LFV decays in theories with Dim-6 operators

(based on arXiv:1408.3565, in collaboration with Adrian Signer)

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Lepton Flavour Violation: a conceptual challenge

The Dim-4 SM provides an accidental flavour symmetry:

- it holds in QCD and EM interactions;
- in the quark sector, it's broken by EW interactions.

The lepton sector strictly conserves the flavour.

At the same time, we have remarkable phenomenological evidences of FV in the neutrino sector, but...

...No evidence of the following phenomenological realisations:

- $l_h^\pm \to \gamma + l_i^\pm$ where $h, i = e, \mu, \tau$,
- $l_h^\pm \to l_i^\pm l_j^\pm l_k^\mp$ where $h, i, j, k = e, \mu, \tau$,
- $Z \to l_h^\pm l_i^\mp$ where $h, i = e, \mu, \tau$,
- $H \to l_h^\pm l_i^\mp$ where $h, i = e, \mu, \tau$.

What the experiments "measured"

MUONIC AND TAUONIC LFV TRANSITIONS - A SELECTION

- BR$(\mu \to 3e) < 1.0 \times 10^{-12}$ at the 90% C.L.
  SINDRUM Collaboration, Nucl. Phys. B **299** (1988) 1;
- BR$(\mu \to \gamma + e) < 5.7 \times 10^{-13}$ at the 90% C.L.
  MEG Collaboration, Phys. Rev. Lett. **110** (2013) 201801;
- BR$(Z \to e + \mu) < 7.5 \times 10^{-7}$ at the 95% C.L.
  ATLAS Collaboration, Phys. Rev. D **90** (2014) 072010;

- BR$(\tau \to 3e) < 2.1 \times 10^{-8}$ at the 90% C.L.
  BELL Collaboration, Phys. Lett. B **687** (2010) 139-143;
- BR$(\tau \to \gamma + \mu) < 4.4 \times 10^{-8}$ at the 90% C.L.
  BaBar Collaboration, Phys. Rev. Lett. **104** (2010) 021802;
- BR$(Z \to \tau + \mu) < 1.2 \times 10^{-5}$ at the 95% C.L.
  DELPHI Collaboration, Z. Phys. C **73** (1997) 243-251;
- BR$(H \to \tau + \mu) < 1.8 \times 10^{-2}$ at the 90% C.L.
  ATLAS/CMS Collaboration, arXiv:1508.03372/arXiv:1502.07400.

Synergy among Low and High Energy Experiments

An extensive long-term programme is undergoing to push the experimental limits both at low and high energy scales.

- Low energy (from m_μ to m_b):
  - Muon: limit on $\mu \to e$ conversion (SINDRUM II), $\mu \to e + \gamma$ (MEG), $\mu \to 3e$ (SINDRUM), $\mu \to e + 2\gamma$ (LAMPF), etc.
  - Tau-lepton: $\tau \to e/\mu + \gamma$ (BaBar, Belle), $\tau \to l_i l_j l_k$ with $i, j, k = e, \mu$ (BaBar, Belle and LHCb).

- High energy (from the EW scale to LHC run 2)
  - Neutral current mediated: $Z \to l_i l_j$ with $i, j = e, \mu, \tau$ (ALEPH, DELPHI, L3, OPAL, UA1).
  - Higgs mediated: $H \to \tau\mu$ (ATLAS&CMS).

A bottom-up approach: dim-n effective theory

Assumptions: SM is merely an effective theory, valid up to some scale Λ. It can be extended to a field theory that satisfy the following requirements:

- its gauge group should contain $SU(3)_C \times SU(2)_L \times U(1)_Y$;
- all the SM degrees of freedom must be incorporated;
- at low energies (i.e. when $\Lambda \to \infty$), it should reduce to SM.

Assuming that such reduction proceeds via decoupling of New Physics (NP), the Appelquist-Carazzone theorem allows us to write such theory in the form:

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda} \sum_k C_k^{(5)} Q_k^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} Q_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right).$$

Dimension 5 operator

Only one dimension 5 operator is allowed by gauge symmetry:

$$Q_{\nu\nu} = \varepsilon_{jk}\varepsilon_{mn}\varphi^j\varphi^m(l_p^k)^T C l_r^n \equiv (\widetilde{\varphi}^\dagger l_p)^T C(\widetilde{\varphi}^\dagger l_r).$$

After the EW symmetry breaking, it can generate neutrino masses and mixing (no other operator can do the job).

Its contribution to LFV has been studied since the late 70s:

- in the context of higher dimensional effective realisations;
  S. T. Petcov, Sov. J. Nucl. Phys. **25** (1977) 340 [Yad. Fiz. **25** (1977) 641]
- in connection with the “see-saw” mechanism.
  P. Minkowski, Phys. Lett. B **67**, 421 (1977)

It will not be considered in the current discussion.

Dimension 6 operators

2-leptons

$$Q_{eW} = (\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu};$$
$$Q_{eB} = (\bar{l}_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}.$$
$$Q^{(1)}_{\varphi l} = (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{l}_p \gamma^\mu l_r)$$
$$Q^{(3)}_{\varphi l} = (\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r)$$
$$Q_{\varphi e} = (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r)$$
$$Q_{e\varphi} = (\varphi^\dagger \varphi)(\bar{l}_p e_r \varphi)$$

4-leptons

$$Q_{ll} = (\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$$
$$Q_{ee} = (\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$$
$$Q_{le} = (\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$$

4-fermions

$$Q^{(1)}_{lq} = (\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$$
$$Q^{(3)}_{lq} = (\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$$
$$Q_{eu} = (\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$$
$$Q_{ed} = (\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$$
$$Q_{lu} = (\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$$
$$Q_{ld} = (\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$$
$$Q_{qe} = (\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$$
$$Q_{ledq} = (\bar{l}^j_p e_r)(\bar{d}_s q^j_t)$$
$$Q^{(1)}_{lequ} = (\bar{l}^j_p e_r)\varepsilon_{jk}(\bar{q}^k_s u_t)$$
$$Q^{(3)}_{lequ} = (\bar{l}^j_p \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}^k_s \sigma^{\mu\nu} u_t)$$

They all provide LFV...

Dim-6 operators: $l_2 \to l_1\gamma$ at the tree level

Only one dim-6 term can produce $l_2 \to l_1\gamma$ at the tree level:
B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, JHEP **1010** (2010) 085

Working in the physical basis, we consider:

$$Q_{eB} \to Q_{e\gamma} c_W - Q_{eZ} s_W ,$$
$$Q_{eW} \to -Q_{e\gamma} s_W - Q_{eZ} c_W ,$$

where $s_W = \sin(\theta_W)$ and $c_W = \cos(\theta_W)$ are the sine and cosine of the weak mixing angle. The term

$$\mathcal{L}_{e\gamma} \equiv \frac{C_{e\gamma}}{\Lambda^2} Q_{e\gamma} + \text{h.c.} = \frac{C_{e\gamma}^{pr}}{\Lambda^2} (\bar{l}_p \sigma^{\mu\nu} e_r)\varphi F_{\mu\nu} + \text{h.c.},$$

where $F_{\mu\nu}$ is the electromagnetic field-strength tensor, is then the only term in the D-6 Lagrangian that induces a $l_2 \to l_1\gamma$ transition at tree level.

Dim-6 operators: $H \to l_i l_j$ at the tree level

Only one dim-6 term provides $H \to l_i l_j$ at the tree level:

$$Q_{e\varphi} = (\varphi^\dagger \varphi)(\bar{l}_p e_r \varphi),$$

that sums to the SM Yukawa sector:

$$\begin{aligned}
\mathcal{L}_{\mathrm{D4}} + \mathcal{L}_{e\varphi} = & \frac{v}{\sqrt{2}} \left(-y_{pr} + \frac{v^2}{2\Lambda^2} C_{e\varphi}^{pr} \right) \bar{e}_p e_r \\
& + \frac{1}{\sqrt{2}} \left(-y_{pr} + \frac{v^2}{2\Lambda^2} C_{e\varphi}^{pr} \right) \bar{e}_p e_r h + \boxed{\frac{v^2}{\sqrt{2}\Lambda^2} C_{e\varphi}^{pr}} \bar{e}_p e_r h \\
& + \frac{i}{\sqrt{2}} \left(-y_{pr} + \frac{v^2}{2\Lambda^2} C_{e\varphi}^{pr} \right) \bar{e}_p e_r \widehat{Z} \\
& + i \left(-y_{pr} + \frac{v^2}{2\Lambda^2} C_{e\varphi}^{pr} \right) \bar{e}_p \nu_r \widehat{W}^+ + [\ldots] \, .
\end{aligned}$$

Other operators that are relevant at the tree level

Other LFV processes such as $Z \to l_i l_j$ or $l_j \to 3l_i$ are phenomenologically present at the tree-level if the following operators appear in the Lagrangian:

2-leptons

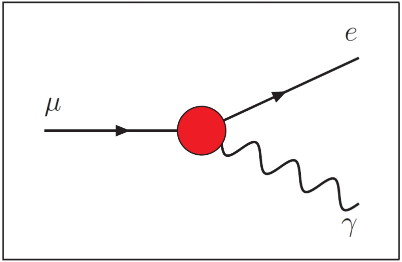
$$
\begin{aligned}
Q_{eW} &= (\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu}; \\
Q_{eB} &= (\bar{l}_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}. \\
Q^{(1)}_{\varphi l} &= (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{l}_p \gamma^\mu l_r) \\
Q^{(3)}_{\varphi l} &= (\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r) \\
Q_{\varphi e} &= (\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r)
\end{aligned}
$$

4-leptons

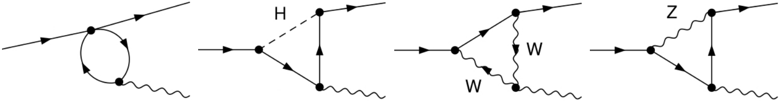
$$
\begin{aligned}
Q_{ll} &= (\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t) \\
Q_{ee} &= (\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t) \\
Q_{le} &= (\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)
\end{aligned}
$$

Dim-6 operators: $\mu(\tau) \to e(\mu/e)\gamma$ at one loop

For good eyes, even a point-like interaction...



...looks like a wild place to explore!



FeynRules

The generation of Feynman Rules was automatised by means of the FeynRules package.

Comput. Phys. Commun. **185** (2014) 2250 [arXiv:1310.1921 [hep-ph]]

At the end of the day, it was rather simple as we had great technical assistance (thanks to C. Duhr and C. Degrande).

The philosophy is straightforward:

- write your operator in a Mathematica notebook,
- press a button,
- print out your Feynman Rules.

Plus, it can also produce a FeynArts/FormCalc model file.

Interaction and branching ratio

Dim-6 operators contribute to the coefficients C_{TL} and C_{TR} of the photon-mediated FV interaction:

$$V^\mu = \frac{1}{\Lambda^2} i\sigma^{\mu\nu} \left(C_{TL}\,\omega_L + C_{TR}\,\omega_R\right) \left(p_\gamma\right)_\nu .$$

Being the partial width of the process $\mu \to e\gamma$

$$\Gamma_{\mu\to e\gamma} = \frac{1}{16\pi m_\mu} |\mathcal{M}|^2 , \quad \text{with} \quad |\mathcal{M}|^2 = \frac{4\left(|C_{TL}|^2 + |C_{TR}|^2\right) m_\mu^4}{\Lambda^4},$$

then the branching ratio is

$$\mathrm{BR}(\mu \to e\gamma) = \frac{\Gamma_{\mu\to e\gamma}}{\Gamma_\mu} = \frac{m_\mu^3}{4\pi\Lambda^4\Gamma_\mu} \left(|C_{TL}|^2 + |C_{TR}|^2\right) .$$

By calculating the dim-6 contributions to C_{TL} and C_{TR} one obtain the connection between effective coefficients and BR.

Dim-6 effective contributions to C_{TL} and C_{TR}

Operator	C_{TL} or $C_{TR}(l_2 \longleftrightarrow l_1)$
$Q_{e\gamma}$	$-C_{e\gamma}\frac{\sqrt{2}m_W s_W}{e}$
Q_{eZ}	$-C_{eZ}\frac{e m_Z}{16\sqrt{2}\pi^2}\left(3-6c_W^2+4c_W^2\log\left[\frac{m_W^2}{m_Z^2}\right]+(12c_W^2-6)\log\left[\frac{m_Z^2}{\lambda^2}\right]\right)$
$Q_{\varphi l}^{(1)}$	$-C_{\varphi l}^{(1)}\frac{e m_1\left(1+s_W^2\right)}{24\pi^2}$
$Q_{\varphi l}^{(3)}$	$C_{\varphi l}^{(3)}\frac{e m_1\left(3-2s_W^2\right)}{48\pi^2}$
$Q_{\varphi e}$	$C_{\varphi e}\frac{e m_2\left(3-2s_W^2\right)}{48\pi^2}$
$Q_{e\varphi}$	$C_{e\varphi}\frac{m_W s_W}{48\sqrt{2}m_H^2\pi^2}\left(4m_1^2+4m_2^2+3m_1^2\log\left[\frac{m_1^2}{m_H^2}\right]+3m_2^2\log\left[\frac{m_2^2}{m_H^2}\right]\right)$
$Q_{lequ}^{(3)}$	$-\frac{e}{2\pi^2}\sum_u m_u\left(C_{lequ}^{(3)}\right)^{21uu}\log\left[\frac{m_u^2}{\lambda^2}\right]$

Operator	C_{TL}	C_{TR}
Q_{le}	$\frac{e}{16\pi^2}\left(m_e C_{le}^{2ee1}+m_\mu C_{le}^{2\mu\mu 1}+m_\tau C_{le}^{2\tau\tau 1}\right)$	$\frac{e}{16\pi^2}(m_e C_{le}^{1ee2}+m_\mu C_{le}^{1\mu\mu 2}+m_\tau C_{le}^{1\tau\tau 2})$

No correlation: limits from some muonic transition

Coefficient	MEG ($\mu \to e\gamma$) $BR \leq 5.7 \cdot 10^{-13}$	ATLAS ($Z \to e\mu$) $BR \leq 7.5 \cdot 10^{-7}$	SINDRUM ($\mu \to 3e$) $BR \leq 1.0 \cdot 10^{-12}$
$C_{eZ}^{\mu e}(m_Z)$	$1.4 \cdot 10^{-13} \frac{\Lambda^2}{[\text{GeV}]^2}$	$5.5 \cdot 10^{-8} \frac{\Lambda^2}{[\text{GeV}]^2}$	$2.8 \cdot 10^{-8} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{\varphi l}^{(1)}$	$2.5 \cdot 10^{-10} \frac{\Lambda^2}{[\text{GeV}]^2}$	$5.5 \cdot 10^{-8} \frac{\Lambda^2}{[\text{GeV}]^2}$	$2.5 \cdot 10^{-11} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{\varphi l}^{(3)}$	$2.4 \cdot 10^{-10} \frac{\Lambda^2}{[\text{GeV}]^2}$	$5.5 \cdot 10^{-8} \frac{\Lambda^2}{[\text{GeV}]^2}$	$2.5 \cdot 10^{-11} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{\varphi e}$	$2.4 \cdot 10^{-10} \frac{\Lambda^2}{[\text{GeV}]^2}$	$5.5 \cdot 10^{-8} \frac{\Lambda^2}{[\text{GeV}]^2}$	$2.6 \cdot 10^{-11} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{e\varphi}^{\mu e}$	$2.7 \cdot 10^{-8} \frac{\Lambda^2}{[\text{GeV}]^2}$		$6.1 \cdot 10^{-6} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{le}^{eee\mu}$	$4.2 \cdot 10^{-8} \frac{\Lambda^2}{[\text{GeV}]^2}$		$2.2 \cdot 10^{-11} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{le}^{e\mu\mu\mu}$	$2.0 \cdot 10^{-10} \frac{\Lambda^2}{[\text{GeV}]^2}$		
$C_{le}^{e\tau\tau\mu}$	$1.2 \cdot 10^{-11} \frac{\Lambda^2}{[\text{GeV}]^2}$		
$C_{ee}^{eee\mu}$			$7.7 \cdot 10^{-12} \frac{\Lambda^2}{[\text{GeV}]^2}$
$C_{ll}^{eee\mu}$			$7.7 \cdot 10^{-12} \frac{\Lambda^2}{[\text{GeV}]^2}$

No correlation: limits from some tauonic transition

Coefficient	BaBar ($\tau \to \mu\gamma$) $BR \leq 4.4 \cdot 10^{-8}$	LEP ($Z \to \tau\mu$) $BR \leq 1.2 \cdot 10^{-5}$	BELL ($\tau \to 3\mu$) $BR \leq 2.1 \cdot 10^{-8}$	ATLAS&CMS ($H \to \tau\mu$) $BR \leq 1.85 \cdot 10^{-2}$
$C_{eZ}^{\tau\mu}(m_Z)$	$1.5 \cdot 10^{-9} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	$2.2 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	$6.1 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	
$C_{\varphi l}^{(1)}$	$1.6 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	$2.2 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	$9.0 \cdot 10^{-9} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	
$C_{\varphi l}^{(3)}$	$1.6 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	$2.2 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	$9.0 \cdot 10^{-9} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	
$C_{\varphi e}$	$1.6 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	$2.2 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	$9.5 \cdot 10^{-9} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	
$C_{e\varphi}^{\tau\mu}$	$1.9 \cdot 10^{-6} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$		$1.1 \cdot 10^{-5} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	$1.3 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$
$C_{le}^{\mu e e\tau}$	$4.7 \cdot 10^{-4} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$			
$C_{le}^{\mu\mu\mu\tau}$	$2.3 \cdot 10^{-6} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$		$8.0 \cdot 10^{-9} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	
$C_{le}^{\mu\tau\tau\tau}$	$1.3 \cdot 10^{-7} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$			
$C_{ee}^{\mu\mu\mu\tau}$			$2.8 \cdot 10^{-9} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	
$C_{ll}^{\mu\mu\mu\tau}$			$2.8 \cdot 10^{-9} \frac{\Lambda^2}{[\mathrm{GeV}]^2}$	

Effective coefficients and energy scale

The result of C_T at one loop can schematically be written as

$$C_T^{(1)} = -\frac{v}{\sqrt{2}} \left(C_{e\gamma} \left(1 + e^2 c_{e\gamma}^{(1)} \right) + \sum_{i \neq e\gamma} e^2 c_i^{(1)} \, C_i \right) .$$

In general, the coefficients $c_{e\gamma}^{(1)}$ and $c_i^{(1)}$ contain UV singularities, *i.e.* a renormalisation of $C_{e\gamma}$ is required.

Such procedure makes the scale dependence explicit via the *anomalous dimensions* of the coefficient.

At the end of the day, the renormalised effective coefficients and the C_{TL} and C_{TR} are running quantities.

A scale dependent limit

MEG sets a limit on $\mu \to e\gamma$ at the $\lambda = m_\mu$ scale; we combine it with the information on the interacting current to obtain:

$$\left.\frac{\sqrt{|C_{TL}(\lambda)|^2 + |C_{TR}(\lambda)|^2}}{\Lambda^2}\right|_{\lambda \ll \Lambda} \leq 4.3 \cdot 10^{-14}\,[\text{GeV}]^{-1}\,.$$

In this formula there are two scale dependencies:

- Λ: this is the scale $\gg \Lambda_{EW}$ at which the theory is defined, according to the decoupling theorem.
- λ: this is the scale at which the coefficient is probed by the experiment.

Next step: let's connect low and high energy scales.

From $\lambda = m_\mu$ to $\lambda = \Lambda_{EW}$

In the assumption that $C_{e\gamma}$ is the dominant coefficient in the energy range $m_\mu < \lambda < m_Z \sim m_H$, its running below the EW scale is QED driven:

$$16\pi^2 \frac{\partial C_{e\gamma}}{\partial \log \lambda} \simeq e^2 \left(10 + \frac{4}{3} \sum_q e_q^2(\lambda) \right) C_{e\gamma}.$$

Applying this to the limit on $C_{e\gamma}^{\mu e}(m_\mu)$ and $C_{e\gamma}^{e\mu}(m_\mu)$, one obtains:

$$\sqrt{\frac{|C_{e\gamma}^{\mu e}(m_Z)|^2 + |C_{e\gamma}^{e\mu}(m_Z)|^2}{2}} < 1.8 \cdot 10^{-16} \frac{\Lambda^2}{[\text{GeV}]^2}.$$

This is the limit that must be used to determine the constraints on the remaining effective coefficients at the scale Λ.

Renormalisation Group Equations

If one consider only the gauge contributions and the top-Yukawa coupling, the evolution of the coefficient $C_{e\gamma}$ is described by a coupled SoDE:

$$16\pi^2 \frac{\partial C^{\mu e}_{e\gamma}}{\partial \log \lambda} \simeq \left(\frac{47e^2}{3} + \frac{e^2}{4c_W^2} - \frac{9e^2}{4s_W^2} + 3Y_t^2 \right) C^{\mu e}_{e\gamma} + 6e^2 \left(\frac{c_W}{s_W} - \frac{s_W}{c_W} \right) C^{\mu e}_{eZ} + 16eY_t C^{(3)}_{\mu e tt},$$

$$16\pi^2 \frac{\partial C^{\mu e}_{eZ}}{\partial \log \lambda} \simeq -\frac{2e^2}{3} \left(\frac{2c_W}{s_W} + \frac{31 s_W}{c_W} \right) C^{\mu e}_{e\gamma} + 2e \left(\frac{3c_W}{s_W} - \frac{5s_W}{c_W} \right) Y_t C^{(3)}_{\mu e tt} + \left(-\frac{47e^2}{3} + \frac{151e^2}{12c_W^2} - \frac{11e^2}{12s_W^2} + 3Y_t^2 \right) C^{\mu e}_{eZ},$$

$$16\pi^2 \frac{\partial C^{(3)}_{\mu e tt}}{\partial \log \lambda} \simeq \frac{7eY_t}{3} C^{\mu e}_{e\gamma} + \frac{eY_t}{2} \left(\frac{3c_W}{s_W} - \frac{5s_W}{3c_W} \right) C^{\mu e}_{eZ} + \left(\frac{2e^2}{9c_W^2} - \frac{3e^2}{s_W^2} + \frac{3Y_t^2}{2} + \frac{8g_S^2}{3} \right) C^{(3)}_{\mu e tt} + \frac{e^2}{8} \left(\frac{5}{c_W^2} + \frac{3}{s_W^2} \right) C^{(1)}_{\mu e tt},$$

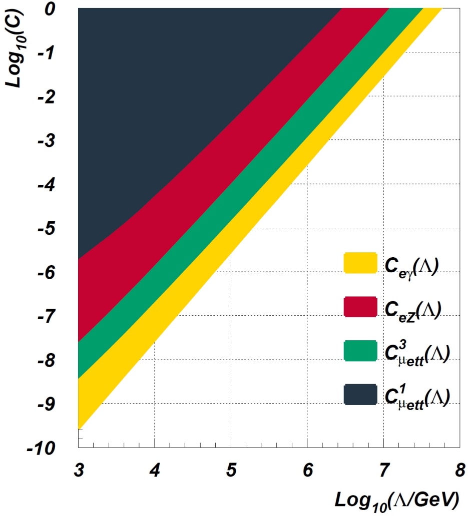
$$16\pi^2 \frac{\partial C^{(1)}_{\mu e tt}}{\partial \log \lambda} \simeq \left(\frac{30e^2}{c_W^2} + \frac{18e^2}{s_W^2} \right) C^{(3)}_{\mu e tt} + \left(-\frac{11e^2}{3c_W^2} + \frac{15Y_t^2}{2} - 8g_S^2 \right) C^{(1)}_{\mu e tt}.$$

Evolution and bounds from low energy

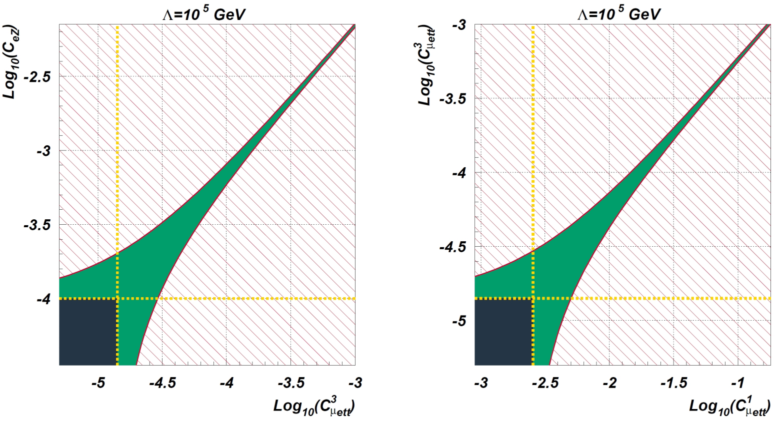
A remarkable set of different constraints on coefficients defined at the decoupling scale Λ!

Behaviour is not completely linear: solutions are not analytically simple.

Bounds on $C^{(1,3)}_{\mu ett}$!



Effects of correlation in the RGE analysis



Cancellations can represent a delicate issue:
naturalness is not a strong argument in effective scenarios!

Limits for coefficients defined at the Λ scale (1)

If no correlation is assumed, one obtains the following limits:

3-P Coefficient	at $\Lambda = 10^3$ GeV	at $\Lambda = 10^5$ GeV	at $\Lambda = 10^7$ GeV
$C_{e\gamma}^{\mu e}$	$2.7 \cdot 10^{-10}$	$2.9 \cdot 10^{-6}$	$3.1 \cdot 10^{-2}$
$C_{eZ}^{\mu e}$	$2.5 \cdot 10^{-8}$	$1.0 \cdot 10^{-4}$	$7.1 \cdot 10^{-1}$
$C_{\mu ett}^{(3)}$	$3.6 \cdot 10^{-9}$	$1.4 \cdot 10^{-5}$	$9.8 \cdot 10^{-2}$
$C_{\mu ett}^{(1)}$	$1.9 \cdot 10^{-6}$	$2.5 \cdot 10^{-3}$	n/a
$C_{\mu ecc}^{(3)}$	$4.8 \cdot 10^{-7}$	$1.9 \cdot 10^{-3}$	n/a
$C_{\mu ecc}^{(1)}$	$2.6 \cdot 10^{-4}$	$3.3 \cdot 10^{-1}$	n/a

TABLE 5: Limits on the Wilson coefficients defined at the scale $\lambda = \Lambda$ for three choices of $\Lambda = 10^3, 10^5, 10^7$ GeV.

Limits from MEG are applied at a fixed scale $\lambda = m_Z$.

Limits for coefficients defined at the Λ scale (2)

If no correlation is assumed, one obtains the following limits:

$\tau \to e\gamma$			
3-P Coefficient	at $\Lambda = 10^3$ GeV	at $\Lambda = 10^4$ GeV	at $\Lambda = 10^5$ GeV
$C_{e\gamma}^{\tau e}$	$2.5 \cdot 10^{-6}$	$2.6 \cdot 10^{-4}$	$2.8 \cdot 10^{-2}$
$C_{eZ}^{\tau e}$	$2.3 \cdot 10^{-4}$	$1.3 \cdot 10^{-2}$	$9.5 \cdot 10^{-1}$
$C_{\tau ett}^{(3)}$	$3.4 \cdot 10^{-5}$	$1.9 \cdot 10^{-3}$	$1.4 \cdot 10^{-1}$
$C_{\tau ett}^{(1)}$	$1.8 \cdot 10^{-2}$	$5.0 \cdot 10^{-1}$	n/a
$C_{\tau ecc}^{(3)}$	$4.6 \cdot 10^{-3}$	$2.5 \cdot 10^{-1}$	n/a
$C_{\tau ecc}^{(1)}$	~ 2.4	n/a	n/a

TABLE 6: Limits on the Wilson coefficients defined at the scale $\lambda = \Lambda$ for three choices of $\Lambda = 10^3, 10^4, 10^5$ GeV.

Limits from BaBar are applied at a fixed scale $\lambda = m_Z$.

Limits for coefficients defined at the Λ scale (3)

If no correlation is assumed, one obtains the following limits:

$\tau \to \mu\gamma$			
3-P Coefficient	at $\Lambda = 10^3$ GeV	at $\Lambda = 10^4$ GeV	at $\Lambda = 10^5$ GeV
$C_{e\gamma}^{\tau\mu}$	$3.0 \cdot 10^{-6}$	$3.1 \cdot 10^{-4}$	$3.2 \cdot 10^{-2}$
$C_{eZ}^{\tau\mu}$	$2.8 \cdot 10^{-4}$	$1.5 \cdot 10^{-2}$	~ 1.1
$C_{\tau\mu tt}^{(3)}$	$4.0 \cdot 10^{-5}$	$2.2 \cdot 10^{-3}$	$1.6 \cdot 10^{-1}$
$C_{\tau\mu tt}^{(1)}$	$2.1 \cdot 10^{-2}$	$5.9 \cdot 10^{-1}$	n/a
$C_{\tau\mu cc}^{(3)}$	$5.4 \cdot 10^{-3}$	$3.0 \cdot 10^{-1}$	n/a
$C_{\tau\mu cc}^{(1)}$	~ 2.8	n/a	n/a

TABLE 8: Limits on the Wilson coefficients defined at the scale $\lambda = \Lambda$ for three choices of $\Lambda = 10^3, 10^4, 10^5$ GeV.

Limits from BaBar are applied at a fixed scale $\lambda = m_Z$.

Conclusion

- √ The motivation to study a dim-5 and dim-6 effective field theory containing LFV couplings was presented.
- √ A systematic approach for the study of LFV observables was presented, and the benchmark process $\mu \to e\gamma$ was analysed at tree level and one loop.
- √ For some relevant low and high energy processes, quantitative limits on dim-6 effective coefficients were provided in a scenario where no correlation among operators is assumed.
- √ The interpretation of LE constraints in terms of HE complementary limits was analysed by means of renormalisation group equations.

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