Leading-order hadronic contribution to the electron and muon $g-2$

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Flavor Changing and Conserving Processes

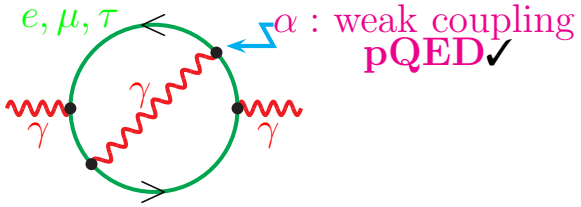
Outline of Talk:

- Introduction: hadronic effects in $g-2$, the limitation to theory
- Leading order HVP to $a_\mu^{\rm had}$ via $\sigma(e^+e^- \to \text{hadrons})$
- Effective field theory: the Resonance Lagrangian Approach
- HVP for the electron anomaly
- HVP subtraction of $R(s)$: a problem of the DR method?
- Theory vs experiment: do we see New Physics?
- Future, Summary

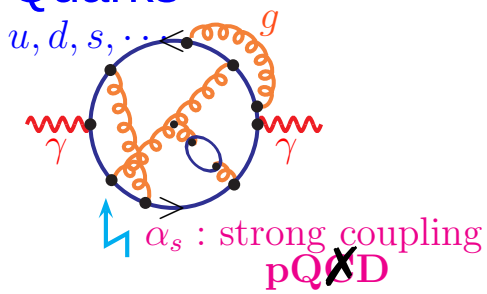
Introduction: hadronic effects in $g-2$, the limitation to theory

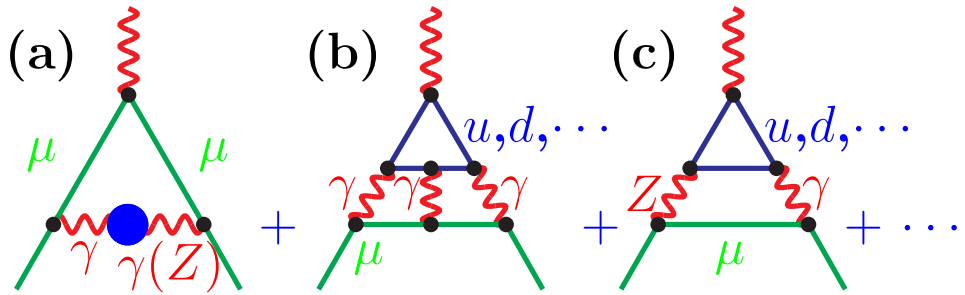
General problem in electroweak precision physics:
contributions from hadrons (quark loops) at low energy scales

Leptons



Quarks





(a) Hadronic vacuum polarization $O(\alpha^2), O(\alpha^3)$ — Light quark loops

(b) Hadronic light-by-light scattering $O(\alpha^3)$ — ↓

(c) Hadronic effects in 2-loop EWRC $O(\alpha G_F m_\mu^2)$ — Hadronic "blobs"

Evaluation of Non-perturbative effects:

- data in conjunction with Dispersion Relations (DR),
- low energy effective modeling, RLA, HLS, ENJL
- lattice QCD

(a) HVP via dispersion integral over $e^+e^- \to$ hadrons-data (1 independent amplitude to be determined by one specific data set), HLS, lattice QCD

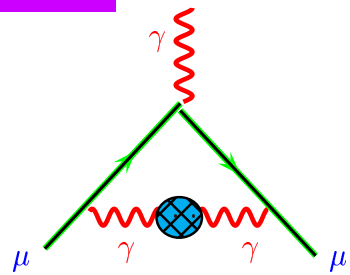
(b) HLbL via Resonance Lagrangian Approach (RLA) (CHPT extended by VDM in accord with chiral structure of QCD), $\gamma\gamma \to$ hadrons-data dispersive approach (32 independent amplitudes to be determined by as many independent data sets), lattice QCD Blum et al

(c) quark and lepton triangle diagrams: VVV = 0 by Furry $\Rightarrow$only VVA (of $\gamma\gamma Z$-vertex) contributes $\Rightarrow$ABJ anomaly is perturbative and non-perturbative simultaneously i.e. leading effects calculable (anomaly cancellation) de Rafael, Knecht, Perottet, Melnikov, Vainshtein

Leading order HVP to $a_\mu^{\rm had}$ via $\sigma(e^+e^- \to \text{hadrons})$

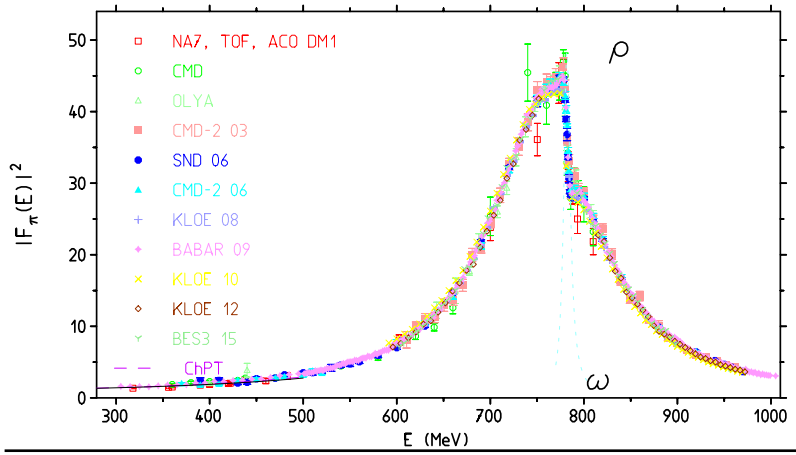
Leading non-perturbative hadronic contributions $a_\mu^{\rm had}$ can be obtained in terms of $R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})/\frac{4\pi\alpha^2}{3s}$ data via **Dispersion Relation** (DR):

$$a_\mu^{\rm had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int_{4m_\pi^2}^{E_{\rm cut}^2} ds \, \frac{R_\gamma^{\rm data}(s)\, \hat{K}(s)}{s^2} + \int_{E_{\rm cut}^2}^{\infty} ds \, \frac{R_\gamma^{\rm pQCD}(s)\, \hat{K}(s)}{s^2} \right)$$



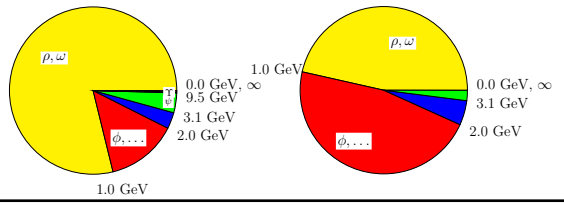
- Experimental error implies theoretical uncertainty!
- Low energy contributions enhanced: $\sim 75\%$ come from region $4m_\pi^2 < m_{\pi\pi}^2 < M_\Phi^2$

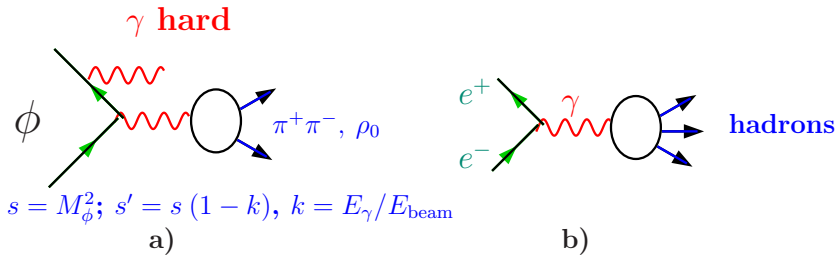
  Data: NSK, KLOE, BaBar, BES3



$$a_\mu^{\rm had(1)} = (688.6 \pm 4.8)[688.9 \pm 3.5]\ 10^{-10}$$

e^+e^-–data based [incl. τ JS11]





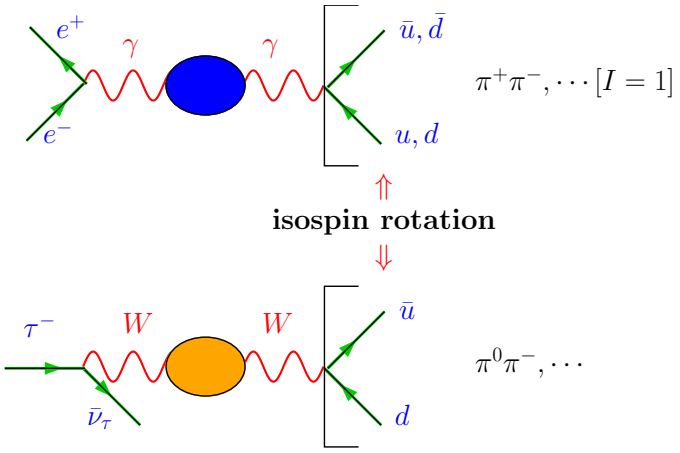
a) Initial state radiation (ISR), b) Standard energy scan.

SCAN: CMD-2, SND (NSK); ISR: KLEO, BaBar, BESIII

New experimental input for HVP: BESIII-ISR,VEPP-2000

[Talks Zhiqing Zhang, Simon Eidelman]

c) τ–decay spectra + isospin breakings

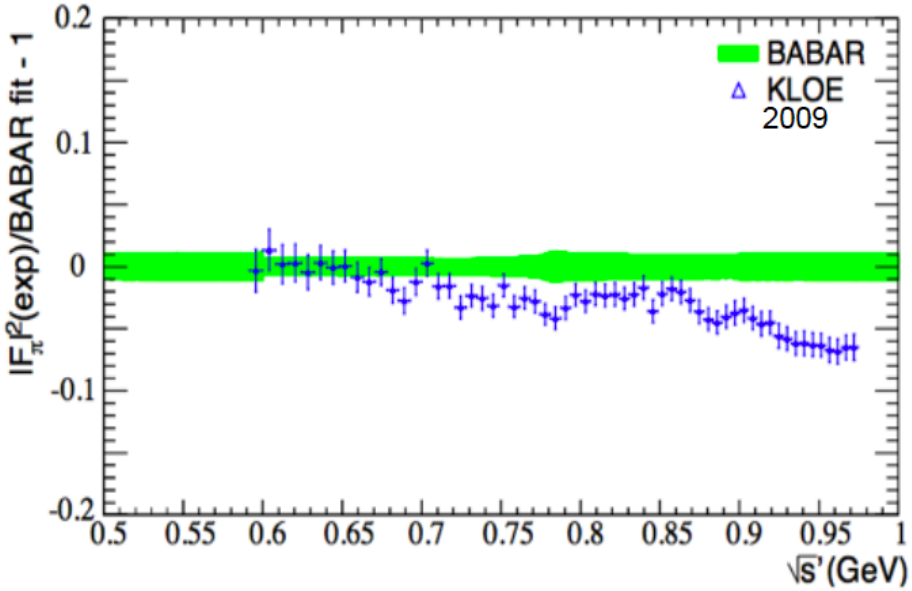


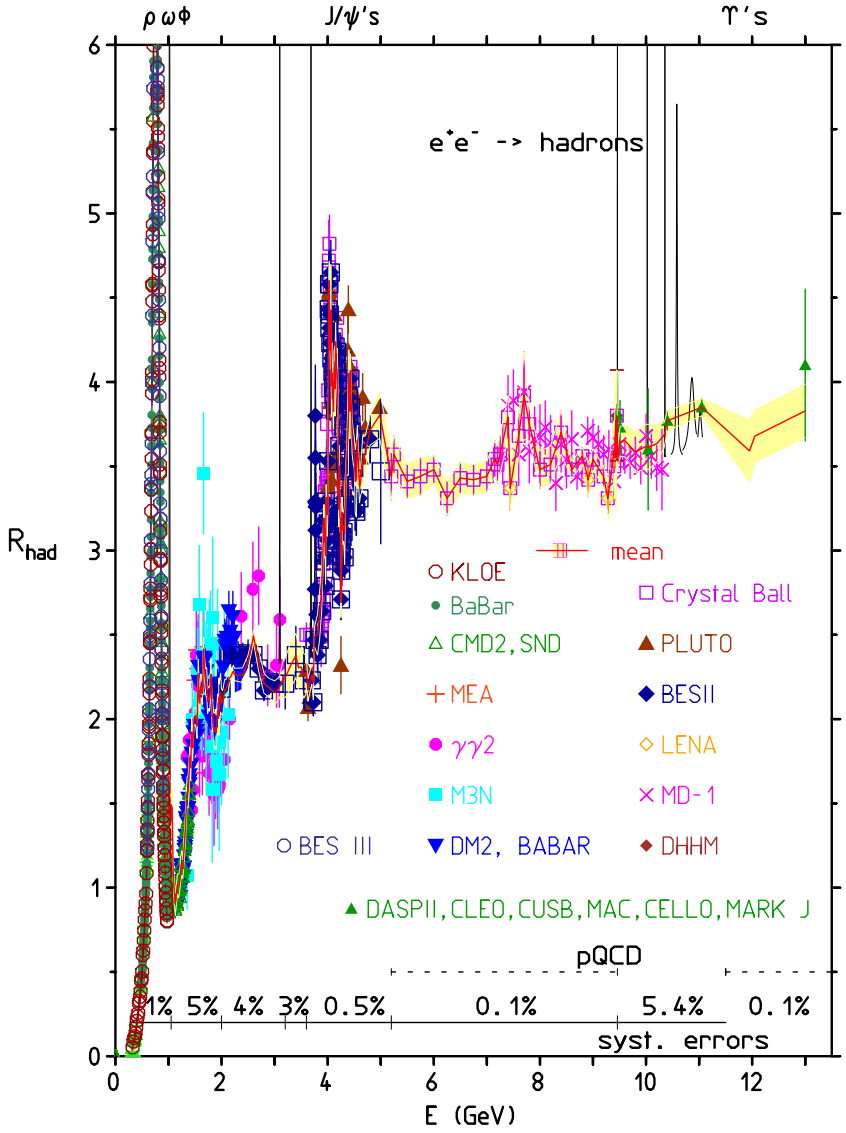
$e^+e^- \to \pi^+\pi^-, \cdots$ vs $\tau^- \to \bar{\nu}_\tau \pi^0 \pi^-, \cdots$

τ–spectra: ALEPH, OPAL, CLEO, Belle

include $I = 1$ $\tau \to \pi\pi\nu_\tau$ in range [0.63-0.96] GeV:

e^+e^-	:	353.82(0.88)(2.17)[2.34]
τ	:	354.25(1.24)(0.61)[1.38]
$e^+e^- + \tau$	:	354.14(0.82)(0.86)[1.19]



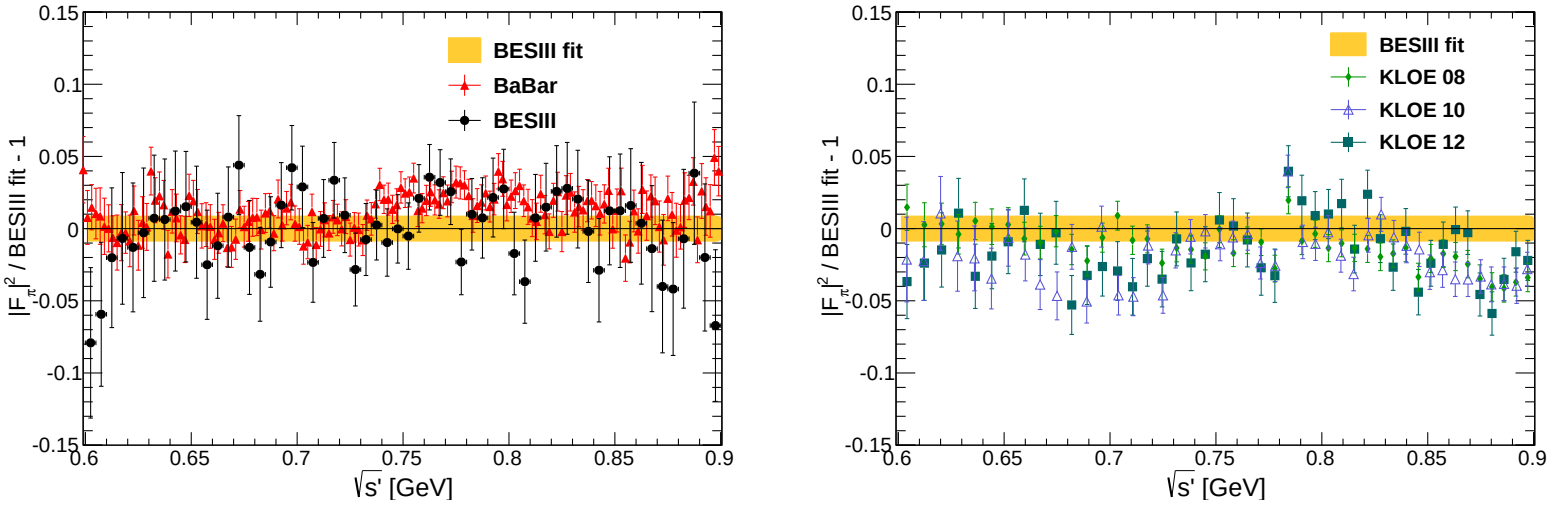


Still: most precise ISR measurements in conflict. BESIII steps to resolve this

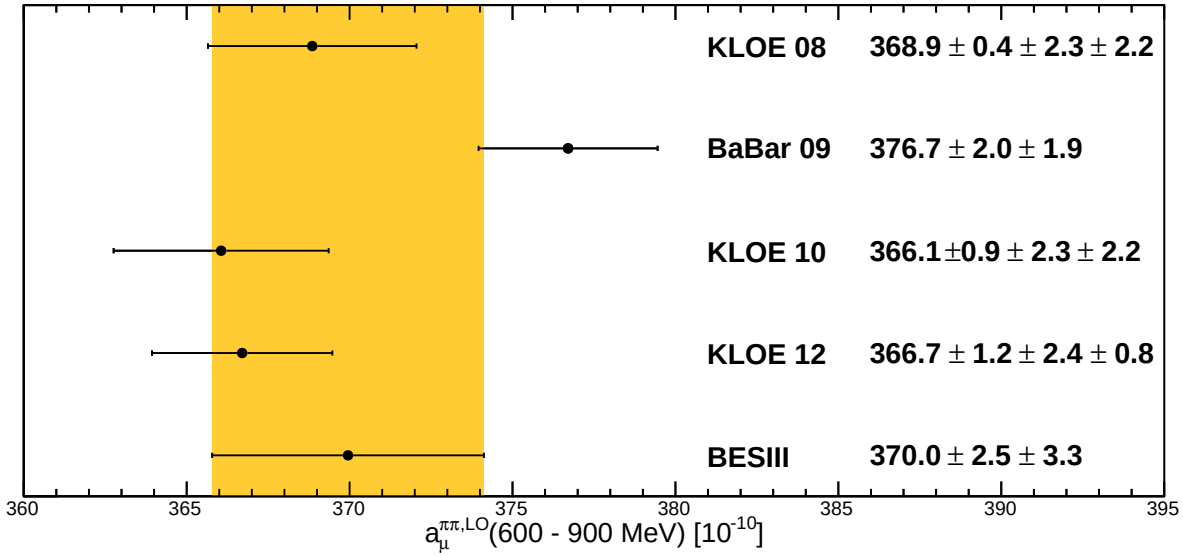
Recent/preliminary results:

- $e^+e^- \to \pi^+\pi^-$ from CMD-3
- $e^+e^- \to \pi^+\pi^-\pi^0$ from Belle
- $e^+e^- \to K^+K^-$ from CMD-3
- $e^+e^- \to K^+K^-$ from SND
- $e^+e^- \to \omega\pi^0 \to \pi^0\pi^0\gamma$ from SND
- $e^+e^- \to \pi^+\pi^-$ from BESIII most recent, most important

New from BESIII



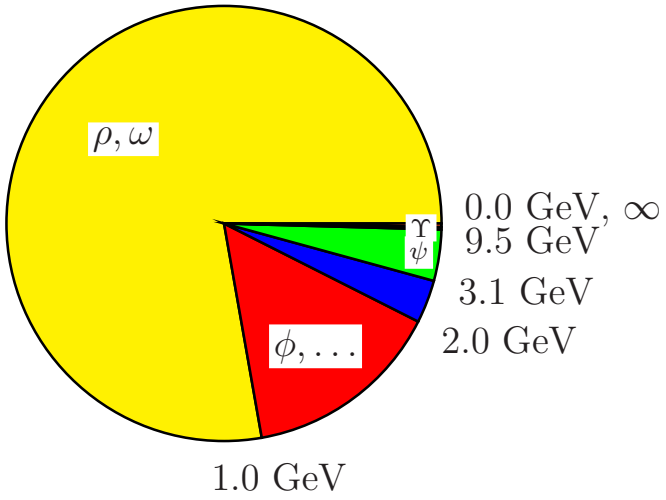
BESSIII vs BaBar and KLOE

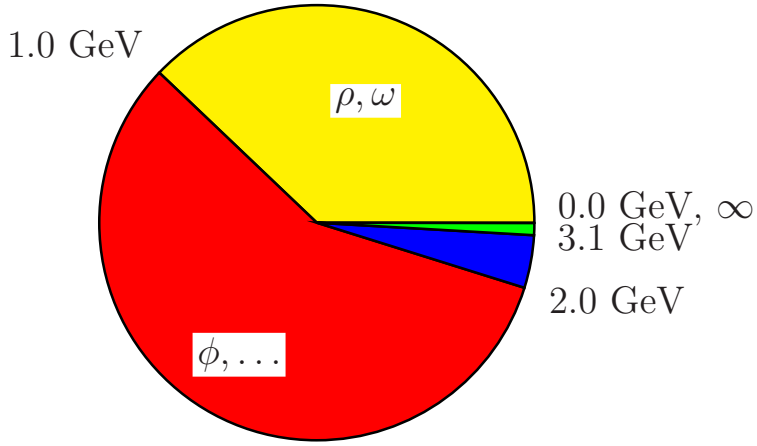


BESIII Collab. arXiv:1507.08188v2

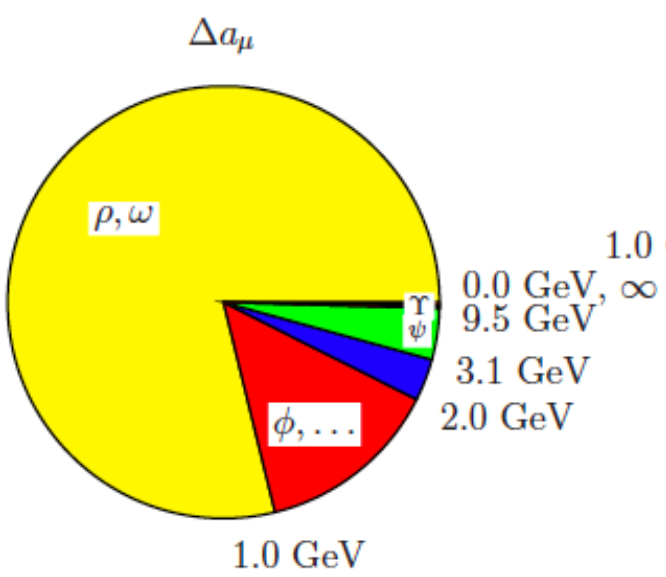
final state	range (GeV)	$a_\mu^{had(1)} \times 10^{10}$ (stat) (syst) [tot]	rel	abs
ρ	(0.28, 1.05)	507.55 (0.39) (2.68)[2.71]	0.5%	39.9%
ω	(0.42, 0.81)	35.23 (0.42) (0.95)[1.04]	3.0%	5.9%
ϕ	(1.00, 1.04)	34.31 (0.48) (0.79)[0.92]	2.7%	4.7%
J/ψ		8.94 (0.42) (0.41)[0.59]	6.6%	1.9%
Υ		0.11 (0.00) (0.01)[0.01]	6.8%	0.0%
had	(1.05, 2.00)	60.45 (0.21) (2.80)[2.80]	4.6%	42.9%
had	(2.00, 3.10)	21.63 (0.12) (0.92)[0.93]	4.3%	4.7%
had	(3.10, 3.60)	3.77 (0.03) (0.10)[0.10]	2.8%	0.1%
had	(3.60, 9.46)	13.77 (0.04) (0.01)[0.04]	0.3%	0.0%
had	(9.46,13.00)	1.28 (0.01) (0.07)[0.07]	5.4%	0.0%
pQCD	(13.0,∞)	1.53 (0.00) (0.00)[0.00]	0.0%	0.0%
data	(0.28,13.00)	687.06 (0.89) (4.19)[4.28]	0.6%	0.0%
total		688.59 (0.89) (4.19)[4.28]	0.6%	100.0%

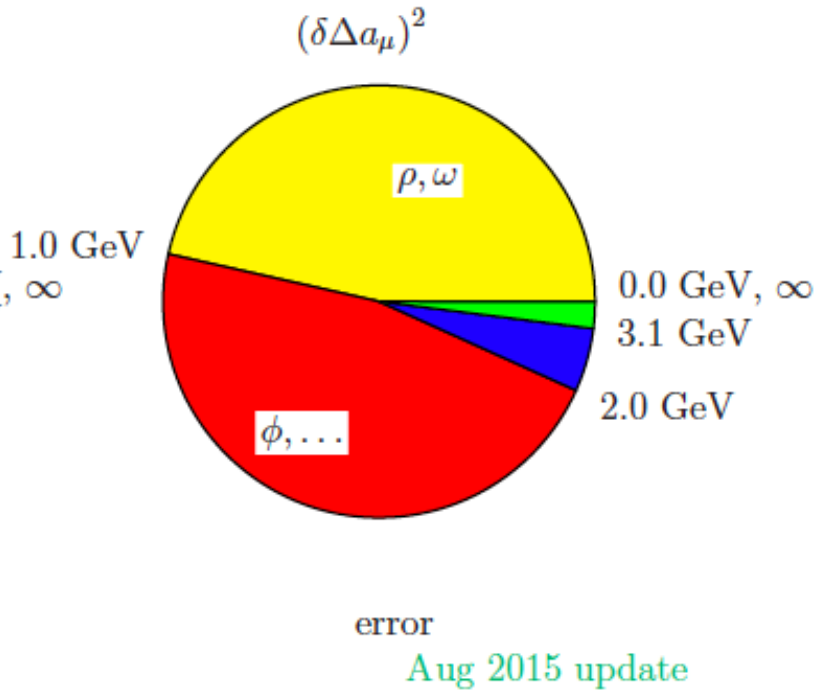
Results for $a_\mu^{had(1)} \times 10^{10}$. Update August 2015, incl SCAN[NSK]+ISR[KLOE10,KLOE12,BaBar,BESIII]





Jun 2007





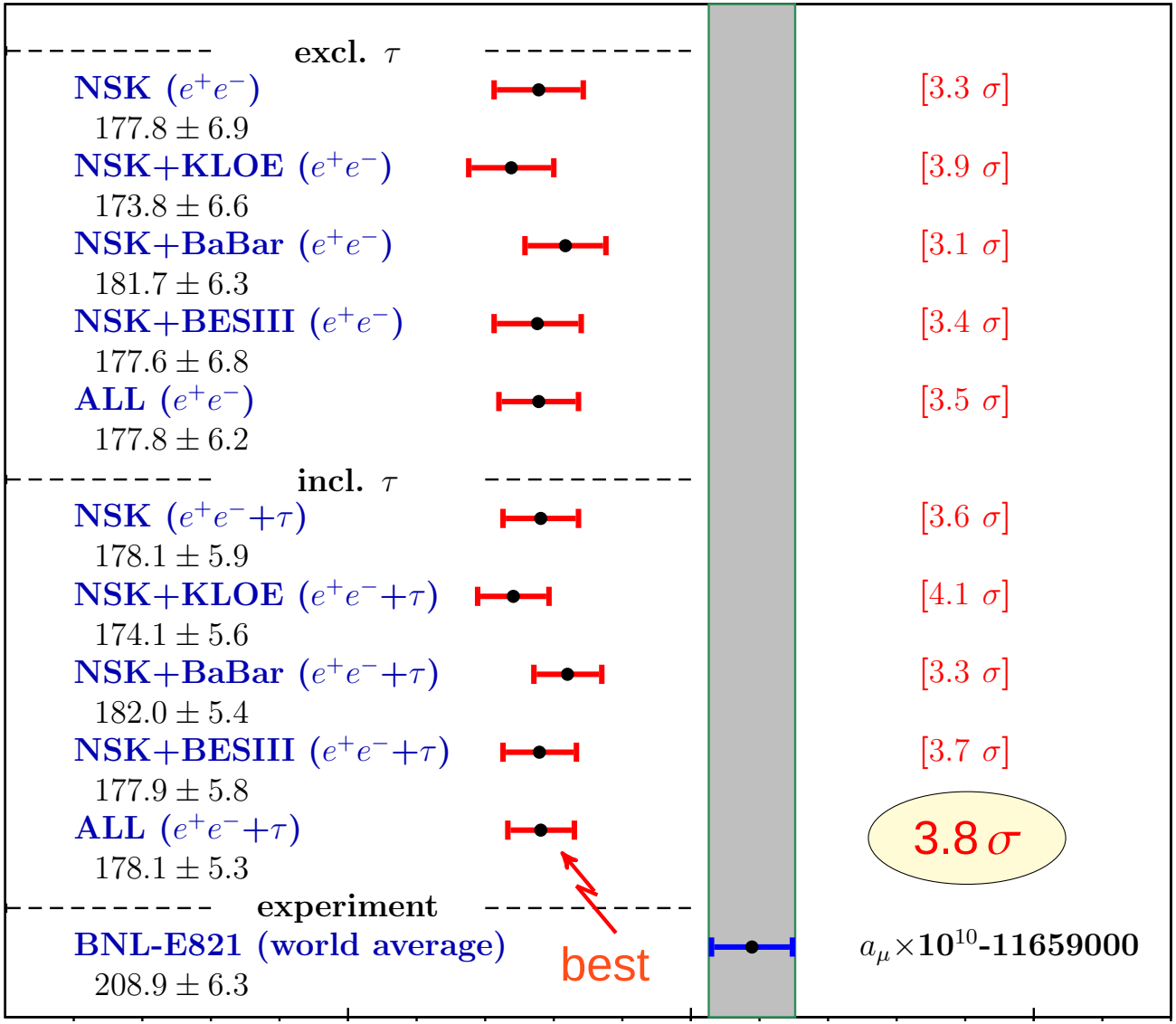
Aug 2015

LO HVP options

Data set	e^+e^-	$e^+e^- + \tau$
NSK	688.59(1.06)(5.63)[5.72]	688.91(1.01)(4.32)[4.43]
NSK+KLOE	684.57(1.00)(5.25)[5.35]	684.89(0.94)(3.95)[4.06]
NSK+BaBar	692.50(0.95)(4.89)[4.98]	692.82(0.89)(3.58)[3.69]
NSK+BESIII	688.41(1.05)(5.42)[5.52]	688.73(1.00)(4.11)[4.23]
ALL	688.59(0.92)(4.72)[4.81]	688.91(0.86)(3.42)[3.52]

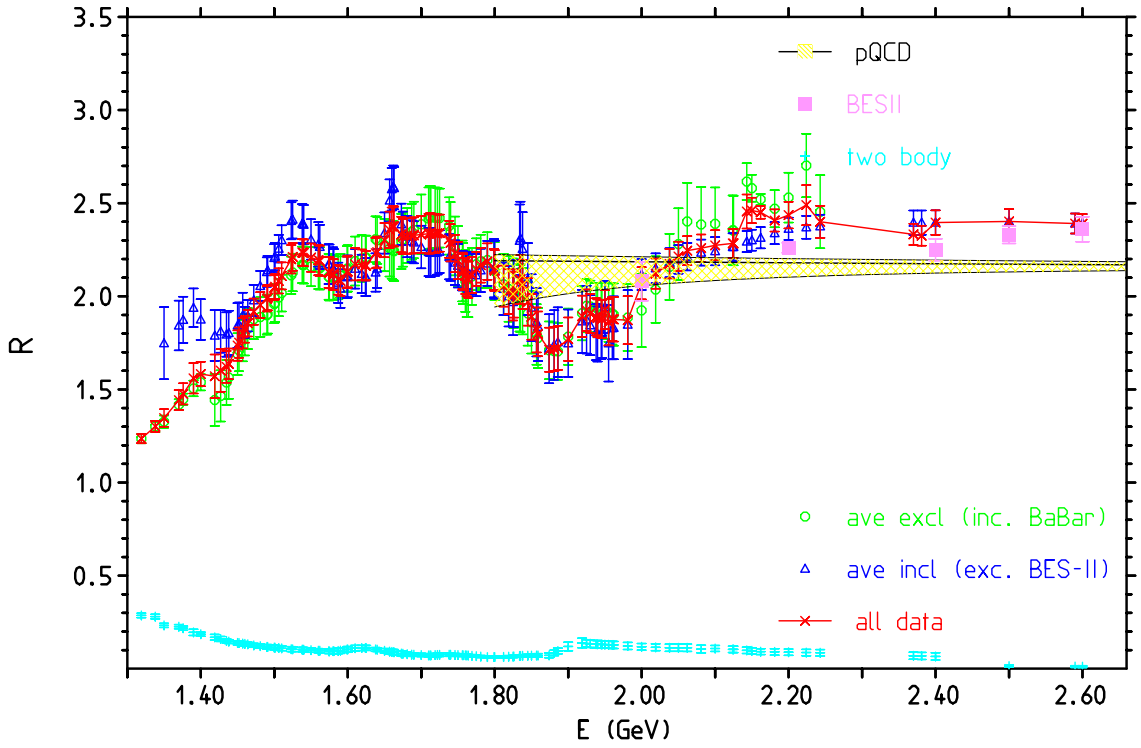
LO, NLO and NNLO:

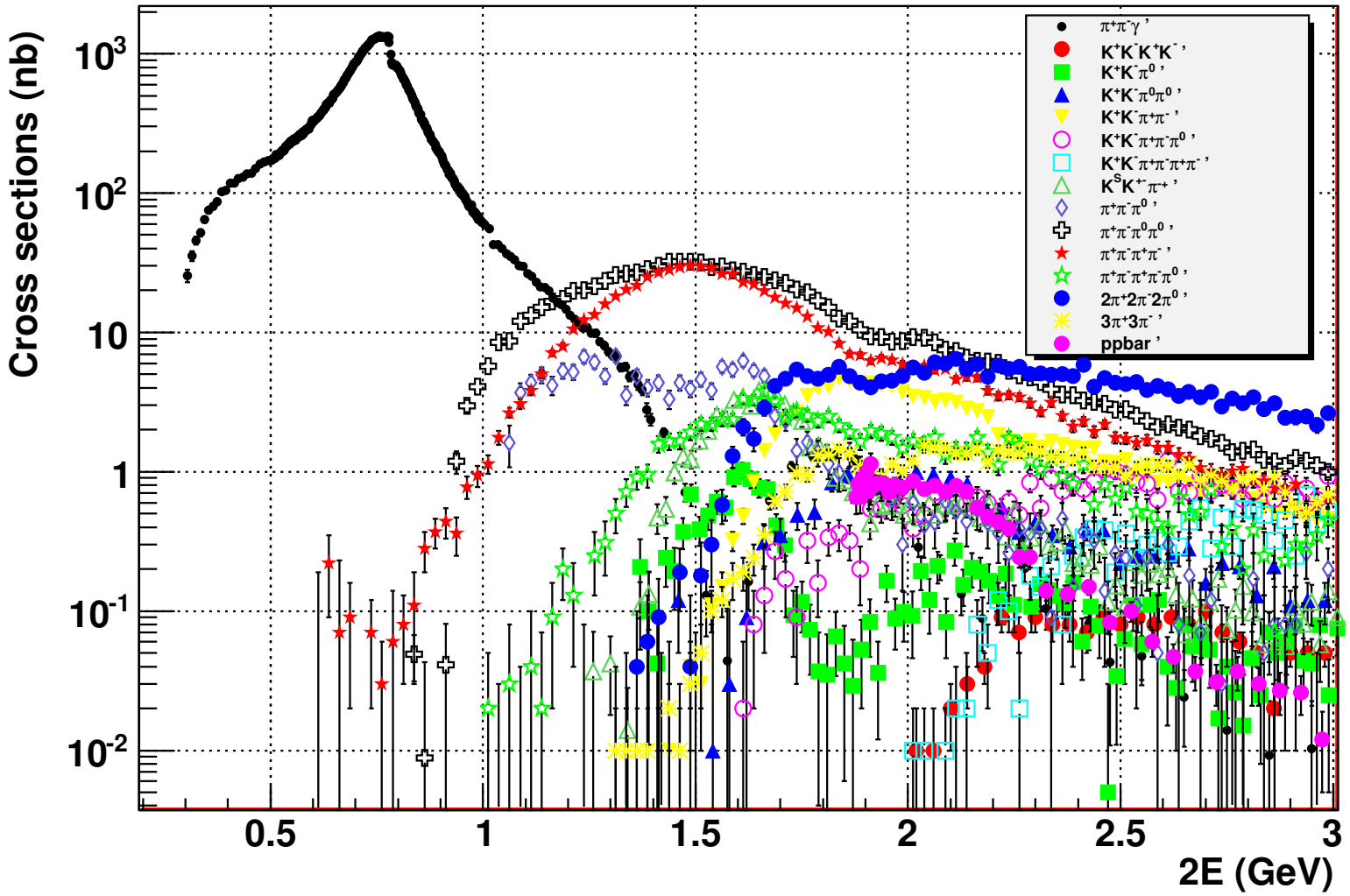
$a_\mu^{\mathrm{had}(1)} = (688.91 \pm 3.52) \, 10^{-10}$ (LO)
$a_\mu^{\mathrm{had}(2)} = (-99.17 \pm 1.00) \, 10^{-10}$ (NLO)
$a_\mu^{\mathrm{had}(3)} = (1.225 \pm 0.012) \, 10^{-10}$ (NNLO) Kurz et al 2014
all e^+e^-–data based [2015 update]


excl. τ
NSK (e^+e^-)
177.8 ± 6.9
[3.3 σ]
NSK+KLOE (e^+e^-)
173.8 ± 6.6
[3.9 σ]
NSK+BaBar (e^+e^-)
181.7 ± 6.3
[3.1 σ]
NSK+BESIII (e^+e^-)
177.6 ± 6.8
[3.4 σ]
ALL (e^+e^-)
177.8 ± 6.2
[3.5 σ]
incl. τ
NSK $(e^+e^-+\tau)$
178.1 ± 5.9
[3.6 σ]
NSK+KLOE $(e^+e^-+\tau)$
174.1 ± 5.6
[4.1 σ]
NSK+BaBar $(e^+e^-+\tau)$
182.0 ± 5.4
[3.3 σ]
NSK+BESIII $(e^+e^-+\tau)$
177.9 ± 5.8
[3.7 σ]
ALL $(e^+e^-+\tau)$
178.1 ± 5.3
3.8 σ
best
experiment
BNL-E821 (world average)
208.9 ± 6.3
$a_\mu \times 10^{10}$-11659000

❒ Main issue in HVP: $R(s)$ in 1.2 to 2.4 GeV

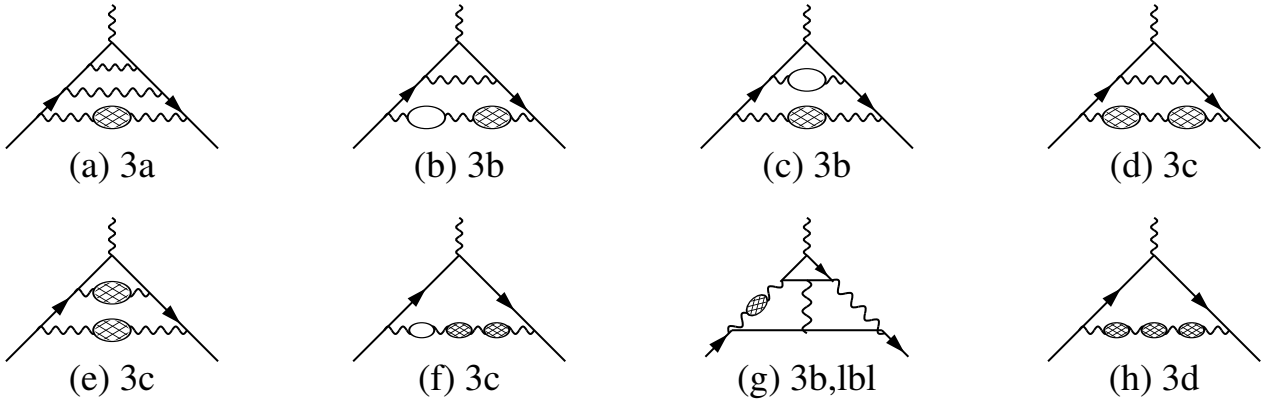
- region 1.2 to 2 GeV bad data; test-ground exclusive vs inclusive R measurements (more than 30 channels!) Who will do it? BES III radiative return!





Eidelman et al 2011

❒ NNLO HVP effects based on 2015 update

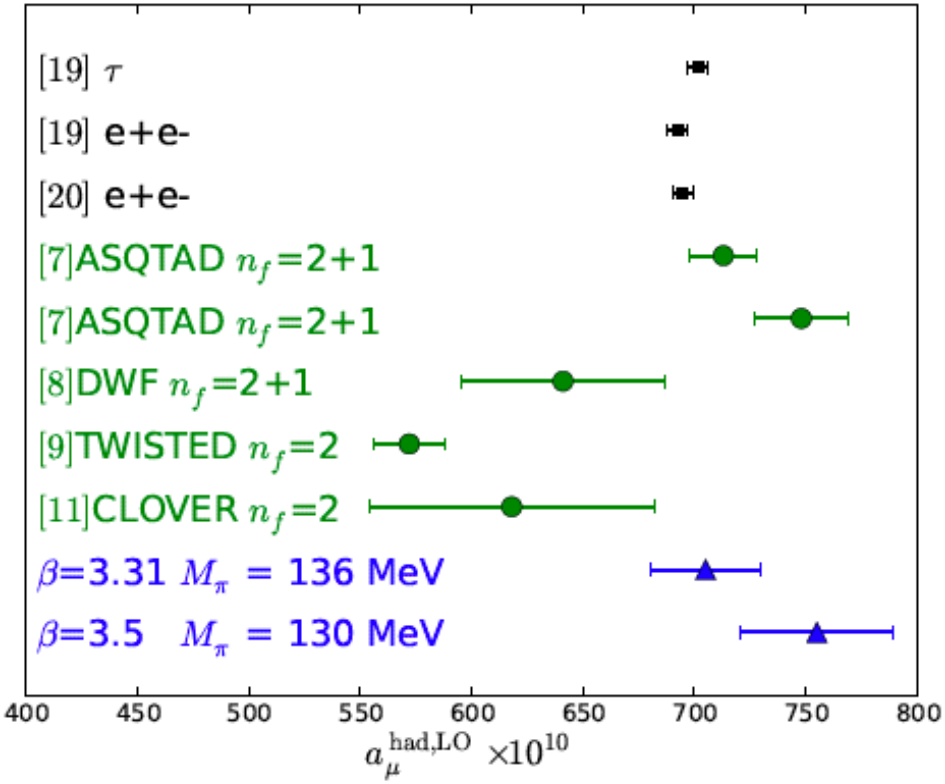


Class		results Kurz et al	my evaluation
$a_\mu^{(3a)}$	=	0.80×10^{-10}	$0.782(77) \times 10^{-10}$
$a_\mu^{(3b)}$	=	-0.41×10^{-10}	$-0.403(37) \times 10^{-10}$
$a_\mu^{(3b,\mathrm{lbl})}$	=	0.91×10^{-10}	$0.900(77) \times 10^{-10}$
$a_\mu^{(3c)}$	=	-0.06×10^{-10}	$-0.0544(7) \times 10^{-10}$
$a_\mu^{(3d)}$	=	0.0005×10^{-10}	$5.22(15) \times 10^{-14}$
$a_\mu^{\mathrm{had,NNLO}}$	=	$12.4(1) \times 10^{-11}$	$12.25(12) \times 10^{-11}$

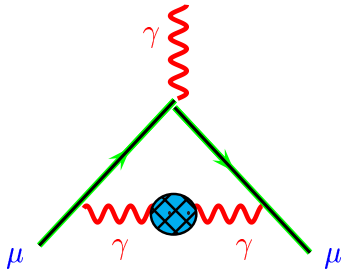
Kurz et al. 2014 [Talk Matthias Steinhauser]

❒ HVP from lattice QCD

Lattice: $\sim$ 3-10% quoted errors, but incomplete, Experiment: 0.6% errors



Plot from Laurent Lellouch



$$a_\mu^{(2)\,\rm had} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty {\rm d}Q^2\, f(Q^2)\, \Pi(Q^2)$$

Very promising in the near future!

Brookhaven, Zeuthen, Mainz, Edinburgh, ...

[Talk Marcus Petschlies]

Effective field theory: the Resonance Lagrangian Approach

HVP dominated by spin 1 resonance physics! need theory of $\rho, \omega, \phi, \cdots$

❒ Principles to be included: Chiral Structure of QCD, VMD & electromagnetic gauge invariance.

❖General framework: resonance Lagrangian extension of chiral perturbation theory (CHPT), i.e. implement VMD model with Chiral structure of QCD. Specific version Hidden Local Symmetry (HLS) effective Lagrangian Bando, Kugo, Yamawaki. First applied to HLbL of muon $g-2$ Hayakawa, Kinoshita, Sanda to HVP Benayoun, David, DelBuono. Equivalent: ENJL model Bijnens, Pallante, Prades [Talk Johan Bijnens]

Global Fit strategy: [Talk Maurice Benayoun]
Data below $E_0 = 1.05$ GeV (just above the ϕ) constrain effective Lagrangian couplings, using 45 different data sets (6 annihilation channels and 10 partial width decays).

- Effective theory predicts cross sections:

$$\pi^+\pi^-,\ \pi^0\gamma,\ \eta\gamma,\ \eta'\gamma,\ \pi^0\pi^+\pi^-,\ K^+K^-,\ K^0\bar{K}^0 \quad (83.4\%),$$

- Missing part:

$$4\pi, 5\pi, 6\pi, \eta\pi\pi, \omega\pi \text{ and regime } E > E_0$$

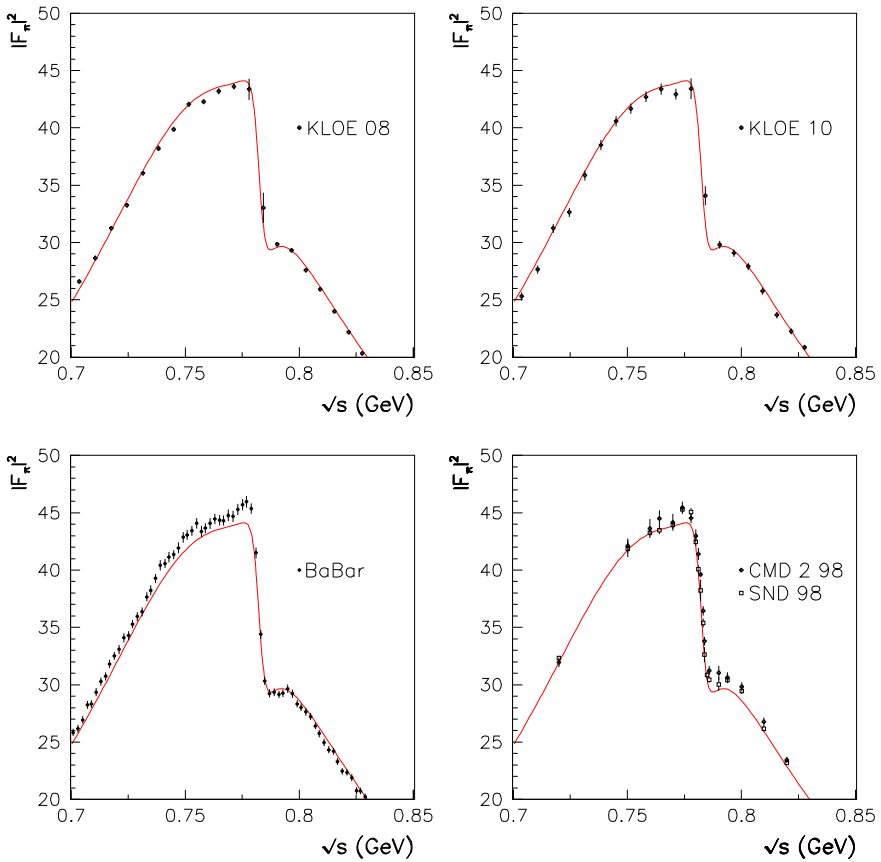
  evaluated using data directly and pQCD for perturbative region and tail
- Including self-energy effects is mandatory ($\gamma\rho$-mixing, $\rho\omega$-mixing ..., decays with proper phase space, energy dependent width etc)
- Method works in reducing uncertainties by using indirect constraints
- Able to reveal inconsistencies in data, e.g. KLOE vs BaBar

Main goal:

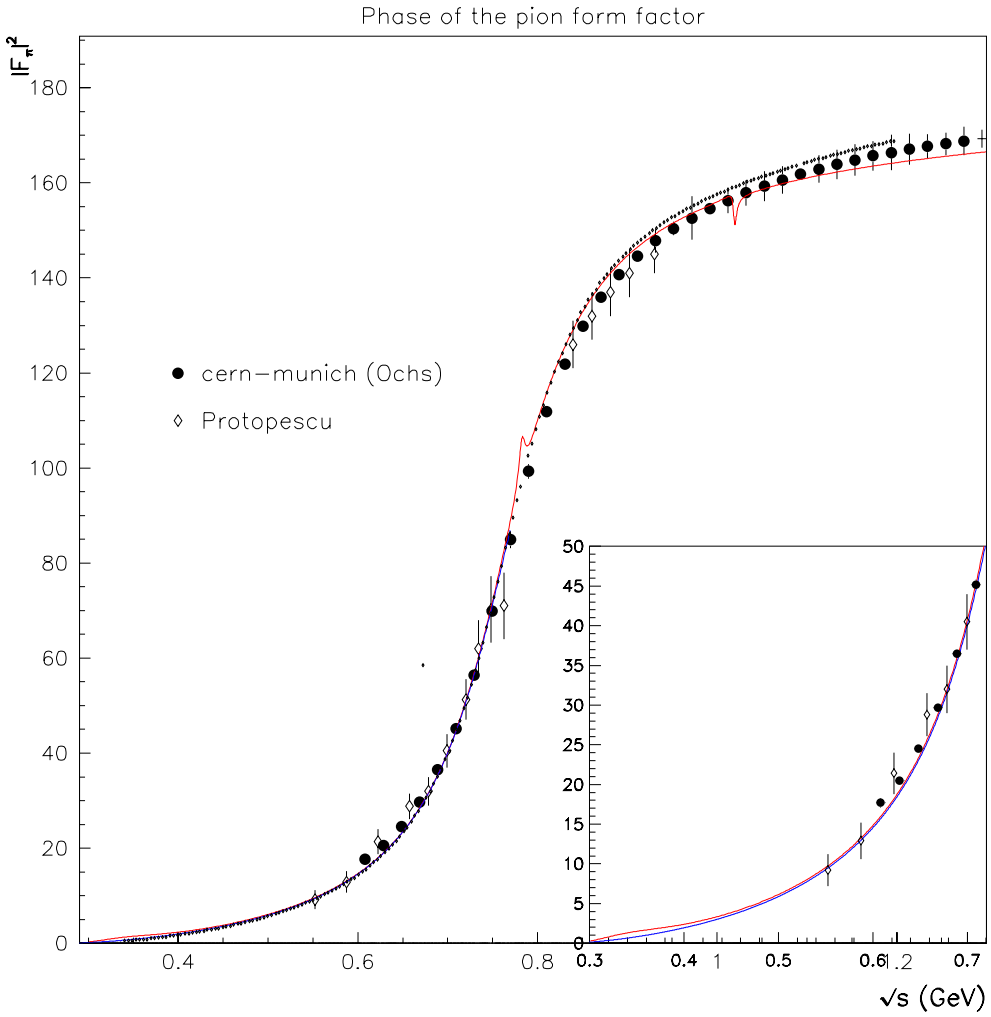
- Single out representative effective resonance Lagrangian by global fit

  is expected to help in improving EFT calculations of hadronic light-by-light scattering (such concept so far missing)
- helps improving uncertainty on hadronic VP, besides e^+e^-, use τ decay data and other experimental information consistently

❑ Fit of τ +PDG vs $\pi^+\pi^-$–data



Benayoun et al 2012/13



The $\pi\pi$ scattering phase of our HLS prediction

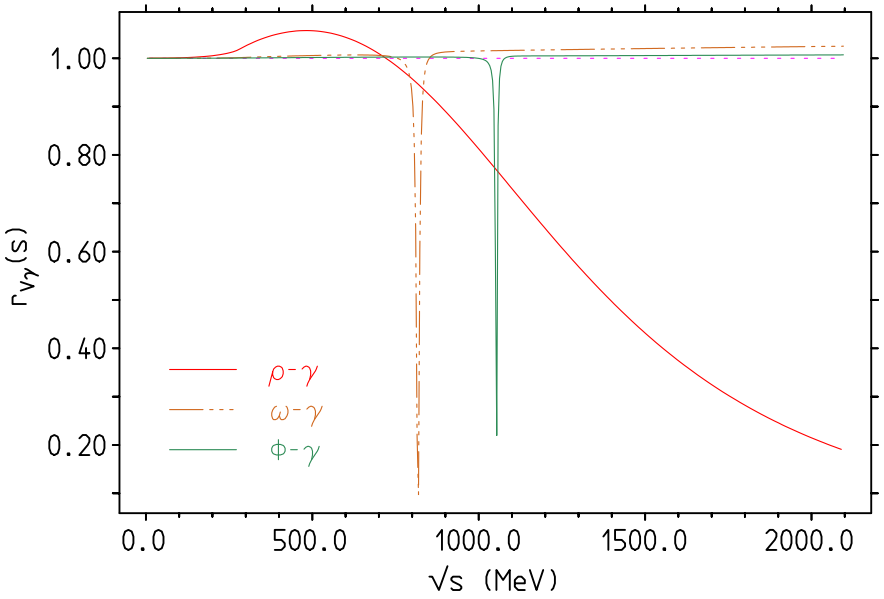
There is no τ (charged channel) vs. e^+e^- (neutral channel) puzzle!

F.J.& R. Szafron, Benayoun et al, $\rho-\gamma$ interference

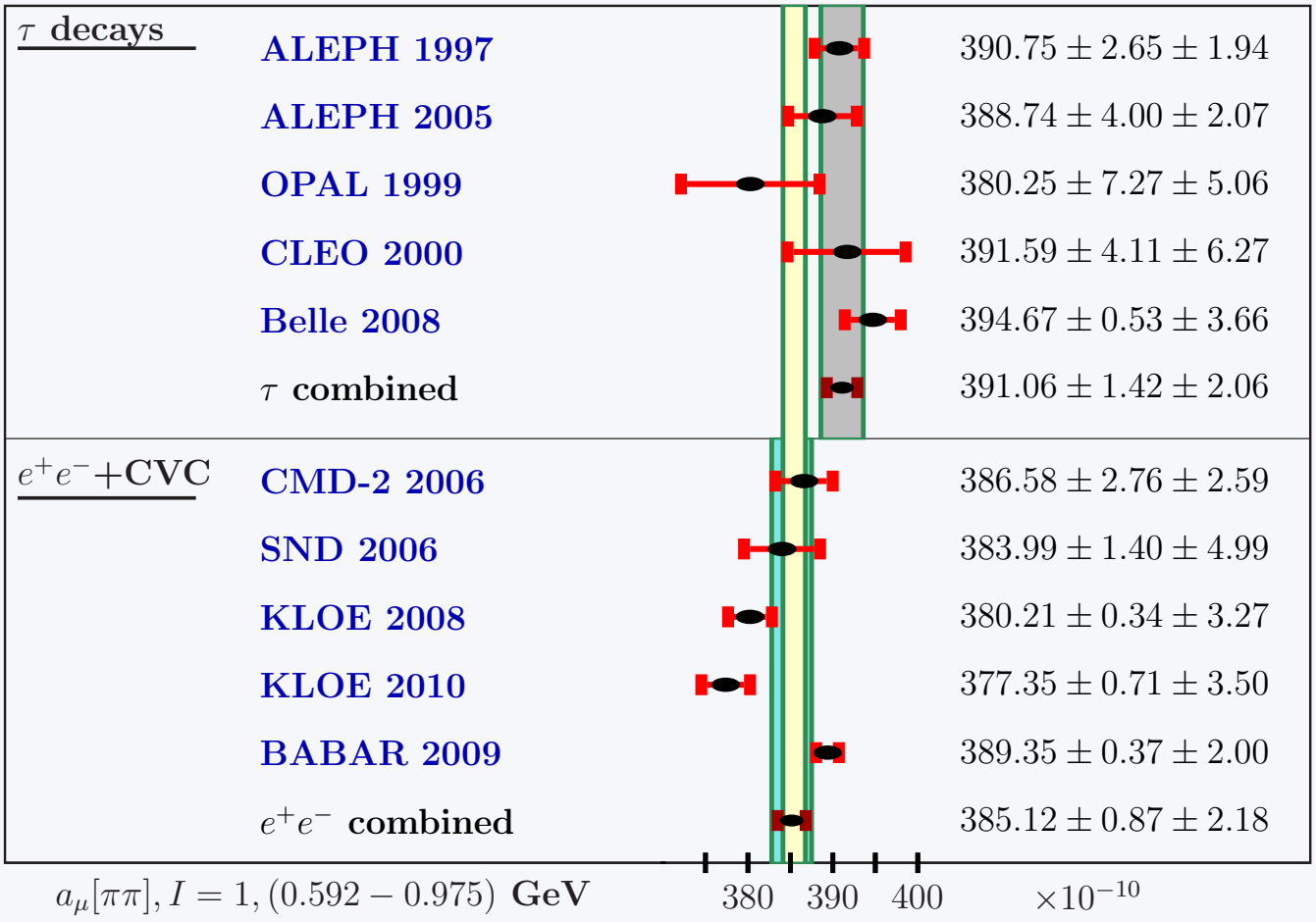
(absent in charged channel): "model" sQED+VMDII $\Rightarrow$fixed by $\Gamma_{\rho ee}$!!!

$$-\mathrm{i}\,\Pi^{\mu\nu\ (\pi)}_{\gamma\rho}(q) =$$  .

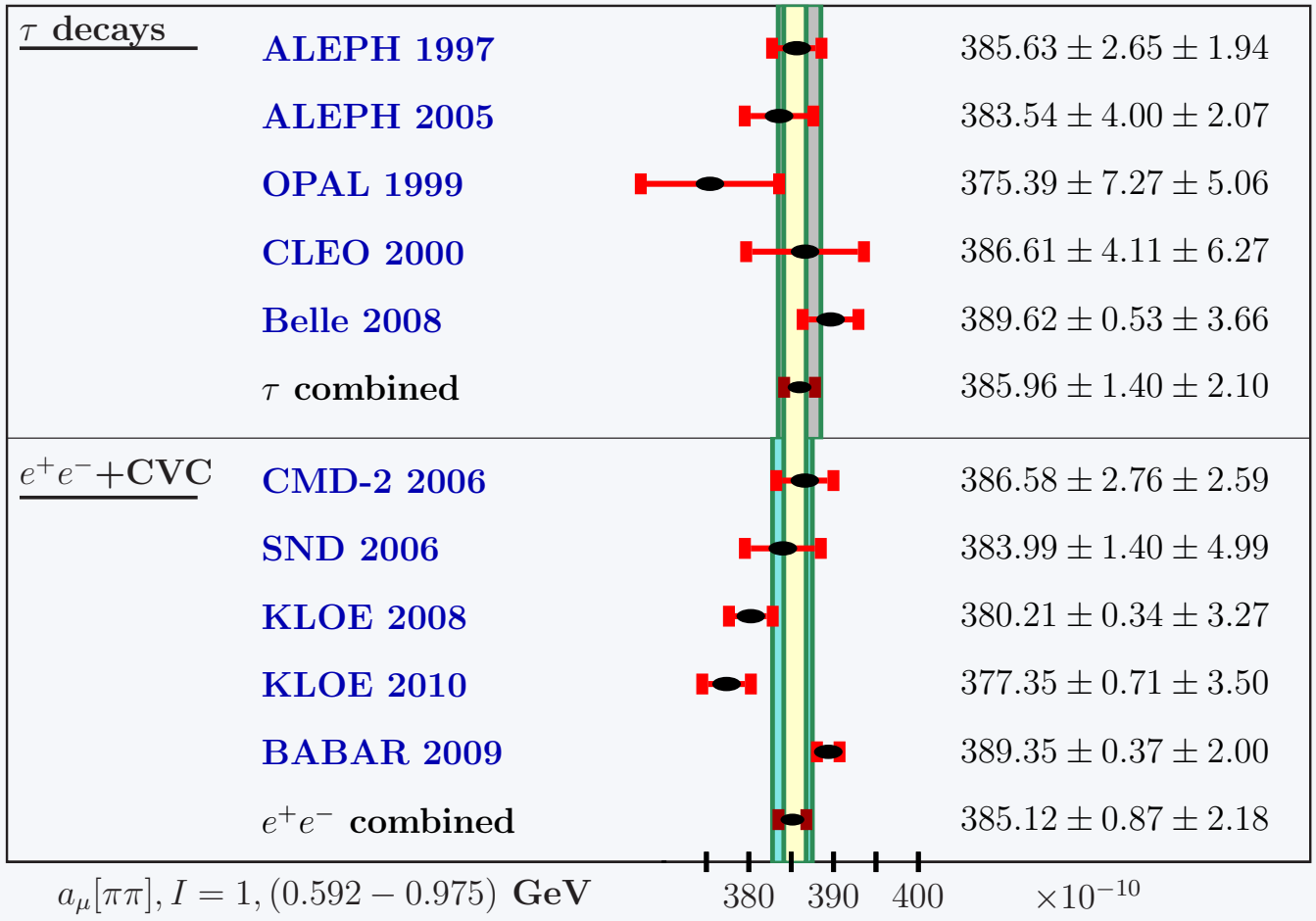
$$v_0(s) = r_{\rho\gamma}(s)\, R_{\rm IB}(s)\, v_-(s)$$



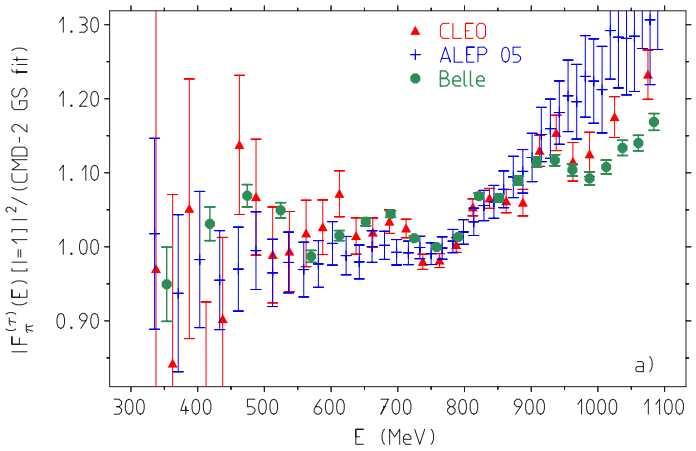
- τ require to be corrected for missing $\rho-\gamma$ mixing!
- results obtained from e^+e^- data is what goes into a_μ
- off-resonance tiny for ω,ϕ in $\pi\pi$ channel (scaled up $\Gamma_V/\Gamma(V\to\pi\pi)$



I=1 part of $a_\mu^{\rm had}[\pi\pi]$

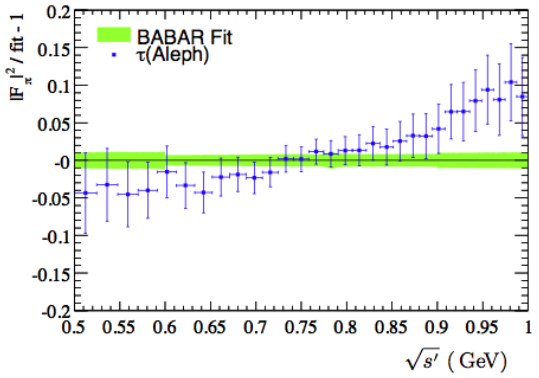


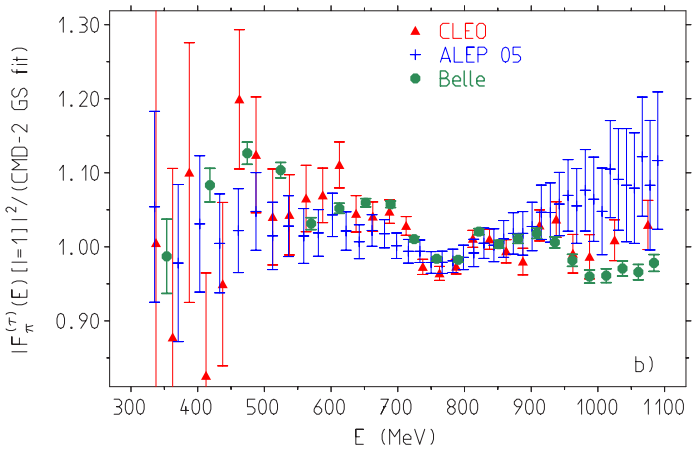
I=1 part of $a_\mu^{\rm had}[\pi\pi]$



$|F_\pi(E)|^2$ in units of e^+e^- I=1 (CMD-2 GS fit)

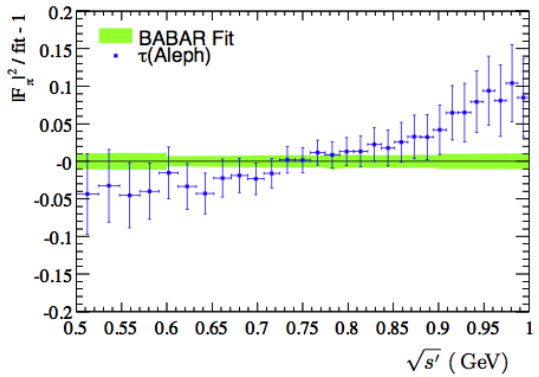
Best “proof”:

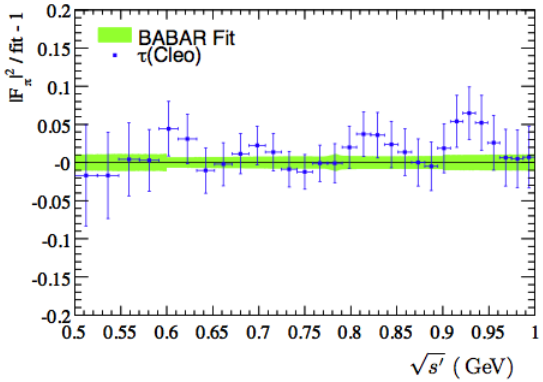


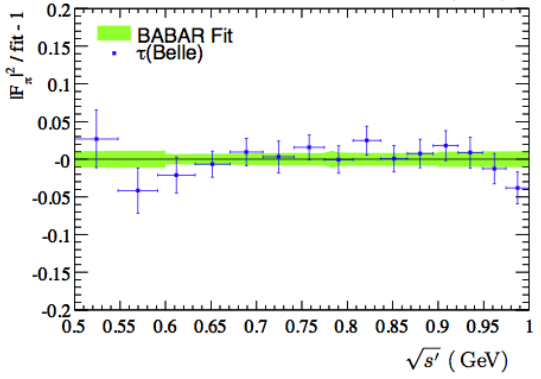


$|F_\pi(E)|^2$ in units of e^+e^- I=1 (CMD-2 GS fit)

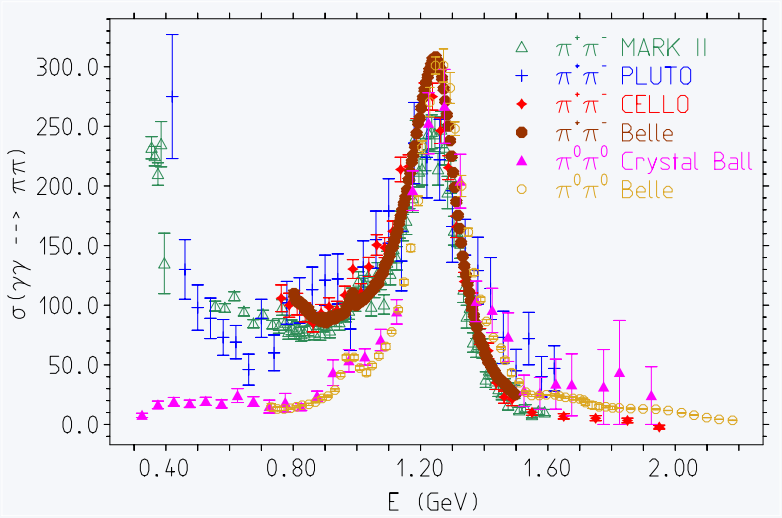
Best “proof”:







❑ Is our model "photons couple to point-like pions" viable?



How photons couple to pions? Use $\gamma\gamma \to \pi^+\pi^-, \pi^0\pi^0$ as a probe: what we see: 1) below about 1 GeV photons couple to pions as point-like objects (i.e. to the charged ones overwhelmingly), 2) at higher energies the photons see the quarks exclusively and form the prominent tensor resonance $f_2(1270)$. Plotted $2\,\sigma(\pi^0\pi^0)$ vs. $\sigma(\pi^+\pi^-)$

Strong tensor meson resonance in $\pi\pi$ channel $f_2(1270)$ with photons directly probe the quarks! Contribution to $a_\mu^{\text{had LbL}}$ (see Pauk, Vanderhaeghen)

Can global fit like HLS discriminate between incompatible data sets?

- The problem of inconsistent data is not a problem of whatever model
- modeling is indispensable for interrelating different data channels
- in HLS fit τ data play a central role as they are simple, i.e. no singlet contribution, no $\gamma, \rho^0, \omega, \phi$– mixing
- τ—spectra supplemented with PDG isospin breaking, provide a good initial shape for most e^+e^--data fits, which are optimized by iteration for a best simultaneous fit
- Why do we get slightly lower results for HVP?
- BaBar data according to Davier et al are in good accord with Belle τ-data, without correcting τ-data for the substantial and quite unambiguous $\gamma - \rho^0$ mixing effects! This is a problem for the BaBar data. They are disfavored by the global fit! BaBar-data rise HVP quite substantially towards the uncorrected τ-data value, which we know is not what enters $g-2$ directly..

- in contrast the NSK, KLOE and new BESIII data are in very good agreement with τ + PDG, so they dominate the fit and give slightly lower HVP!

- besides the e^+e^--data, additional data constrain the HLS Lagrangian and its parameters, which leads to reduced uncertainties and an increased significance

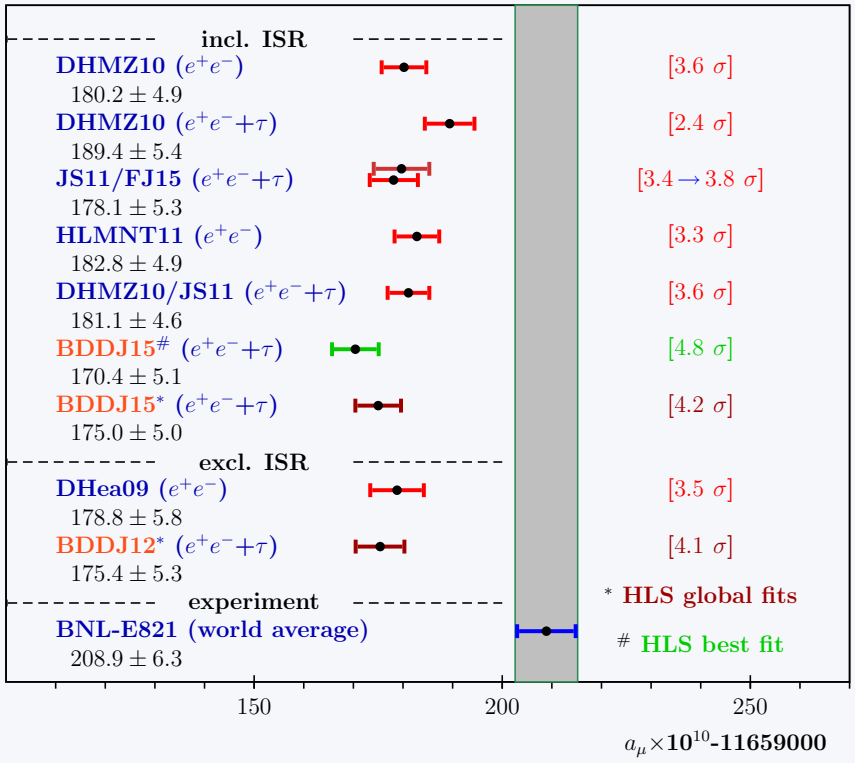
- what are the model (using specifically HLS) errors? Best do a corresponding analysis based on different implementations of the resonance Lagrangian approach. Try to include higher order corrections

- photon radiation on HLS results are now implemented in the CARLOMAT Monte Carlo by Karol Kołodziej

- our analysis is a first step in a direction which should allow for systematic improvements

- specifying a viable phenomenological effective Lagrangian is expected to help for modeling HLbL

Comparison of different estimates and leading uncertainties:



Note: some do not include τ data. HLS best fit (NSK+KLOE10+KLOE12) does not include BaBar data. The JS11/FJ15 is updated to include the new BES III data.

The challenge:

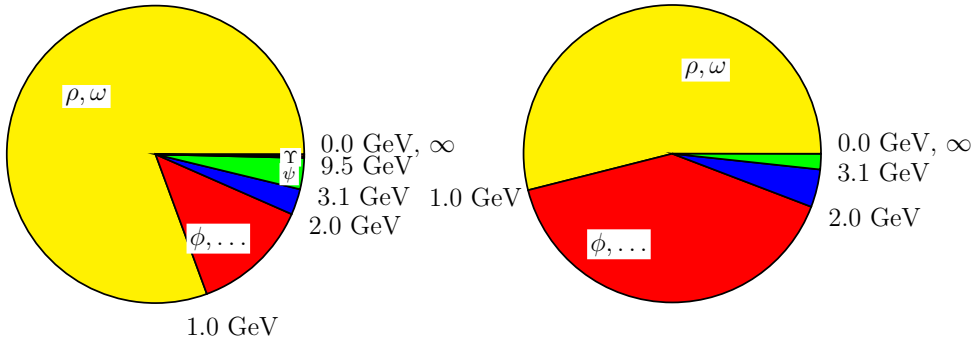
$a_\mu^{\text{had,VP}}[LO]$	$(6889 \pm 35) \times 10^{-11}$	+59.09 ±0.30 ppm
$a_\mu^{\text{had,VP}}[NLO]$	$(-99 \pm 1) \times 10^{-11}$	
$a_\mu^{\text{had,VP}}[NNLO]$	$(12 \pm 0) \times 10^{-11}$	
a_μ^{EW}	$(154 \pm 1) \times 10^{-11}$	
$a_\mu^{\text{had,LbL}}$	$[(105 \div 106) \pm (26 \div 39)] \times 10^{-11}$	+0.90 ±0.28 ppm
$\delta a_\mu^{\text{exp}}$ present	63×10^{-11}	±0.54 ppm
$\delta a_\mu^{\text{exp}}$ future	16×10^{-11}	±0.14 ppm

Next generation experiments require a factor 4 reduction of the uncertainty optimistically feasible is factor 2 we hope

HVP for the electron anomaly

$a_e^{\mathrm{had(1)}} = (185.11 \pm 1.24)\ 10^{-14}$ (LO)
$a_e^{\mathrm{had(2)}} = (-22.15 \pm 0.16)\ 10^{-14}$ (NLO)
$a_e^{\mathrm{had(3)}} = (\ \ 2.80 \pm 0.02)\ 10^{-14}$ (NNLO) Kurz et al 2014
all e^+e^-–data based [2015 update]

The NNLO result compares to the LO hadronic LbL contribution $a_e^{\mathrm{had,LbL)}} = 3.7(5) \times 10^{-14}$ and weak contribution $a_e^{\mathrm{weak}} = 3.0 \times 10^{-14}$.



Aug 2015 update

final state	range (GeV)	$a_e^{\mathrm{had}(1)} \times 10^{14}$ (stat) (syst) [tot]	rel	abs
ρ	(0.28, 1.05)	139.86 (0.12) (0.75)[0.76]	0.5%	45.8%
ω	(0.42, 0.81)	9.38 (0.11) (0.25)[0.28]	3.0%	6.2%
ϕ	(1.00, 1.04)	8.78 (0.12) (0.20)[0.24]	2.7%	4.5%
J/ψ		2.12 (0.10) (0.10)[0.14]	6.6%	1.6%
Υ		0.02 (0.00) (0.00)[0.00]	6.8%	0.0%
had	(1.05, 2.00)	14.95 (0.05) (0.69)[0.69]	4.6%	38.0%
had	(2.00, 3.10)	5.19 (0.03) (0.22)[0.22]	4.3%	4.0%
had	(3.10, 3.60)	0.90 (0.01) (0.02)[0.02]	2.8%	0.0%
had	(3.60, 9.46)	3.25 (0.01) (0.00)[0.01]	0.3%	0.0%
had	(9.46,13.00)	0.30 (0.00) (0.02)[0.02]	5.4%	0.0%
pQCD	(13.0,∞)	0.36 (0.00) (0.00)[0.00]	0.0%	0.0%
data	(0.28,13.00)	184.75 (0.24) (1.09)[1.12]	0.6%	0.0%
total		185.11 (0.24) (1.09)[1.12]	0.6%	100.0%

Results for $a_e^{\mathrm{had}(1)}$ in units 10^{-14}

$$a_e^{\rm QED} = \frac{\alpha}{2\pi} - 0.328\,478\,444\,002\,55(33)\left(\frac{\alpha}{\pi}\right)^2 + 1.181\,234\,016\,816(11)\left(\frac{\alpha}{\pi}\right)^3$$
$$-1.9097(20)\left(\frac{\alpha}{\pi}\right)^4 + 9.16(58)\left(\frac{\alpha}{\pi}\right)^5 .$$

For extracting $\alpha_{\rm QED}$ based on the SM prediction

$$a_e^{\rm SM} = a_e^{\rm QED} + 1.725(12)\times 10^{-12}\ \text{(hadronic \& weak)}$$

Note: weak part reevaluated

$$a_e^{\rm weak} = 0.030\times 10^{-12}$$

replacing 0.039×10^{-12} in JN 2009 Phys. Rep. M. Passera private communication

$$a_e^{\rm exp} = 0.001\,159\,652\,180\,73(28)$$

which yields

$$\alpha^{-1}(a_e) = 137.0359991685(342)(68)(46)(24)[353]\ ,$$

which is close [85 → 57] to the value

$$\alpha^{-1}(a_e) = 137.0359991657(342)[0.25\,\text{ppb}] \quad ,$$

obtained by Aoyama et al 2012

Best test (new physics?) using α from atomic Interferometry:

Input: $\alpha^{-1}(\text{Rb11}) = 137.035999037(91)[0.66\,\text{ppb}]$

$$a_e^{\text{the}} = 0.001\,159\,652\,181\,87(77)$$

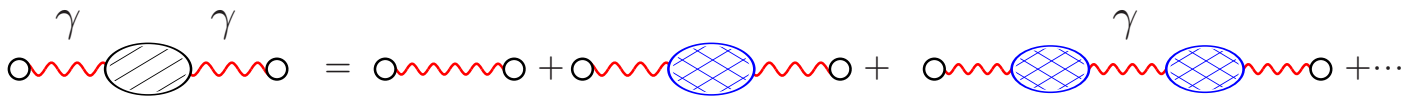
$$a_e^{\text{exp}} - a_e^{\text{the}} = -1.14(0.82) \times 10^{-12}$$

[Talk Saida Guellati]

As we know the better New Physics monitor: muon $g-2$

HVP subtraction of $R(s)$: a problem of the DR method?

Full photon propagator Dyson resummation of 1PI part (blue blob)



$$\begin{aligned} \mathrm{i}\, D'_\gamma(q^2) &\equiv \frac{-\mathrm{i}}{q^2} + \frac{-\mathrm{i}}{q^2}(-\mathrm{i}\Pi_\gamma)\frac{-\mathrm{i}}{q^2} + \frac{-\mathrm{i}}{q^2}(-\mathrm{i}\Pi_\gamma)\frac{-\mathrm{i}}{q^2}(-\mathrm{i}\Pi_\gamma)\frac{-\mathrm{i}}{q^2} + \cdots \\ &= \frac{-\mathrm{i}}{q^2}\left\{1 + \left(\frac{-\Pi_\gamma}{q^2}\right) + \left(\frac{-\Pi_\gamma}{q^2}\right)^2 + \cdots\right\} \\ &= \frac{-\mathrm{i}}{q^2}\left\{\frac{1}{1+\frac{\Pi_\gamma}{q^2}}\right\} = \frac{-\mathrm{i}}{q^2+\Pi_\gamma(q^2)} = \frac{-\mathrm{i}}{q^2}\,\frac{1}{1+\Pi'_\gamma(q^2)}\,. \end{aligned}$$

Including external e.m. coupling

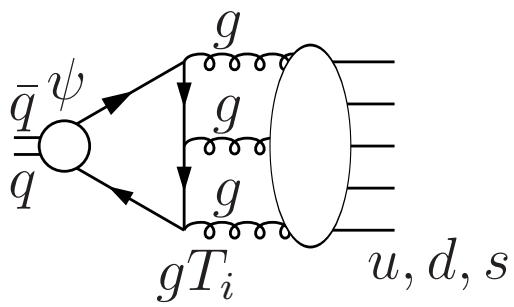
$$\mathrm{i}\, e^2\, D'_\gamma(q^2) = \frac{-\mathrm{i}}{q^2}\,\frac{e^2}{1+\Pi'_\gamma(q^2)}$$

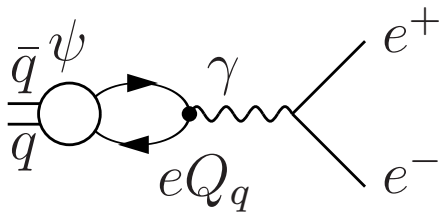
Effective charge

$$\frac{e^2}{1+\Pi'_\gamma(s)} = \frac{e^2}{1-\Delta\alpha(s)} = e^2(s)$$

Usually, $\Delta\alpha(s)$ is a correction i.e $\Delta\alpha(s) \ll 1$ and the Dyson series converges well.

Exceptions: narrow OZI suppressed resonances (below $q\bar{q}$-thresholds)





Γ_{ee} not much smaller than $\Gamma_{\rm QCD}$ (i.e strong decays): $J/\psi, \psi_2, \Upsilon_1, \Upsilon_2, \Upsilon_3$

Note: imaginary parts from narrow resonances, $\mathrm{Im}\,\Pi'(s)) = \frac{\alpha}{3} R(s) = \frac{3}{\alpha}\frac{\Gamma_{ee}}{\Gamma}$ at peak, are sharp spikes and are obtained correctly only by appropriately high resolution scans. For example,

$$\boxed{|1 - \Pi'(s)|^2 - (\alpha/\alpha(s))^2 = (\mathrm{Im}\,\Pi'(s))^2}$$

at $\sqrt{s} = M_R$ is given by

ρ	1.23	$\times 10^{-3}$
ω	2.76	$\times 10^{-3}$
ϕ	1.56	$\times 10^{-2}$
J/ψ	594.81	
ψ_2	9.58	
ψ_3	2.66	$\times 10^{-4}$
Υ_1	104.26	
Υ_2	30.51	
Υ_3	55.58	

- What is measured in an experiment is the full propagator, corresponding to

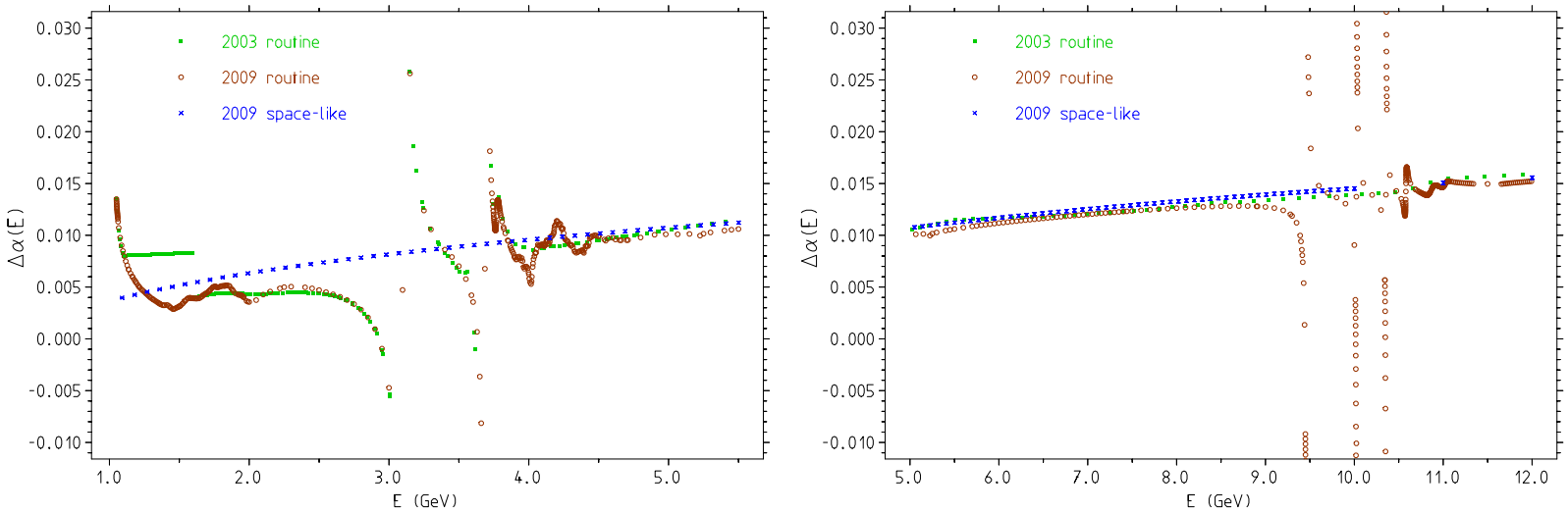
$$\boxed{\frac{1}{1-x}\ ;\quad x \text{ irreducible part}}$$

Object required in the DR:

$$R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})/\frac{4\pi\alpha^2}{3s} \qquad R^{\text{undressed}} = R^{\text{observed}} * |(1-x)|^2$$

Locally near OZI suppressed resonances iterative procedure does not converge!

Way out: utilize smooth space-like charge expected to be OK “in average”



Note that the smooth space-like effective charge agrees rather well with the non-resonant “background” above the Φ (kind of duality)

No proof that this cannot produce non-negligible shifts!

Progress in sight here: KLOE 2015 G. Venanzoni et al first direct measurements of time-like complex running $\alpha_{\rm QED}(s)$! Possible also for J/ψ and other ultra-narrow resonances! Fundamental problem! BES III can and should do it!

Summary HVP

- Dominating $\pi\pi$ channel measured with $< 1\%$ accuracy
  - most precise ISR measurements (KLOE, BABAR, BESIII)
  - conflict between KLOE and BaBar less worrying as BESIII lies in-between
  - BESIII - ISR substantially improves confidence
  - VEPP-2000 aims for unprecedented accuracy 0.3%

- Higher multiplicities dominated by BABAR ISR measurements
  - cross check and improvement by VEPP-2000, BESIII, more to come

- BELLE-II in intermediate future ?!

- Alternative: measuring directly low energy running $\alpha_{\rm QED}(s)$ in spacelike region using $a_\mu^{\rm had} = \frac{\alpha}{\pi}\int_0^1 {\rm d}x\,(1-x)\,\Delta\alpha^{\rm had}\left(-Q^2(x)\right)$ where $Q^2(x) \equiv \frac{x^2}{1-x}m_\mu^2$ is the space–like

  Carloni Calame et al 2015 [Talk Luca Trentadue]

❖ Issues: - Radiative corrections [Talk Guido Montagna]
- Precise formfactor models in MC generators
- FSR modeling

❖ Lattice: Lots of interest, work on hadronic contributions, esp. HVP
[Talk Marcus Petschlies]

- Statistical errors (sub) 1%
- Several groups done/doing physical m_π (m_{quark}) simulations
- Much effort on understanding systematics
- 2-3% total error on connected HVP in 2 years possible
- May be achievable for disconnected too
- Accessing isospin breaking effects crucial, at the beginning $m_d \neq m_u$, e.m. $\rho - \gamma$ mixing etc.

● New tools and experiments to pin down more precisely the hadronic effects are on the way!

a_μ Theory vs Experiment: do we see New Physics?

Contribution	Value	Error	Reference
QED incl. 4-loops+5-loops	11 658 471.8851	0.036	Remiddi, Kinoshita ...
Leading hadronic vac. pol.	688.91	3.52	$e^+e^- + \tau$ data
Subleading hadronic vac. pol.	-9.917	0.100	2015 update
NNLO hadronic vac. pol.	1.240	0.010	2014 KLMS
Hadronic light–by–light	10.6	3.9	evaluation (J&N 09/J 14)
Weak incl. 2-loops	15.40	0.10	CMV06/FJ12/BSS13
Theory	11 659 178.12	5.26	–
Experiment	11 659 209.1	6.3	BNL Updated
Exp.- The. 3.8 standard deviations	30.68	8.20	–

Standard model theory and experiment comparison [in units 10^{-10}]. What represents the 4 σ deviation: ❒ new physics? ❒ a statistical fluctuation? ❒ underestimating uncertainties (experimental, theoretical)?

❖do experiments measure what theoreticians calculate?

New evaluations included:

New contribution	Reference	$a_\mu \cdot 10^{11}$		
NNLO HVP	Kurz et al. 2014	12.4	±	0.1
NLO HLBL	Colangelo et al. 2014	3	±	2
New axial exchange HLBL	Pauk, Vanderhaeghen, F.J. 2014	7.55	±	2.71
Old axial exchange HLBL	Melnikov, Vainshtein 2004	22	±	5
Tensor exchange HLBL	Pauk, Vanderhaeghen 2014	1.1	±	0.1
Total change		+2.1	±	3.4 [← 5]

more on HLbL [Talks Johan Bijnens, Marc Vanderhaeghen, Massimiliano Procura]

Axial exchanges

Landau-Yang Theorem: $\mathcal{A}$ (axial meson $\to \gamma\gamma$)=0

e.g. $Z^0 \not\to \gamma\gamma$, while $Z^0 \to \gamma e^+ e^-$ ✓

Why $a_\mu[a_1, f_1', f_1] \sim 25 \times 10^{-11}$ so large?

❒ untagged $\gamma\gamma \to f_1$ no signal!

❒ single-tag $\gamma^*\gamma \to f_1$ strong peak is $Q^2 \gg m_{f_1}^2$

Contribution from axial mesons: symmetric Melnikov-Vainshtein MV (2004) form-factors violate Landau-Yang, ⇒antisymmetrize ⇒contribution reduced by factor 3, agrees with previous findings by BPP (1995) and Pauk, Vanderhaeghen 2013, FJ 2014

Consequently:

$$a_\mu^{\mathrm{HLbL,\,LO}} = (106 \pm 39) \times 10^{-11}$$

replacing $(116 \pm 39) \times 10^{-11}$ used in JN 2009 Phys. Rep.
for more see [Talks Mark Knecht, Andreas Nyffeler]

Do we see new physics? Possibilities to explain the persisting gap?
[Talks Dominik Stöckinger, Eung Jin Chun, Ketan Patel, Francesca Curciarello]

Future

The big challenge: two complementary experiments: Fermilab with ultra hot muons and J-PARC with ultra cold muons (very different radiation) to come

[Talks David Hertzog, N.N.]

Provided deviation is real $3\sigma \to 9\sigma$ possible? Provided theory and needed cross section data improves the same as the muon $g-2$ experiments!

Key: need substantial progress in non-perturbative QCD

For muon $g-2$:

❖main obstacle: hadronic light-by-light [data, lattice QCD, RLA]

❖progress in evaluating HVP: more data (BaBar, Belle, VEPP 2000, BESIII,...), lattice QCD in reach (recent progress Jansen et al, Wittig et al, Blum et al)

in both cases lattice QCD will be the answer one day,

❖also low energy effective RL and DR approach need be further developed.

Summary

- Muon $g-2$ is one of the most precisely measured quantities in particle physics
- $a_\mu = (g_\mu - 2)/2$ very good monitor for physics beyond the SM:
  about 18 $\simeq$ 40000/2250 more sensitive than a_e
- At the same time it is the quantity showing a persisting discrepancy between theory and experiment $a_\mu^{\rm exp} - a_\mu^{\rm the}$ shows 3-4 σ deviation
- Could turn to about 8 σ after new Fermilab experiment improvement by factor 4, provided theory improves by factor 2 at least
- Low energy precision physics complementary to high energy as LHC
- Non-perturbative hadronic effects are limiting precision of theoretical predictions: vacuum polarization and hadronic light-by-light scattering

- Non-perturbative hadronic modeling requires experimental data to constrain models, challenge for close collaboration between theory and experiment: $e^+e^- \to$ hadrons and $\gamma^{(*)}\gamma^{(*)} \to \pi^0, \pi^+\pi^-,$ other hadronic states

- At the present/future level of precision it depends on all physics incorporated in the SM of particle physics: electromagnetic, weak, and strong interaction effects and beyond that all possible new physics we are hunting for. Excludes light states with “normal size” (perturbatively accessible) couplings. Dark photon still a candidate.

The muon $g-2$ remains an exciting and challenging topic!

Is the muon $g-2$ the ultimate SM killer?

Thank you for your attention!