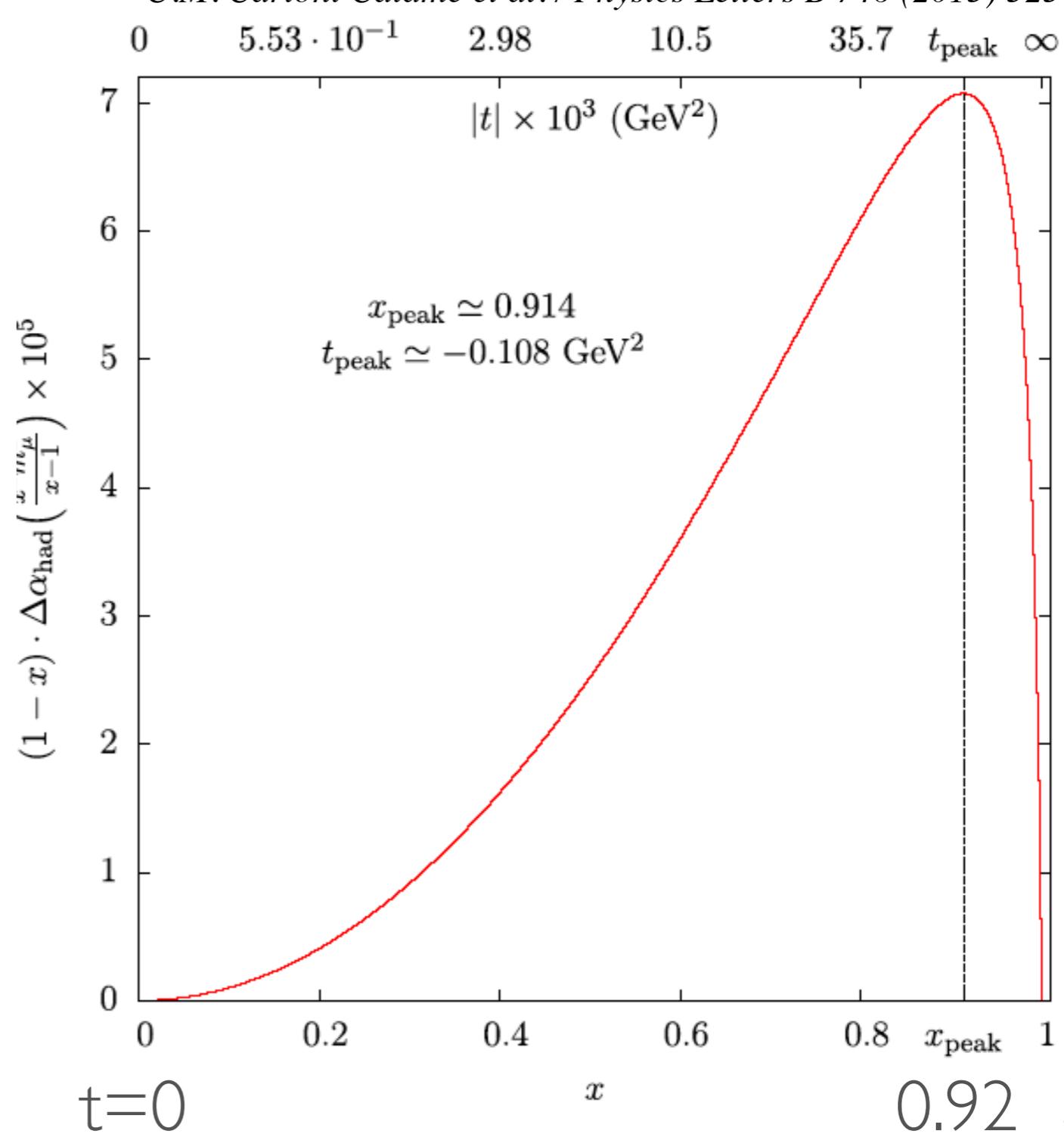
with the “t”
kernel

$$a_{\mu}^{HLO} = -\frac{\alpha}{\pi} \int_0^1 (1-x) \Delta \alpha_{had} \left(-\frac{x^2}{1-x} m_{\mu}^2 \right) dx$$

functional form of the kernel



$\Delta\alpha$ is dominated at low t by the leptonic contribution



$t=0$

x

0.92 8

large t -values are depressed by $x-1$ denominator

The integrand is peaked at $\sim x=0.92$
 $t=-0.11 \text{ GeV}^2$ ($\sim 330 \text{ MeV}$) for which
 $\Delta\alpha_{\text{had}}(0.92) \sim 10^{-3}$

The running of the electromagnetic coupling α in small-angle Bhabha scattering

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Abstract

A method to determine the running of α from a measurement of small-angle Bhabha scattering is proposed and worked out. The method is suited to high statistics experiments at e^+e^- colliders, which are equipped with luminometers in the appropriate angular region. A new simulation code predicting small-angle Bhabha scattering is also presented.

The method to measure the running of α exploits the fact that the cross section for the process $e^+e^- \rightarrow e^+e^-$ can be conveniently decomposed into three factors :

$$\frac{d\sigma}{dt} = \frac{d\sigma^0}{dt} \left(\frac{\alpha(t)}{\alpha(0)} \right)^2 (1 + \Delta r(t)) \quad (3)$$

each one of them known with an accuracy of at least 0.1%

1st factor

$$\frac{d\sigma^0}{dt} = \frac{d\sigma^B}{dt} \left(\frac{\alpha(0)}{\alpha(t)} \right)^2.$$

Born cross section
contains all the soft and
virtual
corrections

Bhabha is a pure QED
processes
quarks enter only
in loops

$$\frac{d\sigma^B}{dt} = \frac{\pi\alpha_0^2}{2s^2} \text{Re}\{B_t + B_s + B_i\},$$

$$B_t = \left(\frac{s}{t} \right)^2 \left\{ \frac{5 + 2c + c^2}{(1 - \Pi(t))^2} + \xi \frac{2(g_v^2 + g_a^2)(5 + 2c + c^2)}{(1 - \Pi(t))} \right. \\ \left. + \xi^2 \left(4(g_v^2 + g_a^2)^2 + (1 + c)^2(g_v^4 + g_a^4 + 6g_v^2g_a^2) \right) \right\}$$

$$B_s = \frac{2(1 + c^2)}{|1 - \Pi(s)|^2} + 2\chi \frac{(1 - c)^2(g_v^2 - g_a^2) + (1 + c)^2(g_v^2 + g_a^2)}{1 - \Pi(s)} \\ + \chi^2 [(1 - c)^2(g_v^2 - g_a^2)^2 + (1 + c)^2(g_v^4 + g_a^4 + 6g_v^2g_a^2)]$$

$$B_i = 2\frac{s}{t}(1 + c)^2 \left\{ \frac{1}{(1 - \Pi(t))(1 - \Pi(s))} \right. \\ \left. + (g_v^2 + g_a^2) \left(\frac{\xi}{1 - \Pi(s)} + \frac{\chi}{1 - \Pi(t)} \right) \right. \\ \left. + (g_v^4 + 6g_v^2g_a^2 + g_a^4)\xi\chi \right\}$$

2nd factor

$$\left(\frac{\alpha(t)}{\alpha(0)}\right)^2$$

Vacuum polarization effects
contains the running of alpha

3rd factor

$$(1 + \Delta r(t))$$

contains all the real and virtual effects not incorporated in the running
of alpha

$$\alpha(q^2) = \frac{\alpha(0)}{1 - \Delta\alpha(q^2)},$$

$\alpha(0)$ is the Sommerfeld
fine structure constant
measured with a precision of
 $O(10^{-9})$

$\Delta\alpha(q^2)$ from loop contributions to the photon propagator

EUROPEAN ORGANISATION FOR NUCLEAR RESEARCH

CERN-PH-EP/2005-014

21 February 2005

Revised 28 June 2005

**Measurement of the running of the
QED coupling in small-angle Bhabha
scattering at LEP**

OPAL Collaboration

arXiv:hep-ex/0505072v3 23 Feb 2006

Abstract

Using the OPAL detector at LEP, the running of the effective QED coupling $\alpha(t)$ is measured for space-like momentum transfer from the angular distribution of small-angle Bhabha scattering. In an almost ideal QED framework, with very favourable experimental conditions, we obtain:

$$\Delta\alpha(-6.07 \text{ GeV}^2) - \Delta\alpha(-1.81 \text{ GeV}^2) = (440 \pm 58 \pm 43 \pm 30) \times 10^{-5},$$

where the first error is statistical, the second is the experimental systematic and the third is the theoretical uncertainty. This agrees with current evaluations of $\alpha(t)$. The null hypothesis that α remains constant within the above interval of $-t$ is excluded with a significance above 5σ . Similarly, our results are inconsistent at the level of 3σ with the hypothesis that only leptonic loops contribute to the running. This is currently the most significant direct measurement where the running $\alpha(t)$ is probed differentially within the measured t range.

All this has been made possible by a very accurate determination of the Luminosity by the OPAL collaboration

A measurement of the Luminosity at 10^{-4} at LEP

Giovanni Abbiendi

INFN - Bologna

Eur. Phys. J. C 45, 1–21 (2006)
Digital Object Identifier (DOI) 10.1140/epjc/s2005-02389-3

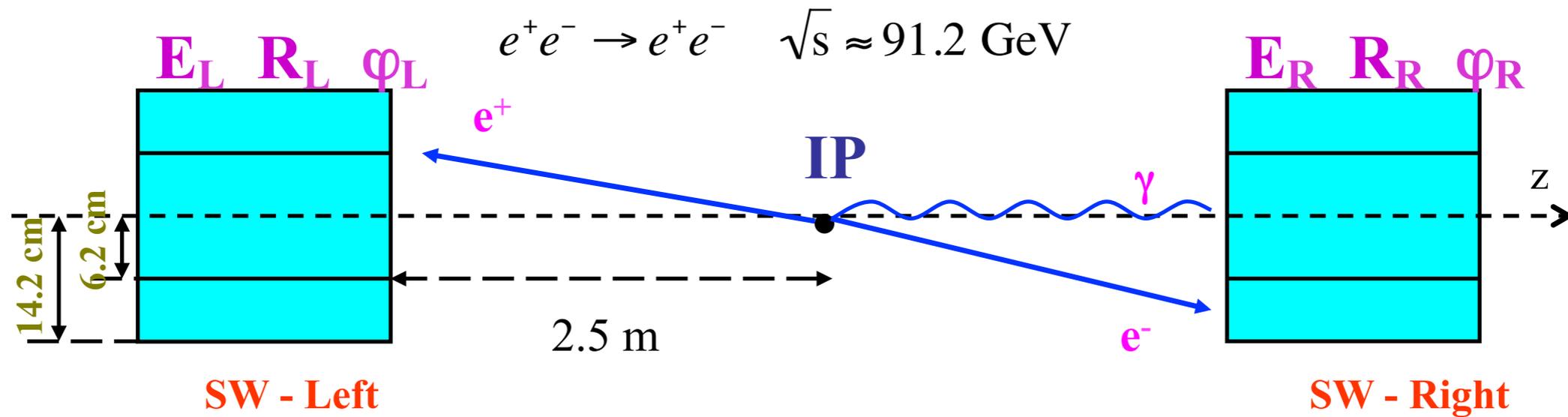
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Measurement of the running of the QED coupling in small-angle Bhabha scattering at LEP

The OPAL Collaboration

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Small-angle Bhabha scattering in OPAL

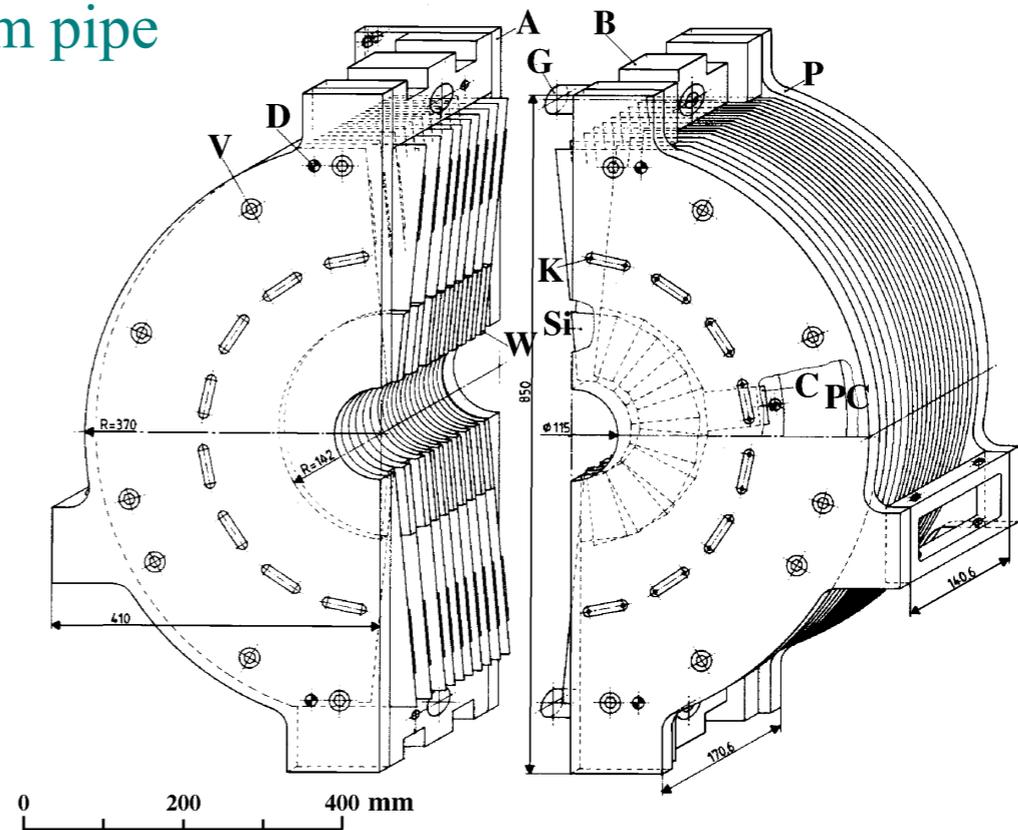


2 cylindrical calorimeters encircling the beam pipe at ± 2.5 m from the Interaction Point

19 Silicon layers Total Depth $22 X_0$
18 Tungsten layers (14 cm)

Each detector layer divided into 16 overlapping wedges

Sensitive radius: 6.2 – 14.2 cm,
 corresponding to **scattering angle**
 of **25 – 58 mrad** from the beam line



Final Error on Luminosity

After all the effort on Radial reconstruction the dominant systematic error is related to Energy (mostly tail in the E response and nonlinearity)

Quantitatively:

(OPAL Collaboration, Eur.Phys.J. C14 (2000) 373)

	Systematic Error ($\times 10^{-4}$)
Energy	1.8
Inner Anchor	1.4
Radial Metrology	1.4

Total Experimental Systematic Error : 3.4×10^{-4}

Theoretical Error on Bhabha cross section: 5.4×10^{-4}

The Method used follows the above parametrization/factorization of the Bhabha cross-section

$$\frac{d\sigma}{dt} = \frac{d\sigma^{(0)}}{dt} \left(\frac{\alpha(t)}{\alpha_0} \right)^2 (1 + \epsilon) (1 + \delta_\gamma) + \delta_Z$$
$$\frac{d\sigma^{(0)}}{dt} = \frac{4\pi\alpha_0^2}{t^2}$$

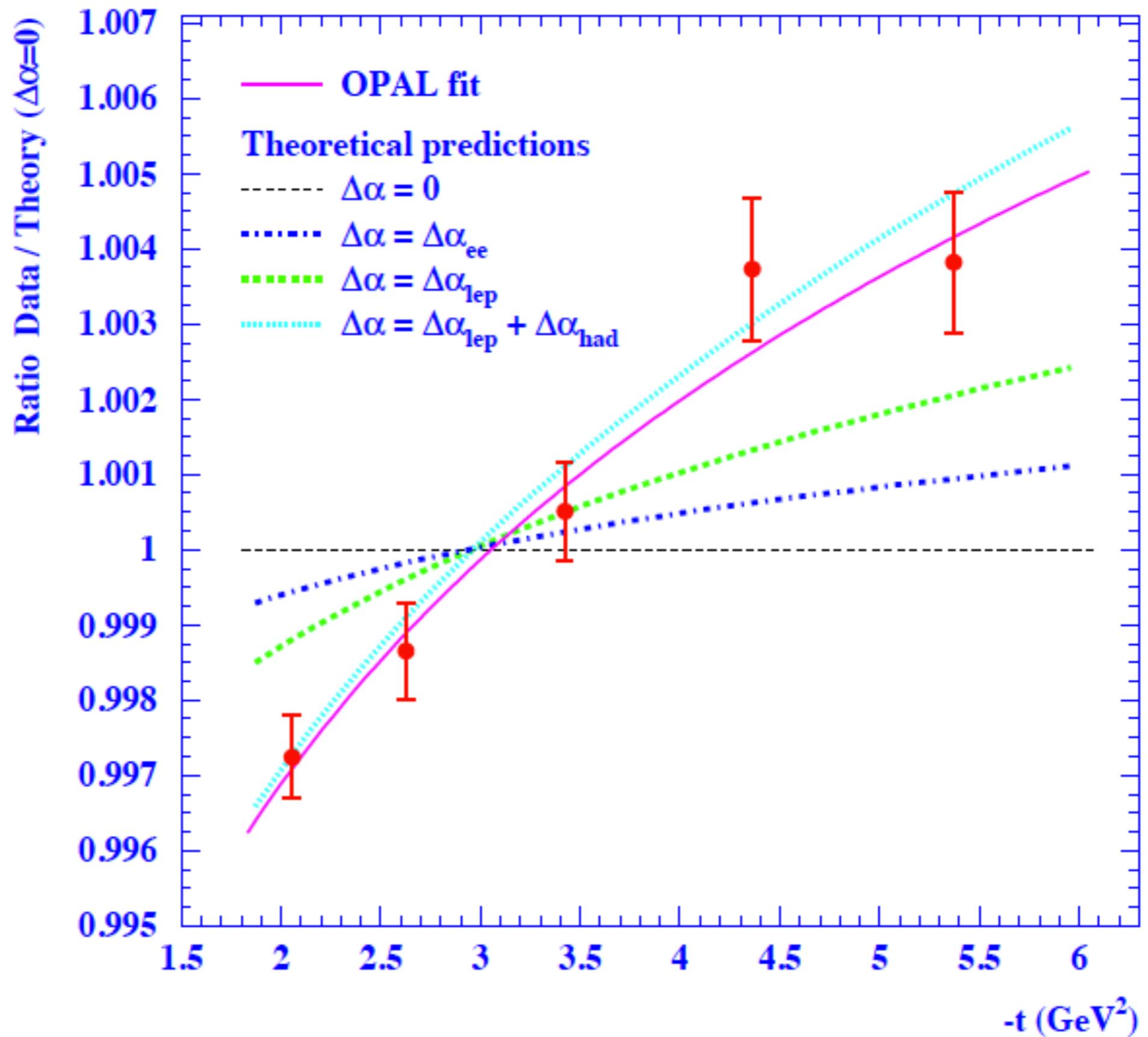
We determined the effective slope of the Bhabha momentum transfer distribution which is simply related to the average derivative of $\Delta\alpha$ as a function of $\ln t$ in the range $2 \text{ GeV}^2 \leq -t \leq 6 \text{ GeV}^2$. The observed t -spectrum is in good agreement with Standard Model predictions. We find:

$$\Delta\alpha(-6.07 \text{ GeV}^2) - \Delta\alpha(-1.81 \text{ GeV}^2) = (440 \pm 58 \pm 43 \pm 30) \times 10^{-5},$$

where the first error is statistical, the second is the experimental systematic and the third is the theoretical uncertainty.

This measurement is one of only a very few experimental tests of the running of $\alpha(t)$ in the space-like region, where $\Delta\alpha$ has a smooth behaviour. We obtain the strongest direct evidence for the running of the QED coupling ever achieved differentially in a single experiment, with a significance above 5σ . Moreover we report clear experimental evidence for the hadronic contribution to the running in the space-like region, with a significance of 3σ .

OPAL



Our proposal therefore has to deal with some
experimental issues....

Experimental Issues

Using Bhabha at small angle (to emphasize t-channel contribution) to extract $\Delta\alpha$:

$$\left(\frac{\alpha(t)}{\alpha(0)}\right)^2 \sim \frac{d\sigma_{ee\rightarrow ee}(t)}{d\sigma_{MC}^0(t)}$$

Where $d\sigma_{MC}^0$ is the MC prediction for Bhabha process with $\alpha(t) = \alpha(0)$ and there are corrections due to RC...

$$\Delta\alpha_{had}(t) = 1 - \left(\frac{\alpha(t)}{\alpha(0)}\right)^{-1} - \Delta\alpha_{lept}(t)$$

since $\Delta\alpha_{lep}(t)$ is theoretically well known

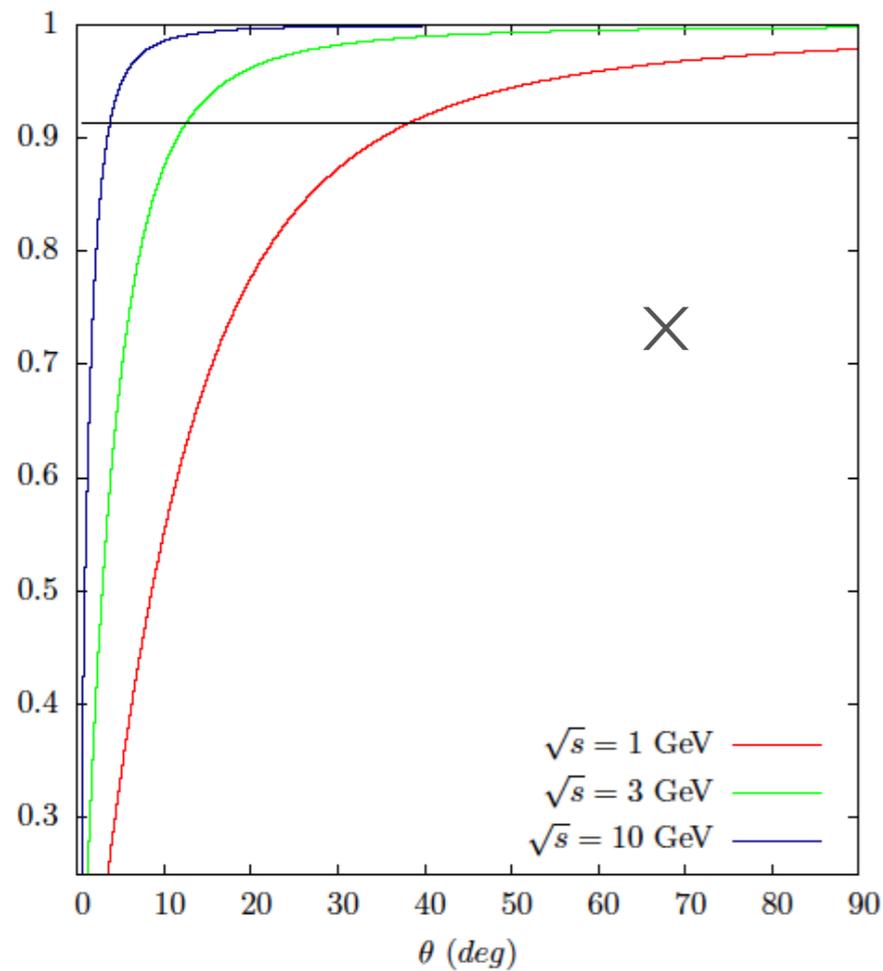
Which experimental accuracy we are aiming at?

$\delta\Delta\alpha_{had} \simeq 1/2$ fractional accuracy on $d\sigma(t)/d\sigma_{MC}^0(t)$.

If we assume to measure $\delta\Delta\alpha_{had}$ at 5% at the peak of the integrand ($\Delta\alpha_{had} \sim 10^{-3}$ at $x=0.92$) fractional accuracy on

$$d\sigma(t)/d\sigma_{MC}^0(t) \sim 10^{-4}$$

Very challenging measurement (one order of magnitude improvement respect to date) due to the systematics



Most of the region (up to $x \sim 0.98$) can be covered with low energy machines as Dafne/VEPP-2000 or tau/charm-B-factories

Covering up to 60° at $\sqrt{s} = 1$ GeV can arrive at $x = 0.95$

A different situation can be obtained at tau/charm/ B-factories (and at future ILC/TLEP machines) where smaller below 20° are needed

Statistics

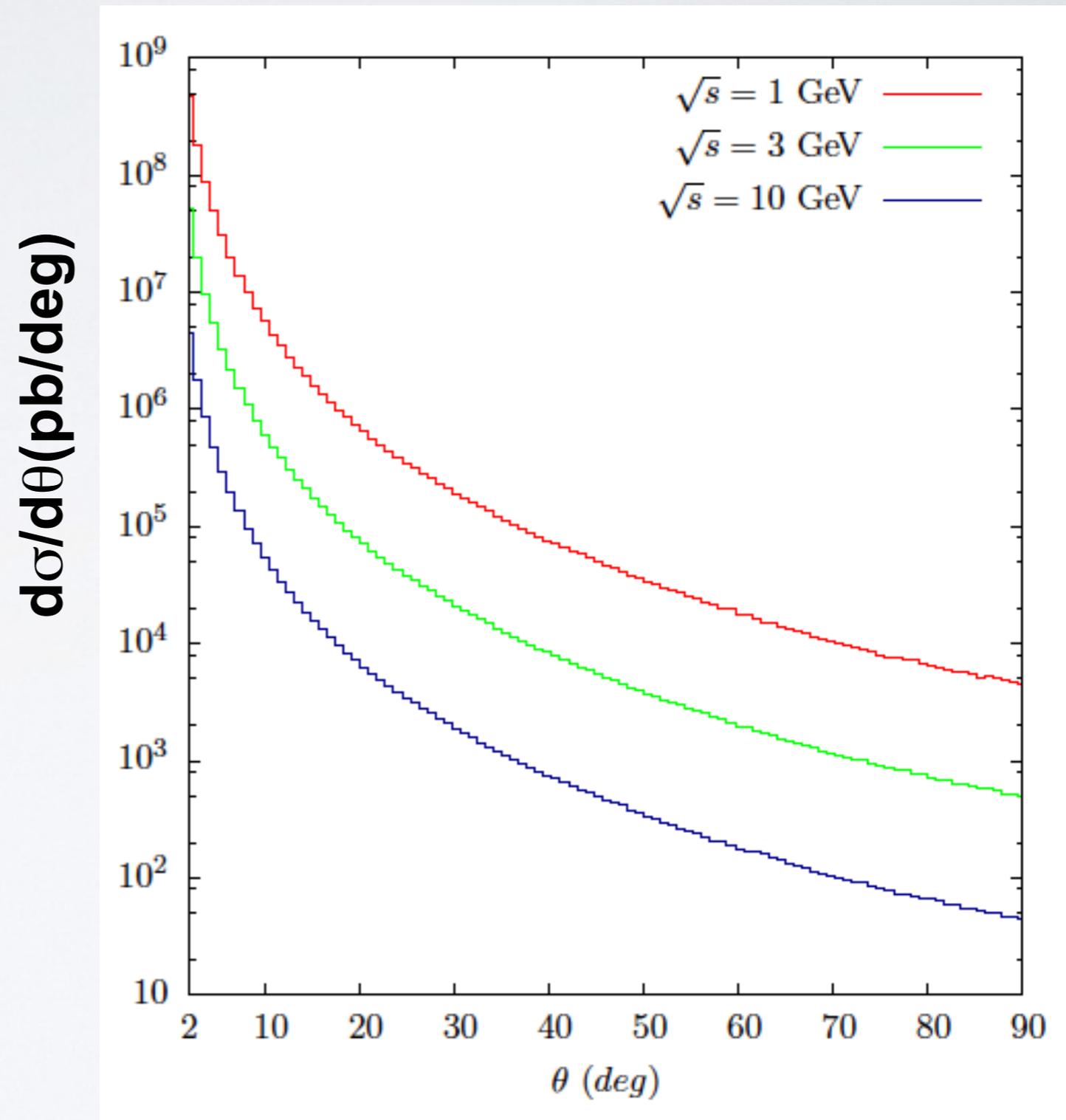
$O(10^{-4})$ accuracies on Bhabha cross section require at least 10^8 events which at 20° mean:

$O(1) \text{ fb}^{-1} @ 1 \text{ GeV}$

$O(10) \text{ fb}^{-1} @ 3 \text{ GeV}$

$O(100) \text{ fb}^{-1} @ 10 \text{ GeV}$

All these luminosities are realistic estimates at flavour factories



Additional considerations: Rad. Corr.

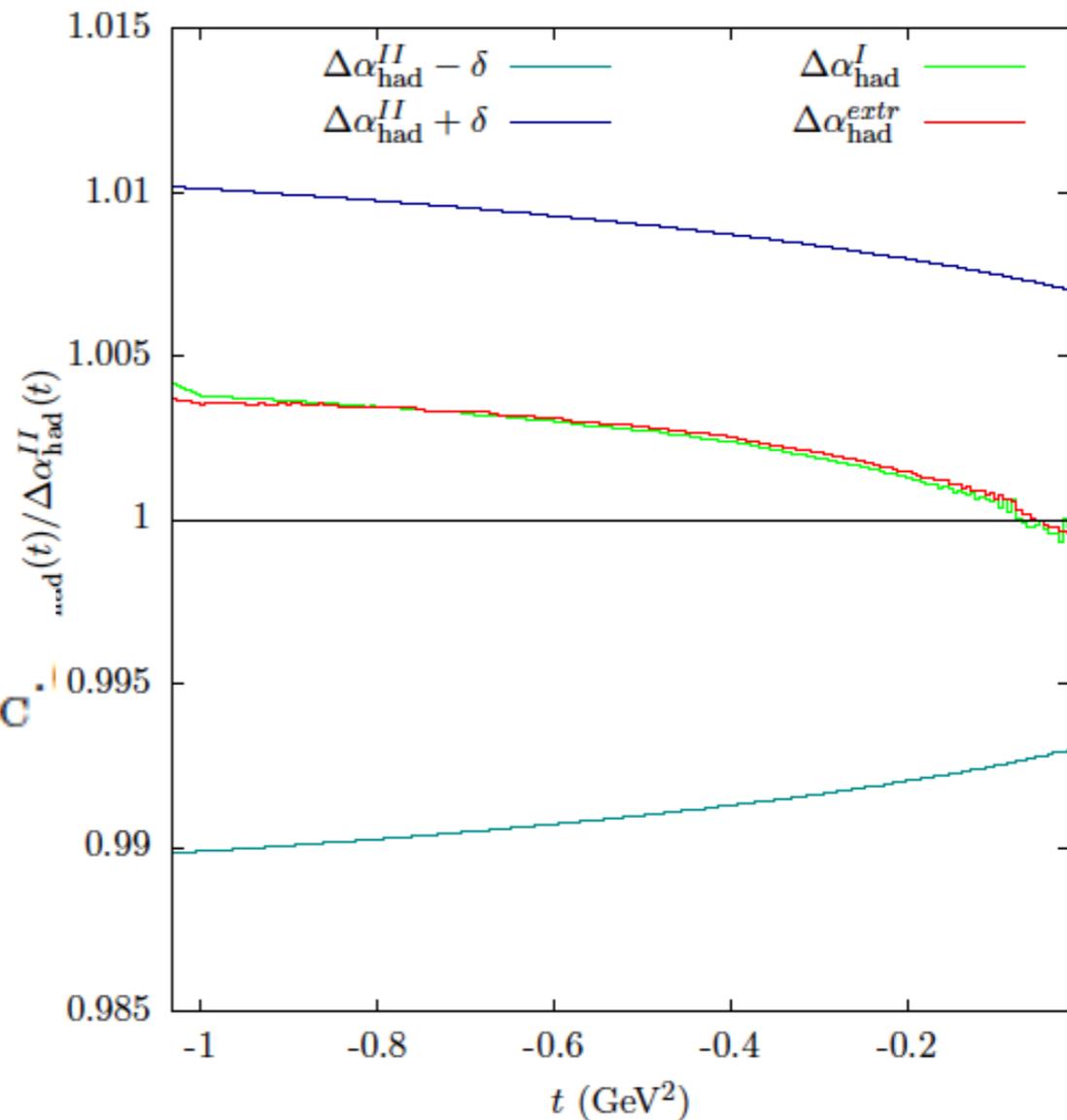
A Monte Carlo procedure has been developed to check if $\Delta\alpha_{\text{had}}(t)$ can be obtained by a minimization procedure with a different $\Delta\alpha_{\text{had}}(t)'$ inside

$$\left. \frac{d\sigma}{dt} \right|_{\text{data}} = \left. \frac{d\sigma}{dt} \left(\alpha(t), \alpha(s) \right) \right|_{\text{MC}},$$

→

$$\left. \frac{d\sigma}{dt} \right|_{j,\text{data}} = \left. \frac{d\sigma}{dt} \left(\bar{\alpha}(t) + \frac{i_j}{N} \delta(t), \alpha(s) \right) \right|_{j,\text{MC}},$$

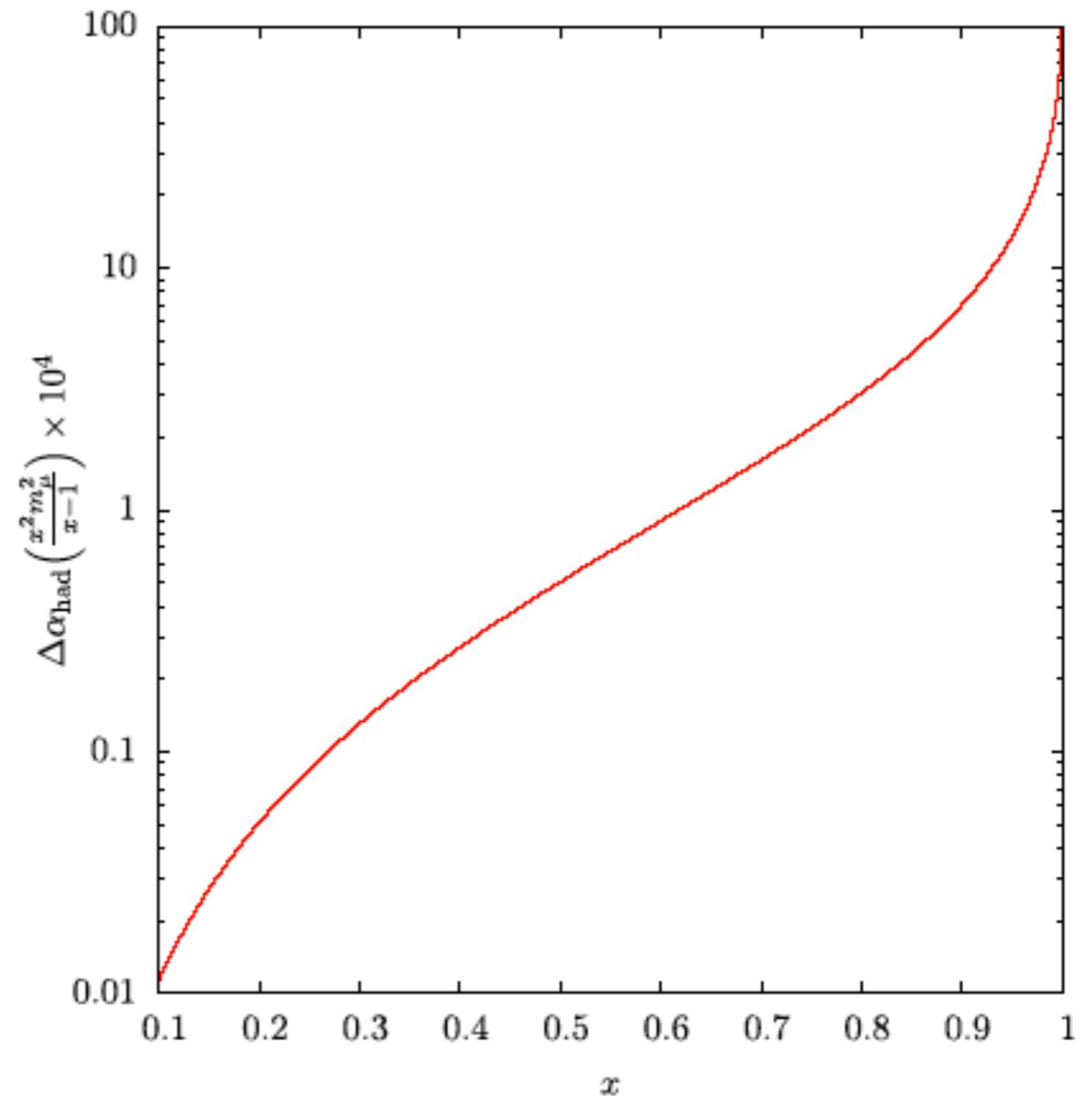
$\Delta\alpha_{\text{had}}(t)$ is obtained
with $< 10^{-4}$ error !



Normalization

Bhabha as a normalization: i.e. the data sample at very small angles $\Delta\alpha_{had} \simeq 10^{-5}$

Use a hadron-free process as $e^+e^- \rightarrow \gamma\gamma$



Measurement of DAFNE Luminosity with KLOE/KLOE-2 at 10^{-4} ?

F. Ambrosino et al [KLOE] Eur. Phys. J. C 47, 589–596 (2006)

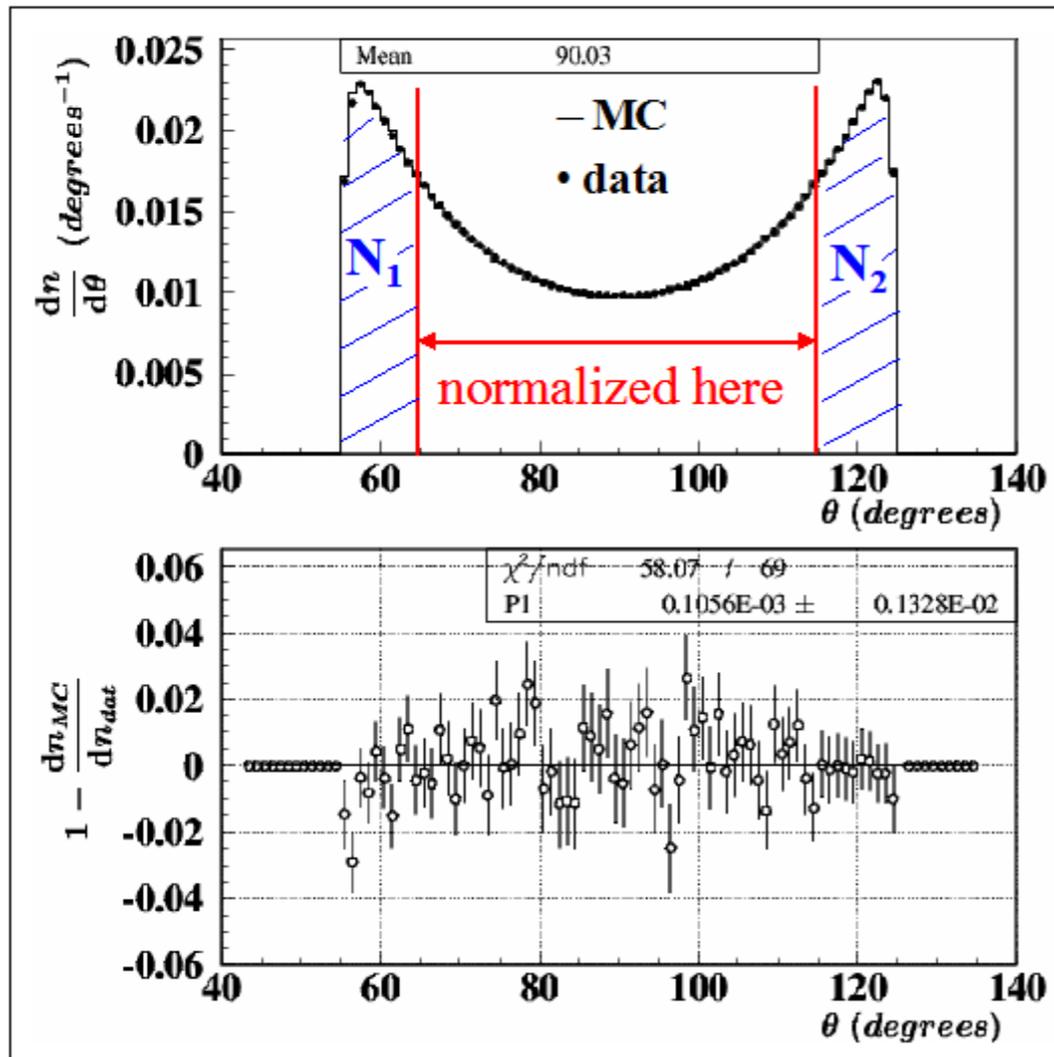
Table 2. Summary of the corrections and systematic errors in the measurement of the luminosity

	correction (%)	systematic error (%)
angular acceptance	+0.25	0.25
tracking	–	0.06
clustering	+0.14	0.11
background	–0.62	0.13
cosmic veto	+0.40	–
energy calibration	–	0.10
center of mass energy	+0.10	0.10
	+0.34	0.32

Adding in quadrature: 0.3 %

(can be improved by a factor 10?)

From F. Nguyen 2006 Polar angle systematics



✓ global agreement is very good

but the cut occurs in a steep region of the distributions
 ⇒ estimate of border mismatches

✓ after normalizing MC to make it coincide with data in the region $65^\circ < \theta < 115^\circ$, we estimate as a systematic error:

$$\frac{N_{[55:65]+[115:125]}^{dat} - N_{[55:65]+[115:125]}^{MC}}{N_{TOT}^{dat}} \sim 0.25\%$$

Can be improved at 10^{-4} ?

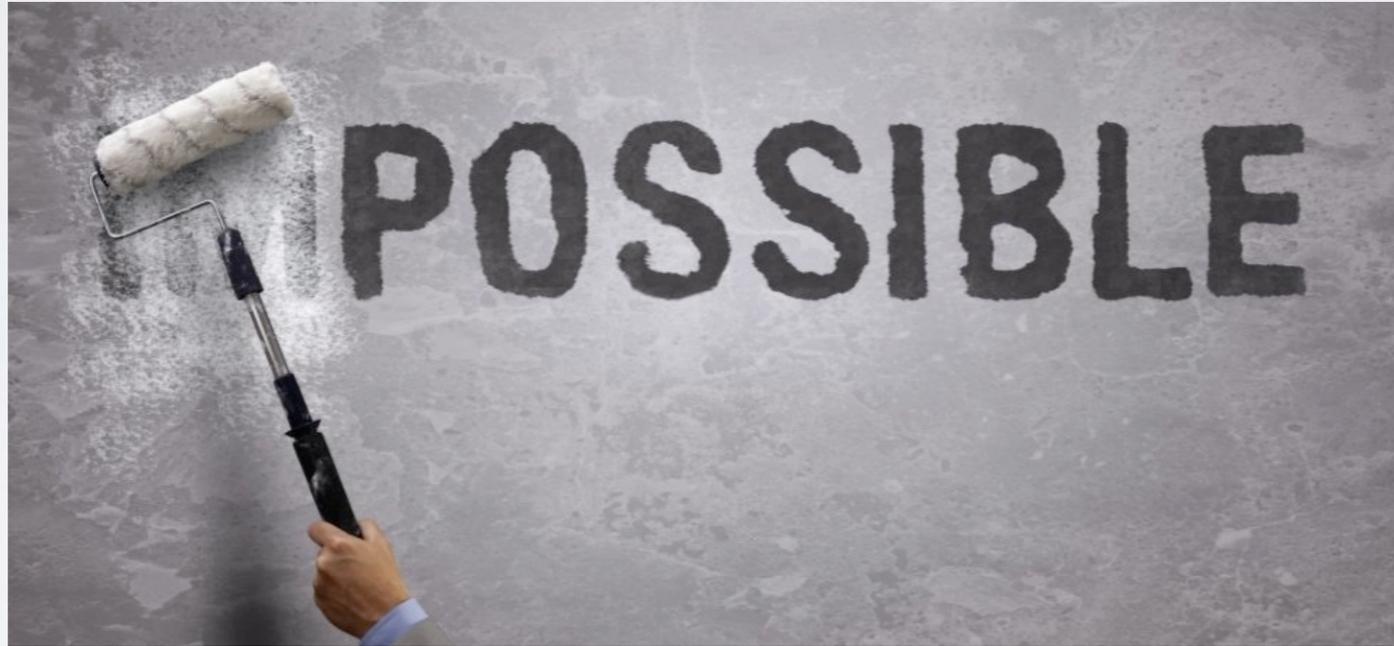
Measuring α_{em} running in space like region appears to be feasible and potentially comparable to the present time like way.

We propose an alternative formula for a_{μ}^{HLO} in spacelike region. at low values of t ($< 1 \text{ GeV}^2$) and can be explored at low energy e^+e^- machines (VEPP2000/DAFNE, τ /charm, B-factories) as well as at high energies ILC/TLEP.

This may be achieved by measuring the Bhabha cross section in a new, poorly explored, region of relatively small angles at accuracy at least of $O(10^{-4})$

Both experimental and theoretical improvements, tests, calculations,have to come out.....

and hopefully.....



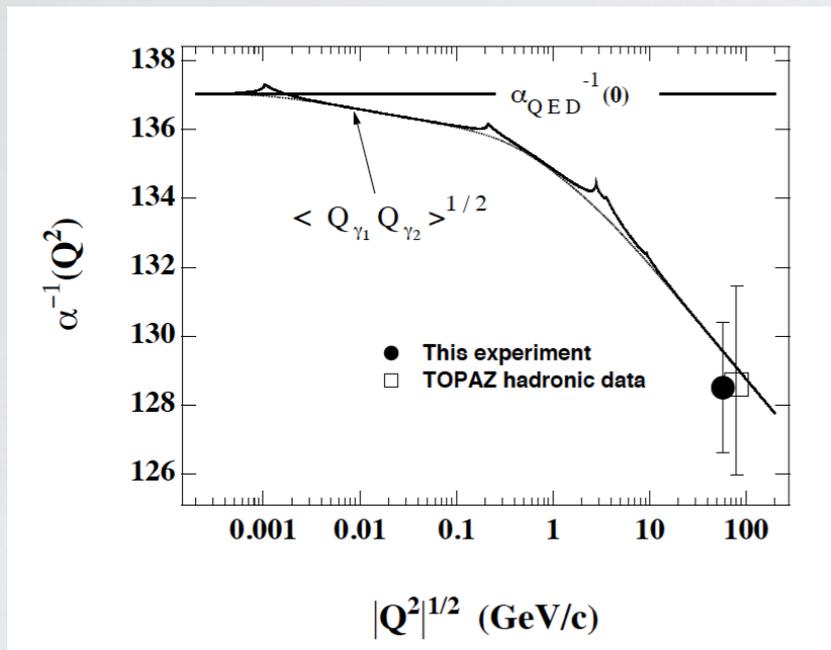
FINIS

 **MISSION:** ?

POSSIBLE

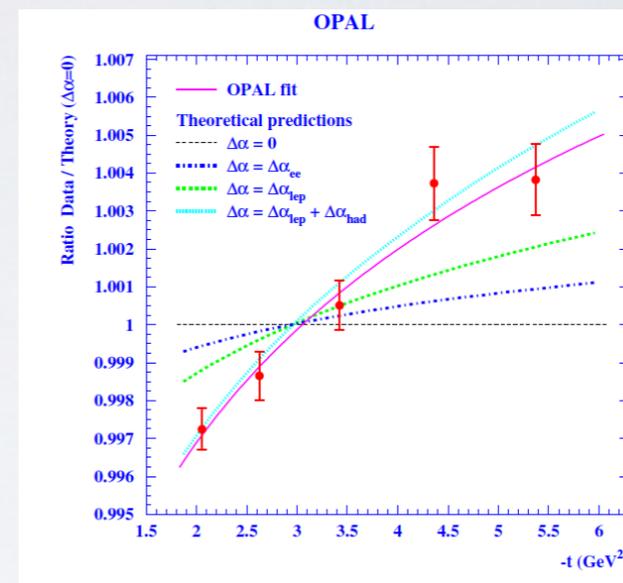
time-like

$\sqrt{s} = 57.8 \text{ GeV}$



$$\frac{e^+e^- \rightarrow \mu^+\mu^-}{e^+e^- \rightarrow e^+e^-\mu^+\mu^-}$$

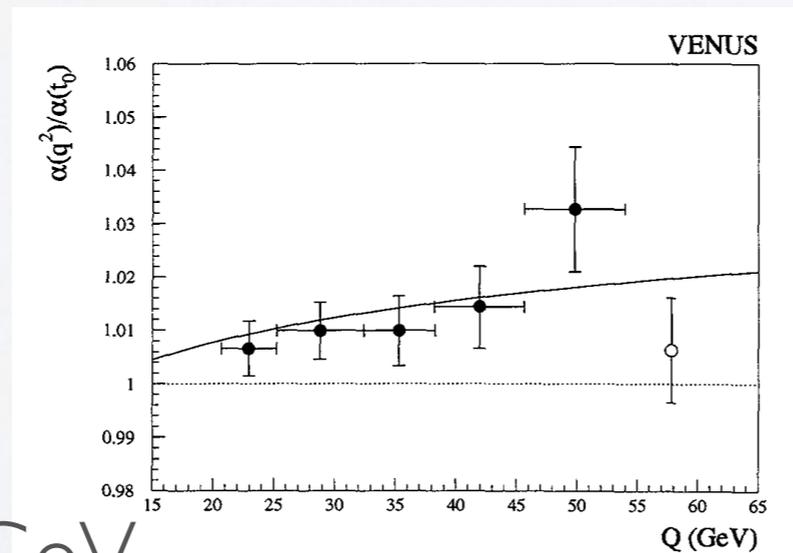
$\sqrt{s} = 189 \text{ GeV}$



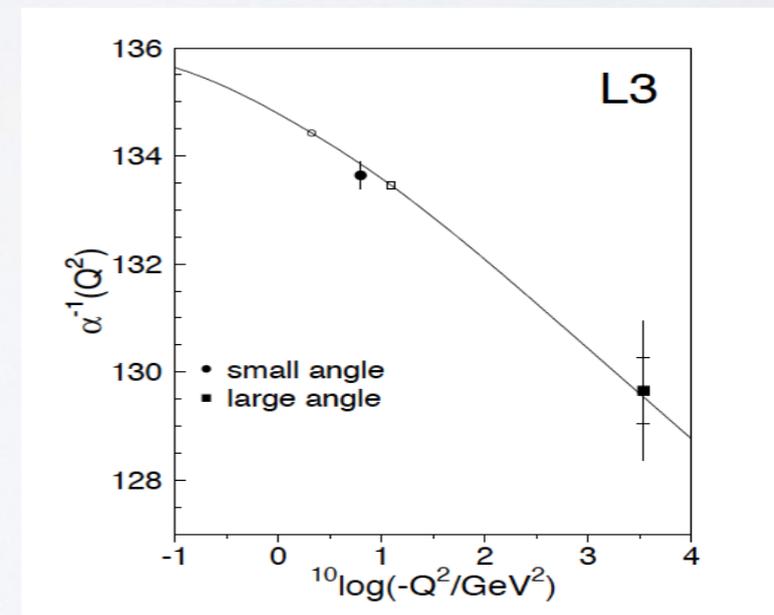
$1.3 < \sqrt{-t} < 2.5 \text{ GeV}$

space-like

$$\frac{e^+e^- \rightarrow e^+e^-}{e^+e^- \rightarrow \mu^+\mu^-}$$

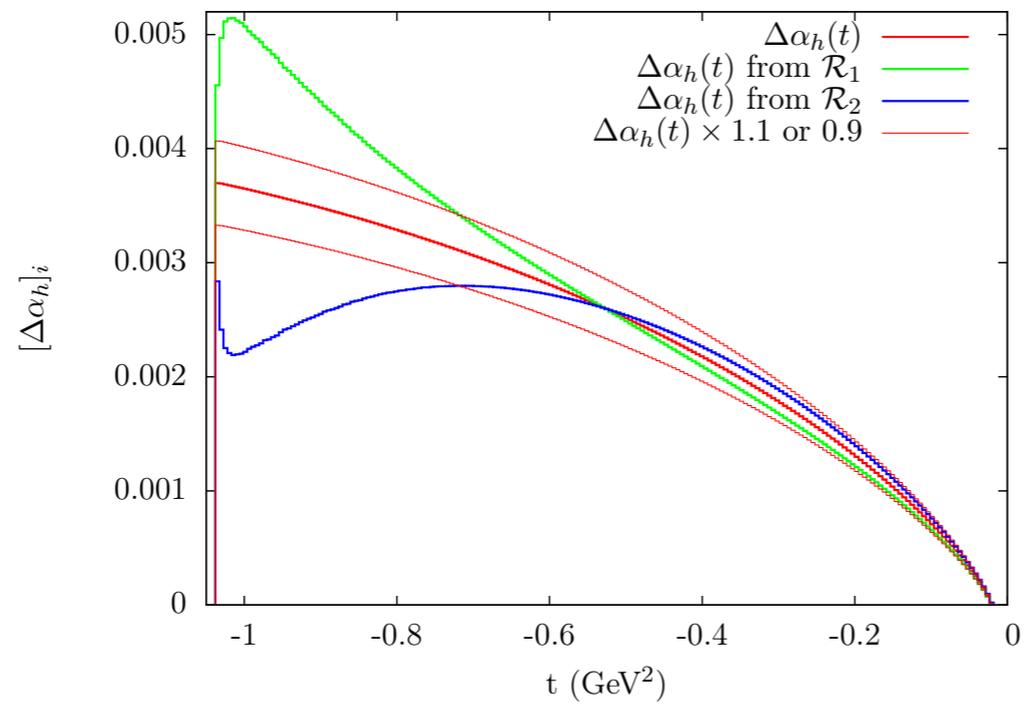


$10 < \sqrt{-t} < 54 \text{ GeV}$

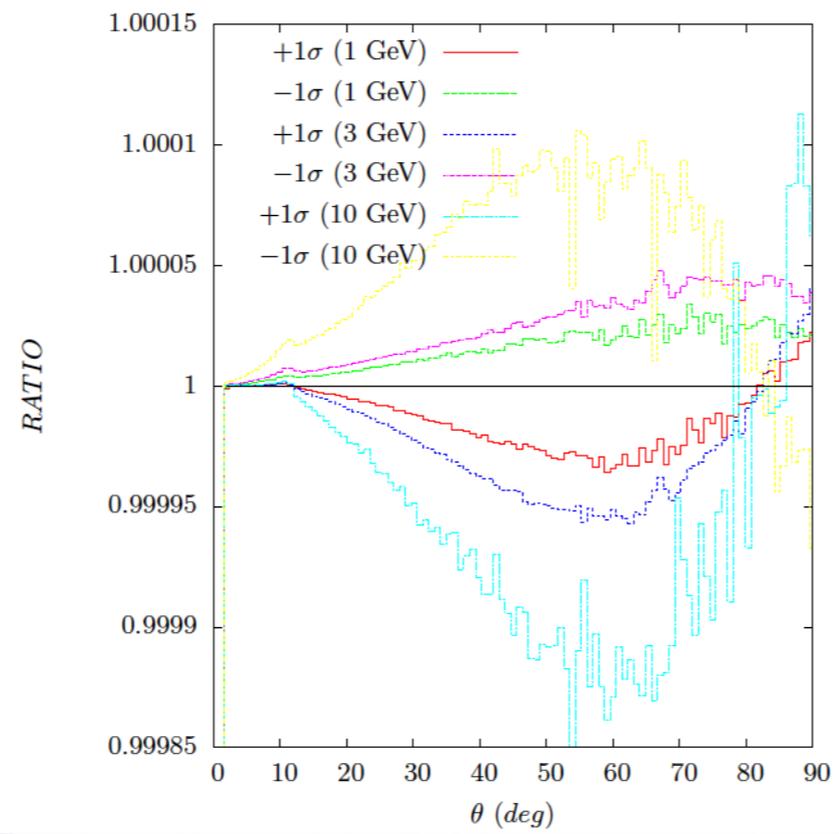


$1.5 < \sqrt{-t} < 2.5 \text{ GeV}$
 $3.5 < \sqrt{-t} < 58 \text{ GeV}$

test



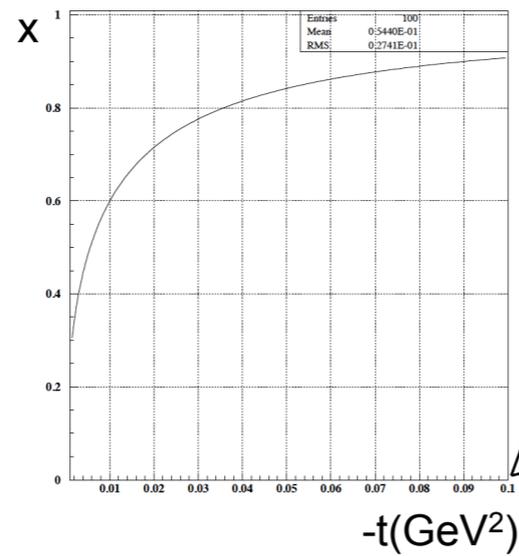
$\Delta\alpha_{em}^{\text{HAD}}(s)$ dependence



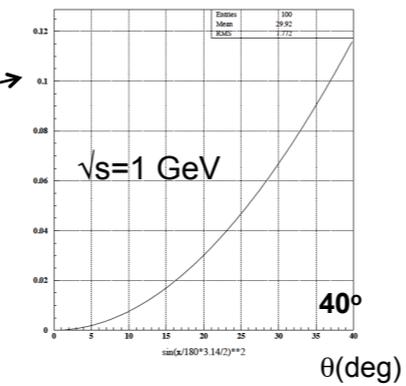
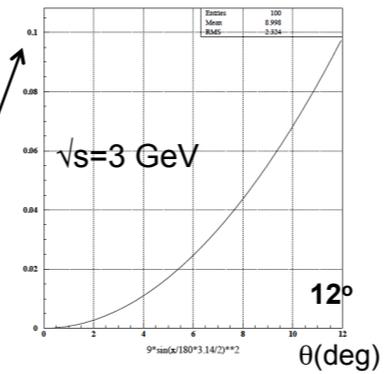
Which is the best energy/angle configuration?

$$x = \frac{t}{2m_\mu^2} \left(1 - \sqrt{1 - \frac{4m^2}{t}}\right)$$

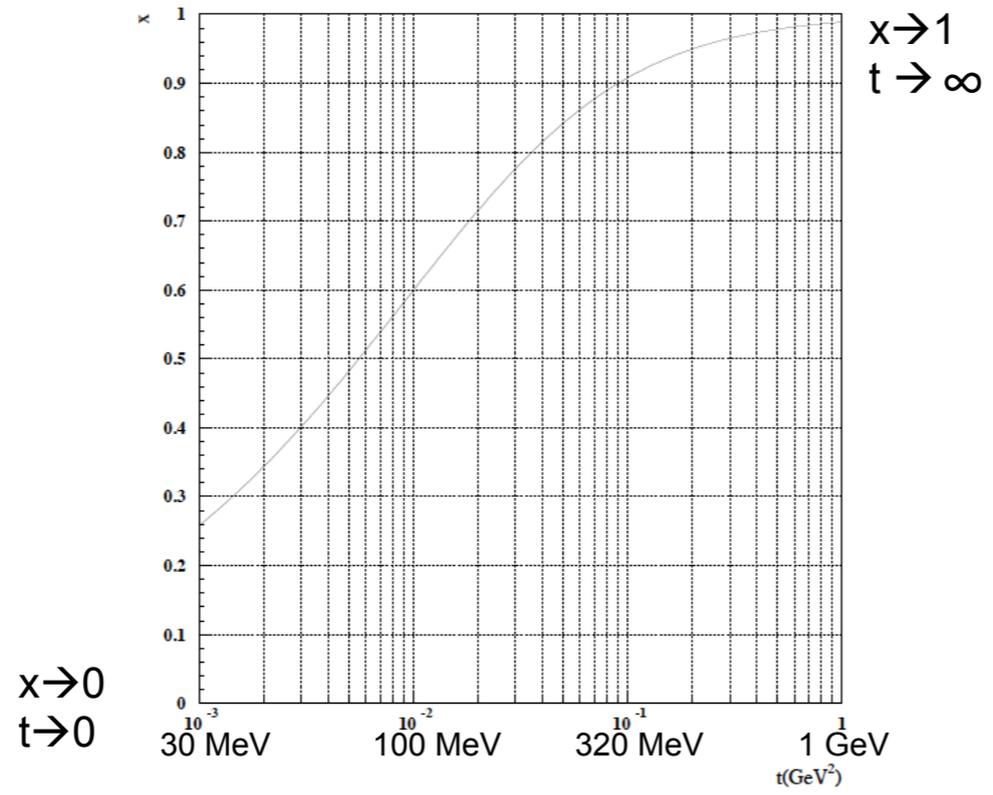
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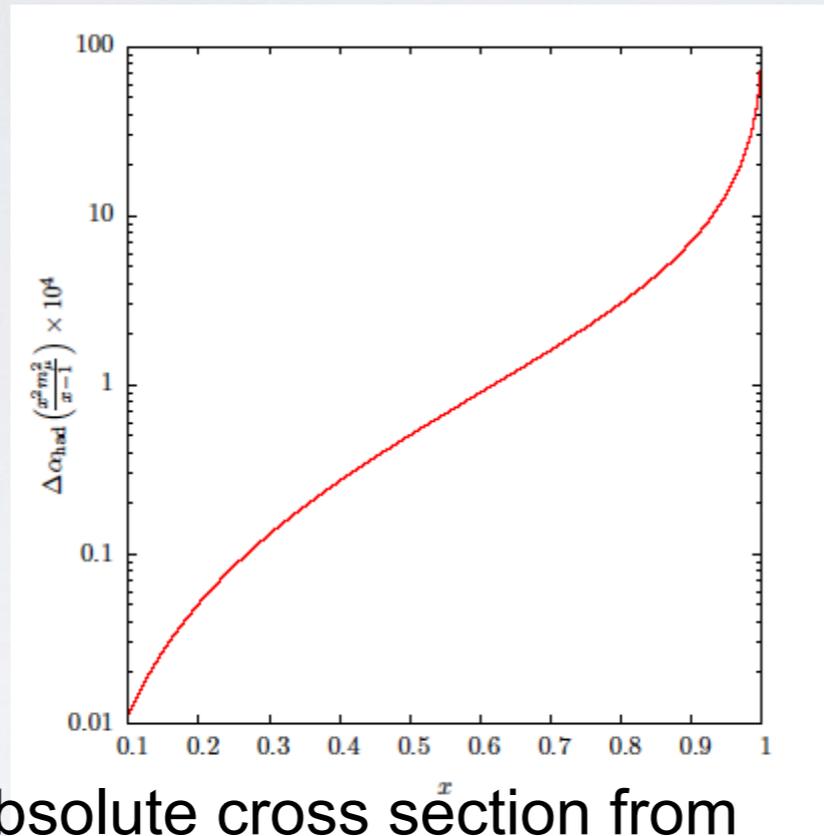
$$-t = 9(1 - \cos\theta)/2$$



x vs t behaviour



Additional consideration: Normalization



To compare Bhabha absolute cross section from data with MC we need Luminosity of the machine.

Two possibilities:

- 1) Use Bhabha at very small angle where the uncertainty on $\Delta\alpha_{had}$ can be neglected (for example at $E_{beam}=1$ GeV and $\theta=5^\circ$, $\Delta\alpha_{had} \sim 10^{-5}$).
- 2) Use a process with $\Delta\alpha_{had}=0$, like $e+e- \gamma\gamma$.
However very difficult to determine it at 10^{-4} accuracy.

Option 1) looks better to us as some of the common systematics cancel in the measurement !

