Hadronic light-by-light contribution to $(g-2)_\mu$ from lattice QCD

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RBC and UKQCD Collaborations

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SM prediction and experimental status of a_μ

Contribution	Value $\times 10^{10}$	Uncertainty $\times 10^{10}$
QED	11 658 471.895	0.008
EW	15.4	0.1
HVP (Leading-order)	*692.3	4.2
HVP (Higher-order)	-9.84	0.06
Hadronic light-by-light	**10.5	**2.6**
Total SM prediction	11 659 180.3	4.9
BNL E821 result	11 659 209.1	6.3
Fermilab E989 target		$\approx$ **1.6**

* $e^+e^- \rightarrow$ hadrons (exp) and dispersion integrals; "3.3σ tension" based on: K. Hagiwara et al., J. Phys. G38 (2011) 085003: $a_\mu^{\mathrm{HAD,\ LO\ VP}} \times 10^{10} \rightarrow 694.91$

** based on Prades, de Raphael, and Vainshtein 2009 "Glasgow White Paper": QCD model including PS meson contribution; Pauk and Vanderhaeghen Eur.Phys.J. C74 (2014) 8, 3008: include AV,S,T meson poles yields $< 1.0 \times 10^{-10}$ shifts in $a_\mu^{\mathrm{HAD,\ LBL}}$

RBC and UKQCD collaboration on the hadronic light-by-light contribution

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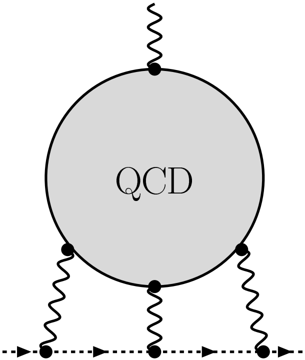
Antonin Portelli (Edinburgh)

Norikazu Yamada (KEK)

For more details, see recent talks at Lattice 2015 by M. Hayakawa, L. Jin, and C.L.

The hadronic light-by-light contribution (HLbL)

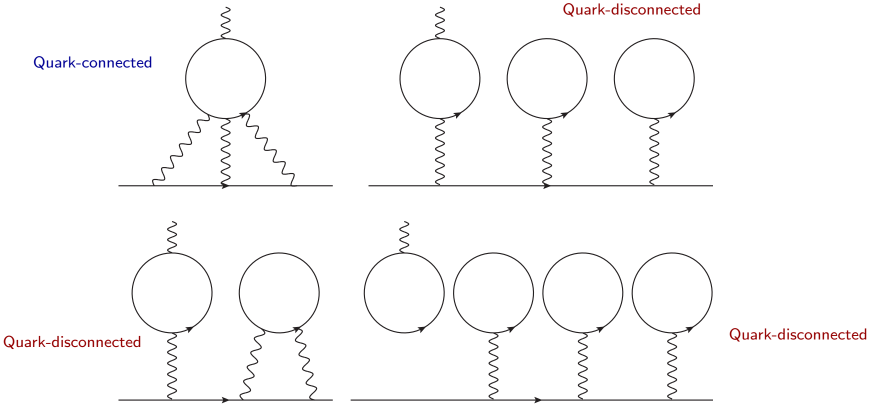


For external photon index μ with momentum q:

$$(-ie)\left[\gamma_\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q^\nu}{2m}F_2(q^2)\right] \quad (1)$$

with $F_2(0) = a_\mu$.

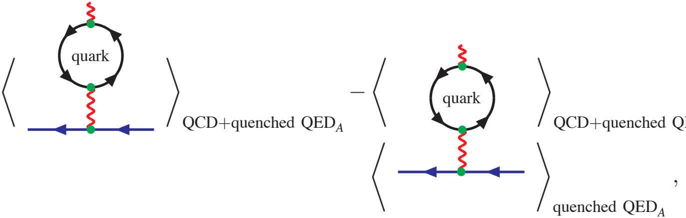
Important lattice terminology – quark-connected diagrams



Representative diagrams with one to four quark loops; gluons not drawn

HLbL – A long-standing problem of interest for our collaboration

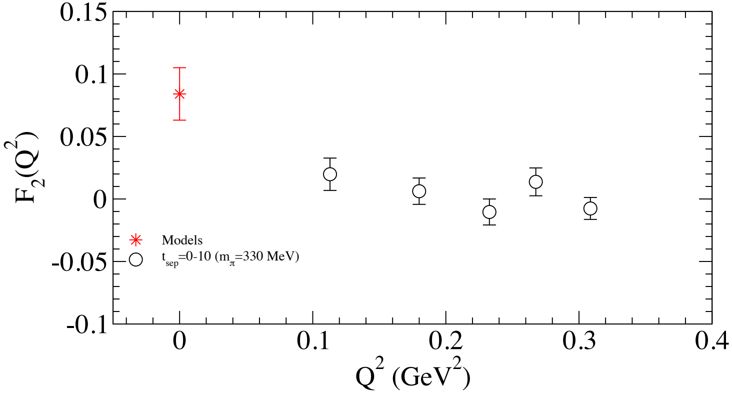
First methodology paper 10 years ago: Blum, Hayakawa, Izubuchi, Yamada: PoS(LAT2005)353; Quark-connected contribution only



Noise control: impose quantum-average properties config-by-config ($e \to -e$, $p \to -p$)

First implementation of this methodology 10 years later:

Blum et al., Phys.Rev.Lett. 114 (2015) 1, 012001: connected diagrams only, $m_\pi = 329$ MeV, $a^{-1} = 1.73$ GeV, $L = 24^3 \times 64$

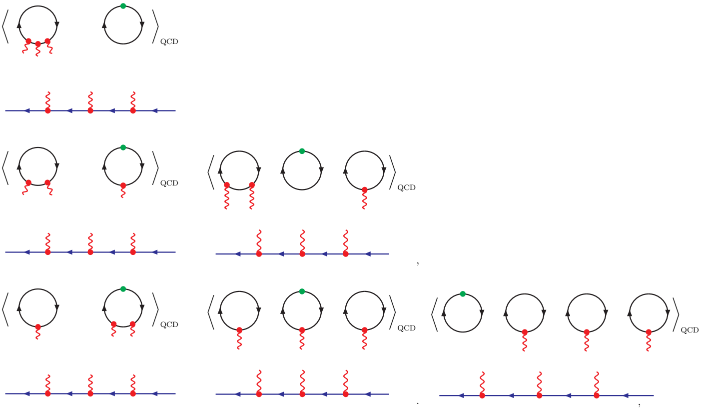


y axis in units of $(\alpha/\pi)^3$

Imperfections that need to be addressed:

- Omission of quark-disconnected diagrams
- Control of large QED finite-volume errors
- Direct evaluation of / extrapolation to F_2 at $Q^2 = 0$
- Control of excited state contributions
- Computation at physical pion mass

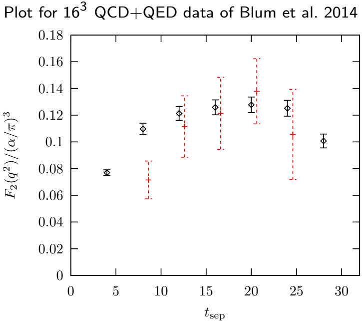
Inclusion of QCD+dynamical QED
Blum, Hayakawa, and Izubuchi, PoS(LATTICE 2013)439; Update: M. Hayakawa Lattice 2015



Addresses disconnected diagrams, however, isolation of signal from noise is challenging

Re-examine statistics

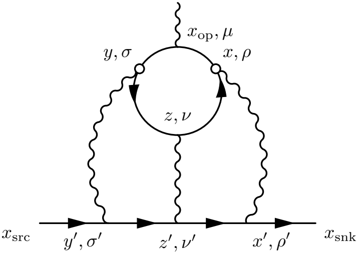
QCD+QED simulations suffer from large statistical uncertainties. We explore a different method here:

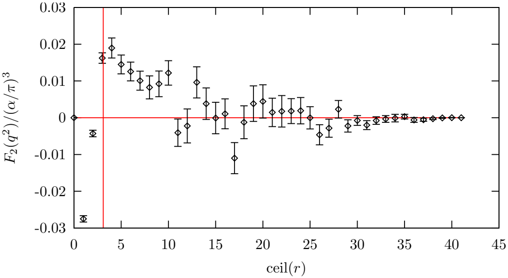


Luchang Jin

Same-cost comparison: red data: old method QCD+quenched QED, black: new stochastic sampling method (Luchang Jin talk at lattice 2015)

New stochastic sampling method



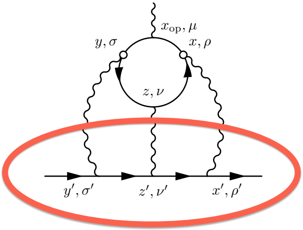


Stochastically evaluate the sum over vertices x and y:

- Pick random point x on lattice
- Sample all points y up to a specific distance $r = |x - y|$, see vertical red line
- Pick y following a distribution $P(|x - y|)$ that is peaked at short distances

Advantage: order of magnitude smaller noise, Disadvantage: disconnected diagrams by hand

QCD + QED on a lattice – finite-volume errors



Need to sum over all displacements between QCD and QED part to control FV errors.

Since muon line does not couple to gluons, this can be done in a straightforward way: C.L. talk at lattice 2015

Direct evaluation of form-factor at $F_2(Q^2 = 0)$

Model of problem: The lattice gives the position-space correlator $C(x)$ whose momentum space version

$$C(q) = \sum_x e^{iqx} C(x) \tag{2}$$

vanishes for $q = 0$ and the observable is related to

$$F = \lim_{q \to 0} \frac{C(q)}{q}, \tag{3}$$

while the lattice only has access to $C(q)$ for finite-volume quantized momenta q.

However: if $C(x) \to 0$ sufficiently fast as $|x| \to \infty$, we can write

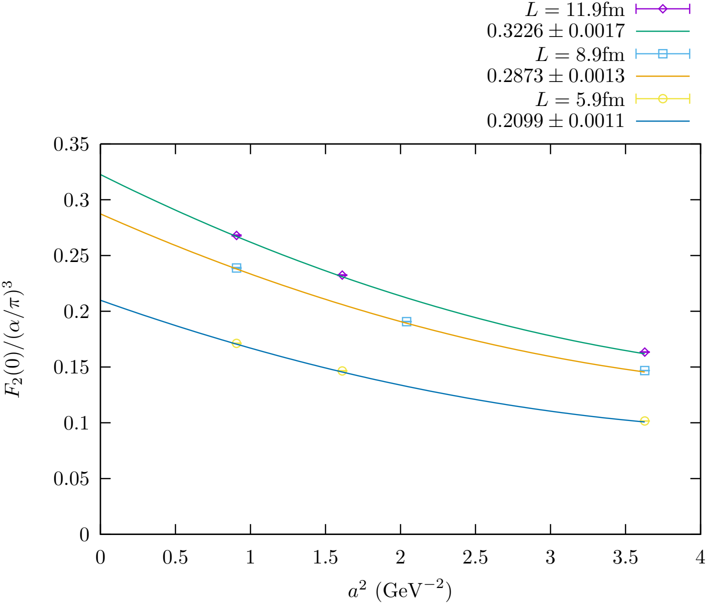
$$F = i \sum_x x\, C(x) \tag{4}$$

with controlled finite-volume errors.

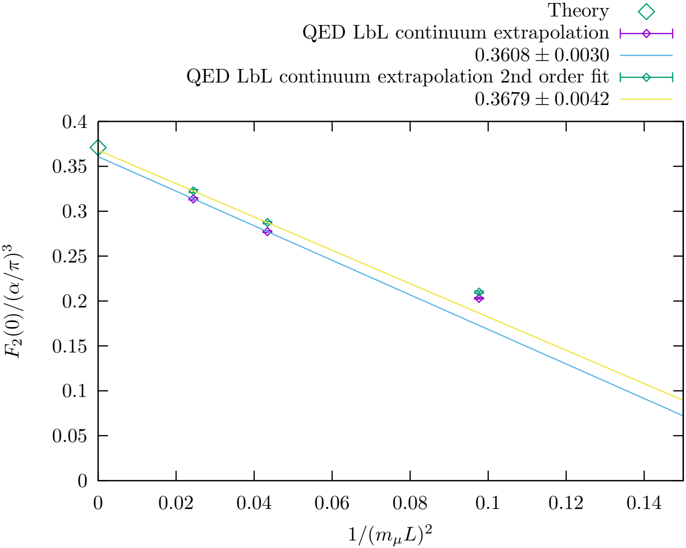
Imperfections that need to be addressed:

- ▶ Omission of quark-disconnected diagrams
- ✓ Control of large QED finite-volume errors
- ✓ Direct evaluation of / extrapolation to F_2 at $Q^2 = 0$
- ✓ Control of excited state contributions
- ✓ Computation at physical pion mass

Demonstration of validity – Replace quark with lepton loop



Demonstration of validity – Replace quark with lepton loop



Lattice result nicely extrapolates to the known analytic theory result;
Note that the difference between the lepton and full computation is merely the quark-propagator used, this is a strong test!

Status of lattice hadronic light-by-light determination:

- Quark-connected diagram seems to be controllable with current methodology
- We are currently running a large-scale computation at Argonne National Laboratory using 175M core hours (≈ 5000 typical laptop years) with precision-target for the quark-connected diagram of $10\% - 20\%$

Work in progress:

- Quark-disconnected diagram strategy needs to be optimized. This is a statistics problem not a systematic one!

Other collaborations have started similar efforts (Mainz group presented a computation of the quark four-point function at lattice 2015).

The lattice community is actively putting its focus on this important quantity.

Lattice methods – who is checking whom?

The comparison of lattice and model computations does not end after the first lattice computation. The beauty of lattice methodology is that it is systematically improvable. Over years more and more lattice collaborations with independent systematics and statistics will repeat and refine the computation.

Thank you

Backup slides

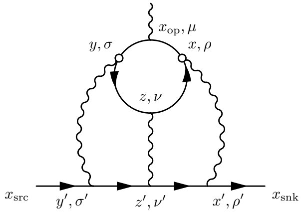
Excited states – A quick reminder of further lattice methodology

The lattice can compute Euclidean-space correlation functions. We extract operator matrix elements by taking large time separations to isolate on-shell contributions. Example:

$$\langle A(t)O(t_{\rm op})B(0)\rangle = \sum_{n,m}\langle A|n\rangle\langle n|O|m\rangle\langle m|B\rangle e^{-E_n(t-t_{\rm op})}e^{-E_m t_{\rm op}}$$

$$\to \langle A|n_0\rangle\langle n_0|O|m_0\rangle\langle m_0|B\rangle e^{-E_{n_0}(t-t_{\rm op})}e^{-E_{m_0} t_{\rm op}}\,.$$

Replacing $O(t_{\rm op}) \to e^{iqt_{\rm op}}$ allows for determination of norm and to extract $\langle n_0|O|m_0\rangle$.



Excited states

- As we go to larger volumes, excited state contributions of $\mu + \gamma$ etc. may be enhanced
- Lattice QED perturbation theory converges well and can be used to construct improved source
- We are exploring this with the *PhySyHCAl* system that also was used for a free-field test of Blum et al. 2014