# Hadronic contributions to the muon anomaly in the Constituent Chiral Quark Model

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10$^{\text{th}}$ September 2015

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Based on JHEP 1207 (2012)

FCCP2015

# The (most) shown table today...

$a_\mu$(E821 - BNL) $= 116592089(54)_{\text{stat}}(33)_{\text{syst}} \times 10^{-11}[0.54$ ppm$]$

Future Experiments:
FNAL with $\pm 0.14$ ppm overall uncertainty (data expected in 2017)
JPARC with similar uncertainty but very different technique

Standard Model contributions

| Contributions | Value in $10^{-11}$ | units |
|---|---|---|
| QED (leptons | 116 584 718.85 | $\pm$ 0.04 |
| HVP (lo)$[e^+e^-]$ | 6 923 | $\pm$ 42 |
| HVP (ho) | -98.4 | $\pm$ 0.7 |
| HLbyL | 105 | $\pm$ 26 |
| EW | 153 | $\pm$ 1 |
| Total SM | 116 591 801 | $\pm$ 49 |

**Persistent $3.6\sigma$ discrepancy between SM theory and Experiment**

J.P. Miller, E. de Rafael, B.L. Roberts, D. Stöckinger,
Annu. Rev. Part. Nucl. Phys. (2012)

## Numerous models to describe QCD contributions...

- Hidden Gauge Symmetry model Hayakawa and Kinoshita '02
- Extended Nambu–Jona-Lasinio model Bijnens-Pallante-Prades '02
- Chiral Quark Model Bartos *et al* '02
- Large Number of Colors Models Knecht and Nyffeler '02, Melnikov and Vainshtein '04, Nyffeler '09
- Holographic Models Cappiello, Cata and D'Ambrosio '11
- Nonlocal Chiral Quark Model Dorokhov-Radzhabov-Zhevlakov ?11
- ...

It would be nice to have a simple "Reference Model"

Our proposal is to use the The Constituent Chiral Quarks Model" (C$\chi$QM)

# The Constituent Chiral Quarks Model

## Extension of the Manohar-Georgi Lagrangian

**Manohar and Georgi '84**

$$\mathscr{L}_{\mathrm{C}\chi\mathrm{QM}} = \underbrace{i\bar{Q}\gamma^\mu\left(\partial_\mu + \Gamma_\mu + iG_\mu\right)Q + \frac{i}{2}g_A\,\bar{Q}\gamma^\mu\gamma_5\xi_\mu Q - M_Q\bar{Q}Q}_{M-G} - \frac{1}{2}\bar{Q}\left(\Sigma - \gamma_5\Delta\right)Q$$

$$+\underbrace{\frac{1}{4}F_\pi^2\,\mathrm{tr}\left[D_\mu U D^\mu U^\dagger}_{M-G} + U^\dagger\chi + \chi^\dagger U\right] - \underbrace{\frac{1}{4}\sum_{a=1}^{8}G_{\mu\nu}^{(a)}G^{(a)\mu\nu}}_{M-G} + e^2 C\,\mathrm{tr}(Q_R U Q_L U^\dagger)$$

$$+ L_5\,\mathrm{tr}\left[D_\mu U^\dagger D^\mu U(\chi^\dagger U + U^\dagger\chi)\right] + L_8\,\mathrm{tr}(U\chi\dagger U\chi\dagger + U^\dagger\chi U^\dagger\chi)$$

where $Q = u, d, s$ quarks and

$$D_\mu U = \partial_\mu U - ir_\mu U + iUl_\mu\ ,\quad l_\mu = v_\mu - a_\mu\ ,\quad r_\mu = v_\mu + a_\mu\ ,\quad U = \xi\xi$$

$$\Gamma_\mu = \frac{1}{2}\left[\xi^\dagger(\partial_\mu - ir_\mu)\xi + \xi(\partial_\mu - il_\mu)\xi^\dagger\right]\ ,\quad \xi_\mu = i\left[\xi^\dagger(\partial_\mu - ir_\mu)\xi - \xi(\partial_\mu - il_\mu)\xi^\dagger\right]$$

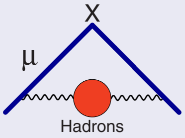
$$\chi = 2B\mathcal{M}\ ,\quad \mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)\ ,\quad \Sigma = \xi^\dagger\mathcal{M}\xi^\dagger + \xi\mathcal{M}^\dagger\xi\ ,\quad \Delta = \xi^\dagger\mathcal{M}\xi^\dagger - \xi\mathcal{M}^\dagger\xi$$

## Some Properties and results

- The CχQM in presence of $SU(3)_L \times SU(3)_R$ sources
  **Espiriu, de Rafael and Taron '90**
- The CχQM is renormalizable in Large N$_c$ **Weinberg '11**
- The number of counter-terms is minimized for $g_A = 1$ **de Rafael '11**

**Greynat and de Rafael '12**
All the relevant Hadronic contributions to $a_\mu$ can be evaluated in the CχQM

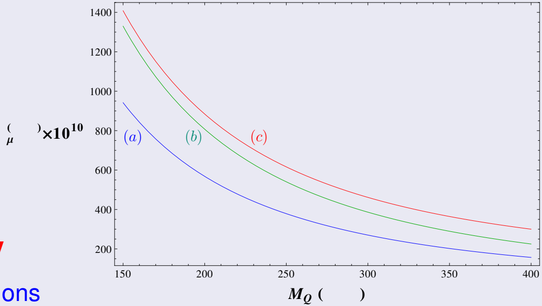
## First step: Fixing the only parameter $M_Q$



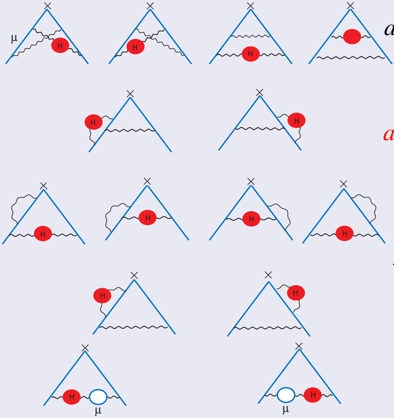

$$a_\mu^{(\text{HVP})} \simeq 653 \cdot 10^{-10}$$

$$M_Q = (240 \pm 10)\ \text{MeV}$$

*(a)* includes $\pi, K$ contributions



## HVP-A: insertions in the fourth order QED vertex diagrams




$$a_\mu^{\text{(HVP-A)}} = \int_{4M_Q^2}^{\infty} \frac{dt}{t} K_4(t/m_\mu^2) \frac{1}{\pi} \mathrm{Im}\,\Pi^{\text{(HVP)}}(t)$$

$$a_\mu^{\text{(HVP-A)}}(M_Q = 240\ \text{MeV}) = -171 \cdot 10^{-11}$$

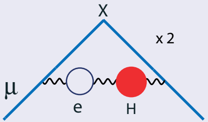
$$-181 \cdot 10^{-11} \leqslant a_\mu^{\text{(HVP-A)}} \leqslant -161 \cdot 10^{-11}$$

$$a_\mu^{\text{(HVP-A)}} \simeq -(207.3 \pm 1.9) \cdot 10^{-11}$$

## HVP-B: insertions in the QED vertex with an electron loop

$$a_\mu^{\text{(HVP-B)}} = \left(\frac{\alpha}{\pi}\right)^2 \int_{4M_Q^2}^{\infty} \frac{dt}{t} \int_0^1 dx \frac{x^2(1-x)}{x^2 + \frac{t}{m_\mu^2}(1-x)}$$

$$\times \left[-2\,\mathrm{Re}\,\Pi^{(e)}\left(-\frac{x^2}{1-x}m_\mu^2\right) \frac{1}{\pi}\mathrm{Im}\,\Pi^{(\mathrm{HVP})}(t)\right]$$

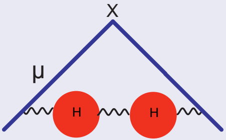



$$a_\mu^{\text{(HVP-B)}}(M_Q = 240\ \text{MeV}) = 88.9 \cdot 10^{-11}$$

$$82.6 \cdot 10^{-11} \leqslant a_\mu^{\text{(HVP-B)}} \leqslant 95.9 \cdot 10^{-11}$$

$$a_\mu^{\text{(HVP-B)}} \simeq (106.0 \pm 0.9) \cdot 10^{-11}$$

## HVP-C: iterated HVP contributions



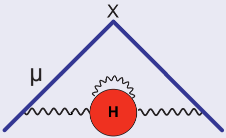

$$a_\mu^{\text{(HVP-C)}}(M_Q = 240\ \text{MeV}) = 2.2 \cdot 10^{-11}$$

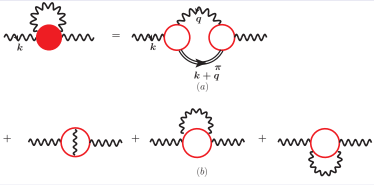
$$1.9 \cdot 10^{-11} \leqslant a_\mu^{\text{(HVP-C)}} \leqslant 2.5 \cdot 10^{-11}$$

$$a_\mu^{(\mathrm{HVP-C})}() = (3.4 \pm 0.1) \times 10^{-11}$$

$$a_\mu^{(\mathrm{HVP-C})}() = (3.0 \pm 0.1) \times 10^{-11}$$

## HVP-D: contributions with HVP corrections at $\mathcal{O}(\alpha)$




$$a_\mu^{(\mathrm{HVP-D})} = a_\mu^{(\pi^0\gamma)} + a_\mu^{(Q,\alpha)}$$

- Contribution from the $\pi^0\gamma$ intermediate state :

$$a_\mu^{(\pi^0\gamma)} = \frac{\alpha}{2\pi}\int_0^1 \frac{dz}{z}(1-z)(2-z)\mathcal{A}^{(\pi^0\gamma)}\left(\frac{z^2}{1-z}m_\mu^2\right)$$

the Adler function is evaluated from the vertex form factor

$$\mathcal{F}^{(\chi\mathrm{QM})}_{\pi^{0*}\gamma^*\gamma^*}\left((k+q)^2, k^2, q^2\right) = -ie^2\frac{N_c}{12\pi^2 F_\pi}\times$$

$$\int_0^1 dxx\int_0^1 dy\frac{2M_Q^2}{M_Q^2 - x(1-x)(1-y)k^2 - x^2y(1-y)(k+q)^2 - xy(1-x)q^2 - i\epsilon}$$

$M_Q$ in $\mathcal{F}^{(\chi\text{QM})}_{\pi^{0*}\gamma^*\gamma^*}$ acts like a UV-regulator such that we recover the Adler-Bell-Jackiw point like coupling

$$\mathcal{F}^{(\chi\text{QM})}_{\pi^{0*}\gamma^*\gamma^*}\left((k+q)^2, k^2, q^2\right) \xrightarrow[M_Q\to\infty]{} -ie^2\frac{N_c}{12\pi^2 f_\pi}$$

We obtain

$$a_\mu^{(\pi\gamma)}(M_Q = 240\text{MeV}) = 2.17\times 10^{-11}$$

with a range

$$2.10\times 10^{-11} \leq a_\mu^{(\pi\gamma)} \leq 2.18\times 10^{-11}$$

Comparing to

$$a_\mu^{(\pi\gamma)} = (44.2 \pm 1.9)\times 10^{-11}$$

This is very bad because CχQM fails to reproduce the contributions from $\eta_0\gamma$ and $\eta_8\gamma$ as intermediate state, here this sub-class of diagrams are important.

- Contribution from the quark loop to $\mathcal{O}(\alpha)$:

$$a_\mu^{(Q,\alpha)} = \int_{4M_Q^2}^{\infty} \frac{dt}{t} K(t/m_\mu^2)\, \frac{1}{\pi} \mathrm{Im}\, \Pi^{(Q,\alpha)}(t)$$

$$a_\mu^{(Q,\alpha)}(M_Q = 240 \text{ MeV}) = 11.4 \times 10^{-11}$$

with a range

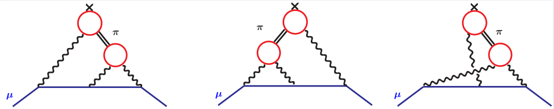
$$10.6 \times 10^{-11} \leq a_\mu^{(Q,\alpha)} \leq 12.3 \times 10^{-11}$$

## Total

| **Class** | **Result in $10^{-11}$ units** |
|---|---|
| A | $-171 \pm 10$ |
| B | $89 \pm 7$ |
| C | $2.2 \pm 0.3$ |
| D($\pi\gamma$) | $2.2 \pm 0.1$ |
| D(Q–loop) | $13.5 \pm 0.5$ |
| Total | $-64 \pm 12$ |

# Hadronic LbL scattering contributions: Two competitive contributions

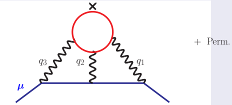
## The $\pi^0$ exchange




Asymptotically:

$$a_\mu^{(\mathrm{HLbyL})}(\pi^0)_{\chi\mathrm{QM}} = \left(\frac{\alpha}{\pi}\right)^3 N_c^2 \frac{m_\mu^2}{16\pi^2 f_\pi^2}\left[\frac{1}{3}\ln^2\frac{M_Q}{m_\pi} + \mathcal{O}\left(\ln\frac{M_Q}{m_\pi}\right) + \mathcal{O}(\mathrm{cte.})\right]$$

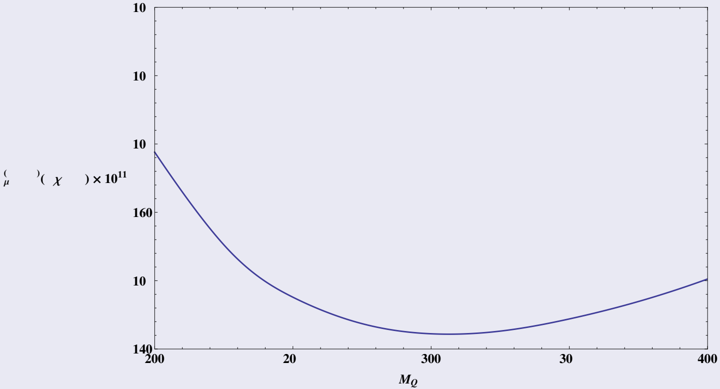
## The Constituent Quark loop:




Asymptotically:

$$a_\mu^{(\mathrm{HLbyL})}(\mathrm{CQL}) = \left(\frac{\alpha}{\pi}\right)^3 N_c \left(\sum_{q=u,d,s} Q_q^4\right)\left\{\left[\frac{3}{2}\zeta(3) - \frac{19}{16}\right]\frac{m_\mu^2}{M_Q^2} + \mathcal{O}\left(\frac{m_\mu^2}{M_Q^2}\log^2\left(\frac{m_\mu^2}{M_Q^2}\right)\right)\right\}$$

## LbL contribution (numerical)




$$a_\mu^{(LbL)}(M_Q = 240\ \text{MeV}) = 153 \cdot 10^{-11}$$

$$143 \cdot 10^{-11} \leqslant a_\mu^{(LbL)} \leqslant 153 \cdot 10^{-11}$$

- Summary: in a syst. error $\sim$30% we are in good agreement

| **Class** | **Result in $10^{-10}$ units** | Phen. |
| --- | --- | --- |
| HVP | $652 \begin{matrix} +47 \\ -42 \end{matrix}$ | $692 \pm 42$ |
| HVP to $\mathcal{O}\left(\frac{\alpha}{\pi}\right)^3$ | $-6.4 \pm 1.2$ | $-9.8 \pm 0.08$ |
| HLbyL | $15.0 \pm 0.3$ | $10.5 \pm 2.6$ |

- The results from the elaborated models which incorporate the known QCD constraints are well digested by a simple C$\chi$QM reference model

- There is interesting work in progress with possible improvements.