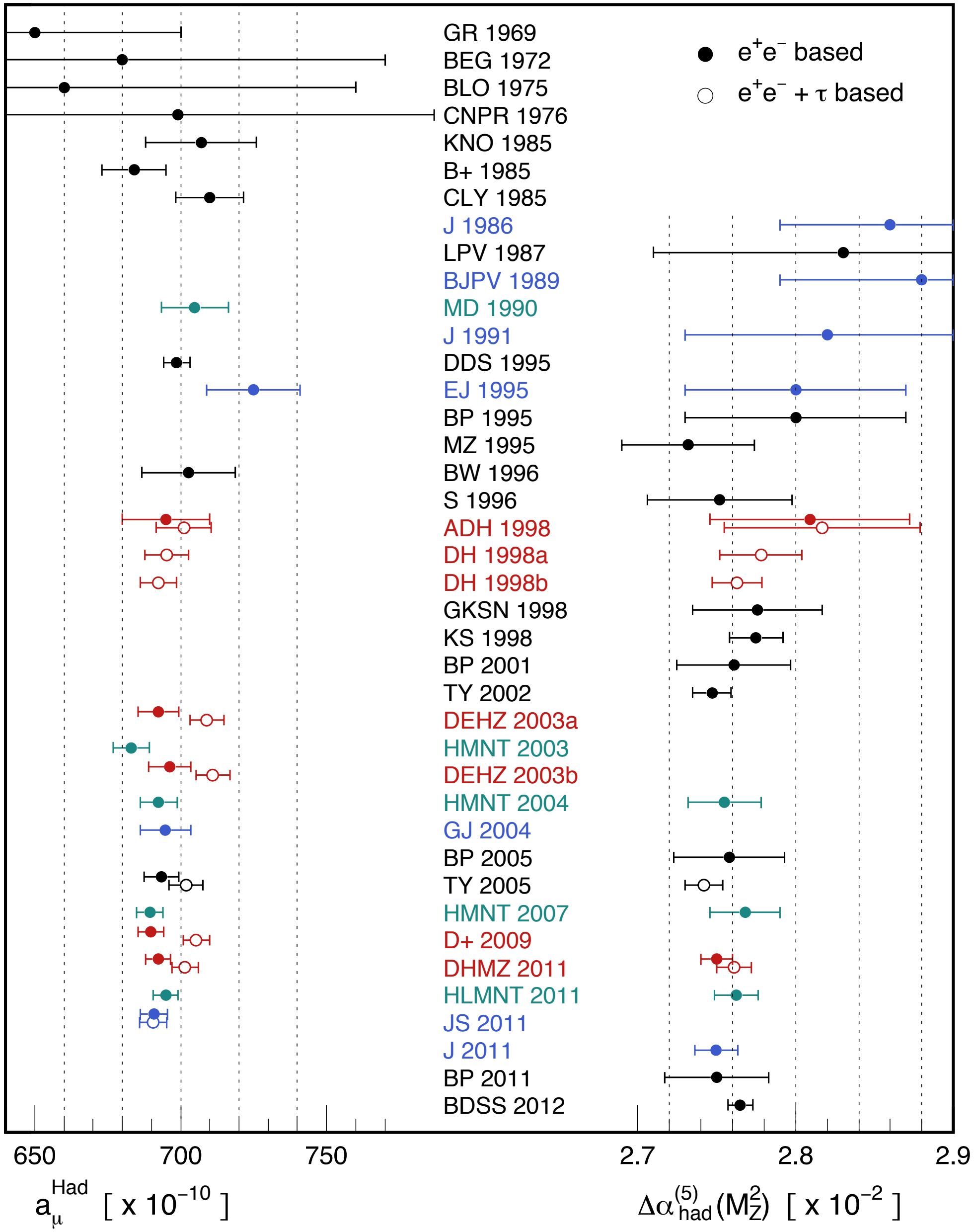
Review of Recent Calculations of the Hadronic Vacuum Polarization Contribution to a_μ

Zhiqing Zhang
In collaboration with
M. Davier, A. Hoecker, B. Malaescu

Outline

- Introduction
- Evaluation of hadronic contribution to a_μ
- Alternative way to evaluate HVP
- Conclusion & perspective

Introduction



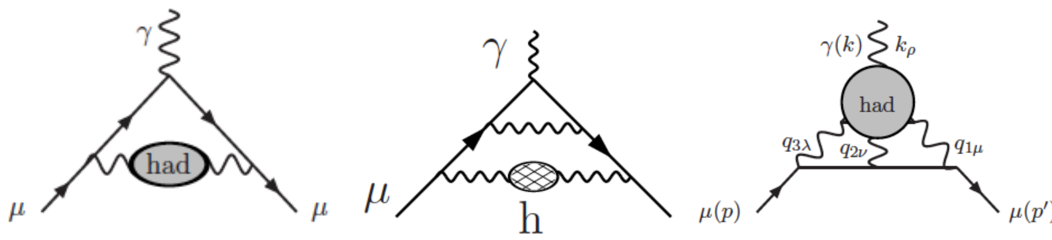
LO Hadronic Vacuum Polarization (HVP) being the most uncertain part for a_μ & $\Delta\alpha$ has been the focus over last 5 decades.

The precision is steadily improving thanks to

- more precise/complete e+e- annihilation (& tau) data
- state of the art techniques for data interpolation, combination and error correlation treatment

Fig. 3, prepared by Davier, Hoecker, Malaescu, Zhang, for "***Standard Theory Essays in the 60th Anniversary of CERN***"

Hadronic Contribution a_μ^{had}



$$\begin{array}{cccc} & \text{Leading-Order} & \text{Higher-Order} & \text{Light-By-Light} \\ a_\mu^{\rm had} = & a_\mu^{\rm had,LO} & + a_\mu^{\rm had,HO} & + a_\mu^{\rm had,LBL} \\ \simeq & 700(\sim 7) & - 9.79(0.09) & + 10.5(2.6) \\ & (\rightarrow \sim 4) & - 9.84\ (0.07) & \end{array}$$

HO: Hagiwara et al., 2007 2011
LBL: Prades-de Rafael-Vainshtein, 2008

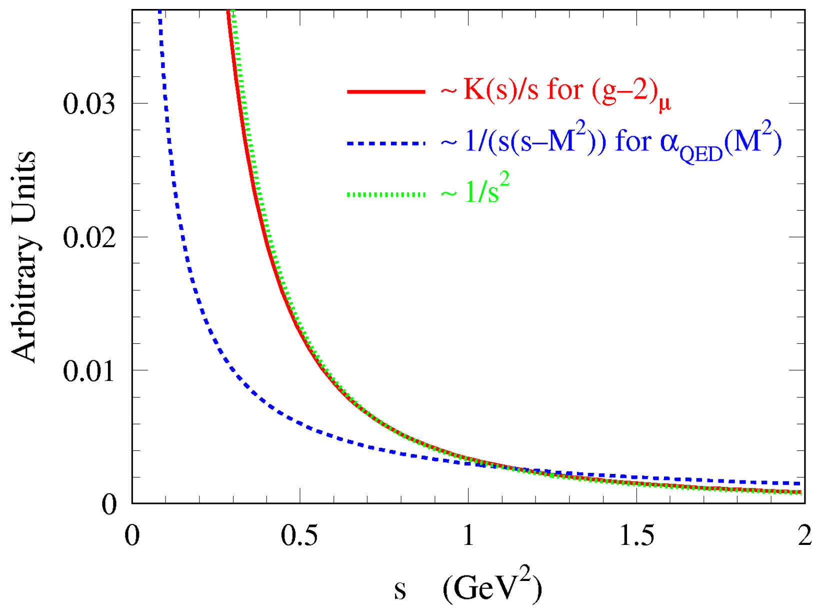
- Hadronic (q & g) loop contributions cannot reliably be calculated from perturbartive QCD (pQCD)
  - → Use dispersion relations with e+e- annihilation data (next slide)
  - ➔ Essentially model independent
- There are however lattice and model based attempts
  - → M. Petschlies & M. Benayoun's talks at this workshop

LO Hadronic Contribution a_μ^{had}

- We focus here on methods which provide precise & model independent estimate
- Use low energy e^+e^- data to calculate the dominant LO contributions:

$$a_\mu^{\rm had} = \frac{\alpha^2(0)}{3\pi^2}\int_{4\pi_\pi^2}^{\infty} ds\,\frac{K(s)}{s}R(s)\,, \qquad R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

Dispersion relation: Bouchiat and Michel, 1961

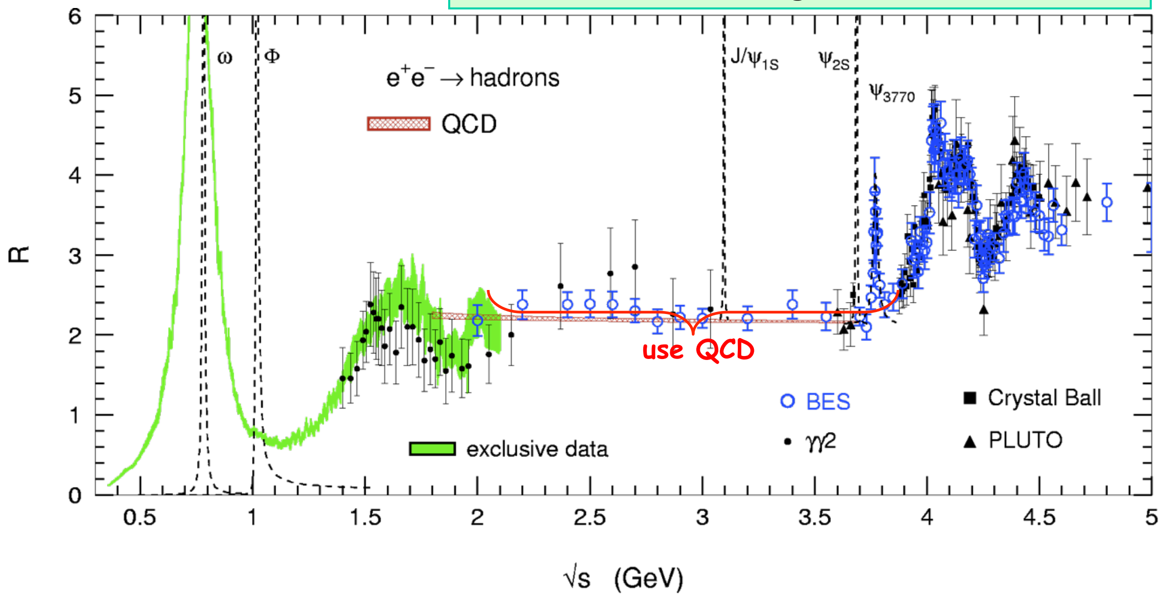


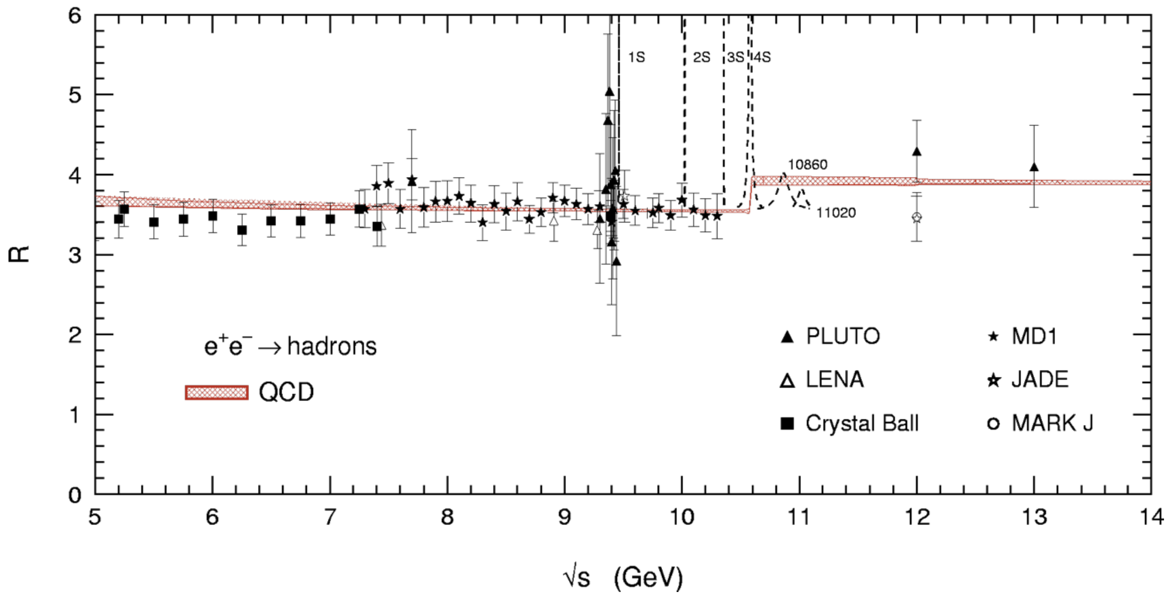
➔ The QED kernel K(s) has such an s dependence that low energy data contribute most

Brodsky, de Rafael, 1968

Input e^+e^- Data in Combination with pQCD

Davier-Hoecker-Zhang, MPR78 (2006) 1043





- [$\pi^0\gamma$-1.8GeV]
  - sum about 22 exclusive channels
  - estimate unmeasured using isospin relations
- [1.8-3.7] GeV
  - good agreement between data and pQCD calcultion → use 4-loop pQCD
  - J/ψ, ψ(2s): Breit-Wigner integral
- [3.7-5] GeV
  use data
- >5GeV
  use 4-loop pQCD calculation

HLMNT 11 have similar treatment

Similar but Different Data Treatments

HVPTools (Davier et al.)	**Clustering method** (Hagiwara et al.)

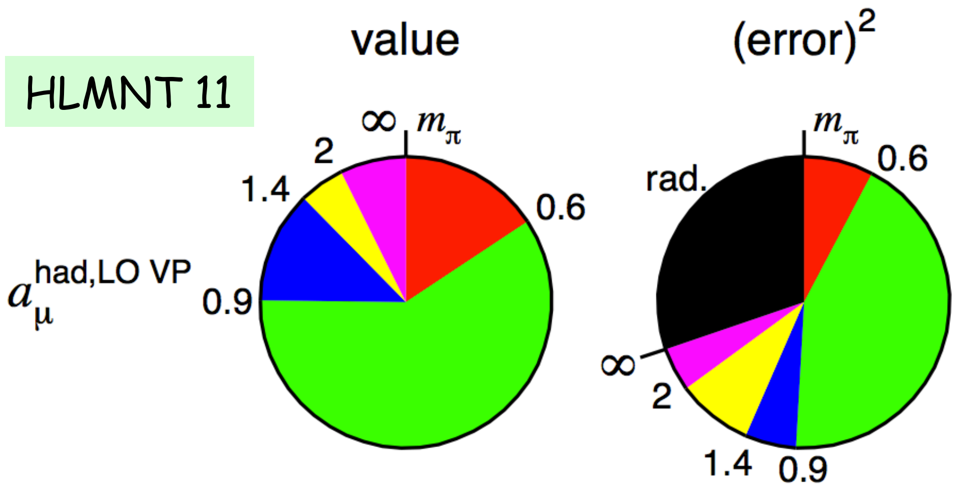
- Use "bare" cross sections (remove ISR, VP, add FSR to some early data)
- Combine different experiments in a same channel before integration

HVPTools	Clustering method
➢ Data interpolation using 2nd order polynomials	Data interpolation using Trapezoidal rule (linear interpolation)
➢ Pseudo-MC generation fluctuates a data point along the original measurement taking into account correlation	Use directly measured data allowing the normalization to float within quoted uncertainty
➢ Fill small (1MeV) bins covered by the data point	Use varying bin/cluster size depending on data density
➢ Weighted average and covariance matrix calculated in the small bins	Average obtained with non-linear χ^2 minimization

- Local χ^2 rescaling correction applied

Relative Contribution of Input Data vs Energy

HLMNT 11

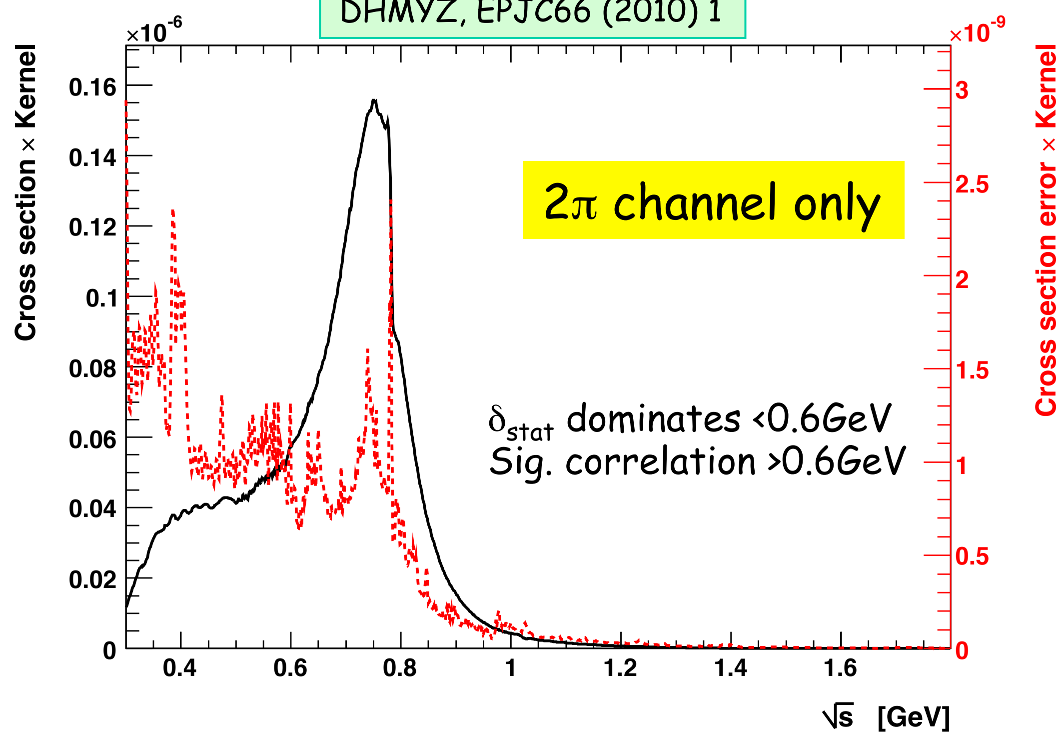


➔ Energy region 0.6-0.9 GeV dominates in both value and uncertainty

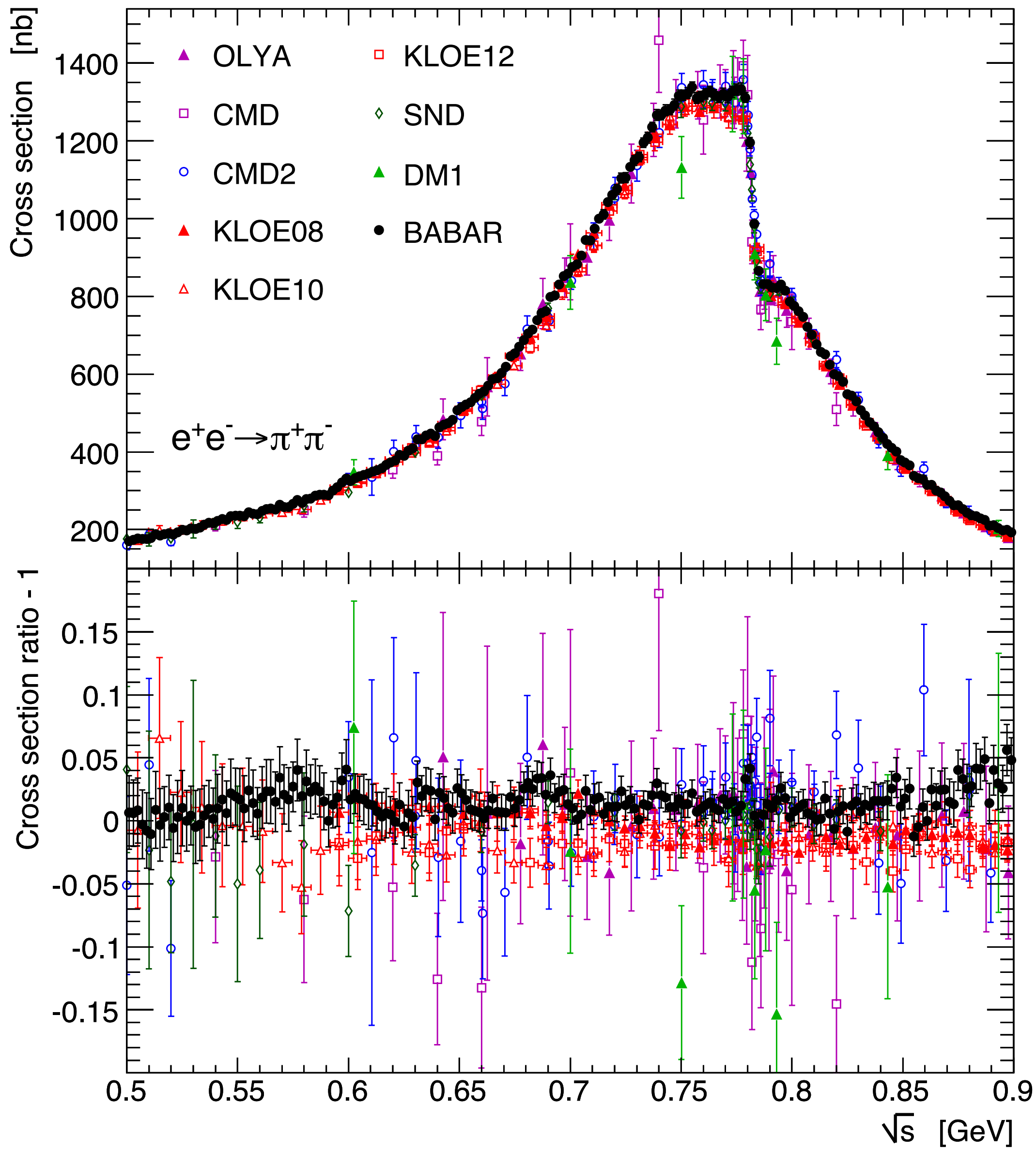
➔ There is a strong energy dependence & different for a_μ & its uncertainty

➔ 2π channel contributes more than 70%

DHMYZ, EPJC66 (2010) 1



A Summary of 2π Input Data



Two main types of experiments:

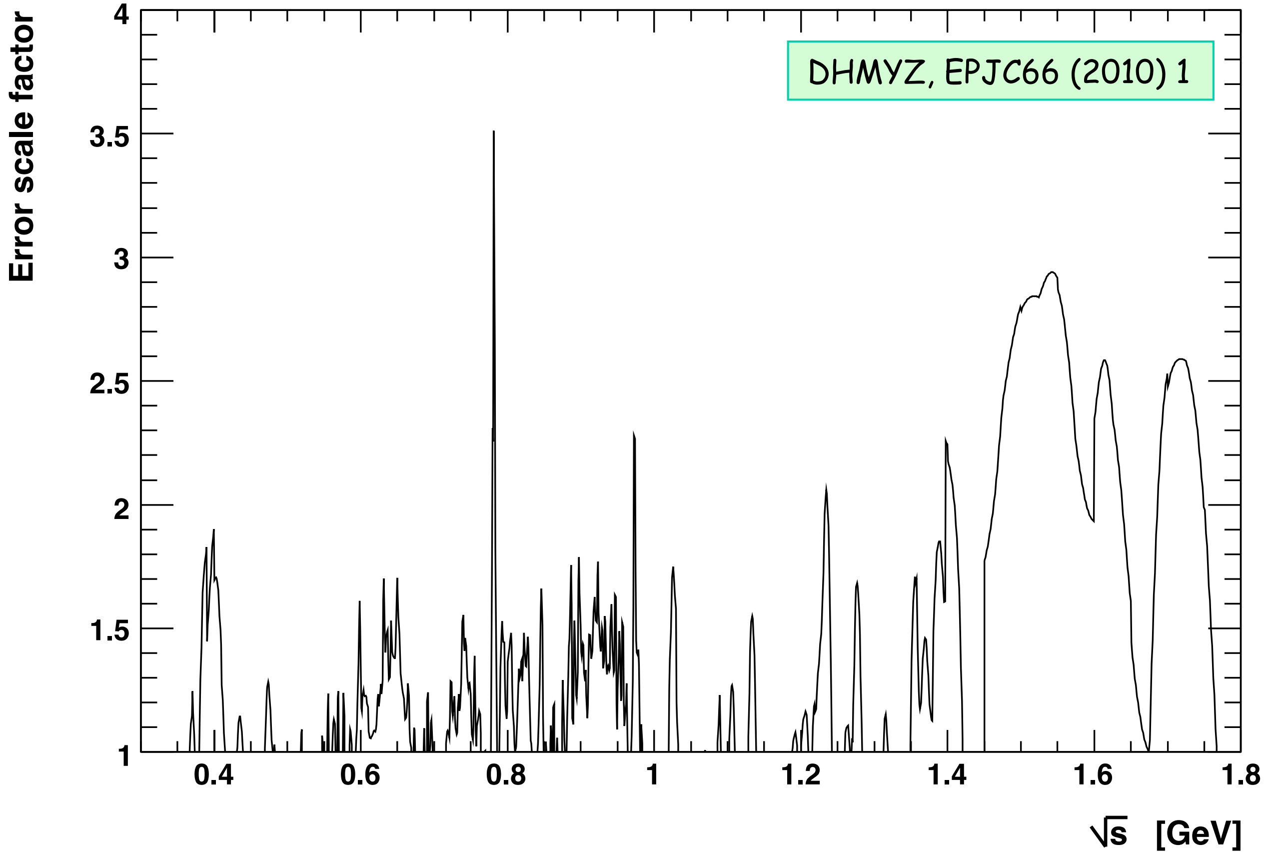
1) Energy scan:
   - CMD(2) (δ_{sy}: 0.8%)
   - SND (δ_{sy}: 1.5%)
   - DM1
   - OLYA

2) ISR based experiments:
   - BABAR (δ_{sy}: 0.5%)
   - KLOE (δ_{sy}: 0.8%)

➔ BABAR has best precision over large energy range
➔ Clear discrepancy between BABAR and KLOE

Fig. 1, prepared by Davier, Hoecker, Malaescu, Zhang for "***Standard Theory Essays in the 60th Anniversary of CERN***"

Discrepancy Affecting the Final Precision

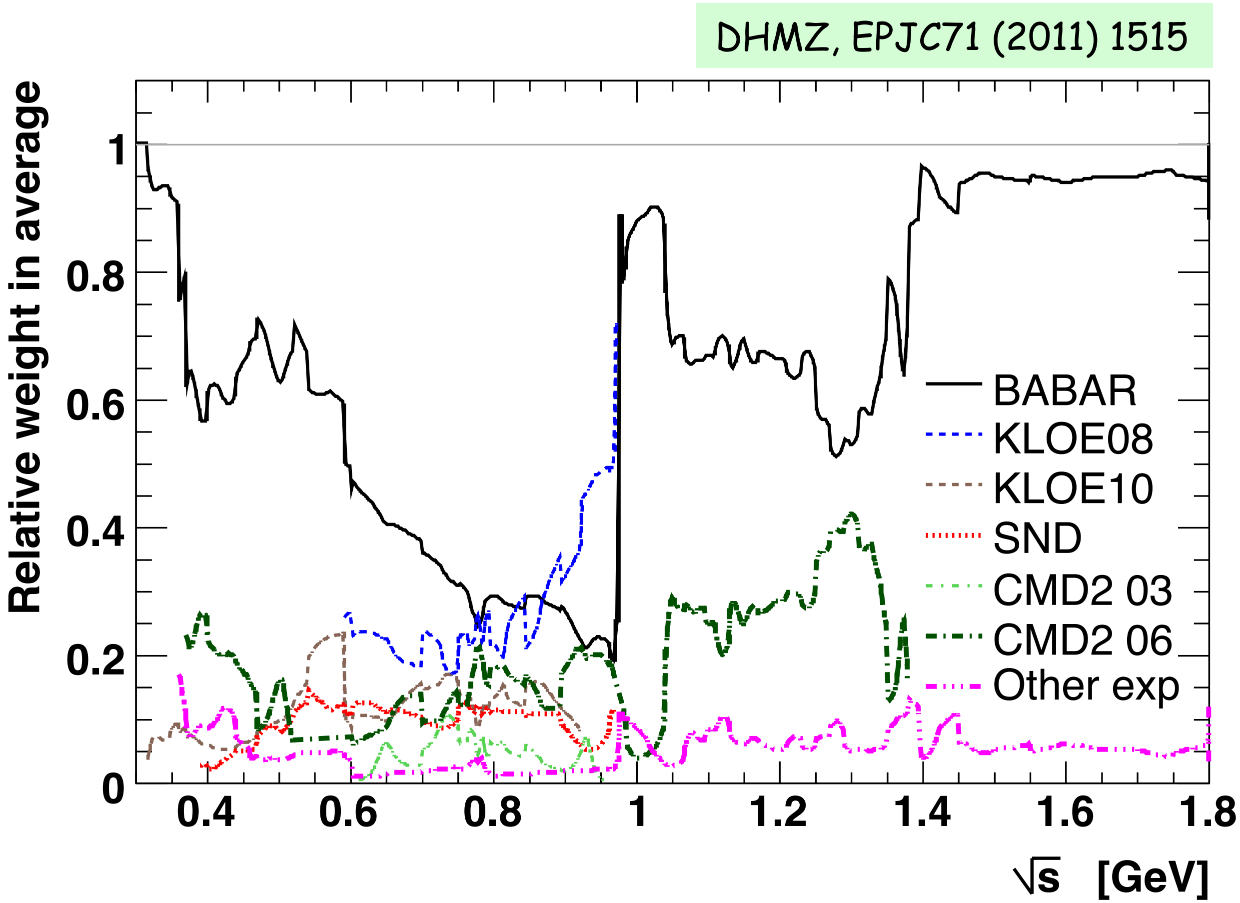


➔ Many data points suffer from significant discrepancy

➔ Major limiting factor for further improving the precision

Zhiqing Zhang (zhang@lal.in2p3.fr, LAL, Orsay)

Relative Impact of Each Data Set



➔ BABAR has dominant weight in the combination except around 0.95GeV

➔ The conclusion remains true when including new KLOE12 data

Comparison HLMNT ('11) and DHMZ ('10)

Channel	HLMNT 11	DHMZ (10) [10]	Difference
$\eta\pi^+\pi^-$	0.88 ± 0.10	1.15 ± 0.19	-0.27
K^+K^-	22.09 ± 0.46	21.63 ± 0.73	0.46
$K^0_S K^0_L$	13.32 ± 0.16	12.96 ± 0.39	0.36
$\omega\pi^0$	0.76 ± 0.03	0.89 ± 0.07	-0.13
$\pi^+\pi^-$	505.65 ± 3.09	507.80 ± 2.84	-2.15
$2\pi^+2\pi^-$	13.50 ± 0.44	13.35 ± 0.53	0.15
$3\pi^+3\pi^-$	0.11 ± 0.01	0.12 ± 0.01	-0.01
$\pi^+\pi^-\pi^0$	47.38 ± 0.99	46.00 ± 1.48	1.38
$\pi^+\pi^-2\pi^0$	18.62 ± 1.15	18.01 ± 1.24	0.61
$\pi^0\gamma$	4.54 ± 0.14	4.42 ± 0.19	0.12
$\eta\gamma$	0.69 ± 0.02	0.64 ± 0.02	0.05
$\eta 2\pi^+2\pi^-$	0.02 ± 0.00	0.02 ± 0.01	0.00
$\eta\omega$	0.38 ± 0.06	0.47 ± 0.06	-0.09
$\eta\phi$	0.33 ± 0.03	0.36 ± 0.03	-0.03
$\phi(\to$ unaccounted$)$	0.04 ± 0.04	0.05 ± 0.00	-0.01
Sum of isospin channels	5.98 ± 0.42	6.06 ± 0.46	-0.08
Total	634.28 ± 3.53	633.93 ± 3.61	0.35

Table 4 from HLMNT, JPG38 (2011) 085003

➔ Different data combination and error treatment

Subleading Channels in terms of Uncertainties

Channel	$a_\mu^{\rm had,LO}$ $[10^{-10}]$	$\Delta\alpha_{\rm had}(M_Z^2)$ $[10^{-4}]$
$\pi^0\gamma$	$4.42 \pm 0.08 \pm 0.13 \pm 0.12$	$0.36 \pm 0.01 \pm 0.01 \pm 0.01$
$\eta\gamma$	$0.64 \pm 0.02 \pm 0.01 \pm 0.01$	$0.08 \pm 0.00 \pm 0.00 \pm 0.00$
$\pi^+\pi^-$	$507.80 \pm 1.22 \pm 2.50 \pm 0.56$	$34.43 \pm 0.07 \pm 0.17 \pm 0.04$
$\pi^+\pi^-\pi^0$	$46.00 \pm 0.42 \pm 1.03 \pm 0.98$	$4.58 \pm 0.04 \pm 0.11 \pm 0.09$
$2\pi^+2\pi^-$	$13.35 \pm 0.10 \pm 0.43 \pm 0.29$	$3.49 \pm 0.03 \pm 0.12 \pm 0.08$
$\pi^+\pi^-2\pi^0$	$18.01 \pm 0.14 \pm 1.17 \pm 0.40$	$4.43 \pm 0.03 \pm 0.29 \pm 0.10$
$2\pi^+2\pi^-\pi^0$ (η excl.)	$0.72 \pm 0.04 \pm 0.07 \pm 0.03$	$0.22 \pm 0.01 \pm 0.02 \pm 0.01$
$\pi^+\pi^-3\pi^0$ (η excl., from isospin)	$0.36 \pm 0.02 \pm 0.03 \pm 0.01$	$0.11 \pm 0.01 \pm 0.01 \pm$ [illegible]
$3\pi^+3\pi^-$	$0.12 \pm 0.01 \pm 0.01 \pm 0.00$	$0.04 \pm 0.00 \pm 0.00 \pm$ [illegible]
$2\pi^+2\pi^-2\pi^0$ (η excl.)	$0.70 \pm 0.05 \pm 0.04 \pm 0.09$	$0.25 \pm 0.02 \pm 0.02 \pm$ [illegible]
$\pi^+\pi^-4\pi^0$ (η excl., from isospin)	$0.11 \pm 0.01 \pm 0.11 \pm 0.00$	$0.04 \pm 0.00 \pm 0.04 \pm$ [illegible]
$\eta\pi^+\pi^-$	$1.15 \pm 0.06 \pm 0.08 \pm 0.03$	$0.33 \pm 0.02 \pm 0.02 \pm$ [illegible]
$\eta\omega$	$0.47 \pm 0.04 \pm 0.00 \pm 0.05$	$0.15 \pm 0.01 \pm 0.00 \pm$ [illegible]
$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.01 \pm 0.00 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$\eta\pi^+\pi^-2\pi^0$ (estimated)	$0.02 \pm 0.01 \pm 0.01 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega\pi^0$ ($\omega \to \pi^0\gamma$)	$0.89 \pm 0.02 \pm 0.06 \pm 0.02$	$0.18 \pm 0.00 \pm 0.02 \pm 0.00$
$\omega\pi^+\pi^-, \omega 2\pi^0$ ($\omega \to \pi^0\gamma$)	$0.08 \pm 0.00 \pm 0.01 \pm 0.00$	$0.03 \pm 0.00 \pm 0.00 \pm 0.00$
ω (non-$3\pi, \pi\gamma, \eta\gamma$)	$0.36 \pm 0.00 \pm 0.01 \pm 0.00$	$0.03 \pm 0.00 \pm 0.00 \pm 0.00$
K^+K^-	$21.63 \pm 0.27 \pm 0.58 \pm 0.36$	$3.13 \pm 0.04 \pm 0.08 \pm 0.05$
$K_S^0K_L^0$	$12.96 \pm 0.18 \pm 0.25 \pm 0.24$	$1.75 \pm 0.02 \pm 0.03 \pm 0.03$
ϕ (non-$K\overline{K}, 3\pi, \pi\gamma, \eta\gamma$)	$0.05 \pm 0.00 \pm 0.00 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$K\overline{K}\pi$ (partly from isospin)	$2.39 \pm 0.07 \pm 0.12 \pm 0.08$	$0.76 \pm 0.02 \pm 0.04 \pm 0.02$
$K\overline{K}2\pi$ (partly from isospin)	$1.35 \pm 0.09 \pm 0.38 \pm 0.03$	$0.48 \pm 0.03 \pm 0.14 \pm 0.01$
$K\overline{K}3\pi$ (partly from isospin)	$-0.03 \pm 0.01 \pm 0.02 \pm 0.00$	$-0.01 \pm 0.00 \pm 0.01 \pm 0.00$
$\phi\eta$	$0.36 \pm 0.02 \pm 0.02 \pm 0.01$	$0.13 \pm 0.01 \pm 0.01 \pm 0.00$
$\omega K\overline{K}$ ($\omega \to \pi^0\gamma$)	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$
J/ψ (Breit-Wigner integral)	6.22 ± 0.16	7.03 ± 0.18
$\psi(2S)$ (Breit-Wigner integral)	1.57 ± 0.03	2.50 ± 0.04
$R_{\rm data}$ $[3.7 - 5.0$ GeV$]$	$7.29 \pm 0.05 \pm 0.30 \pm 0.00$	$15.79 \pm 0.12 \pm 0.66 \pm 0.00$
$R_{\rm QCD}$ $[1.8 - 3.7$ GeV$]_{uds}$	33.45 ± 0.28	24.27 ± 0.19
$R_{\rm QCD}$ $[5.0 - 9.3$ GeV$]_{udsc}$	6.86 ± 0.04	34.89 ± 0.18
$R_{\rm QCD}$ $[9.3 - 12.0$ GeV$]_{udscb}$	1.21 ± 0.01	15.56 ± 0.04
$R_{\rm QCD}$ $[12.0 - 40.0$ GeV$]_{udscb}$	1.64 ± 0.01	77.94 ± 0.12
$R_{\rm QCD}$ $[> 40.0$ GeV$]_{udscb}$	0.16 ± 0.00	42.70 ± 0.06
$R_{\rm QCD}$ $[> 40.0$ GeV$]_t$	0.00 ± 0.00	-0.72 ± 0.01
Sum	$692.3 \pm 1.4 \pm 3.1 \pm 2.4 \pm 0.2_\psi \pm 0.3_{\rm QCD}$	$274.97 \pm 0.17 \pm 0.78 \pm 0.37 \pm 0.18_\psi \pm 0.52_{\rm QCD}$

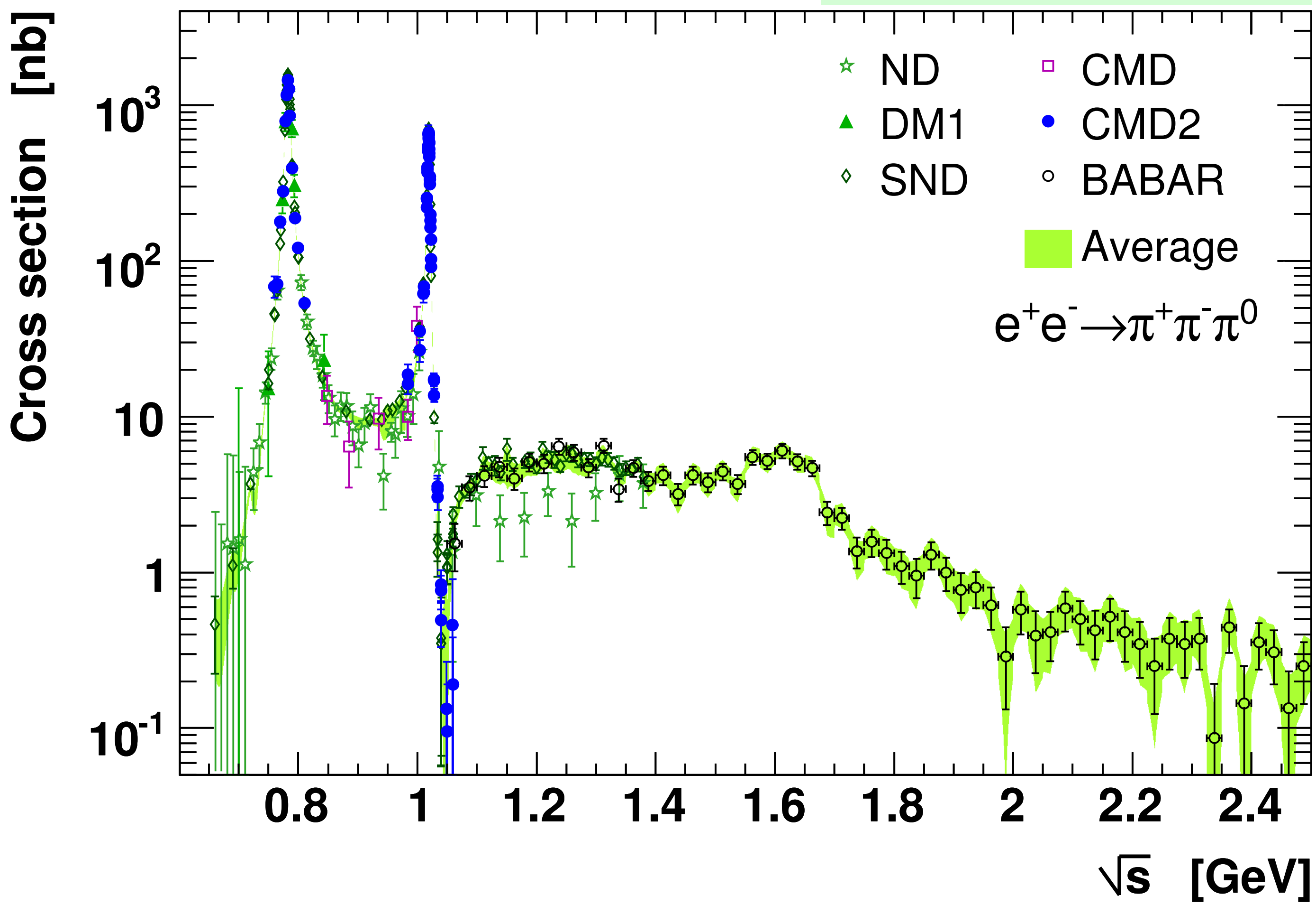
$\pi^+\pi^-\pi^0$	46.00	± 0.42	± 1.03	± 0.98
$2\pi^+2\pi^-$	13.35	± 0.10	± 0.43	± 0.29
$\pi^+\pi^-2\pi^0$	18.01	± 0.14	± 1.17	± 0.40

stat $\quad$ sys$_{\rm ch}$ $\quad$ sys$_{\rm com}$

Table II from DHMZ EPJC71 (2011) 1515, EPJC72 (2012) 1874

The $\pi^+\pi^-\pi^0$ Channel

DHMZ, EPJC71 (2011) 1515



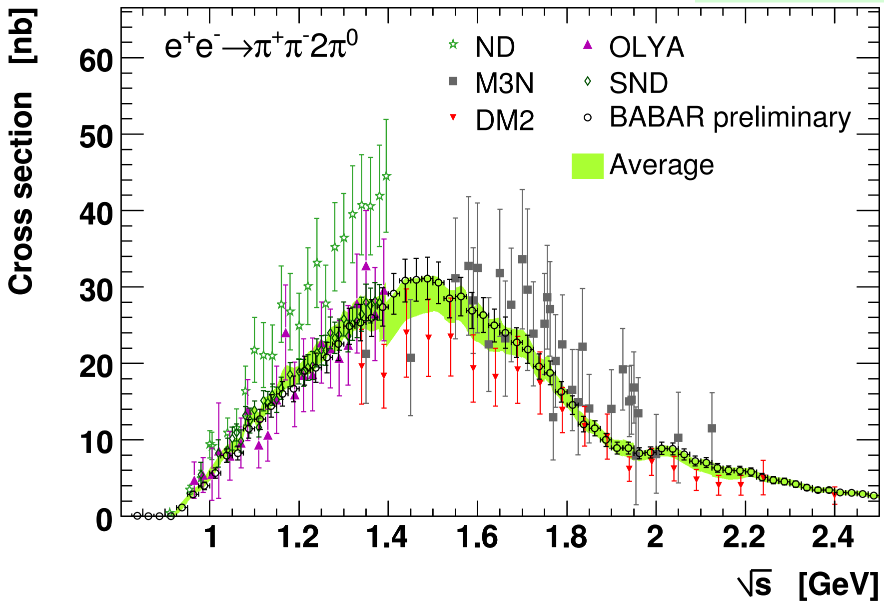
➔ 2nd or 3rd channel has largest error next to $\pi^+\pi^-$ channel
Need more precise data

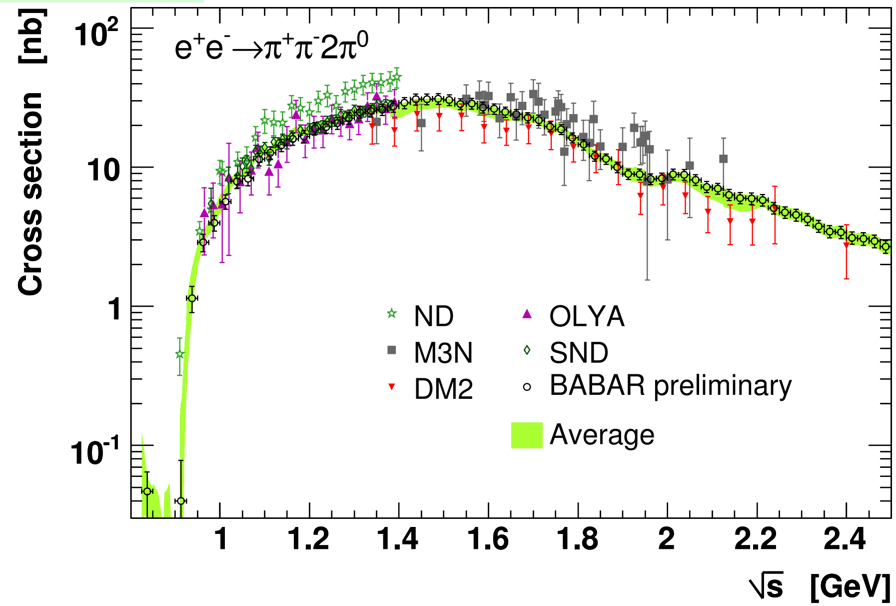
The $\pi^+\pi^-2\pi^0$ Channel

Update 2009: 18.0 ± 1.2 including preliminary ISR BABAR data:
A. Petzold, EPS-HEP (2007)

It was: 16.8 ± 1.3 (Davier-Eidelman-Hoecker-Zhang, 2006)

DHMZ, EPJC71 (2011) 1515





→ Large scattering among e^+e^- data sets

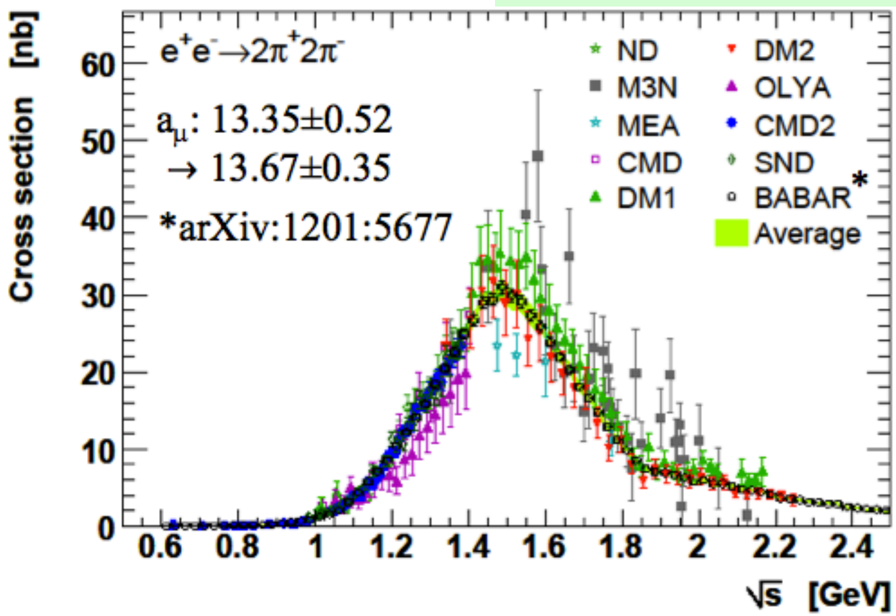
→ Looking forward to the final BABAR data

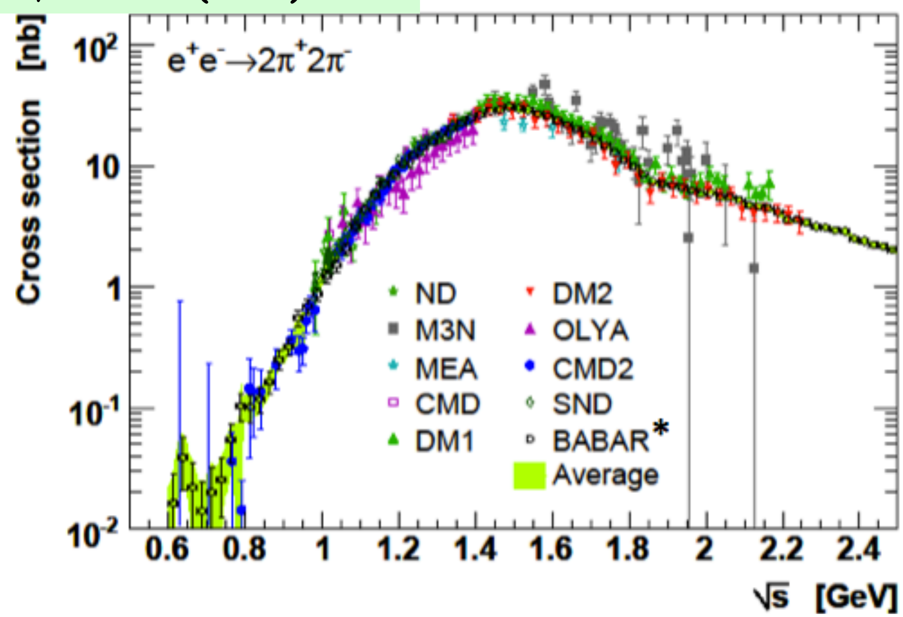
The $2\pi^+2\pi^-$ Channel

Update: 13.67 ± 0.36 using published BABAR data (arXiv:1201.5677)

It was: 13.35 ± 0.53 (Davier-Hoecker-Malaescu-Zhang, 2011)

Updated version of DHMZ, EPJC71 (2011) 1515

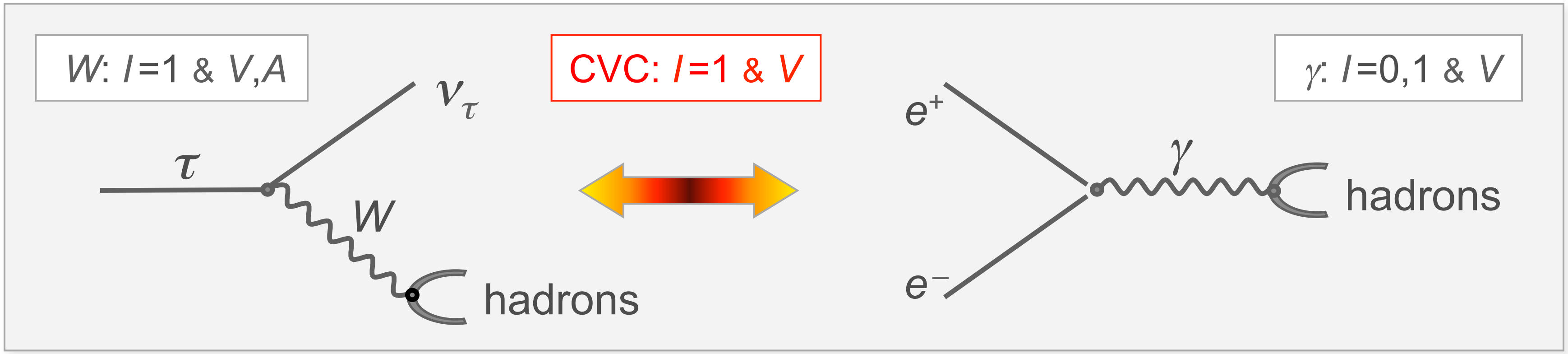




→ The uncertainty substantially improved with final BABAR data

Alternative Way to Evaluate HVP

Proposed by Alemany-Davier-Hoecker, EPJC2 (1998) 123



Hadronic physics factorizes in Spectral Functions :

Isospin symmetry connects I=1 e^+e^- cross section to vector τ spectral functions:

$$\sigma^{(I=1)}\left[e^+e^- \to \pi^+\pi^-\right] = \frac{4\pi\alpha^2}{s}\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]$$

fundamental ingredient relating long distance (resonances) to short distance description (QCD)

$$\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right] \propto \frac{BR\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]}{BR\left[\tau^- \to e^-\bar{\nu}_e\nu_\tau\right]} \frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds} \frac{m_\tau^2}{\left(1-s/m_\tau^2\right)^2\left(1+s/m_\tau^2\right)}$$

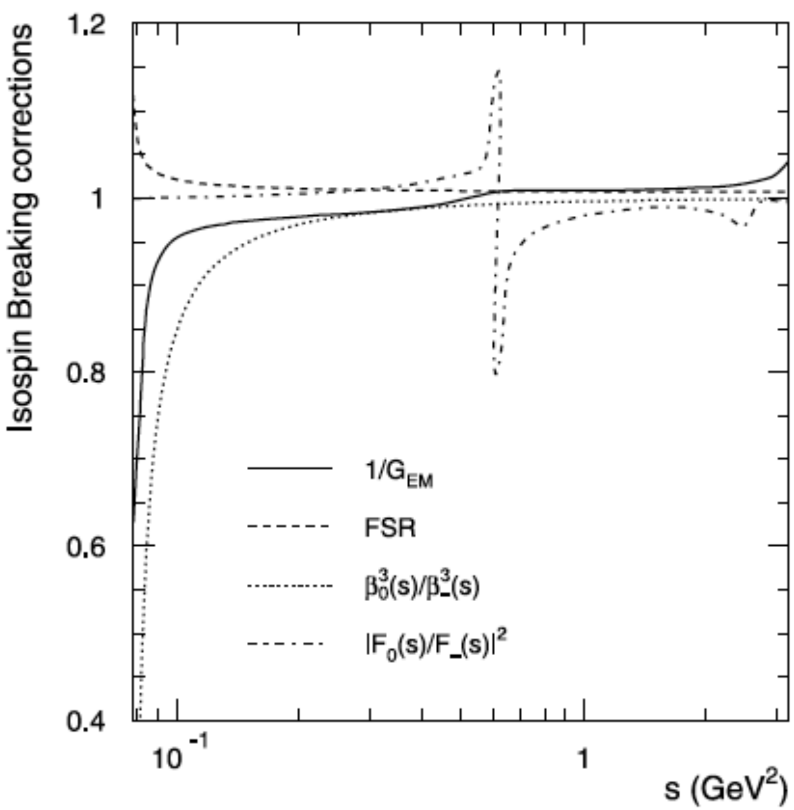
branching fractions — mass spectrum — kinematic factor (PS)

Known Isospin Corrections

Davier et al., EPJC66 (2010) 127

$$v_{1,X^-}(s) = \frac{m_\tau^2}{6|V_{ud}|^2}\frac{\mathcal{B}_{X^-}}{\mathcal{B}_e}\frac{1}{N_X}\frac{dN_X}{ds} \times \left(1-\frac{s}{m_\tau^2}\right)^{-2}\left(1+\frac{2s}{m_\tau^2}\right)^{-1}\frac{R_{IB}(s)}{S_{EW}},$$

$$R_{IB}(s) = \frac{\mathrm{FSR}(s)}{G_{EM}(s)}\frac{\beta_0^3(s)}{\beta_-^3(s)}\left|\frac{F_0(s)}{F_-(s)}\right|^2.$$

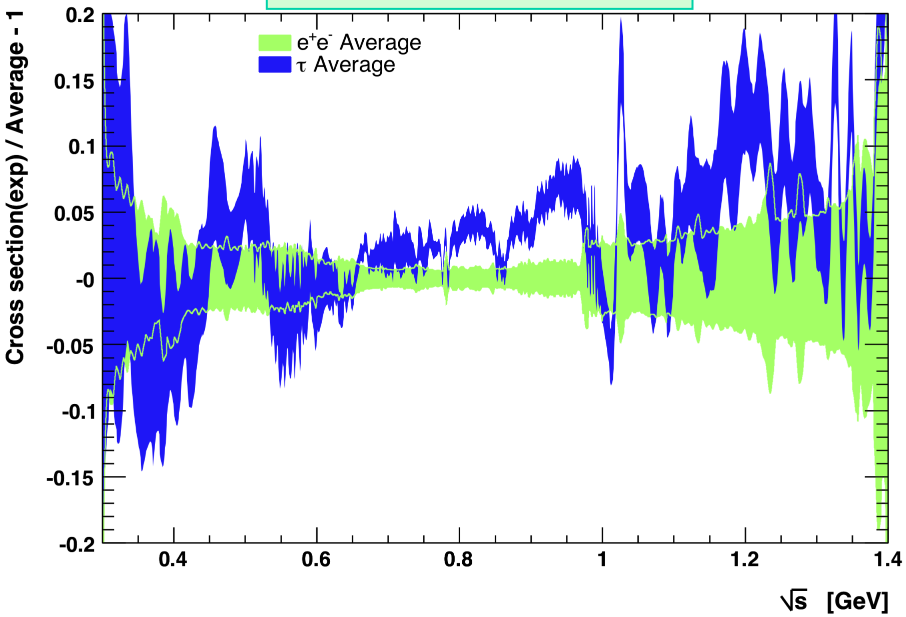


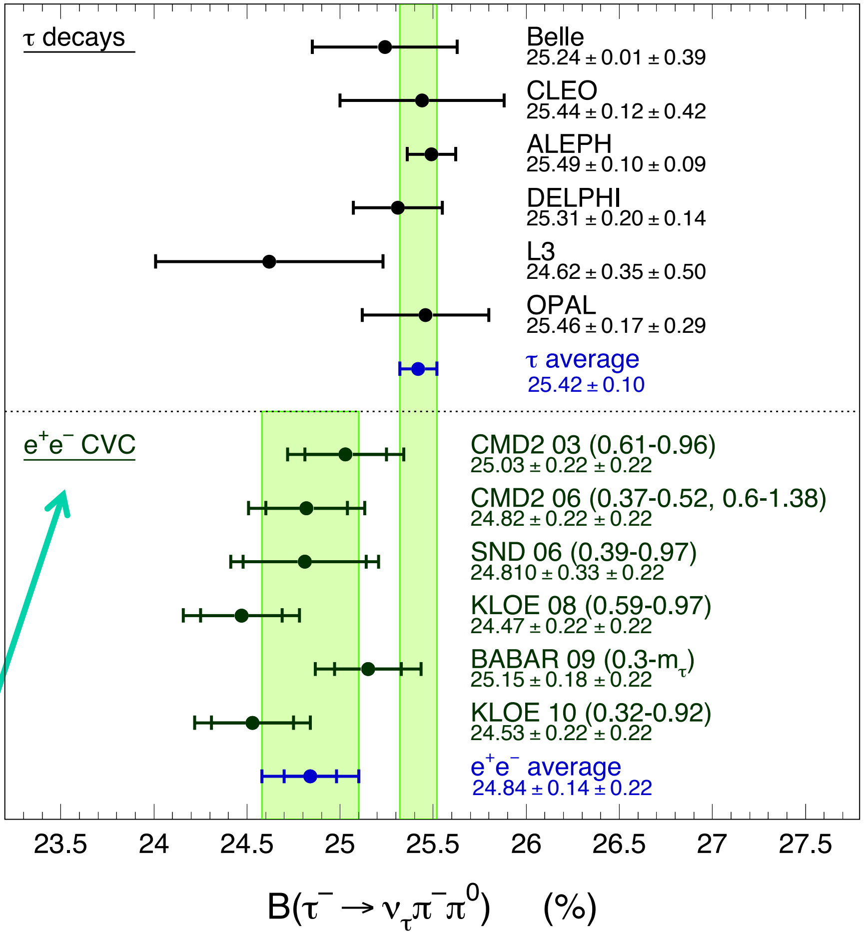
Source	$\Delta a_\mu^{\mathrm{had,LO}}[\pi\pi,\tau]$ (10^{-10})	
	GS model	KS model
S_{EW}	-12.21 ± 0.15	
G_{EM}	-1.92 ± 0.90	
FSR	$+4.67 \pm 0.47$	
ρ–ω interference	$+2.80 \pm 0.19$	$+2.80 \pm 0.15$
$m_{\pi^\pm} - m_{\pi^0}$ effect on σ	-7.88	
$m_{\pi^\pm} - m_{\pi^0}$ effect on Γ_ρ	$+4.09$	$+4.02$
$m_{\rho^\pm} - m_{\rho^0_{\mathrm{bare}}}$	$+0.20^{+0.27}_{-0.19}$	$+0.11^{+0.19}_{-0.11}$
$\pi\pi\gamma$, electrom. decays	-5.91 ± 0.59	-6.39 ± 0.64
Total	-16.07 ± 1.22	-16.70 ± 1.23
	-16.07 ± 1.85	

Open Issue in $\pi^+\pi^-$ Channel (e^+e^- vs. τ)

All known isospin breaking effects taken into account

DHMYZ, EPJC66 (2010) 1





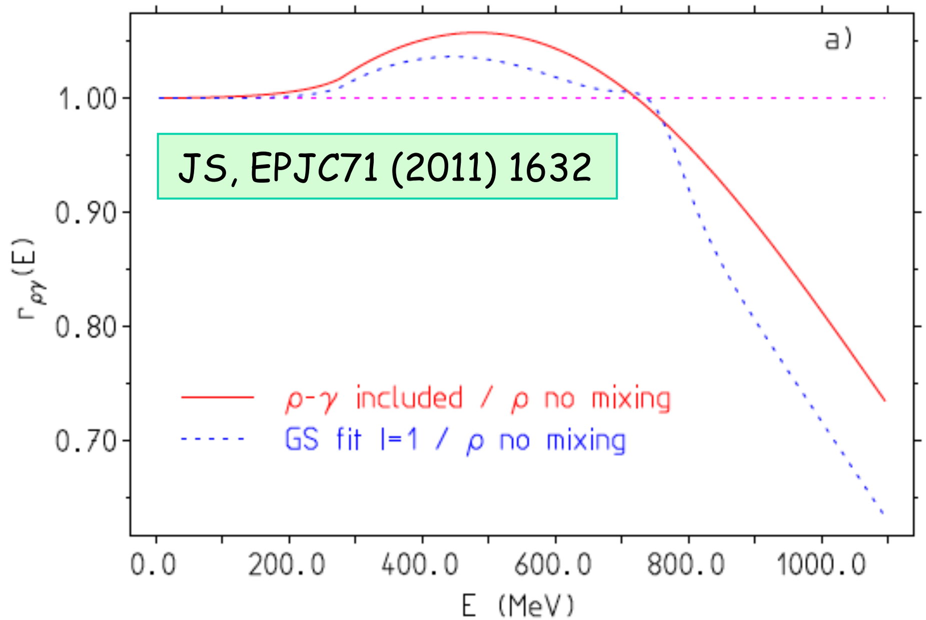
Good agreement between BABAR vs. Belle and CLEO
Conflict between KLOE and Tau data

$$\mathcal{B}_X^{\rm CVC} = \frac{3}{2}\frac{\mathcal{B}_e|V_{ud}|^2}{\pi\alpha^2 m_\tau^2}\int_{s_{\rm min}}^{m_\tau^2} ds\, s\,\sigma^I_{X^0}\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+\frac{2s}{m_\tau^2}\right)$$

e+e- vs. tau discrepancy reduced (2.9σ (2006) → 2.4σ) but not fully resolved

Additional EFT based γ−ρ mixing Correction

Jegerlehner and Szafron proposed to use the missing ρ^0-γ mixing in τ data to explain the remaining e^+e^- and τ difference



a_μ [0.582-0.975]GeV:

ee: $385.21 \pm 0.18 \pm 1.54$

τ: $385.96 \pm 1.40 \pm 1.45 \pm 1.40 \pm 0.5$ (ρ−γ)

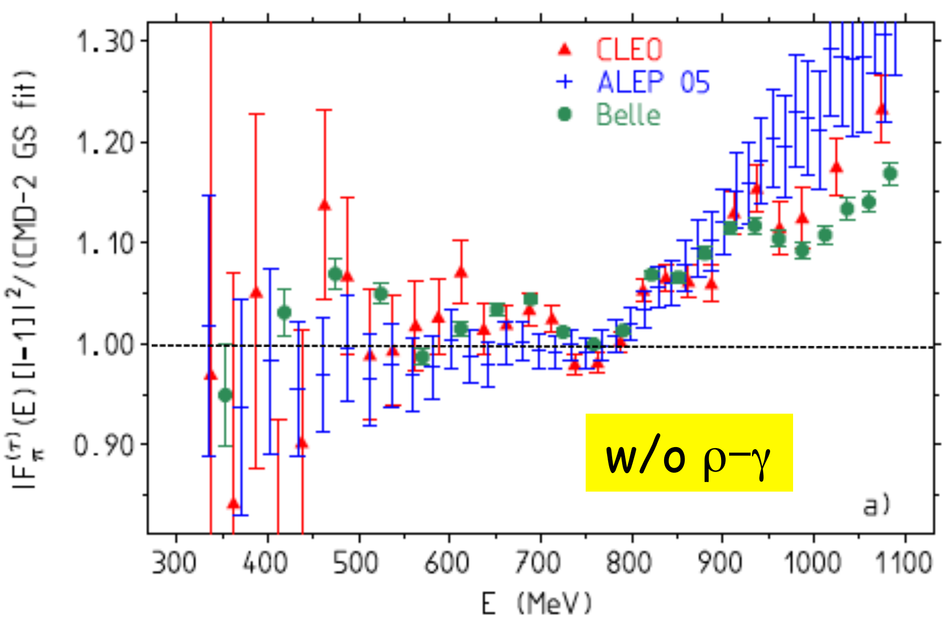
391.06 (w/o ρ−γ)

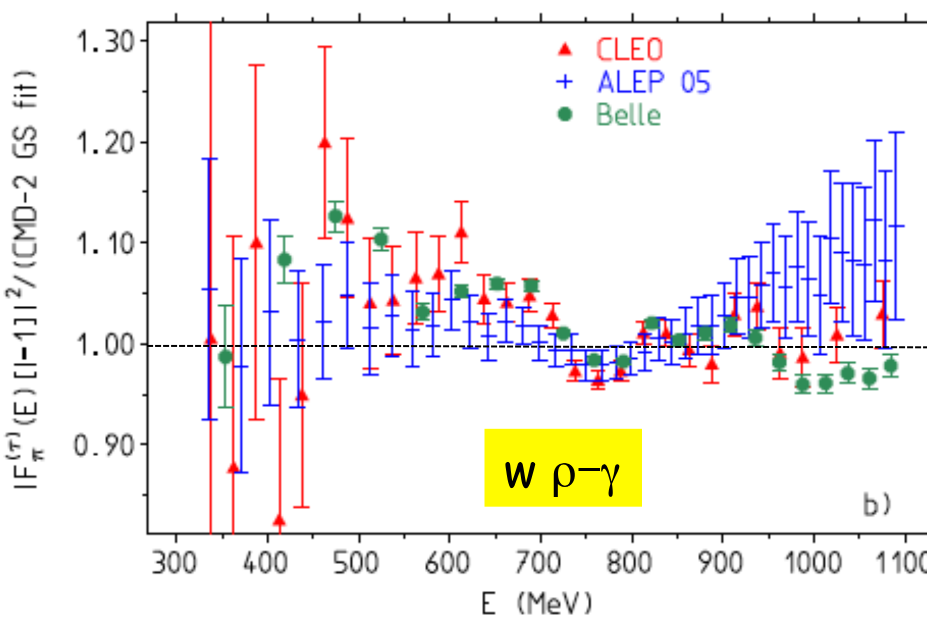
BR[$\pi\pi^0$] (in %):

τ: $25.34 \pm 0.06 \pm 0.08$

ee: $25.20 \pm 0.17 \pm 0.28$ (ρ−γ)

24.58 (w/o ρ−γ)





Conclusion & Perspective

- More precise/complete e+e- (tau) input data +
  Dedicated data treatment and uncertainty correlation studies

  → steadily improving precision for the LO HVP evaluation
  based on dispersion relations

- The long standing e+e-/tau discrepancy in 2π resolved?
  → more in backup

- Improvements are expected in the next years with
  - applying new idea to existing BABAR data
  - collecting new data from CMD3, SND, BESIII, Belle II

Backup: Comparison of IB Corrections

Davier et al., EPJC66 (2010) 127

JS, EPJC71 (2011) 1632

Items	Davier et al. [Δa_μ^{had}]	Jegerlehner et al.
FSR(s)	Yes [+4.67±0.47]	No
GEM(s)	Yes [-1.93±0.90]	Yes but maybe different
$[\beta_0(s)/\beta_-(s)]^3$ due to $\delta m(\pi^- - \pi^0)$	Yes [-7.88]	Yes
$\delta m(\rho^- - \rho^0)$	1.0±0.9MeV [+0.20 …]	0.814MeV
$\delta\Gamma(\rho^- - \rho^0)$	-0.76MeV [+4.09-5.91]	+1.300MeV
$\rho-\omega$ interference	Yes [+2.80±0.19]	Yes
$\gamma-\rho$ mixing	Not explicitly included with GS parameterization	Included with an EFT model valid up to ~1GeV

Zhiqing Zhang (zhang@lal.in2p3.fr, LAL, Orsay)