Strongly Correlated Electrons in High Temperature Superconductors

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Introduction

Model Fermi Surface (FS) for the Cuprates

Renormalization Group Approach

Non-Trivial Fixed Point and Main Instabilities of Hot Spots Model

Renormalization of the FS Induced by Interactions: The Two-Coupled Chains Example

Conclusion

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- In spite of the fact that **HTSC** was discovered nearly 30 years ago in **LaBaCuO** there are still many open problems which limit our understanding about this phenomenon.
- Those compounds are characterized essentially by **High Critical Temperatures** of order $\mathbf{T_c \approx 10\, T_c^{conv}}$
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- Besides, if we change its chemical composition by reducing the number of charge carriers, the **SC phase** is completely destroyed and, at sufficiently low doping, these materials become **Mott insulators**!

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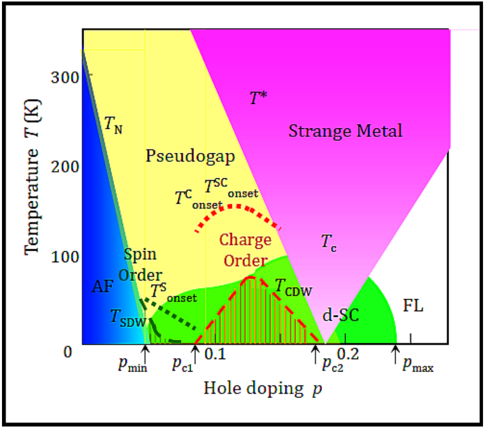
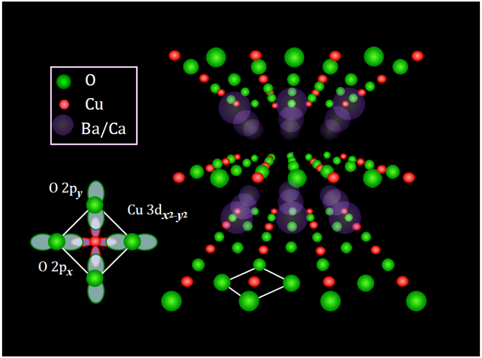

Temperature T (K)
300
200
100
0
T*
Strange Metal
T_N
Pseudogap
T^C_onset
T^SC_onset
Charge
Order
T_c
Spin
Order
AF
T^S_onset
T_CDW
T_SDW
d-SC
FL
p_min
p_c1
0.1
p_c2
0.2
p_max
Hole doping p


O
Cu
Ba/Ca
O 2p$_y$
Cu 3d$_{x^2-y^2}$
O 2p$_x$

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- The origin of the **AF** state is well understood in a representation in which there is a **strong coupling** between **electrons** and the presence of the **"superexchange"** interaction **J** between **localized spins**. This coupling is such that $\mathbf{J} \approx \mathbf{1/U}$ where **U** is the the **Coulomb repulsion**.

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- The **SC** in the **cuprates** has **d-type** symmetry: the **SC wave function** changes its **sign** if we rotate it by **90**° and there are **gapless quasiparticle** excitation modes along certain directions in **k**-**space**.

- In others non-conventional **SCs**, e.g. **heavy fermions** metals ($\mathbf{UGe_2}$, $\mathbf{UPt_3}$,...), the **organic superconductors** ($\mathbf{(BEDT-TTF)_2M}$,...), the **pnictides** ($\mathbf{PuCoGa_5}$,...), the **cobaltes** ($\mathbf{Na_xCoO_4}$,...), the **ruthenates** ($\mathbf{Sr_2Ru_4}$,...), $\mathbf{T_c}$ is easily supressed to **zero** with a small concentration of impurities. **That is not the case in HTSCs**!

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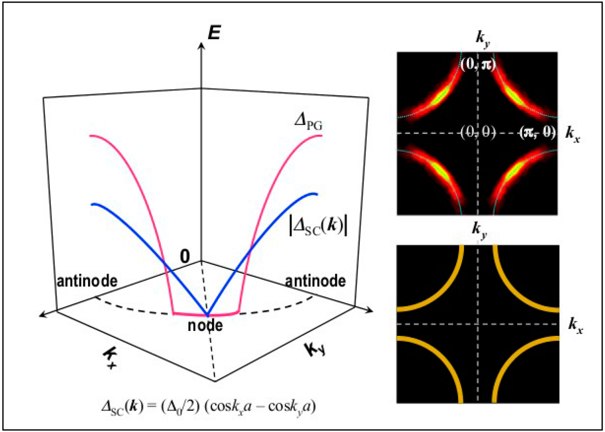
Model Fermi Surface (FS) for the Cuprates

- In the vicinity of **optimal doping**, for $\mathbf{T > T_c}$, **ARPES measurements** indicate that the **Fermi Surface (FS)** of the **cuprates** has the following shape:

B. Keimer *et al* condmat:1409.4673.



- This **FS** is large, hole-like and it can be described by a single band with a **single particle dispersion**

$$\xi_{\mathbf{k}} = -2\mathbf{t}(\cos k_x + \cos k_y) - 4\mathbf{t}' \cos k_x \cos k_y - \mu \tag{1}$$

- where t is the **nearest neighbour hopping**, t' is the **next to nearest neighbour hopping** with $\mathrm{t}' \cong -0.3\mathrm{t}$ and μ is the **chemical potential**.

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- Suppose to begin with that we are dealing with an ordinary **Fermi Liquid (FL)**.
- An appropriate action **S** for this **FL** state is

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S \quad &= \quad \int dt \sum_{\mathbf{k},\sigma} Z\, \psi^\dagger_\sigma(\mathbf{k},t)\,[\, i\partial_t - \xi_{\mathbf{k}} \,]\, \psi_\sigma(\mathbf{k},t) \qquad (2) \\
&- \quad \int dt \sum_{\sigma,\sigma'} \delta(\mathbf{k_1}+\mathbf{k_2}-\mathbf{k_3}-\mathbf{k_4})\, Z^2\, \psi^\dagger_\sigma(\mathbf{k_3},t)\, \psi^\dagger_{\sigma'}(\mathbf{k_4},t) \\
&\qquad\qquad g_B(\{\mathbf{k_i}\})\, \psi_{\sigma'}(\mathbf{k_2},t)\, \psi_\sigma(\mathbf{k_1},t)
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- where g_B is the **4-fermion coupling** and Z is the **quasiparticle renormalization** factor.

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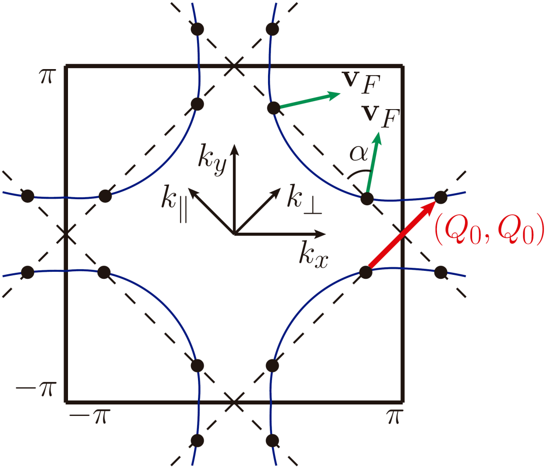
π
$-\pi$
$-\pi$
π
$\mathbf{v}_F$
$\mathbf{v}_F$
α
k_y
$k_\parallel$
$k_\perp$
k_x
(Q_0, Q_0)

- At **half-filling** for $t' = 0$, the **FS** is the so-called **"magnetic zone boundary"**.
- For this **FS** the g_B flows to **strong coupling** and the **FL** is turned into a **Mott Insulator**!
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- Let us analyse this problem from the perspective of the **renormalization group (RG)** approach.
- It is clear that the proximity to the **antiferromagnetic (AF)** phase is a very important feature for the **cuprates**. This should show itself up in our **RG** calculations.
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- This approach was revitalized by **Metlitski and Sachdev** (PRB 82, 075128 (2010)).
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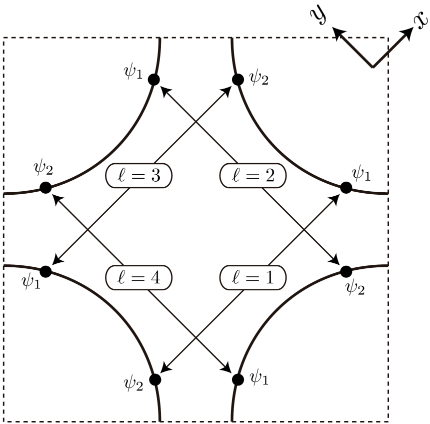

y
x
ψ_1
ψ_2
ψ_2
$\ell = 3$
$\ell = 2$
ψ_1
ψ_1
$\ell = 4$
$\ell = 1$
ψ_2
ψ_2
ψ_1

$$
\begin{aligned}
L \quad = \quad & \frac{1}{2}\,\Psi_1^{\dagger l}\,(\,\partial_\tau \,-\, i\mathbf{v_1}^{\,l}\cdot\nabla\,)\,\Psi_1^l \,+ \\
& \frac{1}{2}\,\Psi_2^{\dagger l}\,(\,\partial_\tau \,-\, i\mathbf{v_2}^{\,l}\cdot\nabla\,)\,\Psi_2^l \,+ \\
& +\lambda\,\boldsymbol{\phi}\cdot(\,\Psi_1^{\dagger l}\,\boldsymbol{\sigma}\,\Psi_2^l \,+\, h.c.\,)
\end{aligned} \tag{3}
$$

- with $\Psi_i^l = \begin{pmatrix} \psi_i^l \\ i\sigma^2\psi_i^{\dagger l} \end{pmatrix}, \qquad l = 1,2,3,4$,

  $\boldsymbol{\phi}$ being a **bosonic field** representing the **SDW** fluctuations and $\boldsymbol{\sigma}$'s are the **Pauli matrices**.

- The single part energies are linearized around the **"hot spots"** and the **Fermi velocities** $\mathbf{v_i}^l$'s are related by simple $\frac{\pi}{2}$ rotations:

$$\mathbf{v_i}^l = \left(\mathbf{R}_{\frac{\pi}{2}}\right)^{l-1} \cdot \mathbf{v_i}^{l=1} \tag{4}$$

- This lagrangian is **SU(2)** symmetric. This **pseudospin** symm indicates that the **d-wave superconducting(d-SC)** ordering is related to a **d-wave bond ordering (BDW)** by a **SU(2)** rotation!

- This led **Efetov et al** (Nature Physics 9, 442 (2013)) to postulate that the **pseudogap state** results from the pre-formation of pairs produced by this combined **d-SC** and **d-CDW excitation**.

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- where

$$\begin{aligned} Z^{\frac{1}{2}}\,\psi^R_\sigma(\mathbf{k},t) &= \psi^B_\sigma(\mathbf{k},t)\,, \\ Z\,v_x^B &= v_x^R + \Delta v_x = Z_{v_x} v_x^R\,, \\ Z\,v_y^B &= v_y^R + \Delta v_y = Z_{v_y} v_y^R\,, \\ Z\,Z_{v_x}\,k^B_{Fx} &= k^R_{Fx} + \Delta k_{Fx}\,, \\ Z\,Z_{v_y}\,k^B_{Fy} &= k^R_{Fy} + \Delta k_{Fy}\,, \quad Z^2\,g_i^B = g_i^R + \Delta g_i \end{aligned} \tag{6}$$

- $$\begin{aligned} L_R &= \sum_{\mathbf{k},\sigma} Z\,\psi^{\dagger\,R}_{\sigma}(\mathbf{k},t)\,[\,i\partial_t - v^B_x\,(k_x - k^B_{Fx}) \\ &\qquad - v^B_y\,(k_y - k^B_{Fy})\,]\,\psi_{\sigma\,R}(\mathbf{k},t) \\ &- \sum_i \sum_{\{\mathbf{k_i}\},\sigma,\sigma'} \delta(\mathbf{k_1}+\mathbf{k_2}-\mathbf{k_3}-\mathbf{k_4})\,Z^2\,g^B_i \\ &\qquad \psi^{\dagger\,R}_{\sigma}(\mathbf{k_3},t)\,\psi^{\dagger\,R}_{\sigma'}(\mathbf{k_4},t)\,\psi^R_{\sigma'}(\mathbf{k_2},t)\,\psi^R_{\sigma}(\mathbf{k_1},t) \end{aligned} \tag{5}$$

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$$\begin{aligned} Z^{\frac{1}{2}}\,\psi^R_\sigma(\mathbf{k},t) &= \psi^B_\sigma(\mathbf{k},t)\ , \\ Z\,v^B_x &= v^R_x + \Delta v_x = Z_{v_x} v^R_x\ , \\ Z\,v^B_y &= v^R_y + \Delta v_y = Z_{v_y} v^R_y\ , \\ Z\,Z_{v_x}\,k^B_{Fx} &= k^R_{Fx} + \Delta k_{Fx}\ , \\ Z\,Z_{v_y}\,k^B_{Fy} &= k^R_{Fy} + \Delta k_{Fy}\ , \quad Z^2\,g^B_i = g^R_i + \Delta g_i \end{aligned} \tag{6}$$

- The renormalized couplings are directly related to their corresponding **one-particle irreducible functions** at the **"hot spots"**:

$$\Gamma^4_{iR}(\mathbf{k_1}, \mathbf{k_2}; \mathbf{k_3}, \mathbf{k_4})\Big|_{HS} = -ig_i^R$$

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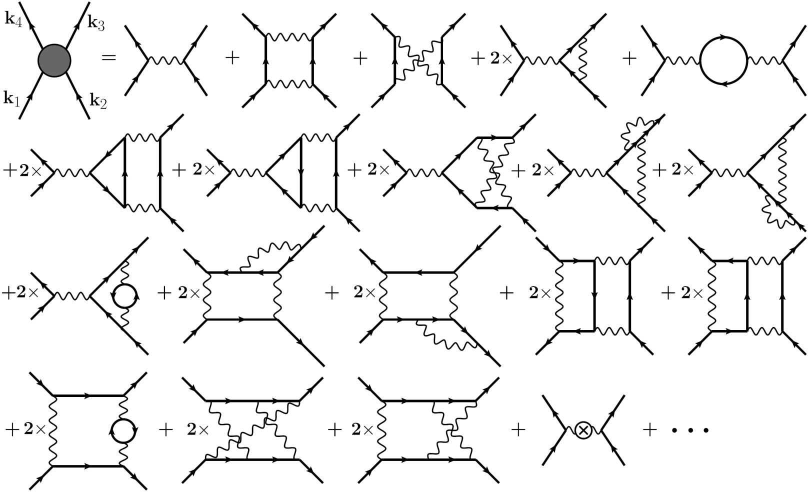
k4
k3
k1
k2
= + + + 2× +
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+2× + 2× + 2× + 2× + 2×
+ 2× + 2× + 2× + + · · ·

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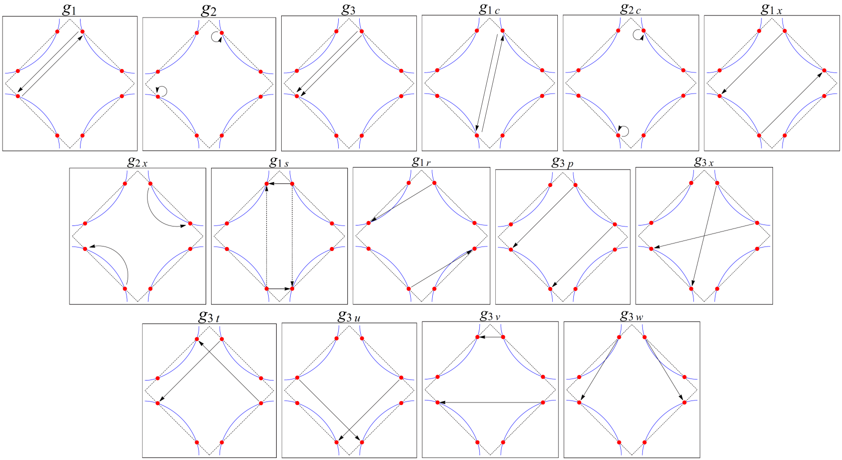
g_1
g_2
g_3
g_{1c}
g_{2c}
g_{1x}
g_{2x}
g_{1s}
g_{1r}
g_{3p}
g_{3x}
g_{3t}
g_{3u}
g_{3v}
g_{3w}

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$$w\frac{dg_i^R(w)}{dw} = 2\gamma(w)\, g_i^R(w) - w\frac{d}{dw}\Delta g_i^R(w) \ , \qquad (7)$$

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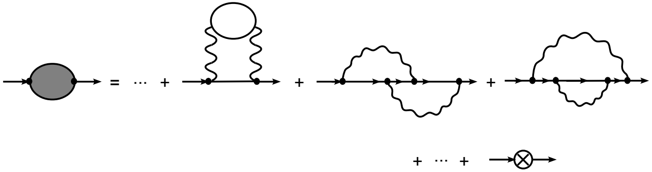
$$\gamma(w) = w\frac{d}{dw}\ln Z(w) \ , \qquad (8)$$

- with $Z(w)$ determined perturbatively from the self-energy corrections:



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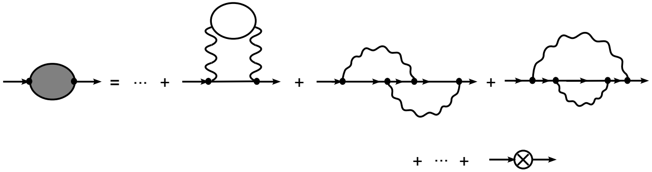

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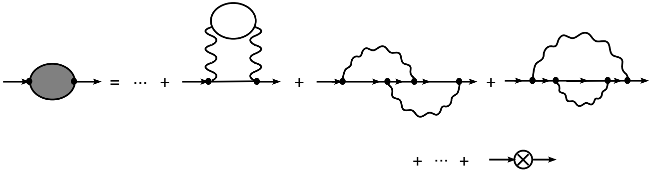
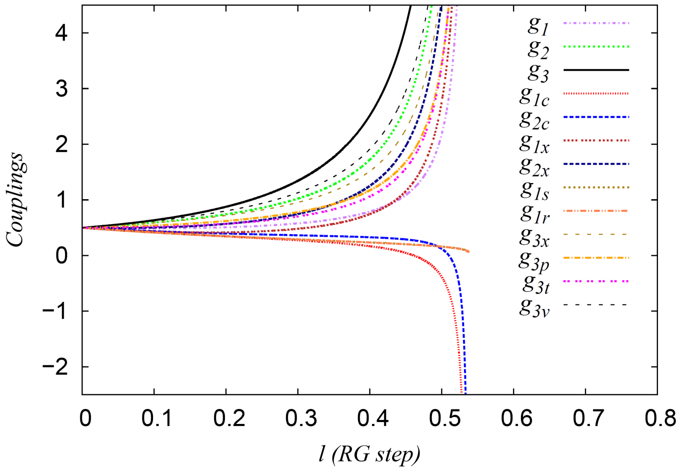


- Following this scheme we display **Carvalho and Freire's** 1 and 2-loop results for the **HSM** (Ann. of Phys. 348, 32 (2014)):


Couplings
l (RG step)
4
3
2
1
0
−1
−2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
g_1
g_2
g_3
g_{1c}
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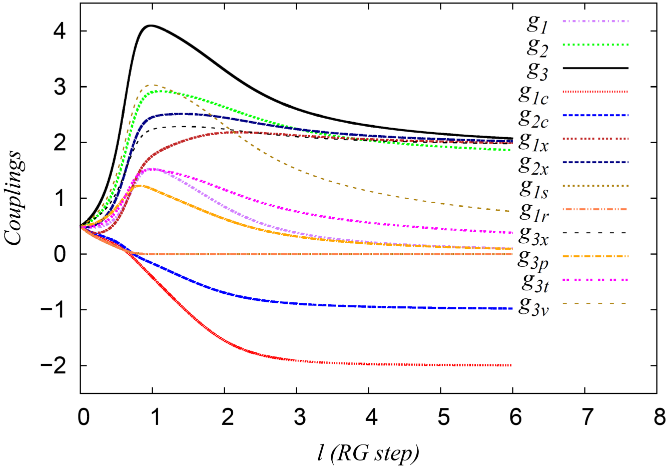

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Non-Trivial Fixed Point and Main Instabilities of Hot Spots Model

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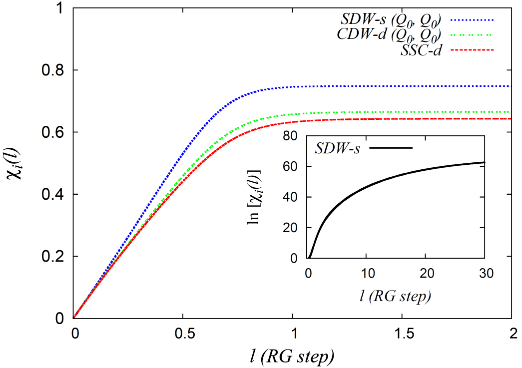

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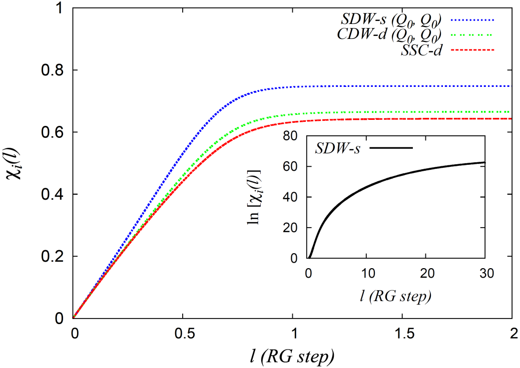
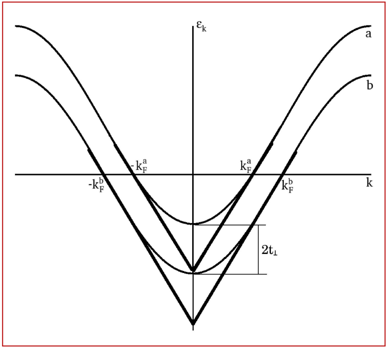


Renormalization of the FS Induced by Interactions: The Two-Coupled Chains Example

- In the absence of interactions the **2 Luttinger chains** coupled by a **transverse hopping** $t_\perp$ can be diagonalized exactly and mapped into a **system of 2–bands**:



- $\mathbf{t}_\perp$ is measured directly by the difference $\Delta k_F = k_F^b - k_F^a$.
- The **TCCM** was imagined as a possible **prototype** of a **Luttinger liquid** in dimension $D > 1$.
- At **very weak coupling** the physical system is driven to a **FL** regime.
- This result was **confirmed** by both **RG** and **bosonization** approaches.
- However **P. W. Anderson** argued that those results were all based on **weak coupling** approximations and he **predicted** a **strong coupling** regime in which $\mathbf{t}_\perp \to \mathbf{0}$ due to interactions.

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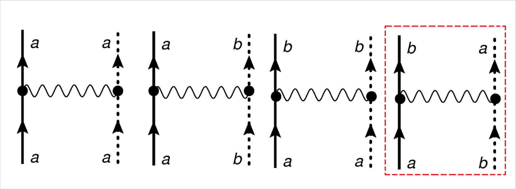
- It turns out that among those 4 the **most relevant** is the so–called **backscattering** $g_{\mathcal{B}}$ (From the left to the right we have g_0, $g_{\mathcal{F}}$, $g_{\mathcal{U}}$ and $g_{\mathcal{B}}$).



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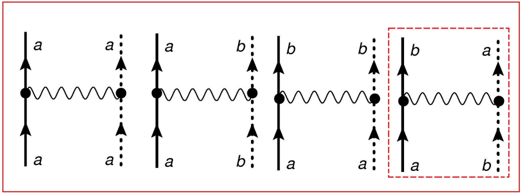

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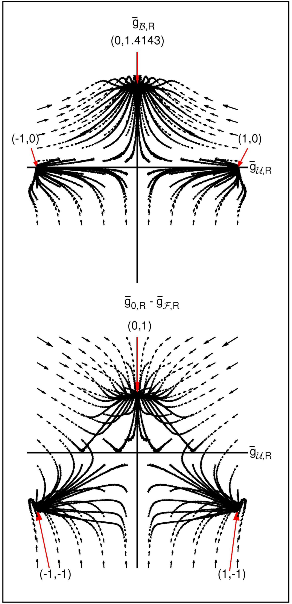

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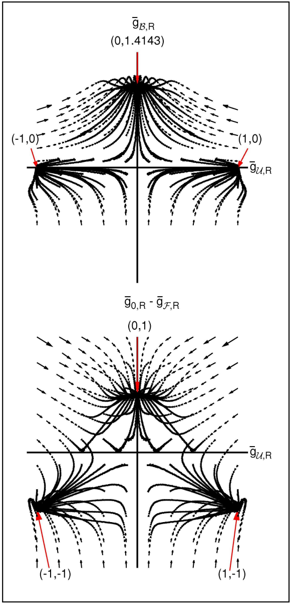

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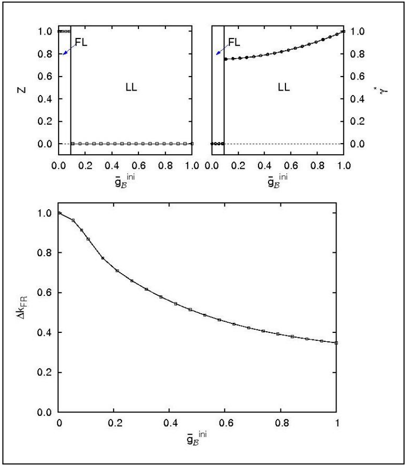

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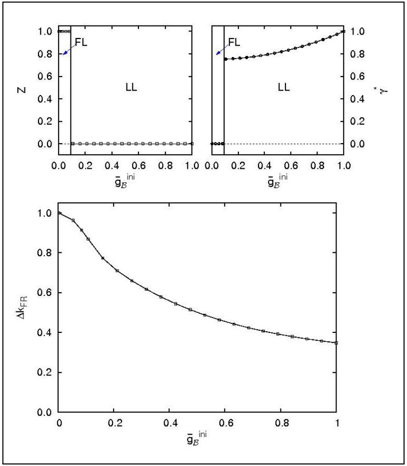

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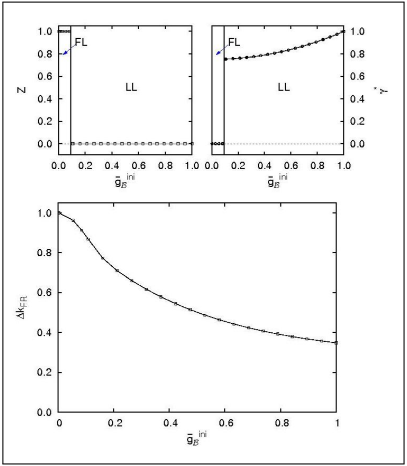

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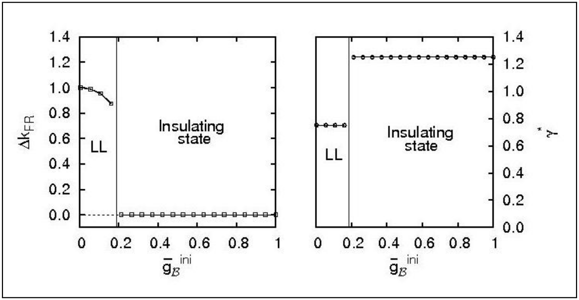

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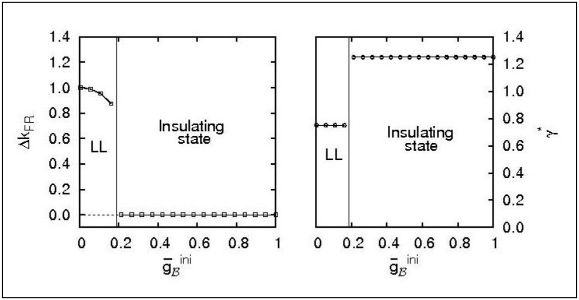

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- For a given set of initial conditions (*i.e.*, initial values of **renormalized couplings** + **underlying FS**) we run all the **RG flows** for the renormalized **coupling functions**, **Fermi velocities**, **Fermi momenta** & the **FS** itself.

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Conclusion

- Although the "**hot spots**" model was able to **correctly predict** the existence of **d-CDW** intertwined with **d-SC instabilities** it is **not able** to determine neither the correct **modulation vector** in agreement with experiments nor to establish what really produces the **pseudogap phase**.

- One possible way out of that is to take into account that the **FS** is **renormalized by interaction**.

- For that we should **introduce new couplings** and **compute** how the "**hot spots**" interact with both **neighbouring** "**luke warm**" as well as with '**cold spots**" points of the **FS**.