Quantum Field Theories:
from fundamental interactions to cold atoms

ettore vicari

Quantum and Statistical Field theories are discussed, emphasizing their general principles, and their striking successes to describe natural phenomena,

such as fundamental interactions, condensed-matter physics, quantum many-body systems, statistical mechanics, critical phenomena, etc...

QFTs and SFTs provide general frameworks and set fundamental paradigma in theoretical physics.

They are the bases of the theoretical studies of the **LAUREA MAGISTRALE**

See the WEB PAGE **PisaTheoryGroup** for information on the research in theoretical physics at the university of Pisa and INFN.

What is a quantum field theory (QFT)?

theoretical framework for quantum mechanical models of subatomic particles in particle physics and quasiparticles or collective modes in condensed-matter physics.

QFT treats particles as excited states of an underlying physical field, thus they are indistinguishable.

For example:

• quantum electrodynamics has an electron field and a photon field

• in metals there is an atomic displacement field that gives rise to phonon particles

QFTs may also be more complex, including extended objects, topological modes, critical modes at quantum transitions, etc...

Quantization of Fields (functions of space-time)

- **Classical Electromagnetic FT** $\longrightarrow$ EM forces among charges and lights, using $A_\mu = [A_0, A_i]$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and $\partial_\mu = [\partial_t, \partial_i]$

$$\vec{E} = -\vec{\nabla} A_0 - \partial_t \vec{A}, \qquad \vec{B} = \vec{\nabla} \times \vec{A},$$
$$\partial_\mu \partial^\mu A_\nu = 0 \quad \rightarrow \quad \text{electromagnetic waves}$$
$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + J_\mu A^\mu \qquad \longrightarrow \qquad \partial_\mu F^{\mu\nu} = J^\nu$$

equivalent to $\vec{\nabla} \cdot \vec{E} = \rho, \quad \vec{\nabla} \times \vec{B} - \partial_t \vec{E} = \vec{J}$, etc...

- **Quantum Mechanics of a Particle**, promoting $x,\ p$ to operators:

$$H = \frac{p^2}{2m} + V(x), \qquad [x, p] = i\hbar$$

- **Quantization of a Field** $\phi(x)$ promoting it and its conjugate momentum $\pi = \partial\mathcal{L}/\partial_t \phi$ to operators and imposing the canonical commutation relations

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi(x,t) \partial^\mu \phi(x,t) - \frac{1}{2} m^2 \phi(x,t)^2, \qquad [\phi(x,t), \pi(y,t)] = i\hbar \delta(x - y)$$

- **Quantization of the EM Field**: from electromagnetic waves to photons

Quantum Field Theories provide a powerful framework for high-energy physics and condensed-matter physics

Statistical Field Theory is generally related by an analytic continuation of the time, from Minkowski to Euclidean space

in quantum mechanics the amplitude $\langle\Psi'|e^{-itH}|\Psi\rangle$ can be obtained by a sum over paths $\longrightarrow$ PATH INTEGRAL

in statistical physics the matrix density $\rho = \langle\Psi'|e^{-\beta H}|\Psi\rangle$ ($Z = \mathrm{Tr}\rho$) is given by an analogous PATH INTEGRAL with

$$t \longrightarrow -i\tau, \qquad \tau = \beta = 1/T$$

QFT and/or SFT provide a unified framework which allows us to investigate issues related to quantum and thermal fluctuations

FUNDAMENTAL INTERACTIONS

- STRONG INTERACTIONS, **QCD** (quarks and gluons)
$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu,a} + \bar{q}_f (i\gamma^\mu \partial_\mu - g\gamma^\mu A_\mu + m_f) q_f$

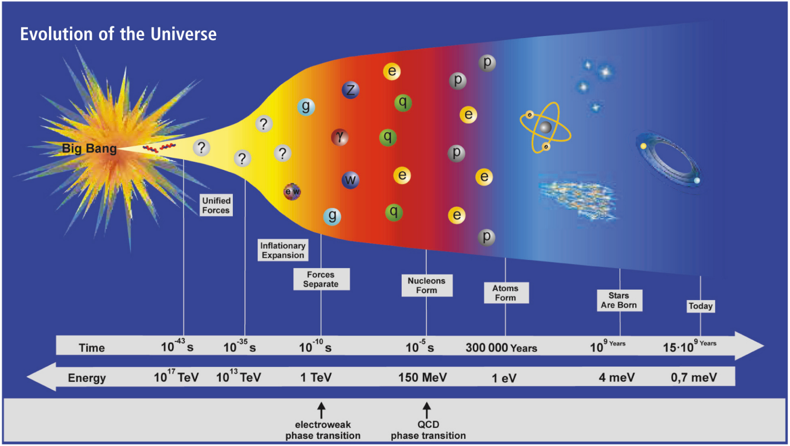
- ELECTRODYNAMICS, **QED** (+ photons, electrons, μ,τ)
$\mathcal{L}_{\mathrm{QED}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}_f (i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu + m_f) \psi_f$

- **ELECTROWEAK** INTERACTIONS (+ neutrinos, W, Z, Higgs)
$\mathcal{L}_{\mathrm{sm}} = -\frac{1}{4} \vec{A}_{\mu\nu} \vec{A}^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{\psi}_f [(i\gamma^\mu \partial_\mu + m_f) \psi_f + e J^\mu_\gamma A_\mu + g_W J^\mu_W W_\mu + g_Z J^\mu_Z Z_\mu + \ldots + (\partial_\mu \partial^\mu + M^2_H) H^2 + \ldots$

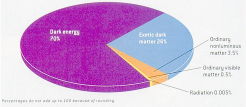
- **GRAVITY** (+ gravitons) $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G (T_{\mu\nu} + \rho_\Lambda g_{\mu\nu})$

$\longrightarrow$ le istruzioni d'uso sono fornite dai corsi della magistrale

The theories of fundamental interactions allow us to sketch the story of the universe from **BIG BANG** to today (after 13.7 billion years)

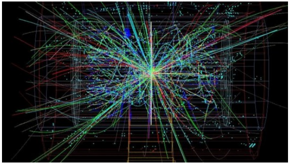


However, only 4% of the universe is made of known particles such as barions, leptons, lights; **26% is DARK MATTER** and **70% is DARK ENERGY**

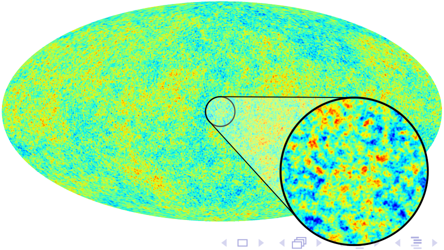


High-energy QFTs perform extremely well

checked to $\ell \approx 10^{-19}$ m in high-energy experiments at colliders



and to large scales by observations of the universe



the muon magnetic moment $\vec{M}_\mu = g_\mu \frac{e}{2m_\mu}\vec{S}$

$$a_\mu \equiv \frac{g_\mu - 2}{2} = 0.001\,165\,918\,0(7) \qquad \text{theory}$$

$$a_\mu \equiv \frac{g_\mu - 2}{2} = 0.001\,165\,920\,9(8) \qquad \text{experiments}$$

the available experimental results are so precise to probe the quantum nature of the theory, requiring high-loop calculations for the comparison with theory (up to five-loop calculations within QED, supplemented with two-loop *weak* contributions, and hadrons contributions to next-to-leading order)

small (significant?) difference $\Delta a_\mu = 2.9(1.1) \times 10^{-9}$

hint of new physics? adding a further 5-dim Pauli interaction

$$\frac{e}{2\Lambda}\bar{\psi}[\gamma_\mu, \gamma_\nu]\psi F^{\mu\nu} \quad \longrightarrow \quad \Lambda \gtrsim 10^8 \text{Gev}$$

Essential features of high-energy QFTs

(a) they describe **infinitely many degrees of freedom**, many-body systems

(b) the fundamental dynamical degrees of freedom are **field operators** $\Phi(\vec{x}, t)$, giving rise to **indistinguishable particles** (excitations over the ground state)

(c) **forces associated with particle exchanges** ex. photons in QED

(d) **interactions are local**, they describe many-body processes with creation and destruction of elementary excitations (particles), **causality**

(e) **Lorentz invariance**: $\longrightarrow$ SPIN AND STATISTICS theorem: **particles of spin-1/2 are fermions, those with integer spin are bosons**
$\longrightarrow$ CPT theorem: **existence of antiparticles**

(f) **based on abelian and nonabelian gauge principles**

(g) **renormalizability** of the lowest-order polynomial interactions ensures predictivity of the perturbative (loop) expansion (for $E \ll \Lambda_{\text{new physics}}$)

QFTs are powerful tools for quantum many-body problems

effective low-energy QFTs allow us to describe many features of the phenomena arising in condensed-matter physics:

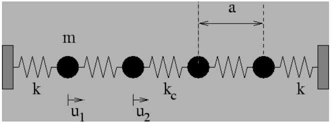
theory of metals, superconductivity, ^{3}He and ^{4}He quantum liquids, quantum Hall effects, optical lattices, etc...

- **universality** $\longrightarrow$ different systems may show common collective behaviors although they differ microscopically
- large number of degrees of freedom, $N = O(10^{23})$, so that problems can be addressed using statistical approaches
- space-time and internal symmetries which entail conservation laws
- **Wilson renormalization-group theory** (RG flow in the Hamiltonian space) further supports the use of QFT
- the degrees of freedom describing the low-energy excitations are often very different from the microscopic fundamental ones.

the sound: from particles to field and then back to particle-like excitations

Dynamics of the ions at the sites of a crystal at macroscopic length scales $\ell \gg a$:

$$H = \sum_{i=1} \left[\frac{p_i^2}{2m} + \frac{\kappa}{2}(x_{i+1} - x_i - a)^2 \right]$$



$\phi_i = x_i - x_{\mathrm{eq},i}$, then continuum limit $a \to 0$, $\phi_i \to \phi(x)$, $\longrightarrow$ classical field theory describing sound excitations with sound speed $v_s \sim \sqrt{\kappa/m}$

$$\mathcal{L} = \pi_i \dot{\phi}_i - H \longrightarrow \frac{1}{2}[\partial_t \phi(x,t)]^2 - \frac{v_s^2}{2}[\partial_x \phi(x,t)]^2 \qquad \longrightarrow \qquad \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi$$

After quantization, analogous to quantization of coupled oscillators, excitations above the ground state appear as massless scalar particles

$$p_\mu p^\mu = 0, \qquad c \to v_s$$

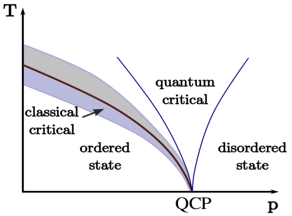
EMERGENCE of a Lorentz invariant QFT: we start from coupled oscillators and we end up with massless particles PHONONS

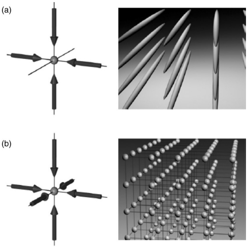
The low-energy physics of many-body systems may qualitatively differ from the microscopic one

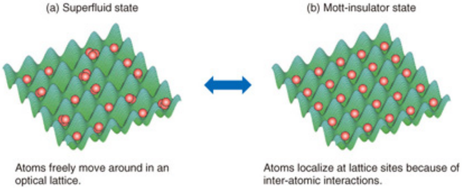
phase transitions driven by thermal and quantum fluctuations

liquid-gas and magnetic transitions, superfluid transitions, deconfinement transition in QCD, etc..., Bose-Einstein condensation in cold-atom gases, Mott transitions, in particular in optical lattices

A quantum $T = 0$ critical point separates different quantum phases. Scaling laws describe the interplay between quantum and thermal fluctuations at the QCP. $\longrightarrow$ new interesting phenomena in quantum magnetism, high-T superconductors, quantum particle systems, new matter states, such as spin liquids







$$H = -t \sum_{\langle \mathbf{xy} \rangle} (c^\dagger_{\sigma \mathbf{x}} c_{\sigma \mathbf{y}} + \text{h.c.}) + U \sum_{\mathbf{x}} n_{\uparrow \mathbf{x}} n_{\downarrow \mathbf{x}}$$

with their (nonrelativistic) QFTs

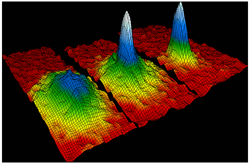
$$\mathcal{L} = \Psi^* \partial_\tau \Psi + \frac{1}{2m} |\partial_x \Psi|^2 - \mu |\Psi|^2 + u |\Psi|^4$$

Bose-Einstein condensation in atomic gases and liquid ^{4}He

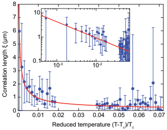
BEC in bosonic gases, when

$$\lambda_{\text{de Broglie}} = \left(\frac{2\pi\hbar^2}{mT}\right)^{1/2} \approx d_{\text{atoms}} = (N/V)^{-1/3} \quad ,$$

recently observed in weakly interacting gases, made of alkali atoms, rubidium, sodium, lithium

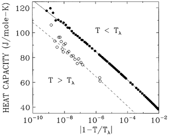


velocity distribution of rubidium atoms $\longrightarrow$



$\longleftarrow$ the correlation length of critical modes at BEC of a trapped atomic gas

$C_{\rm V}$ at the normal-superfluid transition in liquid ^{4}He up to a few nK from T_c $\longrightarrow$



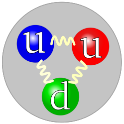
Bose-Einstein condensation $\rightarrow$ quantum amplitude of atoms as order parameter $\longrightarrow$ complex field $\varphi(x)$ with U(1) symmetry

RG theory and universality $\longrightarrow$ QFT: $\mathcal{L} = |\partial_\mu\varphi|^2 + r\,|\varphi|^2 + u\,|\varphi|^4$

		α	ν	η
EXPT	^{4}He	$-0.0127(3)$	**0.6709(1)**	
QFT	6,7-loops MZM	$-0.011(4)$	**0.6703(15)**	0.035(3)
	5-loop $O(\epsilon^5)$ exp	$-0.004(11)$	**0.6680(35)**	0.038(5)
Lattice	MC+HT	$-0.0151(3)$	**0.6717(1)**	0.0381(2)
	MC	$-0.0151(9)$	**0.6717(3)**	

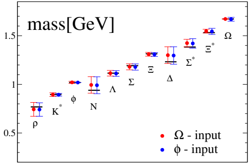
important developments from interchange of ideas among high-energy, condensed-matter and statistical theory through the QFT framework

- **RG theory and nonperturbative formulation of QFTs**



QCD defined from the critical regime of 4D statistical systems

from quarks to baryons



- **symmetry breaking and Goldstone bosons** • Goldstone particles: massless excitations in broken phases (pions in hadronic physics) • analogous excitations in magnets, superfluids, etc...

- **Higgs mechanism** • model of superconductivity in the case of U(1) gauge theories • mechanism to provide mass to particles in standard model

- **RG theory of critical phenomena and effective field theories** physical interpretation of renormalizability, suppressed *new-physics* in terms of a large scale Λ, analogously to irrelevant perturbations at continuous transitions

- **topological structures** such as solitons, instantons, vortices, etc

- **conformal field theory**, developed within QED and string theory, and used to described 2D critical phenomena, more recently holography approach

- **anomalies** $\pi_0 \to \gamma + \gamma$, topological insulators • **large-N expansions**

Summarizing .....

Quantum and Statistical Field theories play a central role in our description of natural phenomena

Their applications range from fundamental interactions to many-body physics in condensed matter, and critical phenomena

Is QFT completely understood? NO! several issues within QFTs are not understood yet! research projects to develop their methods are still important: wide potentiality and applications

Nonperturbative methods require further developments, summation of perturbative expansions, bound states, extended topological objects, off-equilibrium quantum real-time evolutions, ...

..., numerical approaches such as those to simulate QCD and fermionic condensed-matter theories, sign problem

Cold-atom experiments to analogically simulate theories may provide some progress (already achieved in the case of particle systems described by Hubbard and Bose-Hubbard models)

open problems and interesting issues which may be solved within QFT frameworks

dark matter are they light axions or heavy WIMPs?

dark energy why is it so small? is it constant?

inflation after Big Bang? what drives it?

new physics? what is the large scale Λ parametrizing the high-order polynomial interactions in the standard model?

quantum gravity to go beyond the actual nonrenormalizable theory?

phase diagram of quark matter quark-gluon plasma

θ dependence of QCD does the topological parameter θ vanish?

neutrino mass is it a Majorana or a Dirac particle?

new quantum state of matters such as spin liquid, deconfined transitions, topological phases, etc...

high-T superconductors related to quantum phase transitions?

emergence? fermions and gauge bosons emerging from more fundamental structures? string theory? ideas borrowed from condensed matter?