Perspectives on the hadronic contributions to the muon g-2

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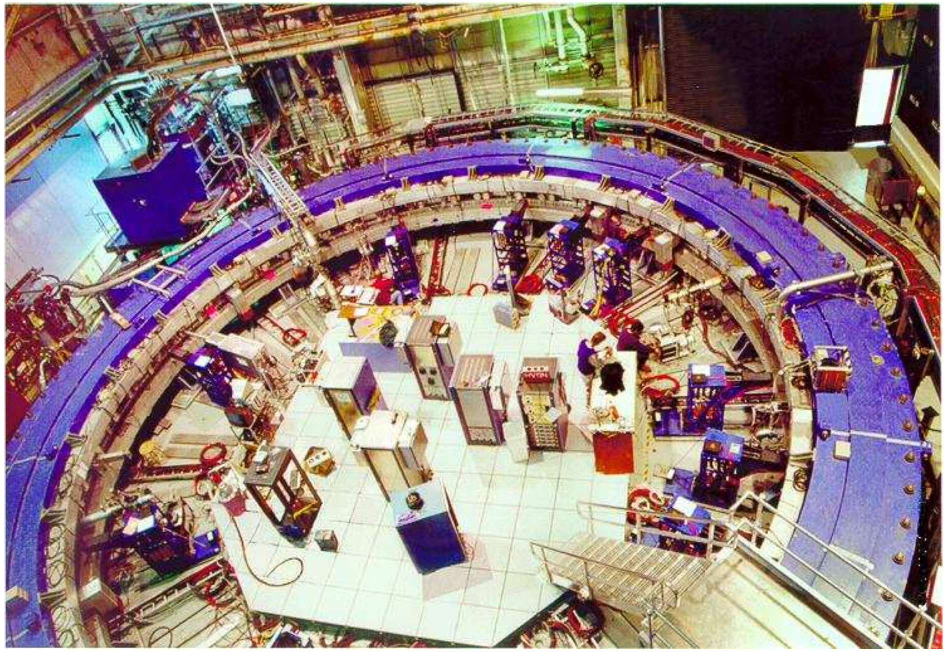
OUTLINE

- Introduction
- Hadronic vacuum polarization
- Hadronic light-by-light
- Conclusion/perspectives

Introduction

$$\tau_\mu = (2.19703 \pm 0.00004) \times 10^{-6}\ \text{s}$$

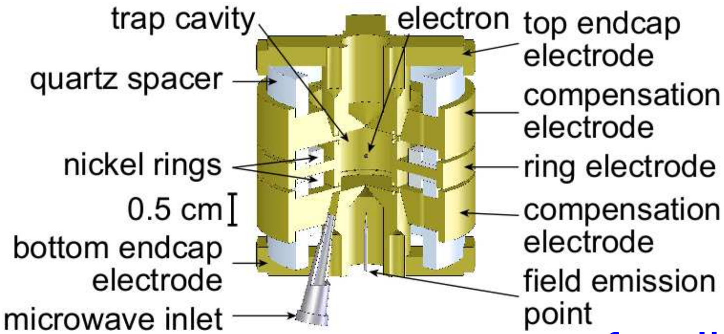
$$\gamma \sim 29.3,\ p \sim 3.094\ \text{GeV/c}$$



$$a_\mu^{\text{exp}} = 116\,592\,089(63) \cdot 10^{-11}$$

$$\Delta a_\mu^{\text{exp}} = 6.3 \cdot 10^{-10}\ \text{[0.54ppm]}$$

[G. W. Bennett et al, Phys Rev D 73, 072003 (2006)]



$$a_e^{\text{exp}} = 1\,159\,652\,180.73(0.28) \cdot 10^{-12}$$

$$\Delta a_e^{\text{exp}} = 2.8 \cdot 10^{-13}\ \text{[0.24ppb]}$$

[D. Hanneke et al, Phys. Rev. Lett. 100, 120801 (2008)]

$\longrightarrow$ cf. talk by S. Guellati-Khelifa

$\tau\ [\tau_\tau = (290.6 \pm 1.1) \times 10^{-15}\ \text{s}] \longrightarrow$ cf. talk by M. Fael

Theory (in units of 10^{-10})

QED	$+11658471.9$	[T. Aoyama et al. (2015)]
HVP-LO	$\begin{cases}+692.3(4.2) \\ +694.9(4.3)\end{cases}$	[M. Davier et al. (2011)] [K. Hagiwara et al. (2011)]
HVP-NLO	$-9.84(7)$	[K. Hagiwara et al. (2011)]
HVP-NNLO	$+1.24(1)$	[A. Kurz et al. (2014)]
HLxL	$\begin{cases}+10.5(2.6) \\ +11.5(4.0)\end{cases}$	[J. Prades et al. (2009)] [F. Jegerlehner, A. Nyffeler (2009)]
EW 1 loop	$+19.48(1)$	[(1972)]
EW 2 loops	$-4.12(10)$	[C. Gnendiger et al. (2013)]

$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (27.4 \pm 8.0) \cdot 10^{-10}$ $[3.4\sigma]$ for $a_\mu^{\text{HLxL}} = (10.5 \pm 2.6) \cdot 10^{-10}$, $a_\mu^{\text{HVP-LO}} = 692.3 \pm 4.2 \cdot 10^{-10}$

$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (23.7 \pm 8.6) \cdot 10^{-10}$ $[2.8\sigma]$ for $a_\mu^{\text{HLxL}} = (11.6 \pm 4.0) \cdot 10^{-10}$, $a_\mu^{\text{HVP-LO}} = 694.9 \pm 4.3 \cdot 10^{-10}$

- QED provides more than 99.99% of the total value, without uncertainties at this level of precision

- hadronic corrections provide the second important contribution, and the bulk of the uncertainty

- theory and experiment give comparable contributions to the total error on $a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$

- the present situation remains unconclusive as to the presence of BSM degrees of freedom

- forthcoming experiments at FNAL (E989) and at J-PARC (E34) plan to increase the experimental precision by a factor of 4 $\longrightarrow$ talk by D. Hertzog

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Two issues (at least)

- are the values given in the table reliable (central values and uncertainties)?

- is it possible, in view of the planed experiments at FNAL and J-PARC, to reduce the dominant theoretical uncertainties (HVP-LO and HLxL)?

Remarks about the QED and EW contributions (the realm of perturbation theory)

- the QED contributions are known to five loops

$$a_\ell^{\rm QED} = \sum_n C_\ell^{(2n)} \left(\frac{\alpha}{\pi}\right)^n$$

$$C_\mu^{(2)} = 1/2 \quad C_\mu^{(4)} = 0.765\,857\,425(17)$$

$$C_\mu^{(6)} = 24.050\,509\,96(32) \quad C_\mu^{(8)} = 130.879\,6(63) \quad C_\mu^{(10)} = 753.29(1.04)$$

[J. Schwinger, Phys. Rev. 73, 416L (1948)]
[C. M. Sommerfield, Phys. Rev. 107, 328 (1957); Ann. Phys. 5, 26 (1958)]
[A. Petermann, Helv. Phys. Acta 30, 407 (1957)]
[S. Laporta, E. Remiddi, Phys. Lett. B265, 182 (1991); B356, 390 (1995); B379, 283 (1996)]
[S. Laporta, Phys. Rev. D 47, 4793 (1993); Phys. Lett. B343, 421 (1995)]
[T. Aoyama et al., Phys. Rev. Lett. 109, 111808 (2012)] $\longrightarrow$ talk by M. Steinhauser

- Uncertainties on the coefficients $C_\mu^{(2n)}$ not relevant for a_μ at the present (and future) level of precision

- Drastic increase with n in the coefficients $C_\mu^{(2n)}$ [$\pi^2 \ln(m_\mu/m_e) \sim 50$!]

- Estimate of $\mathcal{O}(\alpha^6)$ contributions with these enhancement factors

$$\delta a_\mu \sim A_2^{(6)}(m_\mu/m_e;{\rm LxL}) \left[\frac{2}{3}\ln\frac{m_\mu}{m_e} - \frac{5}{9}\right]^3 \cdot 10 \left(\frac{\alpha}{\pi}\right)^6 \sim 0.6 \cdot 10^4 \cdot \left(\frac{\alpha}{\pi}\right)^6 \sim 1 \cdot 10^{-12}$$

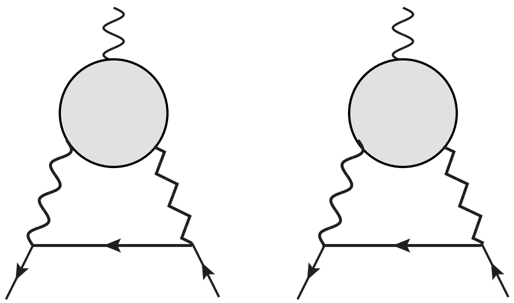
- No sign of substantial contribution to a_μ from higher order QED

Remarks about the QED and EW contributions (the realm of perturbation theory)

- The knowledge of the mass of the EW scalar boson has reduced the uncertainty in the one-loop EW correction

- The discrepancy between theory and experiment is about twice the size of the one-loop + two-loop EW correction (which also includes the complete three-loop short-distance leading logarithms)

- There are also hadronic contributions at two-loops, e.g. those [enhanced by $\ln(m_Z/m_\mu)$ and not suppressed by $1 - 4\sin^2\theta_w$] involving the $\langle VVA \rangle$ three-point function



[A. Czarnecki, W.J. Marciano, A. Vainshtein, Phys. Rev. D 67, 073006 (2003). Err.-ibid. D 73, 119901 (2006)]

[M. K., S. Peris, M. Perrottet, E. de Rafael, JHEP11, 003 (2002)]

- Difficult to expect any substantial reduction of the discrepancy from hadronic and/or higher order effects in EW corrections

Hadronic vacuum polarization

(in units of 10^{-10})

HVP-LO	$\begin{cases} +692.3(4.2) \\ +694.9(4.3) \end{cases}$	[M. Davier et al. (2011)] [K. Hagiwara et al. (2011)]
HVP-NLO	$-9.84(7)$	[K. Hagiwara et al. (2011)]
HVP-NNLO	$+1.24(1)$	[A. Kurz et al. (2014)]

- Occurs first at order $\mathcal{O}(\alpha^2)$

- NLO and NNLO corrections also known $\longrightarrow$ talk by M. Steinhauser

[B. Krause, Phys. Lett. B 390, 392 (1997)]
[A. Kurz et al., B **734**, 144 (2014)]

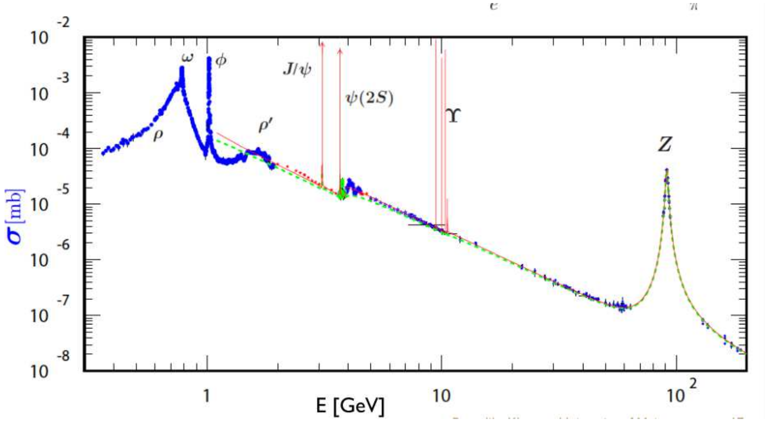
- Provides the largest hadronic contribution to a_μ

- Is dominated by the low-energy region (non-perturbative)

- Can be evaluated using available experimental input

$$a_\ell^{\text{HVP-LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4M_\pi^2}^{\infty} \frac{dt}{t} K(t) R^{\text{had}}(t) \qquad K(t) = \int_0^1 dx \, \frac{x^2(1-x)}{x^2 + (1-x)\,\frac{t}{m_\ell^2}}$$

[C. Bouchiat, L. Michel, J. Phys. Radium 22, 121 (1961)]
[L. Durand, Phys. Rev. **128**, 441 (1962); Err.-ibid. **129**, 2835 (1963)]
[M. Gourdin, E. de Rafael, Nucl. Phys. B 10, 667 (1969)]



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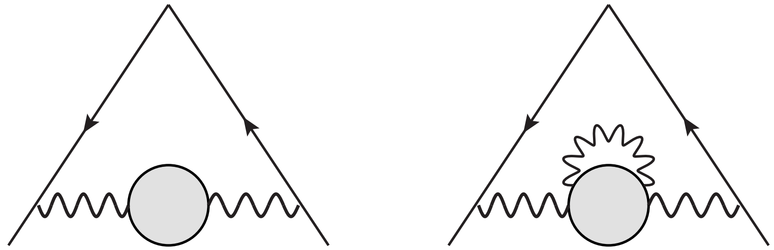


- Some order $\mathcal{O}(\alpha^3)$ corrections included

- exchange of virtual photons between final state hadrons

- some radiative exclusive modes, e.g. $\pi^0\gamma$

$$a_\mu^{\pi^0\gamma}(600\ \text{MeV} - 1030\ \text{MeV}) = 4.4(1.9)\cdot 10^{-10}$$

- The two most recent determinations are in good agreement (being based on the same data sets, this should not be a surprise) and give a relative precision of 0.6%

- Some tension between, for instance, the high-precision data collected in the region of the ρ resonance by BaBar and KLOE/KLOE-2

Experiment	$a_\mu^{\mathrm{HVP-LO}\,2\pi}(600-900\ \mathrm{MeV})$
BaBar	376.7(2.0)(1.9)
KLOE 08	368.9(0.4)(2.3)(2.2)
KLOE 10	366.1(0.9)(2.3)(2.2)
KLOE 12	366.7(1.2)(2.4)(0.8)

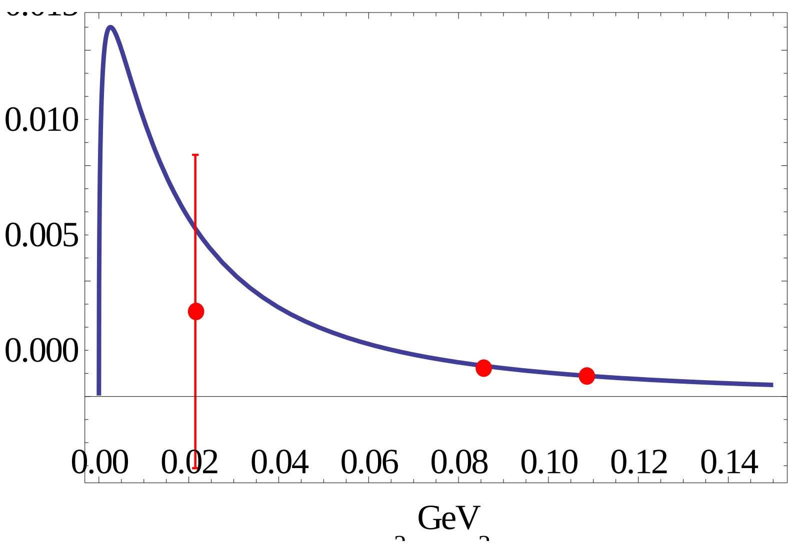
- These tensions need to be resolved in order to achieve higher precision
$\longrightarrow$ new data (KLOE-2, BaBar, VEPP-2000, BESIII,...)

Experiment	$a_\mu^{\mathrm{HVP-LO}\,2\pi}(600-900\ \mathrm{MeV})$	
BESIII (prel.)	374.4(2.6)(4.9)	[B. Kloss, PHOTON 2015 Conf.]

$\longrightarrow$ cf. talks by S. Eidelman, Z. Zhang, F. Jegerlehner, M. Benayoun

- Possibility to extract HVP from Bhabha scattering? $\longrightarrow$ cf. talk by L. Trentadue

- HVP from lattice QCD $\longrightarrow$ difficulties to reach the (euclidian) low-Q^2 region (finite volume)



[C. Aubin et al, Int. J. Mod. Phys. Conf. Ser. 35, 1460418 (2014)]

- Various strategies are considered

- twisted boundary conditions [C. Aubin, T. Blum, M. Golterman and S. Peris, Phys. Rev. D 88, 074505 (2013)]

- time moments and Padé approximants [B. Chakraborty et al, Phys. Rev. D 89, 114501 (2014)]

[M. Golterman, K. Maltman and S. Peris, Phys. Rev. D 90, 074508 (2014)]

- Moments of the MB transform [E. de Rafael, Phys. Lett. B 736, 522 (2014)]

- . . .

- Which precision can be achieved on $a_\mu^{\mathrm{HVP-LO}}$? 5% already interesting, 0.5% challenging

$\longrightarrow$ cf. talk by M. Petschlies, C. Lehner

Hadronic light-by-light

(in units of 10^{-10})

HLxL	$\left\{ \begin{array}{l} +10.5(2.6) \\ +11.5(4.0) \end{array} \right.$	[J. Prades et al. (2009)] [F. Jegerlehner, A. Nyffeler (2009)]

- Occurs at order $\mathcal{O}(\alpha^3)$
- Involves the fourth-rank vacuum polarization tensor

$$\mathrm{F.T.}\ \langle 0|T\{VVVV\}|0\rangle \longrightarrow \Pi_{\mu\nu\rho\sigma}(q_1,q_2,q_3,q_4) \quad q_1+q_2+q_3+q_4=0$$

$$\begin{aligned}
\bar{\mathrm{u}}(p')\Gamma_\rho^{\mathrm{HLxL}}(p',p)\mathrm{u}(p) &= \bar{\mathrm{u}}(p')\left[\gamma_\rho F_1^{\mathrm{HLxL}}(k^2) + \frac{i}{2m}\,\sigma_{\rho\tau}k^\tau F_2^{\mathrm{HLxL}}(k^2)\right]\mathrm{u}(p) \\
&= -ie^6 \int \frac{d^4q_1}{(2\pi)^4} \int \frac{d^4q_2}{(2\pi)^4} \frac{1}{q_1^2\, q_2^2\, (q_1+q_2-k)^2} \\
&\quad \times \frac{1}{(p'-q_1)^2-m^2}\,\frac{1}{(p'-q_1-q_2)^2-m^2} \\
&\quad \times \bar{\mathrm{u}}(p')\gamma^\mu(\not{p}'-\not{q}_1+m)\gamma^\nu(\not{p}'-\not{q}_1-\not{q}_2+m)\gamma^\lambda \mathrm{u}(p) \\
&\quad \times k^\sigma \frac{\partial}{\partial k^\rho}\,\Pi_{\mu\nu\lambda\sigma}(q_1,q_2,k-q_1-q_2,-k)
\end{aligned}$$

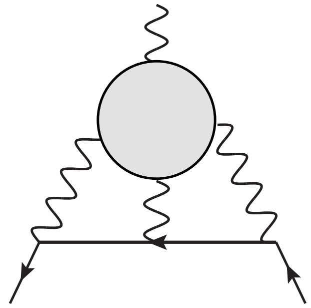
Project on the Pauli form factor, and take the limit $k^\mu \to (0,0,0,0)$

- Theory vs. experiment discrepancy $\sim$ 3 times a_μ^{HLxL}
- Estimate of higher order HLxL corrections: $a_\mu^{\mathrm{HLxL;HO}} \sim 0.3(0.2)\cdot 10^{-10}$

[G. Colangelo et al., Phys. Lett. B **735**, 90 (2014)]

- Cannot be related, as a whole, to an experimental observable

? →



- Individual contributions can be identified

single meson poles, two-meson intermediate states (loops),. . .

- Need some organizing principle: ChPT, large-N_c (turns out to be most relevant in practice)

[E. de Rafael, Phys. Lett. B 322, 239 (1994)]

$$a_\mu^{\mathrm{HLxL}} = N_c \left(\frac{\alpha}{\pi}\right)^3 \frac{N_c}{F_\pi^2} \frac{m_\mu^2}{48\pi^2} \left[\ln^2 \frac{M_\rho}{M_\pi} + c_\chi \ln \frac{M_\rho}{M_\pi} + \kappa\right] + \mathcal{O}(N_c^0)$$

[M. Knecht, A. Nyffeler, Phys. Rev. D 65, 073034 (2002)]

[M. Knecht, A. Nyffeler, M. Perrottet, E. de Rafael, Phys. Rev. Lett. 88, 071802 (2002)]

M. J. Ramsey-Musolf, M. B. Wise, Phys. Rev. Lett. 89, 041601 (2002)]

[J. Prades, E. de Rafael, A. Vainshtein, Glasgow White Paper (2008)]

- Impose QCD short-distance properties

[K. Melnikov, A. Vainshtein, Phys. Rev. D, 113006 (2004)]

- Only two (so far) attempts at a global evaluation

$$
\begin{aligned}
a_\mu^{\mathrm{HLxL}} &= +8.3(3.2)\cdot 10^{-10} \quad [\mathrm{BPP}] \\
a_\mu^{\mathrm{HLxL}} &= +8.96(1.54)\cdot 10^{-10} \quad [\mathrm{HKS}]
\end{aligned}
$$

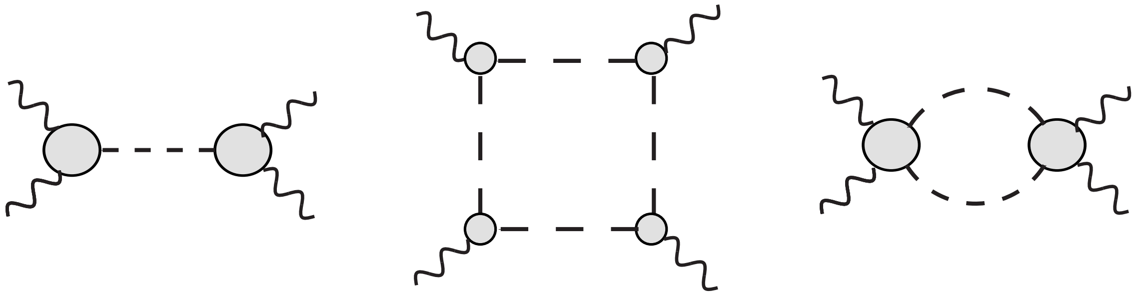
[J. Bijnens, E. Pallante, J. Prades, Phys. Rev. Lett. 75, 1447 (1995) [Err.-ibid. 75, 3781 (1995)]; Nucl. Phys. B 474, 379 (1995); Nucl. Phys. B 626, 410 (2002)]
[M. Hayakawa, T. Kinoshita, A. I. Sanda, Phys. Rev. Lett. 75, 790 (1995); Phys. Rev. D **54**, 3137 (1996)]
[M. Hayakawa, T. Kinoshita, Phys. Rev. D 57, 365 (1998) [Err.-ibid. 66, 019902(E) (2002)]

[BPP]: ENJL model $\longrightarrow$ cf. talk by J. Bijnens

- Some individual (pole) contributions have been reevaluated since then

- More recently: dispersive approaches

- for $\Pi_{\mu\nu\rho\sigma}$



$$\Pi = \Pi^{\pi^0,\eta,\eta' \text{ poles}} + \Pi^{\pi^\pm,K^\pm \text{ loops}} + \Pi^{\pi\pi} + \Pi^{\text{residual}}$$

[G. Colangelo, M. Hoferichter, M. Procura, P. Stoffer, JHEP09, 091 (2014); arXiv:1506.01386 [hep-ph]]

$\longrightarrow$ cf. talk by M. Procura

Needs input from data (transition form factors,...) $\longrightarrow$ cf. talks by A. Kupsc and A. Nyffeler

[G. Colangelo, M. Hoferichter, B. Kubis, M. Procura, P. Stoffer, Phys. Lett. B 738, 6 (2014)]

Main unanswered issues:

- how will short-distance constraints be imposed?
- how will Π^{residual} be estimated? Cf. axial vectors (leading in large-N_c) $\rightarrow 3\pi$ channel

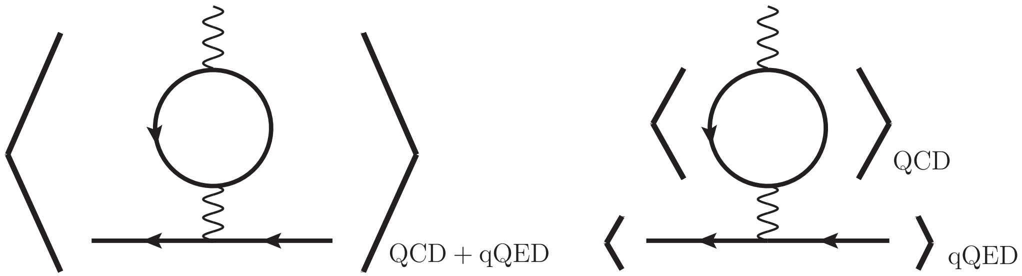
- for $F_2^{\text{HLxL}}(k^2)$

only pion pole with VMD form factor (two-loop graph) reconstructed this way so far

$\longrightarrow$ cf. talk by M. Vanderhaeghen

- Lattice QCD

Preparatory phase



[T. Blum, S. Chowdhury, M. Hayakawa and T. Izubuchi, Phys. Rev. Lett. 114, 012001 (2015)]

- Other strategies are being considered

- Goal: relative uncertainty $\sim 10\%$, but with systematics under control (continuum limit, infinite volume limit, physical quark masses, quark-disconnected contributions)

Conclusions/perspectives

- Two experiments are planed at FNAL and J-PARC, with the aim of reducing the experimental uncertainty on a_μ by a factor of four. This is a real challenge

- Reducing the theoretical uncertainties (by a factor of two), dominated by those on the hadronic contributions, is equally challenging

- HVP:

- present obstacle: conflicting data from different experiments

- forthcoming data may (hopefully) bring clarification and allow to reach a relative precision below the $\sim 0.5\%$ level

- a lot of activity on the front of lattice QCD (several groups, different strategies to overcome challenging difficulties)
Precision goal? a few percents with controlled systematics is already interesting

● Two experiments are planed at FNAL and J-PARC, with the aim of reducing the experimental uncertainty on a_μ by a factor of four. This is a real challenge

● Reducing the theoretical uncertainties (by a factor of two), dominated by those on the hadronic contributions, is equally challenging

● HLxL:

- $a_\mu^{\rm exp} - a_\mu^{\rm SM}$ represents about three times the size of this contribution

- could we have missed some important physics? Quite unlikely

- simple models quite useful to have a good idea of the magnitude of this contribution
$\longrightarrow$ talks by D. Greynat (CχQM) and L. Cappiello (AdS/QCD)

- program based on dispersive approach: requires data on transition form factors to “feed” the dispersion relations, needs to implement short-distance constraints and to provide a reliable bound on the “residual” contributions

- recent lattice developments look promising if systematics can be brought under control (quark-disconnected contributions)

• Two experiments are planed at FNAL and J-PARC, with the aim of reducing the experimental uncertainty on a_μ by a factor of four. This is a real challenge

• Reducing the theoretical uncertainties (by a factor of two), dominated by those on the hadronic contributions, is equally challenging

• Keep an eye on electron $g-2$

- naively less sensitive to BSM physics that a_μ by a factor $(m_\mu/m_e)^2 \sim 40\,000$, but measured $2\,300$ more accurately, and moreover naive scaling does not hold in all BSM scenarios

- room for improvement in the measurement of both g_e and α

$\longrightarrow$ cf. talk by S. Guellati-Khelifa